Projectile Motion

(NSW HSC Syllabus: 9.2.2.2.2)

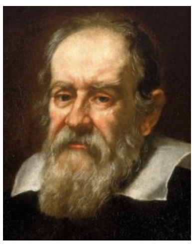
Medieval understanding of projectile motion was based on the ideas of Aristotle. Aristotle thought that projectiles were driven by a force of the air, however later medieval scholars believed that an internal force, called impetus, was responsible for their motion. Projectiles were thought to move in a straight line until they expended all of their impetus, and then fall straight to the ground.

During the Renaissance, the resurgence interest in Greek and Roman culture led to a rebirth of both art and learning. One aspect of this was the re-emergence of the scientific approach to problem solving, first conceived of by the ancient Greeks.

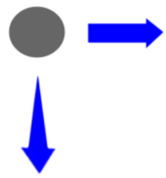
The growing military importance of cannons in warfare at this time led to greater interest in the study of projectile motion. It was realised from careful observation that projectiles followed a curved path, rather than moving in two straight lines. The nature of the curve, however, was not known. A precise mathematical description of the trajectory of a projectile was highly desirable.

Galileo Galilei 1564 to 1642:

(NSW HSC Syllabus: 9.2.2.2.2)



Following his work on gravity Galileo became interested in projectile motion. Using the inclined planes he had used to study gravity, Galileo carried out experiments in which an inked bronze ball was rolled down an inclined plane, then across a horizontal table and off the edge. Where the ball landed on the floor, it left a mark, allowing the horizontal distance travelled by the ball after it rolled off the table to be measured. Galileo varied the horizontal velocity of the ball, and the vertical height of the table, and these experiments led him to the understanding that projectiles follow a parabolic trajectory.



Based on his experiments Galileo realised that the motion of a projectile has two components, a constant horizontal velocity, and a constant vertical acceleration due to gravity.

Galileo, by making precise measurements, was able to determine that the vertical displacement of a falling body was related to the square of the time. This relationship is expressed in the equation:

$$s = \frac{1}{2} g t^2$$

Where: s = displacement (m)

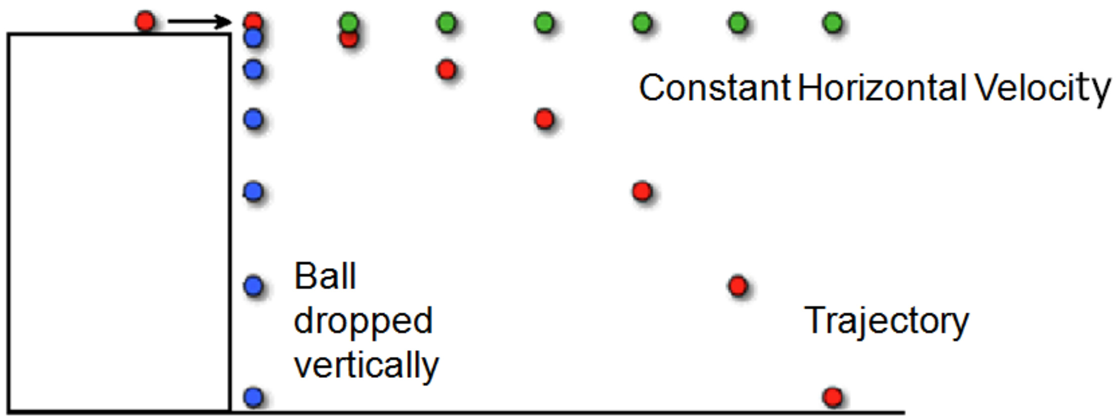
g = acceleration due to gravity (ms^{-2})

t = time (s)

Vertical Velocity:

In Galileo's experiments the projectiles were launched horizontally from a table.

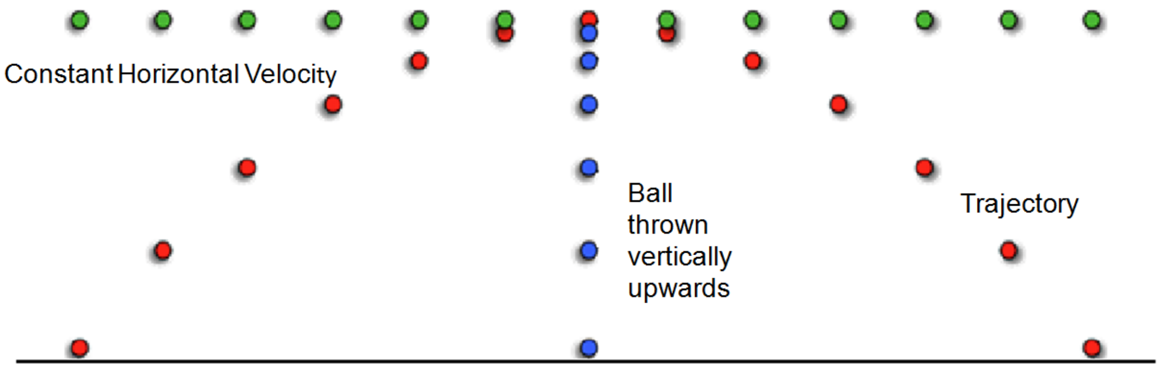
The balls followed a trajectory with a constant horizontal velocity, a constant vertical acceleration, and an initial vertical velocity of zero.



Galileo realised that this trajectory was parabolic.

For a projectile launched from the ground, and landing at the same elevation, the vertical velocity decreases to zero at the top of the projectile's flight, then accelerates as it falls back to Earth, behaving in the same way as a ball thrown vertically upwards.

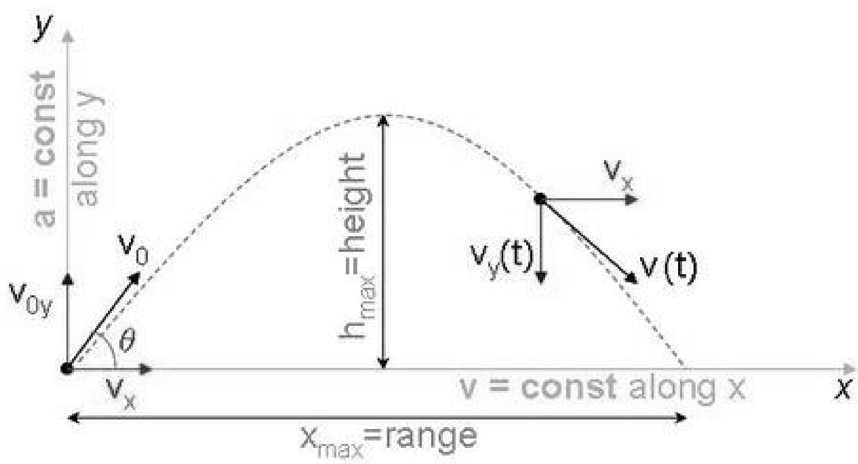
The horizontal velocity remains constant.



Trajectory:

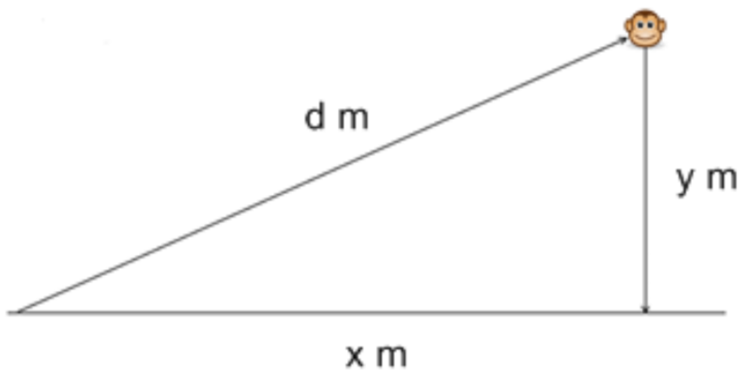
(NSW HSC Syllabus: 9.2.2.2.1)

As Galileo discovered, projectiles follow a parabolic path, the only force acting on a projectile after launch is the force due to gravity, which is directed downward, towards the centre of the Earth. Projectiles have a constant vertical acceleration, and a constant horizontal velocity.

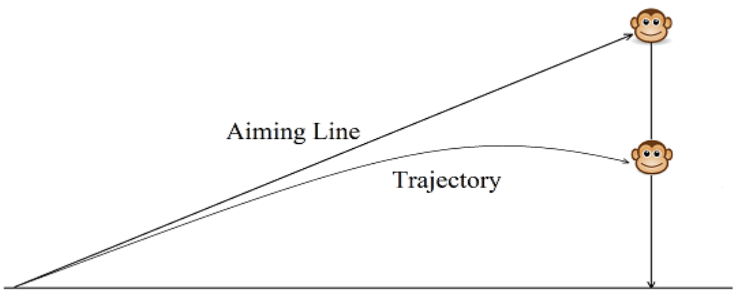


The Monkey and the Gun

A scientist researching monkeys sees a monkey in a tree, some distance away and some distance above the ground. They fire a tranquiliser dart from a gun directly at the monkey. The sound of the gun startles the monkey, which drops from the tree the instant the gun is fired, will the dart hit the monkey?



According to Galileo's analysis of projectile motion the answer is yes. Both the tranquiliser dart and the monkey are subject to the same acceleration due to gravity, so fall an equal distance in an equal time.

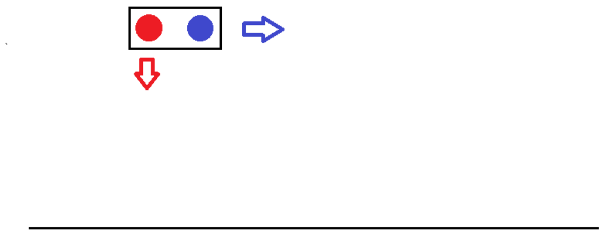


In the absence of gravity the dart would travel in a straight line, the aiming line, shown above. In the time the dart takes to reach the monkey it has fallen, relative to this line, the same distance as the monkey has fallen vertically.

Problems:

1, (Based on 2009 HSC Question 4)

Two identical balls (red and blue) are released from a device simultaneously; the red ball is dropped vertically, while the blue ball is launched in a horizontal direction, with velocity v, from the same height using the apparatus shown.



What happens?

(A) Red hits the ground before blue as it is closer to the launch site.

(B) Blue hits the ground before red as it has a higher launch velocity.

(C) Red and blue hit the ground simultaneously with the same velocity.

(D) Red and blue hit the ground simultaneously with different velocities.

2, (Based on 2010 HSC Question 2)

Which of these statements best describes Galileo's analysis of projectile motion?

(a) A projectile launched with enough momentum would escape the Earth's gravity.

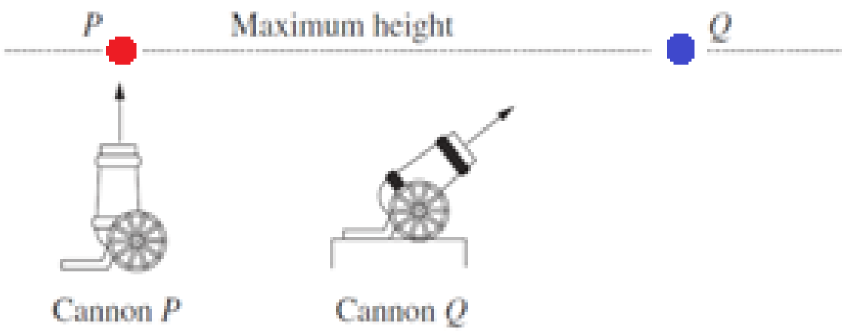
(b) A projectile travels in a straight line until it runs out of momentum, then falls vertically.

(c) A projectile launched towards the east from the equator with enough momentum would orbit Earth.

(d) A projectile travels in a parabolic path as it has constant horizontal velocity and constant vertical acceleration.

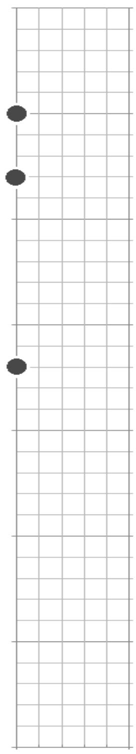
3, (Based on 2014 HSC Question 30)

Cannonballs P and Q are fired so that they leave their barrels from the same height. Cannonball P is fired vertically upwards while cannonball Q is fired at an angle as shown. Both cannonballs take 3 seconds to reach the same maximum height.

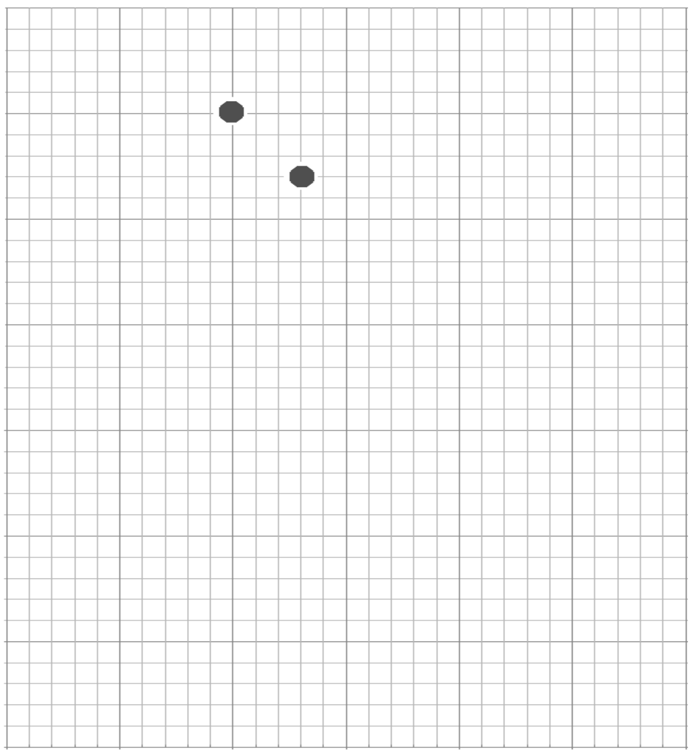


(a) Explain how the motion of the cannonballs supports Galileo's analysis of projectile motion.

(b) The position of cannonball P shown at the 3rd, 4th and 5th seconds of its flight, and cannonball Q for the 3rd and 4th seconds of its flight. Plot the positions of both cannonballs at each second for the rest of their flight. Show calculations.



Cannon P



Cannon Q

Answers to Problems

1, (Based on 2009 HSC Question 4)

Answer: D

2, (Based on 2010 HSC Question 2)

Answer: D

3, (Based on 2014 HSC Question 30)

Answers:

(a) Galileo used experimental evidence and geometry to demonstrate that projectiles follow a parabolic path. Galileo proposed that the motion of a projectile consists of two separate components, a constant horizontal velocity, and a constant vertical acceleration due to gravity, and that these components operate independently of one another. In the vertical direction both balls will start off with the same vertical velocity, and will decelerate under the influence of gravity, until they reach the same maximum height, and they will then accelerate back towards the ground, again under the influence of gravity, striking the ground at the same time. Cannon ball Q will continue moving in a horizontal direction with its initial horizontal velocity, ignoring air resistance, following a parabolic trajectory until it strikes the ground.

3, (b).

The position of cannonball P shown at the 3rd, 4th and 5th seconds of its flight, and cannonball Q for the 3rd and 4th seconds of its flight. Plot the positions of both cannonballs at each second for the rest of their flight. Show calculations.

$\Delta s = 12$

$\Delta t = 2s$

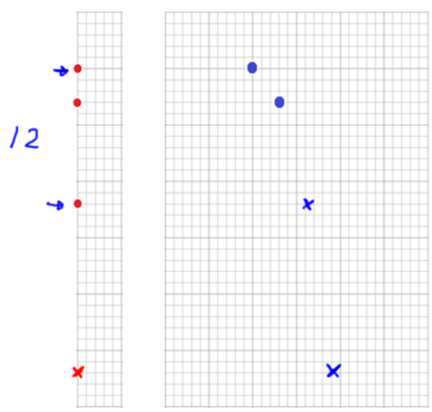
$a = ?$

6th second

$S_y = \frac{1}{2} a_y t^2$

$= \frac{1}{2} \times 6 \times 3^2$

$= 27$ units



Cannon P Cannon Q

$S_y = u_y t + \frac{1}{2} a_y t^2$

$u_y = 0$

$\therefore S_y = \frac{1}{2} a_y t^2$

$2 S_y = a_y t^2$

$2 S_y / t^2 = a$

$a = \frac{2 \times 12}{2^2}$

$= 6$ units/s^2