

Chapter 10 Comparing Two Populations or Groups

10.1 Comparing 2 Proportions

Question	I have no idea, you will see me outside of class.	I can get it with help or my notes.	I can do it, I can't explain it.	I could teach this stuff.	
					DETERMINE whether the conditions for performing inference are met.
					CONSTRUCT and INTERPRET a confidence interval to compare two proportions.
					PERFORM a significance test to compare two proportions.
					INTERPRET the results of inference procedures in a randomized experiment.

10.2 Comparing 2 Means

Question	I have no idea, you will see me outside of class.	I can get it with help or my notes.	I can do it, I can't explain it.	I could teach this stuff.	
					DESCRIBE the characteristics of the sampling distribution of the difference between two sample means
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					CALCULATE probabilities using the sampling distribution of the difference between two sample means
					DETERMINE whether the conditions for performing inference are met
					USE two-sample t procedures to compare two means based on summary statistics or raw data
					INTERPRET computer output for two-sample t procedures
					PERFORM a significance test to compare two means
					INTERPRET the results of inference procedures

10.1 Comparing 2 Proportions

Population or treatment	Parameter	Statistic	Sample size
1	p_1	\hat{p}_1	n_1
2	p_2	\hat{p}_2	n_2

A SAMPLING DISTRIBUTION of sample proportion has the following characteristics:

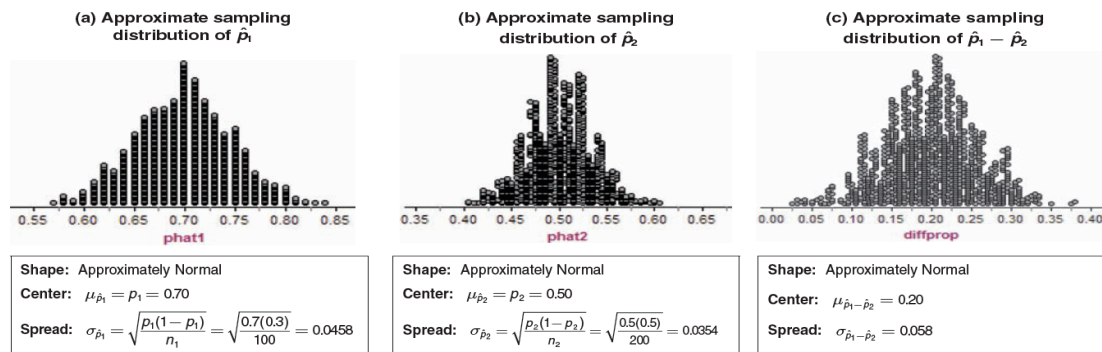
Shape:

Center:

Spread:

The DIFFERENCE in two Sampling Distributions: What if...

At School 1, 70% of students did their homework last night – at school 1, we took an SRS of _____
 At School 2, 50% of students did their homework last night – at school 2, we took an SRS of _____



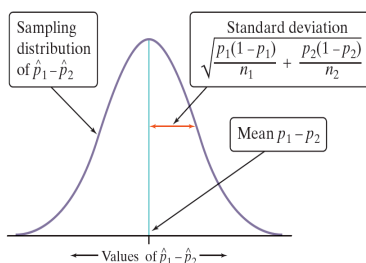
What do you notice about the shape, center and spread the sampling distribution of $\hat{p}_1 - \hat{p}_2$

Shape:

Center:

Spread:

The Sampling Distribution of a Difference Between Two Proportions



Choose an SRS of size n_1 from Population 1 with proportion of successes p_1 and an independent SRS of size n_2 from Population 2 with proportion of successes p_2 .

Shape:

Center:

Spread:

Example: Who Does More Homework?

Suppose that there are two large high schools, each with more than 2000 students, in a certain town. At School 1, 70% of students did their homework last night. Only 50% of the students at School 2 did their homework last night. The counselor at School 1 takes an SRS of 100 students and records the proportion that did homework. School 2's counselor takes an SRS of 200 students and records the proportion that did homework. School 1's counselor and School 2's counselor meet to discuss the results of their homework surveys. After the meeting, they both report to their principals that $p_1 - p_2 = 0.10$

a) Describe the shape, center, and spread of the sampling distribution of $\hat{p}_1 - \hat{p}_2$.

b) Find the probability of getting a difference in sample proportions $\hat{p}_1 - \hat{p}_2$ of 0.10 or less from the two surveys.

c) Does the result in part (b) give us reason to doubt the counselors' reported value?

Confidence Intervals for $p_1 - p_2$

When data come from **2 random samples** OR **2 groups** in a randomized experiment, the statistic _____ is our best guess for the value of _____.



We will use the same formula as the previous 2 chapters:

statistic \pm (critical value) \cdot (standard deviation of statistic)

When the independent condition is met, the standard deviation of $\hat{p}_1 - \hat{p}_2$ is:

Since we don't know parameters (p_1 or p_2), we replace with sample proportions, \hat{p}_1, \hat{p}_2

Standard deviation becomes _____ with formula:

When the normal condition is met, find the critical value _____

Confidence Interval for $p_1 - p_2$ is

Conditions for 2-sample proportion z-interval:

Random:

Normal:

Independent:

Example: Teens and Adults on Social Networks

As part of the Pew Internet and American Life Project, researchers conducted two surveys in late 2009. The first survey asked a random sample of 800 U.S. teens about their use of social media and the Internet. A second survey posed similar questions to a random sample of 2253 U.S. adults. In these two studies, 73% of teens and 47% of adults said that they use social-networking sites. Use these results to construct and interpret a 95% confidence interval for the difference between the proportion of all U.S. teens and adults who use social-networking sites.

State: Our parameters of interest are p_1 = the proportion of all U.S. teens who use social networking sites and p_2 = the proportion of all U.S. adults who use social-networking sites. We want to estimate the difference $p_1 - p_2$ at a 95% confidence level.

Plan: We should use a two-sample z interval for $p_1 - p_2$ if the conditions are satisfied.

Significance Tests for $p_1 - p_2$

An observed difference between two sample proportions can reflect an actual difference in the parameters, or it may just be due to chance variation in random sampling or random assignment. Significance tests help us decide which explanation makes more sense. The null hypothesis has the general form:

We will only look at cases where we are trying to find a difference, so

H_0 :

H_a :

To do a test:

Standardize _____ to get a _____

Test statistic = $z =$

We need a way to estimate p , so it makes sense to combine the data from the two samples. This POOLED (or combined) sample Proportion is:

$$\hat{p}_c = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Pooled SE



Use \hat{p}_c in place of $\frac{c}{n}$ and $\sqrt{\hat{p}_c(1-\hat{p}_c)}$ in the denominator of the test statistic.

$z =$

Data must again be $\frac{c}{n}$, $\frac{c}{n}$, $\frac{c}{n}$

Example: Presidential approval

Many news organizations conduct polls asking adults in the United States if they approve of the job the president is doing. How did President Obama's approval rating change from August 2009 to September 2010? A CNN poll of 1010 randomly selected U.S. adults on August 28-30, 2009 showed that 53% approved of Obama's job performance. According to a CNN poll of 1024 randomly selected U.S. adults on September 1-2, 2010, 50% approved of Obama's job performance. Determine if there has been a change in President Obama's approval rating. Use a significance level of 0.10.

Notes: This test is two-tailed one tailed proportions means
This test is 1-sample paired data 2-sample
We will use z-score t-score

Example: Presidential Approval

Create a 90% confidence interval to confirm our findings from the significance test.

Example: All drugs created Equal?

Suppose the Acme Drug Company develops a new drug, designed to prevent colds. The company states that the drug is equally effective for men and women. To test this claim, they choose a simple random sample of 100 women and 200 men from a population of 100,000 volunteers. At the end of the study, 38% of the women caught a cold; and 51% of the men caught a cold. Based on these findings, can we reject the company's claim that the drug is equally effective for men and women? Use a 0.05 level of significance.

Notes: This test is _____ two-tailed _____ one tailed _____ proportions _____ means
 This test is _____ 1-sample _____ paired data _____ 2-sample
 We will use _____ z-score _____ t-score

Example: Significance Test in an Experiment

High levels of cholesterol in the blood are associated with higher risk of heart attacks. Will using a drug to lower blood cholesterol reduce heart attacks? The Helsinki Heart Study recruited middle-aged men with high cholesterol but no history of other serious medical problems to investigate this question. The volunteer subjects were assigned at random to one of two treatments: 2051 men took the drug gemfibrozil to reduce their cholesterol levels, and a control group of 2030 men took a placebo. During the next five years, 56 men in the gemfibrozil group and 84 men in the placebo group had heart attacks. Is the apparent benefit of gemfibrozil statistically significant? Perform an appropriate test to find out.

Notes: This test is _____ two-tailed _____ one tailed _____ proportions _____ means
 This test is _____ 1-sample _____ paired data _____ 2-sample
 We will use _____ z-score _____ t-score

What are the implications of a Type I Error?

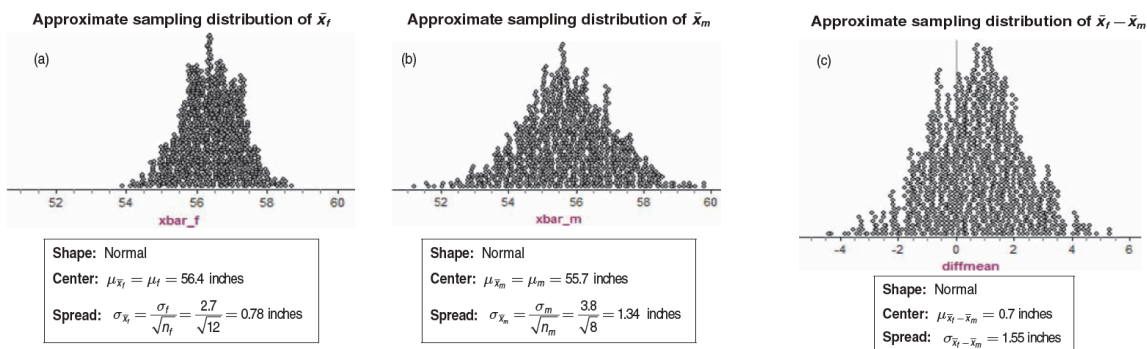
What are the implications of a Type II Error?

10.2 Comparing Two Means

Population or treatment	Parameter	Statistic	Sample size
1	μ_1	\bar{X}_1	n_1
2	μ_2	\bar{X}_2	n_2

Difference of Two Means:

To explore the sampling distribution of the difference between two means, let's start with two Normally distributed populations having known means and standard deviations. Based on information from the U.S. National Health and Nutrition Examination Survey (NHANES), the heights (in inches) of ten-year-old girls follow a Normal distribution $N(56.4, 2.7)$. The heights (in inches) of ten-year-old boys follow a Normal distribution $N(55.7, 3.8)$. Suppose we take independent SRSs of 12 girls and 8 boys of this age and measure their heights.



Shape:

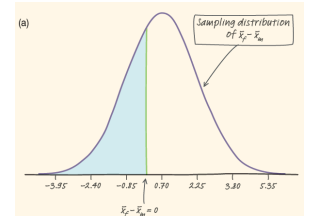
Center:

Spread:

Example: Who's Taller at Ten, Boys or Girls?

Based on information from the U.S. National Health and Nutrition Examination Survey (NHANES), the heights (in inches) of ten-year-old girls follow a Normal distribution $N(56.4, 2.7)$. The heights (in inches) of ten-year-old boys follow a Normal distribution $N(55.7, 3.8)$. A researcher takes independent SRSs of 12 girls and 8 boys of this age and measures their heights. After analyzing the data, the researcher reports that the sample mean height of the boys is larger than the sample mean height of the girls.

- (a) Describe the shape, center, and spread of the distributions.



- (b) Find the probability of getting a difference in sample means $\bar{X}_f - \bar{X}_m$ that is less than 0.

- (c) Does the result in part (b) give us reason to doubt the researchers' stated results?

Two Sample t-statistic

When the independent condition is met, the standard deviation of the statistic $\bar{X}_1 - \bar{X}_2$ is:

Standard Error:

If the Normal condition is met, we standardize the observed difference to obtain a t statistic that tells us how far the observed difference is from its mean in standard deviation units:

$$t =$$

The two-sample t statistic has approximately a t distribution, we can use a conservative approach, using the SMALLER of $n_1 - 1$ and $n_2 - 1$ for the degrees of freedom.

Confidence Intervals for 2 sample Means:

When the random, normal, and independent conditions are met, the approximate level C confidence interval is:
statistic \pm critical value \cdot standard error

Where t^* is the critical value for confidence level C for the t -distribution with appropriate degrees of freedom.

Random:

Remember! You must write out your conditions!

Normal:

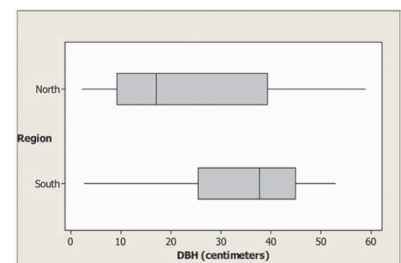


Independent:

Big Trees, Small Trees, Short Trees, Tall Trees

The Wade Tract Preserve in Georgia is an old-growth forest of longleaf pines that has survived in a relatively undisturbed state for hundreds of years. One question of interest to foresters who study the area is “How do the sizes of longleaf pine trees in the northern and southern halves of the forest compare?” To find out, researchers took random samples of 30 trees from each half and measured the diameter at breast height (DBH) in centimeters. Comparative boxplots of the data and summary statistics from Minitab are shown below. Construct and interpret a 90% confidence interval for the difference in the mean DBH for longleaf pines in the northern and southern halves of the Wade Tract Preserve.

Notes: This test is _____ two-tailed _____ one tailed _____ proportions _____ means
This test is _____ 1-sample _____ paired data _____ 2-sample
We will use _____ z-score _____ t-score



Descriptive Statistics: North, South

Variable	N	Mean	StDev
North	30	23.70	17.50
South	30	34.53	14.26

State: Our parameters of interest are μ_1 = the true mean DBH of all trees in the southern half of the forest and μ_2 = the true mean DBH of all trees in the northern half of the forest. We want to estimate the difference $\mu_1 - \mu_2$ at a 90% confidence level.

Plan:

Do:

Upper-tail probability p			
df	.10	.05	.025
28	1.313	1.701	2.048
29	1.311	1.699	2.045
30	1.310	1.697	2.042
	80%	90%	95%
Confidence level C			

Conclude:

Example: Plastic Grocery Bags

Do plastic bags from Target or plastic bags from Walmart hold more weight? A group of AP Statistic students decided to investigate by filling a random sample of 5 bags from each store with common grocery items until the bags ripped. Then they weighed the contents of items in each bag to determine its capacity. Here are their results, in grams:

Target:	12,572	13,999	11,215	15,447	10,896
Walmart:	9552	10,896	6983	8767	9972

Problem:

(a) Construct and interpret a 99% confidence interval for the difference in mean capacity of plastic grocery bags from Target and Walmart.

(b) Does your interval provide convincing evidence that there is a difference in the mean capacity among the two stores?

Significance Tests for $\mu_1 - \mu_2$

An observed difference between two sample means can reflect an actual difference in the parameters, or it may just be due to chance variation in random sampling or random assignment. Significance tests help us decide which explanation makes more sense. The null hypothesis has the general form:

For situations in which the hypothesized hypothesis is 0:

test statistic = _____ for difference of means: $t =$

To find degrees of freedom:

Example: Calcium and Blood Pressure

Does increasing the amount of calcium in our diet reduce blood pressure? Examination of a large sample of people revealed a relationship between calcium intake and blood pressure. The relationship was strongest for black men. Such observational studies do not establish causation. Researchers therefore designed a randomized comparative experiment. The subjects were 21 healthy black men who volunteered to take part in the experiment. They were randomly assigned to two groups: 10 of the men received a calcium supplement for 12 weeks, while the control group of 11 men received a placebo pill that looked identical. The experiment was double-blind. The response variable is the decrease in systolic (top number) blood pressure for a subject after 12 weeks, in millimeters of mercury. An increase appears as a negative response Here are the data:

Group 1 (calcium): 7 -4 18 17 -3 -5 1 10 11 -2

Group 2 (placebo): -1 12 -1 -3 3 -5 5 2 -11 -1 -3

Notes: This test is _____ two-tailed _____ one tailed _____ proportions _____ means
This test is _____ 1-sample _____ paired data _____ 2-sample
We will use _____ z-score _____ t-score

Example: We can estimate the difference in the true mean decrease in blood pressure for the calcium and placebo treatments using a two-sample t interval for $\mu_1 - \mu_2$. To get results that are consistent with the one-tailed test at $\alpha = 0.05$ from the example, we'll use a 90% confidence level. The conditions for constructing a confidence interval are the same as the ones that we checked in the example before performing the two-sample t test.

Example: The stronger picker-upper?

In commercials for Bounty paper towels, the manufacturer claims that they are the “quicker picker-upper.” But are they also the stronger picker upper? Two AP Statistics students, Wesley and Maverick, decided to find out. They selected a random sample of 30 Bounty paper towels and a random sample of 30 generic paper towels and measured their strength when wet. To do this, they uniformly soaked each paper towel with 4 ounces of water, held two opposite edges of the paper towel, and counted how many quarters each paper towel could hold until ripping, alternating brands. Here are their results:

Bounty: 106, 111, 106, 120, 103, 112, 115, 125, 116, 120, 126, 125, 116, 117, 114
118, 126, 120, 115, 116, 121, 113, 111, 128, 124, 125, 127, 123, 115, 114

Generic: 77, 103, 89, 79, 88, 86, 100, 90, 81, 84, 84, 96, 87, 79, 90
86, 88, 81, 91, 94, 90, 89, 85, 83, 89, 84, 90, 100, 94, 87

Problem:

(a) Display these distributions using parallel boxplots and briefly compare these distributions. Based only on the boxplots, discuss whether or not you think the mean for Bounty is significantly higher than the mean for generic.

(b) Use a significance test to determine if there is convincing evidence that wet Bounty paper towels can hold more weight, on average, than wet generic paper towels.

Using the Two-Sample t Procedures: The Normal Condition

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Cautions when using 2-sample t procedures:

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