H

National
Qualifications
2018

2018 Mathematics

Higher - Paper 1

Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	$\bullet^1$ find mid-point of PQ $\bullet^2$ find gradient of median $\bullet^3$ determine equation of median	$\bullet^1$ $(1,2)$ $\bullet^2$ 2 $\bullet^3$ $y=2x$	**3**

Notes:

1. $\bullet^2$ is only available to candidates who use a midpoint to find a gradient.
2. $\bullet^3$ is only available as a consequence of using the mid-point and the point R, or any other point which lies on the median, eg $(2,4)$.
3. At $\bullet^3$ accept any arrangement of a candidate's equation where constant terms have been simplified.
4. $\bullet^3$ is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - Perpendicular Bisector of PQ		**Candidate B - Altitude through R**	
$M_{PQ}(1,2)$	$\bullet^1$ ✓	$m_{PQ}=-\frac{2}{3}$	$\bullet^1$ ^
$m_{PQ}=-\frac{2}{3}\Rightarrow m_{\perp}=\frac{3}{2}$	$\bullet^2$ ✗	$m_{\perp}=\frac{3}{2}$	$\bullet^2$ ✗
$2y=3x+1$	$\bullet^3$ $\boxed{\checkmark\ 2}$	$2y=3x+3$	$\bullet^3$ $\boxed{\checkmark\ 2}$
For other perpendicular bisectors award 0/3			

Candidate C - Median through P		**Candidate D - Median through Q**	
$M_{QR}(3\cdot5,3)$	$\bullet^1$ ✗	$M_{PR}(0\cdot5,5)$	$\bullet^1$ ✗
$m_{PM}=-\frac{2}{11}$	$\bullet^2$ $\boxed{\checkmark\ 1}$	$m_{QM}=-\frac{10}{7}$	$\bullet^2$ $\boxed{\checkmark\ 1}$
$11y+2x=40$	$\bullet^3$ $\boxed{\checkmark\ 2}$	$7y+10x=40$	$\bullet^3$ $\boxed{\checkmark\ 2}$

Question	Generic scheme	Illustrative scheme	Max mark
2.	**Method 1**	**Method 1**	3
	$\bullet^1$ equate composite function to x	$\bullet^1$ $g\left(g^{-1}(x)\right)=x$	
	$\bullet^2$ write $g(g^{-1}(x))$ in terms of $g^{-1}(x)$	$\bullet^2$ $\frac{1}{5}g^{-1}(x)-4=x$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	
	Method 2	**Method 2**	
	$\bullet^1$ write as $y=\frac{1}{5}x-4$ and start to rearrange	$\bullet^1$ $y+4=\frac{1}{5}x$	
	$\bullet^2$ express x in terms of y	$\bullet^2$ eg $x=5(y+4)$ or $x=\frac{(y+4)}{\frac{1}{5}}$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	
	Method 3	**Method 3**	
	$\bullet^1$ interchange variables	$\bullet^1$ $x=\frac{1}{5}y-4$	
	$\bullet^2$ express y in terms of x	$\bullet^2$ eg $y=5(x+4)$ or $y=\frac{(x+4)}{\frac{1}{5}}$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	

Notes:

1. $y=5(x+4)$ does not gain $\bullet^3$.
2. At $\bullet^3$ stage, accept g^{-1} written in terms of any dummy variable eg $g^{-1}(y)=5(y+4)$.
3. $g^{-1}(x)=5(x+4)$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$x \rightarrow \frac{1}{5}x \rightarrow \frac{1}{5}x-4=g(x)$

$\div 5 \rightarrow -4$

$\therefore\ +4 \rightarrow \times 5$ — $\bullet^1$ ✓ awarded for knowing to perform inverse operations in reverse order

$5(x+4)$ — $\bullet^2$ ✓

$g^{-1}(x)=5(x+4)$ — $\bullet^3$ ✓

Candidate B - BEWARE		**Candidate C**	
$g'(x)=\ldots$	$\bullet^3$ ✗	$g^{-1}(x)=5x+4$	**with no working** **Award 0/3**

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ start to differentiate $\bullet^2$ complete differentiation $\bullet^3$ evaluate derivative	$\bullet^1$ $-3\sin 2x\ldots$ stated or implied by $\bullet^2$ $\bullet^2$ $\ldots\times 2$ $\bullet^3$ $-3\sqrt{3}$	3

Notes:

1. Ignore the appearance of $+c$ at any stage.
2. $\bullet^3$ is available for evaluating an attempt at finding the derivative at $\frac{\pi}{6}$.
3. For $h'\left(\frac{\pi}{6}\right)=3\cos\left(2\times\frac{\pi}{6}\right)=\frac{3}{2}$ award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$-3\sin 2x\ldots$	$\bullet^1$ ✓	$3\sin 2x\ldots$	$\bullet^1$ ✗	$3\sin 2x\ldots$	$\bullet^1$ ✗
$\ldots\times\frac{1}{2}$	$\bullet^2$ ✗	$\ldots\times 2$	$\bullet^2$ ✓	$\ldots\times\frac{1}{2}$	$\bullet^2$ ✗
$-\frac{3\sqrt{3}}{4}$	$\bullet^3$ $\boxed{✓\ 1}$	$3\sqrt{3}$	$\bullet^3$ $\boxed{✓\ 1}$	$\frac{3\sqrt{3}}{4}$	$\bullet^3$ $\boxed{✓\ 1}$

Candidate D		Candidate E		Candidate F	
$\pm 6\cos 2x$	$\bullet^1$ ✗	$\pm 3\cos 2x\ldots$	$\bullet^1$ ✗	$6\sin 2x$	$\bullet^1$ ✗
	$\bullet^2$ ✗	$\ldots\times 2$	$\bullet^2$ $\boxed{✓\ 1}$		$\bullet^2$ ✓
± 3	$\bullet^3$ $\boxed{✓\ 1}$	± 3	$\bullet^3$ $\boxed{✓\ 1}$	$3\sqrt{3}$	$\bullet^3$ $\boxed{✓\ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
4.	$\bullet^1$ state centre of circle $\bullet^2$ find gradient of radius $\bullet^3$ state gradient of tangent $\bullet^4$ state equation of tangent	$\bullet^1$ $(6,3)$ $\bullet^2$ -4 $\bullet^3$ $\frac{1}{4}$ $\bullet^4$ $y=\frac{1}{4}x-7$	**4**

Notes:
1. Accept $-\frac{8}{2}$ for $\bullet^2$. 2. The perpendicular gradient must be simplified at the $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be available. 3. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient. 4. At $\bullet^4$ accept $y-\frac{1}{4}x+7=0$, $4y=x-28$, $x-4y-28=0$ or any other rearrangement of the equation where the constant terms have been simplified.
Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
5. (a)	$\bullet^1$ state ratio explicitly	$\bullet^1$ $4:1$	**1**
Notes:			
1. The only acceptable variations for $\bullet^1$ must be related explicitly to AB and BC. For $\frac{BC}{AB}=\frac{1}{4}$, $\frac{AB}{BC}=\frac{4}{1}$ or $BC:AB=1:4$ award 1/1. 2. For $BC=\frac{1}{4}AB$ award 0/1.			
Commonly Observed Responses:			
(b)	$\bullet^2$ state value of t	$\bullet^2$ 8	**1**
Notes:			
3. The answer to part (b) **must** be consistent with a ratio stated in part (a) unless a valid strategy which does not require the use of their ratio from part (a) is used.			
Commonly Observed Responses:			
Candidate A $1:4$ $\bullet^1$ ✗ $t=8$ $\bullet^2$ ✗	**Candidate B** $1:4$ $\bullet^1$ ✗ $t=5$ $\bullet^2$ $\boxed{✓1}$		

Question	Generic scheme	Illustrative scheme	Max mark
6.	•¹ apply $m\log_5 x = \log_5 x^m$	•¹ $\log_5 8^{\frac{1}{3}}$	3
	•² apply $\log_5 x - \log_5 y = \log_5 \dfrac{x}{y}$	•² $\log_5\left(\dfrac{250}{8^{\frac{1}{3}}}\right)$	
	•³ evaluate log	•³ 3	

Notes:

1. Each line of working must be equivalent to the line above within a valid strategy, however see Candidate B.
2. Do not penalise the omission of the base of the logarithm at •¹ or •².
3. For '3' with no working award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B	
$\log_5 250 - \log_5 \dfrac{8}{3}$	•¹ ✗	$\dfrac{1}{3}\log_5(250 \div 8)$	
$\log_5 \dfrac{250}{\frac{8}{3}}$	•² $\boxed{\checkmark\ 1}$	$\dfrac{1}{3}\log_5 \dfrac{125}{4}$	
$\log_5 \dfrac{375}{4}$	•³ $\boxed{\checkmark\ 2}$	$\log_5\left(\dfrac{125}{4}\right)^{\frac{1}{3}}$	Award 1/3 $\boxed{\checkmark\ 1}$ ✗ ^
			•¹ is awarded for the final two lines of working

Question		Generic scheme	Illustrative scheme	Max mark
7.	(a)	•¹ state coordinates of P	•¹ $(0,5)$	**1**

Notes:

1. Accept ‘ $x=0,\ y=5$ ’.
2. ‘ $y=5$ ’ alone or ‘5’ does not gain •¹.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•² differentiate	•² $3x^2-6x+2$	**3**
		•³ calculate gradient	•³ 2	
		•⁴ state equation of tangent	•⁴ $y=2x+5$	

Notes:

3. At •⁴ accept $y-2x=5$, $2x-y+5=0$, $y-5=2x$ or any other rearrangement of the equation where the constant terms have been simplified.
4. •⁴ is only available if an attempt has been made to find the gradient from differentiation.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
7.	(c)	•5 set $y_{\text{line}} = y_{\text{curve}}$ and arrange in standard form •6 factorise •7 state x-coordinate of Q •8 calculate y-coordinate of Q	•5 $x^3 - 3x^2 = 0$ •6 $x^2(x-3)$ •7 3 •8 11	4

Notes:

5. •5 is only available if '$=0$' appears at either •5 or •6 stage.
6. •7 and •8 are only available as a consequence of solving a cubic equation and a linear equation simultaneously.
7. For an answer of $(3,11)$ with no working award 0/4.
8. For an answer of $(3,11)$ verified in both equations award 3/4.
9. For an answer of $(3,11)$ verified in both equations along with a statement such as 'same point on both line and curve so Q is $(3,11)$' award 4/4.
10. For candidates who work with a derivative, no further marks are available.
11. $x = 3$ must be supported by valid working for •7 and •8 to be awarded.

Commonly Observed Responses:

Candidate A $x^3 - 3x^2 = 0$ •5 ✓ $x - 3 = 0$ •6 ✓ $x = 3$ •7 ✓ $y = 11$ •8 ✓ Dividing by x^2 is valid since $x \neq 0$ at •6	

Question	Generic scheme	Illustrative scheme	Max mark
8.	•¹ determine the gradient of the line •² determine the angle	•¹ $m=\sqrt{3}$ or $\tan\theta=\sqrt{3}$ •² 60° or $\frac{\pi}{3}$	2

Notes:

1. Do not penalise the omission of units at •².
2. For 60° or $\frac{\pi}{3}$ without working award 2/2.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$y=\sqrt{3}x+5$	Ignore incorrect processing of the constant term	$m=\sqrt{3}$	•¹ ✓
$m=\sqrt{3}$	•¹ ✓	$\theta=\tan\sqrt{3}$	•² ✗
60°	•² ✓	$\theta=60°$	
		Stating $\tan$ rather than $\tan^{-1}$ See general marking principle (g)	

Question		Generic scheme	Illustrative scheme	Max mark
9.	(a)	•¹ identify pathway	•¹ $-\mathbf{t}+\mathbf{u}$	**1**

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•² state an appropriate pathway	•² eg $\frac{1}{2}\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AD}$ stated or implied by •³	**2**
		•³ express pathway in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$	•³ $-\frac{1}{2}\mathbf{t}-\frac{1}{2}\mathbf{u}+\mathbf{v}$	

Notes:

1. There is no need to simplify the expression at •³. Eg $\frac{1}{2}(-\mathbf{t}+\mathbf{u})-\mathbf{u}+\mathbf{v}$.
2. •³ is only available for using a valid pathway.
3. The expression at •³ must be consistent with the candidate's expression at •¹.
4. If the pathway in •¹ is given in terms of a single vector $\mathbf{t}$, $\mathbf{u}$ or $\mathbf{v}$, then •³ is not available.

Commonly Observed Responses:

Candidate A $\overrightarrow{MD}=-\frac{1}{2}\mathbf{t}+\mathbf{v}-\mathbf{u}$ •² ^ •³ ✗	

Question	Generic scheme	Illustrative scheme	Max mark
10.	•[1] know to and integrate one term •[2] complete integration •[3] substitute for x and y •[4] state equation	•[1] eg $2x^3\ldots$ •[2] eg $\ldots-\frac{3}{2}x^2+4x+c$ •[3] $14=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$ •[4] $y=2x^3-\frac{3}{2}x^2+4x-4$ stated explicitly	4

Notes:

1. For candidates who make no attempt to integrate to find y in terms of x award 0/4.
2. For candidates who omit $+c$, only •[1] is available.
3. Candidates must attempt to integrate both terms containing x for •[3] and •[4] to be available. See Candidate B.
4. For candidates who differentiate any term, •[2] •[3] and •[4] are not available.
5. •[4] is not available for '$f(x)=\ldots$'.
6. Candidates must simplify coefficients in **<u>their</u>** final line of working for the last mark available in that line of working to be awarded.

Commonly Observed Responses:

Candidate A		Candidate B - partial integration	
$y=2x^3-\frac{3}{2}x^2+4x+c$	•[1] ✓ •[2] ✓	$y=2x^3-\frac{3}{2}x^2+4+c$	•[1] ✓ •[2] ✗
$y=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$		$14=2(2)^3-\frac{3}{2}(2)^2+4+c$	•[3] $\boxed{✓ 1}$
$c=-4$	•[3] ✓ substitution for y implied by $c=-4$ •[4] ^	$c=0$	
		$y=2x^3-\frac{3}{2}x^2+4$	•[4] $\boxed{✓ 1}$

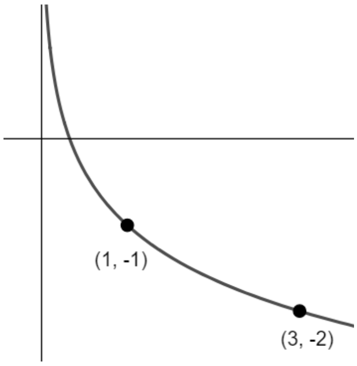
Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•¹ curve reflected in x-axis and translated 1 unit vertically •² accurate sketch	•¹ a generally decreasing curve above the x-axis for $1<x<3$ •² asymptote at $x=0$ **and** passing through $(3,0)$ **and** continuing to decrease for $x\geq 3$	2

Notes:

1. For any attempt which involves a horizontal translation or reflection in the y-axis award 0/2.
2. For a single transformation award 0/2.
3. For any attempt involving a reflection in the line $y=x$ award 0/2

Commonly Observed Responses:

Candidate A



Award 1/2

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•³ set ‘ $y=y$ ’ •⁴ start to solve •⁵ state x coordinate	•³ $\log_3 x = 1 - \log_3 x$ •⁴ $\log_3 x = \frac{1}{2}$ or $\log_3 x^2 = 1$ •⁵ $\sqrt{3}$ or $3^{\frac{1}{2}}$	3

Notes:

4. •³ may be implied by $\log_3 x = \frac{1}{2}$ from symmetry of the curves.
5. Do not penalise the omission of the base of the logarithm at •³ or •⁴.
6. For a solution which equates a to $\log_3 a$, the final mark is not available.
7. If a candidate considers and then does not discard $-\sqrt{3}$ in their final answer, •⁵ is not available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	$\bullet^1$ find components	$\bullet^1$ $\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	1

Notes:

1. Accept $6\mathbf{i}-3\mathbf{j}+(4+p)\mathbf{k}$ for $\bullet^1$.
2. Do not accept $\begin{pmatrix} 6\mathbf{i} \\ -3\mathbf{j} \\ (4+p)\mathbf{k} \end{pmatrix}$ or $6\mathbf{i}-3\mathbf{j}+4\mathbf{k}+p\mathbf{k}$ for $\bullet^1$. However $\bullet^2$, $\bullet^3$ and $\bullet^4$ are still available.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	$\bullet^2$ find an expression for magnitude	$\bullet^2$ $\sqrt{6^2+(-3)^2+(4+p)^2}$	3
	$\bullet^3$ start to solve	$\bullet^3$ $45+(4+p)^2=49 \Rightarrow (4+p)^2=4$ or $p^2+8p+12=0$	
	$\bullet^4$ find values of p	$\bullet^4$ $p=-2,\ p=-6$	

Notes:

3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{6^2+-3^2+(4+p)^2}$ or $\sqrt{6^2-3^2+(4+p)^2}$ leading to $\sqrt{45+(4+p)^2}$, $\bullet^2$ is awarded.
4. $\bullet^4$ is only available for two distinct values of p.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓	$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓
$\sqrt{6^2-3^2+(4+p)^2}$	$\bullet^2$ ✗	$\sqrt{6^2+(-3)^2+p^2}$	$\bullet^2$ ✗
$27+(4+p)^2=49$		$45+p^2=49$	$\bullet^3$ $\boxed{\checkmark 2}$
$(4+p)^2=22$	$\bullet^3$ $\boxed{\checkmark 1}$	$p=\pm 2$	$\bullet^4$ $\boxed{\checkmark 1}$
$p=-4\pm\sqrt{22}$	$\bullet^4$ $\boxed{\checkmark 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
13. (a) (i)	•¹ find the value of $\cos x$ •² substitute into the formula for $\sin 2x$ •³ simplify	•¹ $\frac{\sqrt{7}}{\sqrt{11}}$ stated or implied by •² •² $2\times\frac{2}{\sqrt{11}}\times\frac{\sqrt{7}}{\sqrt{11}}$ •³ $\frac{4\sqrt{7}}{11}$	**3**
(ii)	•⁴ evaluate $\cos 2x$	•⁴ $\frac{3}{11}$	**1**

Notes:

1. Where a candidate substitutes an incorrect value for $\cos x$ at •², •² may be awarded if the candidate has previously stated this incorrect value or it can be implied by a diagram.
2. •³ is only available as a consequence of substituting into a valid formula at •².
3. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•⁵ expand using the addition formula •⁶ substitute in values •⁷ simplify	•⁵ $\sin 2x\cos x+\cos 2x\sin x$ stated or implied by •⁶ •⁶ $\frac{4\sqrt{7}}{11}\times\frac{\sqrt{7}}{\sqrt{11}}+\frac{3}{11}\times\frac{2}{\sqrt{11}}$ •⁷ $\frac{34}{11\sqrt{11}}$	**3**

Notes:

4. For any attempt to use $\sin(2x+x)=\sin 2x+\sin x$, •⁵ •⁶ and •⁷ are not available

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
14.	$\bullet^1$ write in integrable form $\bullet^2$ start to integrate $\bullet^3$ complete integration $\bullet^4$ process limits $\bullet^5$ evaluate integral	$\bullet^1$ $(2x+9)^{-\frac{2}{3}}$ $\bullet^2$ $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\ldots$ $\bullet^3$ $\ldots\times\dfrac{1}{2}$ $\bullet^4$ $\frac{3}{2}(2(9)+9)^{\frac{1}{3}}-\frac{3}{2}(2(-4)+9)^{\frac{1}{3}}$ $\bullet^5$ 3	5

Notes:

1. For candidates who differentiate throughout, only $\bullet^1$ is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/5.
3. $\bullet^2$ may be awarded for the appearance of $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$ in the line of working where the candidate first attempts to integrate. See Candidate F.
4. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
5. For $\bullet^2$ to be awarded the integrand must contain a non-integer power.
6. Do not penalise the inclusion of '$+c$'.
7. $\bullet^4$ and $\bullet^5$ are not available to candidates who substitute into the original function.
8. The integral obtained must contain a non-integer power for $\bullet^5$ to be available.
9. $\bullet^5$ is only available to candidates who deal with the coefficient of x at the $\bullet^3$ stage. See Candidate A.

Commonly Observed Responses:

Candidate A		Candidate B	
$(2x+9)^{-\frac{2}{3}}$	$\bullet^1$ ✓	$(2x+9)^{\frac{2}{3}}$	$\bullet^1$ ✗
$\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$	$\bullet^2$ ✓ $\bullet^3$ ^	$\dfrac{(2x+9)^{\frac{5}{3}}}{\frac{5}{3}}\times\dfrac{1}{2}$	$\bullet^2$ $\boxed{✓ 1}$ $\bullet^3$ ✓
$3(2(9)+9)^{\frac{1}{3}}-3(2(-4)+9)^{\frac{1}{3}}$	$\bullet^4$ $\boxed{✓ 1}$	$\frac{3}{10}(2(9)+9)^{\frac{5}{3}}-\frac{3}{10}(2(-4)+9)^{\frac{5}{3}}$	$\bullet^4$ $\boxed{✓ 1}$
6	$\bullet^5$ $\boxed{✓ 2}$ see note 9	$\dfrac{363}{5}$	$\bullet^5$ $\boxed{✓ 1}$

Commonly Observed Responses:	
Candidate C $(2x+9)^{-\frac{2}{3}}$ $\bullet^1$ ✓ $-\frac{5}{3}(2x+9)^{-\frac{5}{3}}\times\frac{1}{2}$ $\bullet^2$ ✗ $\bullet^3$ ✓ $-\frac{5}{6}(2(9)+9)^{-\frac{5}{3}}-\left(-\frac{5}{6}(2(-4)+9)^{-\frac{5}{3}}\right)$ $\bullet^4$ ✓ 1 $\frac{605}{729}$ $\bullet^5$ ✓ 1	**Candidate D** $(2x+9)^{-\frac{2}{3}}$ $\bullet^1$ ✓ $\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\times 2$ $\bullet^2$ ✓ $\bullet^3$ ✗ $6(2(9)+9)^{\frac{1}{3}}-6(2(-4)+9)^{\frac{1}{3}}$ $\bullet^4$ ✓ 1 12 $\bullet^5$ ✓ 1
Candidate E $(2x+9)^{-\frac{3}{2}}$ $\bullet^1$ ✗ $\frac{(2x+9)^{-\frac{1}{2}}}{-\frac{1}{2}}\times\frac{1}{2}$ $\bullet^2$ ✓ 1 $\bullet^3$ ✓ $\left(-(2(9)+9)^{-\frac{1}{2}}\right)-\left(-(2(-4)+9)^{-\frac{1}{2}}\right)$ $\bullet^4$ ✓ 1 $-\frac{1}{\sqrt{27}}+1$ $\bullet^5$ ✓ 1	**Candidate F** $1\times(2x+9)^{-\frac{2}{3}}$ $\bullet^1$ ✓ $x\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\times\frac{1}{2}$ $\bullet^2$ ✓ $\bullet^3$ $\bullet^4$ and $\bullet^5$ are not available

Question	Generic scheme	Illustrative scheme	Max mark
15.	•[1] root at $x = -4$ identifiable from graph •[2] stationary point touching x-axis when $x = 2$ identifiable from graph •[3] stationary point when $x = -2$ identifiable from graph •[4] identify orientation of the cubic curve and $f'(0) > 0$ identifiable from graph	•[1] •[2] •[3] •[4] 	**4**

Notes:

1. For a diagram which does not show a cubic curve award 0/4.
2. For candidates who identify the roots of the cubic at '$x = -4, -2$ and 2' or at '$x = -2, 2$ and 4' •[4] is unavailable.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]