

THE EMPEROR HAS NO CLOTHES: MUSIC AS IT ACTUALLY IS ON THE PIANO KEYBOARD

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INTRODUCTION

I approached the piano as an adult beginner interested in trying to play jazz, which eventually led me to wonder how jazz pianists can improvise in terms of a music notation that seemed to me to be misleadingly complex for the piano.¹ I thought improvisation must tap into deeper musical structures that are obscured by this notation. Music notation has stood the test of time and is here to stay for piano music, even if for no more reason than the huge legacy of piano music written in it. But the piano has also stood the test of time without needing variable-pitch piano keys to play notes exactly as specified by music notation (e.g., C# vs Db). In effect, “the emperor has no clothes” — piano music is much simpler than the “clothes” of this notation make it seem.

Trying to understand the deep structure of piano music without the obscuring clothes of music notation became an absorbing hobby that eventually led me to a simple system I call **PKP** (standing for “Picturing Keyboard Patterns”). The scope of PKP is tonal music in which a melody line is harmonized at selected points by chords formed of clusters of piano keys. Before you stop reading because of my admitted lack of musical credentials, consider the opinions of PKP of a couple of music professionals. Musical theorist Paul Steinbeck: “The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music.” Jazz pianist/composer/teacher Taylor Eigsti: “... a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano.”

PKP differs from music notation in putting parallel modes on notational center stage, determined by words from a 6-letter, DNA-like alphabet, instead of relative modes, determined by the key signatures of music notation. “Modes” are scales defined by interval sequences: parallel modes have the same tonic; relative modes have the same notes going up from different tonics. Parallel modes place the focus on the home octave of a piece defined by a single home tonic. This enables exploiting two simple facts to yield a simple picture: every octave on the keyboard has the same shape (12 half tones), independently of its mix of black and white keys; and the half tones of overlapping octaves are aligned.

The alphabet is analogous to the alphabet of biological DNA in the sense that the words identify deep structure in terms of elemental **building blocks** (primarily fifths, fourths and tritones). The words are **signatures** of parallel modes. Sequences of letters or short words from the signatures represent the flow of building blocks in melody and harmony. Conversely, letters in this flow identify the signatures of contributing modes.

The alphabet represents building blocks relative to a single conceptual home octave that's

¹ This chord progression for the haunting E ♭ blues *Goodbye Porkpie Hat* written by Mingus as a tribute to Lester Young, is a “poster child” for the complexity with which music notation represents things that are simple on the keyboard. It's presented here only as an example of the need for a simpler view. The progression is E ♭ 7#9—B9(13)—**EM9**—A7#11—**D ♭ 9sus**—B9(13)—**D ♭ 7sus**—E ♭ 7—**A ♭ m11**—B7(13)—Fm7 ♭ 5—B ♭ 7#5#9—C13#11—F7(13)—B7—**EM7**—A7(13)—A ♭ 7—B ♭ 7—D ♭ 7—E ♭ 7#9—B7—EM7#11—A7#11. There are 24 chords, an average of 2 per bar for 12 bars, only 5 of which (highlighted in blue) don't contain tritones. The tritone content is the basis of deep structure of startling simplicity that is the subject of this document.

harmonically equivalent to a stack of home octaves on the keyboard. Many pieces of music have an easily recognizable home tonic: it's the piano key on which the melody line comes to rest at the end, or starts, or both. Even if there's more than one possible home tonic, it's always possible with tonal music to select one, leaving the others as secondary tonics to be understood by reference to it.

PKP represents shapes of scales and chords in the same terms, enabling simple annotations on the written music to provide contextual cues that help in learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. This turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

Is this combination of simplicity and depth only a serendipitous side effect of the organization of the piano keyboard? Or is it a fundamental property of music that's obscured by music notation? Either way, it works in practice. To discover it, I had to enter uncharted territory. Nothing I read said anything about it and no expert I approached knew anything about it. The piano is not just for experts, but the conventional approach to teaching and learning it tends to make it so: understanding is expected to emerge by osmosis from the same extensive practicing that develops "chops." The idea that understanding can be had independently of practicing, in a form that guides practicing, is unconventional.

What qualifies me, an amateur with no formal musical training, to write about these matters? Arguably, the lack of training is itself a qualification because the absence of received wisdom about music notation helped in seeing that the "emperor has no clothes." Add my experience as a university professor developing notations to deal with complexity in another field, computer software — notations that seemed to me to offer insight by analogy. Computer programs are like music notation in the sense of using abstract notations to describe things to be performed on "hardware." Add long-standing curiosity about how music works and why its notation is so complex. Add training in math and physics that made me confident I could find an interval-based representation of piano music that's a dual of the note-based representation of music notation, analogous to dual representations in math and physics such as frequency-response/time-response and wave/particle. Add time to indulge my curiosity after retiring from being a university professor not long after I took up the piano.

GUIDE TO READERS

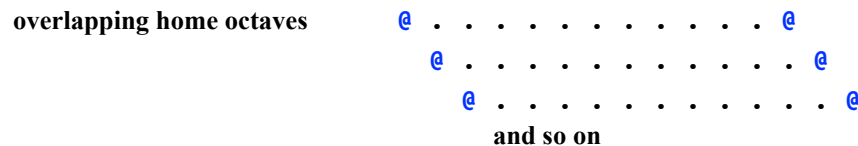
This document began as notes to myself to explain the ideas I was developing, in way that I would have understood when I started out. It should be read sitting in front of a piano keyboard on which to try out the unfamiliar concepts and notations. The potential audience includes novices like I was when I started out, pop and jazz musicians who are not pianists but want to explore harmony on the piano, music teachers interested in a different approach to teaching piano music, and "wannabe" experts interested in understanding music at a higher conceptual level than notes.

Chapter 2 develops the basic concepts and notation. Chapter 3 provides examples of a few basic pieces represented in these terms. Chapter 4 explores the building-block world in depth. Chapter 5 provides some more advanced pieces that initially puzzled me and I suspect might puzzle anyone (the first example is the piece with the footnoted chord progression on the previous page). Chapter 6 provides observations and conclusions. References, acknowledgements and comments from some readers follow. Appendix A summarizes unfamiliar terminology, Appendices B-E are about scales (B), chords (C), non-classical parallel modes (D) and the importance of hidden symmetries (E).

CHAPTER 2: CONCEPTS & NOTATION

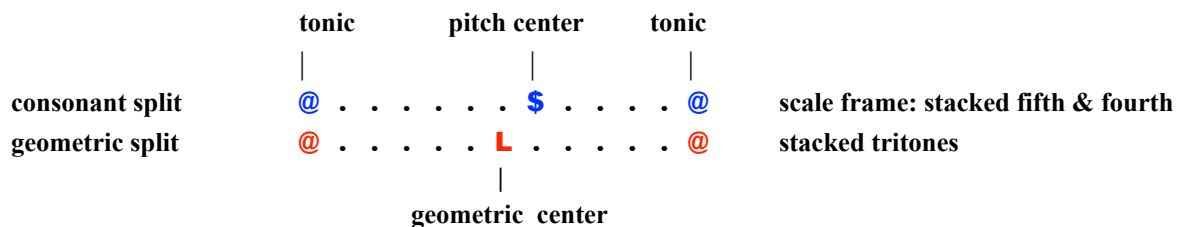
A SIMPLE CONCEPTUAL MODEL

A simple but accurate conceptual model of the home octave on the emperor-has-no-clothes piano is pictured below. Each possible home octave is represented by a horizontal line split into twelve equal parts representing twelve musically equal half tones. The top and bottom notes are home tonics an octave apart symbolized by @. Twelve overlapped potential home octaves are offset by half tones, thus vertically aligning all the shared notes represented by the dots. A particular piece of music has a single home octave identified by the assignment of @ to a piano key. A single conceptual home octave represents a stack of visually identical, harmonically equivalent home octaves on the keyboard. This is the jumping off point for PKP. The payoff is a high level view of musical changes that's independent of the particular home tonic of a piece and of music notation. Different home octaves look very different on the keyboard but the differences are independent of music notation and are understood in a simple way.



Pitch sizes of the half tones increase within the octave to make the pitch of the top note double that of the bottom note. The increasing pitch sizes are represented by equal divisions of the line because they're musically equal. The fundamental difference between the piano and music notation for it is the sharing of intervals between overlapped octaves, symbolized by the vertically aligned dots. The reality is the dots cannot be exactly vertically aligned if the half tones within each octave are exactly musically equal. The piano forces alignment by providing the same piano key for all the vertical dots. Musical ears are relatively insensitive to the resulting slight errors in the pitch sizes of half tones, because they're dissonant intervals. Equal temperament tuning minimizes the effect on larger intervals formed of sums of half tones. This must be good enough because the piano has stood the test of time without adjustable piano keys to play the slightly different pitches.

Fundamental **building blocks** of music emerge from splitting the conceptual home octave in half in two ways, as shown next. This simple picture turns out to be the basis for a simple way of voicing chord progressions, in context, as sequences of shapes that morph from one to the next.



The pitch center **\$** is shared by most tonic scales, making the scale **frame** formed by it and the tonic a useful reference for all tonic scales (tonic scales without a pitch center exist but they're understood by reference to scales with pitch centers). The letter **L** that identifies the geometric center is one of the letters of PKP's mode-identifying alphabet. Other letters will emerge as we go along.

Two kinds of building blocks are determined by these splits. Tritones (6 half tones, or 3 whole tones) are one of a kind: fundamentally dissonant intervals when their notes are sounded together. Fifths and fourths (sizes 7 and 5 half tones) are two of a kind: fundamentally consonant intervals that are mutual inversions. This small set of building blocks, in different keyboard positions, is sufficient for most purposes because smaller and larger intervals emerge as inner or outer intervals of shapes of scales or chords formed from them. The smaller or larger intervals are occasionally needed as independent building-block-like quantities but the building-block notation to come covers this possibility in a simple way that doesn't require explicitly including them in the set of basic building blocks.

The sounds of tritones, fifths and fourths **sliding** to different keyboard positions and **morphing** into each other within the home octave as the music moves forward are fundamental to much music. Even with no additional information, this seems plausible because of the visibly fundamental nature of the octave splits that produce them. The notes may be spread out in the music but the essence of their musical contribution is captured by their harmonically equivalent presence within the home octave. The tritone-tritone split morphs into the fifth-fourth split, and vice-versa by altering one note by a half tone. The bottom fifth and the bottom tritone morph into each other in the same way, as do the top fourth and the top tritone. One may easily imagine any of these intervals sliding to other keyboard positions, where different morphings may be performed.

Tritones are structurally fundamental to this conceptual view because they are the only intervals of fixed size on the keyboard (a tritone inverts into a tritone). There are only six tritones with different notes on the keyboard. All the possible fifths or fourths on the keyboard may be understood as morphed from these tritones. This warrants a new term, **fifo**, to identify the result as a building block of known type but unspecified size. The traditional terms are used when the size is known but the new term enables leaving the size to context. This provides deep notational simplification at a higher conceptual level than that of music notation.

THE ALPHABET

As illustrated next, the alphabet **PADMIL** provides six letters identifying the **anchors** of the six possible different tritones relative to the home tonic (counting opposite inversions as the same tritone). The anchors are also of fifos morphed from the tritones but the default interpretation is tritones. A tritone anchor identifies the position of its bass note relative to the home tonic. Letter **L** identifies the geometric center of the home octave as the anchor of a tritone going up from it. The other letters identify tritones anchored successive half tones down from this. The opposite end of the tritone requires no symbol because it's determined by the fixed tritone size of 6 half tones. There are no anchors in the top fourth because inversions of tritones anchored in the bottom fifth provide tritones with bass notes (not anchors) in the top fourth.

conceptual home octave

@ P A D M I L \$ x x x x @

This particular set of anchor symbols is useful because it provides a connection to music notation via the classical modes that determine the scales of key signatures. The letters are the first letters of the names **P**hyrgian, **A**eolian, **D**orian, **M**ixolydian, **I**onian and **L**ydian/**L**ocrian of parallel modes that contain tritones with anchors in these relative positions. Letter **L** covers two modes with the same

tritone in opposite inversions. There is no ambiguity because knowing one means means knowing the other (all non-tritone notes are different and context determines which is meant). Signatures of parallel classical modes are of the form **//X**, where **X** is an alphabet letter identifying a tritone anchor and the prefix stands for “parallel.” The two modes for tritone **L** are identified by signatures **//L** (Lydian) and **alt-L** (Locrian).

The connection of the alphabet letters to the classical modes of music notation is at once one of the most powerful features of PKP and a source of possible confusion, due to some letters also being used in standard music theory and notation (letter notes A and D, Roman Numeral I for the tonic as a chord root, letter M for “major” in chord notation). Any set of six symbols would do in principle but this notation is too evocative to discard. Possible confusion is made manageable by the special font (Arial Black) and by avoiding mixing the notations together.

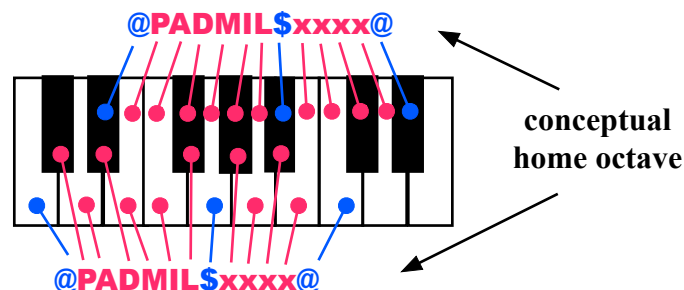
The new thing here is recognizing that the uniqueness of tritones in classical modes makes them identifiers of parallel modes. This is not even imaginable in music notation for two reasons: every tritone originates in a different key signature, making anything involving different tritones complex; and the tritones cannot be independent identifiers of the modes because they can’t even be written down without first knowing the modes as sequences of notes relative to a key signature.

The alphabet is a circular loop in the sense that traversing it in one direction goes off one end to proceed in the same direction from the other end. Each step in such a traversal identifies a tritone offset from the previous one by a half tone. This is true even between opposite ends of the alphabet because, opposite inversions of the **L** and **P** tritones are offset by a half tone.

Anyone familiar with classical modes will recognize that the **PADMIL** ordering is unconventional. The conventional ordering is **IDPLMAL**, identifying successive relative modes of the Ionian mode. In this order, **L** appears twice because the Lydian and Locrian relative modes have different starting notes. The **PADMIL** order is different because it’s based on parallel modes. It’s also more practically useful because it puts parallel modes that differ by one note adjacent in the list.

One classical pianist with whom I discussed my ideas wondered why I bothered with “all the old church modes” because, he said, composers seldom write in modes other than Ionian or Aeolian that define the conventionally accepted major and minor modes of key signatures. The answer is twofold: all the modes come into play indirectly via accidentals; and the mode signatures of non-classical modes that originate in no key signature are determined by words formed of the same letters that determine the classical modes.

The mapping of the conceptual home octave to the keyboard is illustrated next for two of the twelve possible home octaves on the keyboard. The twelve equal divisions of the octave are of the half tones played by adjacent piano keys, not the linear distances between them on the keyboard.



The same conceptual home octave looks very different in different places on the keyboard, but the difference is manageable because it's independent of music notation and depends only on six easily recognizable tritones. When playing a particular piece — whether learning it or revisiting it — keeping the piano key of its home tonic fixed in the mind is essential, to avoid inadvertently interpreting things relative to the different home tonic of some recently played piece that's still in the mind. Putting a temporary label (e.g., a small stick-on circle) on the home-tonic piano key can be helpful.

Fifos from Tritones

Different meanings of any letter **X** are determined either by context or by color coding: **X** for tritones, **X** for fifths and **X** for fourths. A tritone may morph into four possible fifos on three different anchors. For example, tritone **M** (pronounced “em-tri”) may morph into a fifth **M** (“em-fi”) or a fourth **M** (“em-fo”) on the same anchor; or into a fifth **D** (“dee-fi”) or a fourth **I** (“eye-fo”) on different anchors a half tone down or up. The ability to refer to all these possibilities so simply greatly simplifies much thinking. Context at different points in a piece of music generally narrows the choices down, often to only one. The concept of fifos morphed from tritones is independent of the presence or absence of the tritone in particular scales because the alphabet is a property of the home octave as a whole.

There's a mismatch that can be confusing between the conventional meanings of “fifth” or “fourth” as 5 or 4 scale steps in a classical mode, and their meanings as sizes in half tones. A fifth has 7 half tones, not 5; a fourth has 5 half tones, not 4; a major third has 4 half tones.

Inversions of Building Blocks

The default meaning of an anchor is a building block going up from it. An underlined letter means a building block of the indicated size going down from it. For example, symbols **L** (“el-tri”) and **L** (“el-tri-down”) indicate tritones going up and down from the anchor to the top or bottom of the home octave. For tritones only, an underlined letter indicates an inversion because a tritone is the same size in either inversion. Inverting a tritone is the same as transposing it by a tritone. Inverting a mode tritone may therefore be interpreted as transposing a mode by a tritone, providing a simple basis for understanding parallel mode changes and same-mode tonic changes as “two sides of the same coin.”

The underlining notation is the same for fifos but the interpretation as inversions is different because opposite inversions of fifos are different sizes. For example, **M** (“em-fi”) indicates a fifth going up from an anchor and **M** (“em-fo-down”) indicates its opposite inversion, namely a fourth going down.

There are no anchors in the top fourth of the scale frame because building blocks with bass notes in the top fourth are determined by the six anchors in the bottom fifth. **This convention makes tritone anchors unambiguous:** a tritone has one anchor that determines it; the other end of the tritone may be a bass note but never an anchor. Making tritone anchors unambiguous does the same for mode signatures.

ASSEMBLING BUILDING BLOCKS INTO MUSICAL SHAPES

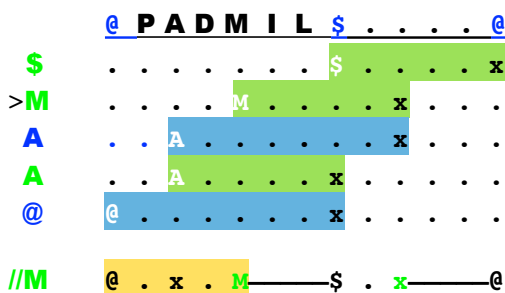
Experimenting with shapes formed from building blocks immediately introduces parallel modes and chords from them in a very simple way. Let's start with shapes that have no dissonance in them, meaning no half tones and no tritones.

Pentatonic Modes

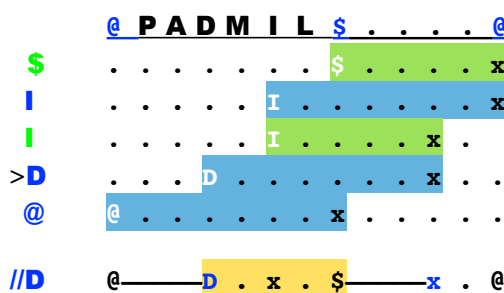
The following two assemblies of five fifos, pictured in Lego-like terms, determine foundation parallel modes of music called pentatonic major and pentatonic minor. These modes are foundation

modes of music in many cultures worldwide because anyone with a musical ear can sing melodies in them and harmonizing the melodies is simple because there's no dissonance. The alphabet letters in contrasting white text inside the building blocks are the fifo anchors. In the bodies of tables such as this, both here and later, entries are in a fixed width font (Courier) that enables creating aligned columns in text; there's no change of meaning.

pentatonic major

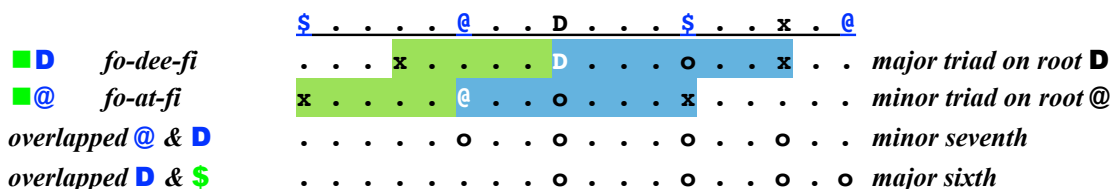


pentatonic minor



The scale frame and one fifo anchor (identified by ">") are visibly sufficient to identify each mode, given the constraint of no half tones. The mode signatures at bottom left are the identifying fifo anchors with prefix "//" (standing for "parallel"). The scale frame provides 2 notes, the fifo of the mode signature provides 2 notes, and the constraint of no half tones determines a symmetric split into two whole tones of the scale segments highlighted in yellow. The horizontal lines bring forward a characteristic feature of these modes, namely the presence of two minor third intervals (3 half tones). The terms "mode" and "scale" mean different things here: a mode is defined by a mode signature; a scale is defined by a note sequence. In music notation, the note sequence is represented by note symbols. The note sequences here are understood as keyboard shapes, without reference to note symbols, but can, obviously, be translated directly into them. The thrust here is understanding keyboard shapes independently of the notes of which they're formed.

This way of understanding parallel modes goes beyond just identifying them. It also offers harmonic sequences from them. **Core harmony** is provided by sequences of building blocks, identified by **anchor lines** formed of sequences of anchor letters. An example is **D-@** (*dee-fi, at-fi*) from the pentatonic minor. Useful stepping stones to chords are **octave shapes** centered on the anchor line, as illustrated next for the **D-@** sequence. The box prefixes identify octave-completion fourths (think of them as attached underneath the anchor ends of the fifths). Add one internal scale note in the fifths and you have triad chords with the top note doubled an octave down. Minor seventh and major sixth chords are overlapped building blocks from these shapes. This is a simple example of very general concepts.



Pentatonic major and minor modes formed of all five black piano keys are easiest to see on the keyboard. The tonic of the all-black-key pentatonic-major mode is the black key at the geometric center of the C octave. The tonic of the all-black-key pentatonic-minor mode is the next black key down. A lot of fun can be had, and insight gained, by improvising pentatonic melody and harmony on the black keys. This leads straightforwardly to blues because basic blues modes add one note to these modes.

BLUES MODES

Pentatonic modes lead directly to both blues modes and classical modes. The path to blues modes is worth exploring first because it's simpler. Blues is complex in music notation because its scales are different in kind from the classical modes that determine the scales of key signatures. Blues is simple when understood as a simple extension of pentatonic modes.

As shown next, basic 6-note blues modes taught to beginners as “the” blues scales add one note the pentatonic modes. In each case, the added note is a tritone anchor because it and a scale note form a tritone (red text). The mode signature is therefore a combination of the fifa anchor that provides the pentatonic mode and the added tritone anchor. The fifa anchor distinguishes this signature from the single-tritone signature that identifies a classical mode. These modes share the same interval sequence starting from different positions a minor third apart. The interval sequence is *3h-2h-h-h-3h-2h* going up from the tonic of the minor blues mode, and up from a minor third below that for the major blues mode.

major blues (actually minor-major)

@ P A D M I L \$ @
//M @ . x . **M** — \$. **x** — @
//DM @ . x **D** **M** — \$. **x** — @

minor blues

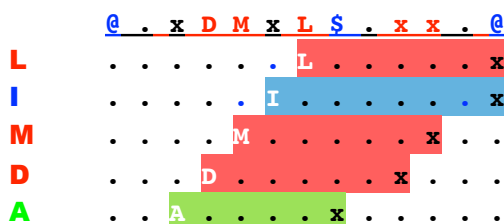
@ P A D M I L \$ @
//D @ — **D** . x . \$ — **x** . @
//DL **x** — **D** . x **L** \$ — **x** . **x**

A powerful way of creating new modes is by **mashups** of more basic modes. A mashup, as the term is used in the music business, means combining different musical things to make a new musical thing that's different in kind. A mashup of parallel modes combines all the notes to form a new parallel mode. Creating new modes this way only makes sense if the scales are nearby ones that share many notes, such that their combination has no more than 9-10 notes, and is organized in a coherent way. The blues mashups shown next are like that.

	@ P A D M I L \$ @	
//M	@ . x . M — \$. x — @	pentatonic major (5 notes)
//D	@ — D . x . \$ — x . @	pentatonic minor (5 notes)
//DM	@ . x D M x . \$. x x . @	mashup (8 notes)
<hr/>		
//DM	@ . x D M — \$. x — @	major blues (6 notes)
//DL	x — D . x L \$ — x . x	minor blues (6 notes)
//DM.L	@ . x D M x L \$. x x . @	mashup (9 notes)

The 8-note mashup of the two pentatonic modes may be understood to follow from singers of simple pentatonic tunes “bending” selected notes by a half tone to give a sad twist to the major mode or a happy twist to the minor mode. The effect is to switch to the opposite pentatonic mode. The mashup of the major and minor blues modes adds one note, often called the “flatted 5th” because it’s a half tone down from the 5th note of a classical mode. This note is a logical extension of singers bending the notes of pentatonic modes. Blues modes are different in kind from classical modes — different numbers of notes, multiple tritones, multiple half tones (including adjacent half tones forbidden in classical modes). I learned them in conventional terms some years ago in a blues piano workshop course at the then *Jazz School* in Berkeley (now the *Jazz Institute*). The novelty here is knowing them by simple, tritone-only mode signatures that are independent of specific home tonics and of music notation.

That the tritones are sufficient identifiers of the modes is illustrated by the following Lego-like picture of the main building blocks. The 3 tritones provide 6 notes (including the tonic), the scale frame adds 1 note (the pitch center) and the original pentatonic modes add 2 notes that anchor scale fifos, for a total of 9 notes. Think of the fifth **I** as morphed from tritone **I** and of the fourth **A** as morphed from tritone **A**, neither of which is in the scale.

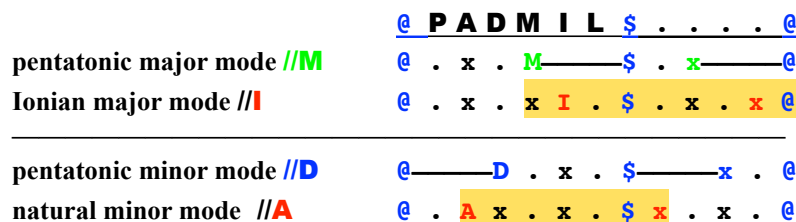


This picture suggests core harmony sequences just as an earlier picture suggested them for pentatonic modes. The difference here is tritones dominate the core harmony, which is therefore very simple because tritones are very simple on the keyboard.

As before, octave shapes based on core harmony provide stepping stones to chords. Examples are left to Chapters 3 and 5.

CLASSICAL MODES

The path from pentatonic modes to classical modes is different in kind. It’s one of refinement rather than mashup. Looking ahead, mashups of classical modes enter the picture as sources of non-classical major and minor modes. The refinement that creates classical modes in the first place is splitting the minor third intervals of pentatonic modes into a half tone and a whole tone, as illustrated next for the Ionian and Aeolian modes that provide the default tonic scales of key signatures. Symmetric shapes (yellow highlighting) are determined by tritones (red text) that follow from the splits.



The complete set of parallel classical modes constructed this way is summarized next. There's a lot of information here, provided for reference, but the meaning is very simple: a tritone determines a primary mode (signatures of the form **//X**); mode changes are determined by tritone changes; changes between adjacent primary modes alter one note, provided by the new mode tritone; knowing a primary means knowing its **alt** mode (all non-tritone notes are different); only **alt-L** is a true parallel mode because only its tritone contains the home tonic; counting relative modes, all possible key-signature scales with tonics provided by the twelve piano keys of a home octave are covered.

		<u>@ P A D M I L \$ @</u>	
Lydian	//L	@ . x . x . L \$. x . x @	major tonality
Locrian	alt-L	@ x . x . x L . x . x . @ x	minor
Ionian	//I	@ . x . x I . \$. x . x @	major
	alt-I	. x . x . I x . x . x x .	minor
Mixolydian	//M	@ . x . M x . \$. x x . @	major
	alt-M	. x . x M . x . x . x x .	minor-major
Dorian	//D	@ . x D . x . \$. x x . @	minor
	alt-D	. x . D x . x . x x . x .	minor-major
Aeolian	//A	@ . A x . x . \$ x . x . @	minor
	alt-A	. x A . x . x . x x . x .	major
Phrygian	//P	@ P . x . x . \$ x . x . @	minor
	alt-P	. P x . x . x \$. x . x .	major

The highlighted symmetric shapes determine the modes. The shapes follow from the symmetric arrangement of half tones relative to the mode tritone (both inside it or both outside it). Symmetric shapes formed by and around tritones are a fundamental feature of the PKP way of knowing scales. Symmetry is well known in math and physics to be a cornerstone for understanding complexity, so it's logical that it might do so for music, which is complex by any measure. The only hurdle to overcome is the unfamiliar notation. The notation is symbolic but there's no math here and everyone has an intuitive understanding of symmetry from everyday life.

The **alt** modes are “tritone substitute modes” (same tritones, tonics a tritone apart). The term is inspired by the term “tritone substitutes” for chords with the same tritone and roots a tritone apart. Learning that such chords are often used by jazz pianists for chord substitutions in improvisations provided my first glimpse of the possibility that tritones might be fundamental to an interval-based view of piano music. That they are fundamental is demonstrated by the fact that they keep popping up as such in every interval-based representation of modes and chords.

Why include **alt** modes that are not true parallel modes in a table of parallel modes? The answer is they and the primary modes provide the means of seeing parallel mode changes and tonic changes as “two sides of the same coin.”

A Building-Block View of the Master Ionian Mode

The building blocks of the master Ionian mode are summarized next in Lego-like terms. As for pentatonic modes and blues modes, the summary provides several deep insights in a simple way. It provides a menu of available building blocks for constructing chords. It brings forward core harmonic sequences to the eye. It illustrates in a graphic way a big reason for introducing the concept of a fifo, namely mutual substitutability.

		@	P	A	D	M	I	L	\$	@
frame	\$	\$	@
core	I	I	@
	I	I	x	.
	M	.	.	.	M	x	.
secondary	M	.	.	.	M	x	.
	A	.	.	A	x	.
	A	.	.	A	\$
frame	@	\$

The main core harmonic sequence is **I-I-M** (*eye-fi*, *eye-tri*, *em-fi*) because the tritone is core by definition and the two fifths are morphed from it, putting them a half tone apart, which makes them mutually dissonant and therefore functionally different. The fifth **M** is a resolution fifth because it establishes major tonality, automatically making the fifth **I** a non-resolution fifth. The sequence **I-I** anticipates resolution in the mode of the tritone and the sequence **I-M** provides the resolution.

The lightweight fifo concept exploits the substitutability of fifths and fourths in harmony. All of the above fifos except the fifth **I** are harmonically ambiguous and mutually substitutable. They're harmonically ambiguous because they're consonant with both core fifos. They're mutually substitutable because they're consonant with each other. Fifos are a source of both harmonic richness in music and notational complexity in music notation. The lightweight notation keeps the former and avoids the latter.

As before, octave shapes based on core harmony provide stepping stones to chords, as illustrated next for the **I-I-M** core of the Ionian mode.

notation	\$	@	P	A	D	M	I	L	\$	@
■ I	x	I	x
■ I	x	I	x
■ M	x	.	.	.	M	x
pronunciation																			
																			<i>fo-eye-fi</i>
																			<i>tri-eye-tri</i>
																			<i>fo-em-fi</i>

Octave shapes are easily morphed into 3-note shapes from context that have all notes different. The shapes may be stacks of building blocks, in which case the same notation applies. For example, the sequence **■ I-■ I-■ M** morphs into **■ I-■ I-■ M** which the final two shapes are "all fourths" (counting the tritone as an augmented fourth). This sequence is very simple on the keyboard: one note drops a scale step and then all notes drop a scale step. It's simple on the keyboard but not so simple in chord notation (Chapters 3 and 5 and Appendix C provide examples).

Chromaticism

Successive parallel classical modes bring chromaticism (deviation from a written key signature) into the domain of classical modes in a very simple way: one alphabet step (up or down) in a mode signature brings in a new tritone that alters one scale note; two steps alters two scale notes; three steps alters three scale notes; and so on. In each step, one altered note is provided by the tritone (the other tritone note is in both modes). The new modes are chromatic relative to the key signature of the original mode.

The chromaticism of such changes is identified in music notation by accidentals in melody lines and chromatic chords in harmony, without any explicit indication of its origin, which may be classical modes, non-classical modes, or ornamentation that has no scale implications. Explicit key-signature changes indicate the origin is in classical modes, provided there are no accidentals in the melody and no chords that go outside the key signatures, but much strongly chromatic music is written with a single key signature. Even the simplest classical mode changes can be complex in music notation.

“How strange the change from major to minor” — these words from the song *Every Time We Say Goodby* (covered in Chapter 5) express, incidentally, the startlingly complex nature of a simple parallel mode change such as **//I-//A** in music notation. Suppose the home tonic is the first black key above C, which is either C# or Db in music notation, depending on context. Then this change is from 5 flats for Db major to 4 sharps for C# minor. The change naturalizes 5 notes and then sharps 4 notes — 9 symbol changes to alter 3 notes! A particularly confusing feature of this change is the different symbols for the minor tonic (C#) and the major tonic (Db) that seem to imply slightly different pitches for the same tonic. They don’t — what they actually imply is different pitches for the notes C and D, when used as references for sharps or flats. The pitch differences are real independently of the piano but not for the piano, and so overcomplicate written piano music. The piano has stood the test of time without variable pitch piano keys that would enable pianists to play the slight pitch differences. The PKP way of understanding parallel modes does an end run around this misleading complexity.

Going beyond classical modes requires thinking of mode signatures as a kind of functional generalization of key signatures. The contribution of PKP is understanding all such changes in a simple way in terms of tritone changes.

These are deep and complex waters in music notation. PKP concepts and notation enable probing these depths in a way but that provides insight without becoming overwhelmed by details. That said, the probing remains challenging because music is challenging, as illustrated by the advanced examples in Chapter 5.

Two Sides of the Same Coin

Parallel mode changes for classical modes and same-mode tonic changes were said earlier to be two sides of the same coin. Both are interpretations of the same tritone change. For example, the tritone change **I-D** down a whole tone may be interpreted as a parallel Ionian-Dorian mode change or as tonic change down a whole tone to a relative Ionian mode. The parallel change alters 2 notes and the relative change alters zero notes. The same tritone appears in opposite inversions.

	<u>@ P A D M I L \$ @</u>	
//I	@ . x . x I . \$. x . x @	Ionian
//D	@ . x D . x . \$. x x . @	the new Ionian tritone is D
	↑ tonic of relative Ionian mode down a whole tone	

A change to a new Ionian tonic that's not in the primary parallel mode of the new tritone requires an **alt** mode with all non-tritone notes different. In the following continuation of the same example, the tonic is a tritone up from a whole tone down, putting it a major third above the home tonic. The mode change alters 4 notes and the tonic change alters zero notes.

@ **P A D M I L** \$ @

//I @ . x . x **I** . \$. x . x @ Ionian

alt-D . x . **D** **x** . x . x **x** . x . the new Ionian tritone is **D**

↑ tonic of relative Ionian mode up a major third

The same simple principles cover all possible mode and tonic changes, as summarized in a notation-free way in the following simple table. The changes in the left column are from an established mode (parallel or alt) identified by a tritone anchor position in the alphabet, to a parallel mode identified by a different position a given number of steps up (+) or down (-) in the alphabet as a circular loop (steps off one end wrap around to the other end). Steps in opposite directions add up to a tritone. The yellow highlights identify changes that alter the fewest notes. This is sufficient to identify the mode signature of a result because changes between primary modes and alt modes alter the most notes.

SAME-MODE CHANGES TO TONICS A TRITONE APART				
<u>anchor</u>	<u>tonic</u>	<u>altered</u>	<u>tonic</u>	<u>altered</u>
<u>change</u>	<u>change</u>	<u>notes</u>	<u>change</u>	<u>notes</u>
+3	-3	3	+3	3
+2	-4	4	+2	2
+1	-5	1	+1	5
0	0	0	± 6	5
-1	+5	1	-1	5
-2	+4	4	-2	2
-3	+3	3	-3	3

← example above

The table generalizes simply and directly to non-classical modes, but is also less important for them because they're already so chromatic that extra chromaticism provided by tonic changes tends to be avoided.

NON-CLASSICAL MODES

Any scale containing one or more tritones (which means all scales with 7 or more notes) is identified by a **mode signature** provided by a word formed of one or more letters of **PADMIL**. With the occasional exception of a fifo anchor added to resolve an ambiguity, the letters of a word identify only tritones. The tonic and pitch center of the home octave — a half tone down from bottom of the alphabet and a half tone up from the top — are never tritone anchors but may be fifo anchors. The full alphabet identifies the chromatic scale, with two notes contributed by each of the six tritones. Words formed by pruning letters from this word identify sub-scales of the chromatic scale. The default meaning of pruning a letter is morphing an anchored tritone into a fifo anchored a half tone above or below it that shares its other note with a mode tritone. Pruning all but one letter leaves the single-letter mode signature of a classical mode. There's nothing in music notation comparable to the range of useful mode signatures this yields.

Major & Minor Families

Major and minor family modes follows from a mashup of the **//A** and **//I** classical modes. The immediate result of the mashup is a 4-tritone, 10-note parent mode identified by **//ADMI**. The mixed minor-major tonality of this mode provides a solid basis for sub-modes of one or the other tonality. These sub-modes follow from morphing either the **D** or **M** tritone into a scale fifo with the same top note. The result is pair of 3-tritone, 9-note modes of minor or major tonality identified by a compound tritone/fifo signature, in which a single fifo anchor identifies the fifo morphed from the missing tritone (the fifth **D** is morphed from missing tritone **M**, and the fourth **M** is morphed from missing tritone **D**). The only difference between these modes is the highlighted tonality in the bottom fifth. The double appearances in the signatures of the same anchor as both a tritone and a fifo anchor are accurate but I prefer the equally accurate but simpler all-tritone signatures with a plus superscript that symbolizes filling in the top fourth.

	<u>@ P A D M I L \$ @</u>	
//ADMI	@ . A D M I . \$ x x x x @	//I + //A mashup
//ADD.I or //AD.I⁺	@ . x x . x . \$ x x + x @	minor family
//A.MMI or //A.MI⁺	@ . x . x x . \$ x + x x @	major family

The forms of the family modes say something important about tonality, namely that it's determined by the bottom fifth of the home octave, leaving the top fourth to context. Tritones **P** and **L** and the corresponding classical modes are excluded from this picture but often appear ornamentally in music from these scales as functional substitutes for tritones **A** and **I** (e.g., in sequences such as **P-A** or **L-I**).

A sampling of important sub-modes is provided below (see Chapter 4 for more).

	<u>@ P A D M I L \$ @</u>	
minor //AD.I	@ . A D . I . \$ x x . x @	“bebop” melodic minor (8 notes)
//D.I	@ . x D . I . \$. x . x @	melodic minor
//AD.I	@ . A D . I . \$ x — x @	harmonic minor
major //A.MI	@ . A . M I . \$ x . x x @	major mode corresponding to //AD.I
//A.M	@ . A . M x . \$ x . x . @	parallel mode of the melodic minor
//A.MI	@ . A . M I . \$ x — x @	harmonic major

The melodic and harmonic modes are like classical modes in having seven notes and no adjacent half tones. They appear in pieces of music as sub-modes of not only major and minor families but also of the blues family seen earlier. The modes are very simple, each in its own way. The melodic modes are simple because they're almost whole tone scales (yellow highlighting brings forward sequences of four whole tones). The harmonic modes are simple because their double tritones have circular symmetry manifested as a stack of three minor thirds. An empty outer minor third (identified by a horizontal line) establishes the opposite minor third as the determiner of the master tonality (identified by a major or minor fifo anchor). The rest of the mode follows from the scale frame and the rule of no adjacent half tones.

These modes are the only non-classical modes that have been formally developed in music notation

into a set of 7 parallel modes that are analogous to the 7 parallel classical modes. The book *Modality* develops these modes in conventional terms, with complex results that were part of my motivation for developing PKP (see Appendix D for more). A simple view of these modes is developed in Chapter 4, based on an 8-note mashup of the harmonic minor and major modes that may be called “harmonic minor-major.” This mode determines a total of 8 parallel modes (instead of $2 \times 7 = 14$), and context generally determines the corresponding 7-note mode, if needed.

CHROMATIC SCALE

The use of note symbols has been avoided so far to develop a view of music based on intervals as musical objects that may be understood independently of the pairs of notes of which they’re formed. Developing the same concepts in terms of note symbols bogs down in complexity. However note symbols are needed for melody lines and tonic changes.

It may seem ironic that an interval-based view of music requires a note-based representation of melody lines, but there’s a good reason for it. Purely interval-based representations of melody lines are ruled out by the difficulty of mentally tracking successive note positions without making errors. The chromatic scale coming up adds no significant complexity to the enterprise because its 12 symbols for 12 piano keys are in direct correspondence with the symbols of the conceptual home octave. A glance at one gives the meaning in the other.

Music notation does not provide a 12-symbol chromatic scale. I found a suitable one in a chord root notation used in Mehegan’s jazz piano instruction book, which I adapted. Mehegan’s notation uses Roman Numerals **I-VII** for the seven roots from the master Ionian mode of a tonic, and flatted Roman Numerals — using the flat symbols of music notation — for the five chromatic roots in the five whole tone intervals. As shown next, the adaption replaces the Roman Numerals by numbers **1-7** and the flat symbol by prefix **p** standing for “phlat,” meaning “next piano key down.”

chromatic scale of the home octave	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
conceptual home octave	@ P A D M I L \$ x x x x @

The replacement of the flat symbols by prefix **p** avoids the confusion that arises with Mehegan’s notation when sharps or mixed sharps and flats enter the picture. It’s not a conventional flat symbol, but a position indicator. It only applies to five numbers (**p1** and **p4** are *not* alternate symbols for **7** and **3**), and there are no symbols corresponding to sharps. The direct correspondence between the chromatic scale and the conceptual home octave enable symbols in either to be understood in terms of the other in a direct and simple way (e.g., the tritone anchored by **M** is **3-p7**).

Why not avoid the additional alphabet notation by identifying anchors by scale symbols? The answer is the powerful concept of an anchor is different in kind and needs its own notation. Scale symbols used for anchors would have to be identified as such. The alphabet symbols are PKP’s way of doing this.

The beauty of this chromatic scale notation is it mirrors the look of the C-octave on the piano: the un-prefixed symbols are the white keys and the prefixed ones are the black keys, giving visibility to the musical functions of the notes in any octave by mental reference to the C octave. The notation is simple enough to annotate on the staff next to note symbols of a written melody line.

Mode Scales

A first step in understanding a piece in PKP terms is annotating chromatic scale symbols next to melody notes on a staff. The annotations are an aid to interpretation. Melody modes are understood by

marking the chromatic scale positions so identified in a 12-column table of the kind seen many times so far. Such a table provides a basis for identifying successive parallel modes in a melody line. As already explained, parallel mode changes may be understood as tonic changes if the melody indicates such changes. The notes are the same, only the order is different.

Skeleton Melody Lines

A separate **skeleton melody line** formed of the annotated chromatic-scale symbols provides a useful basis for linking harmony to melody without becoming bogged down in details: it's independent of both music notation and the actual home tonic on the keyboard. This is the approach taken in the examples of Chapters 3 and 5. This is only for presentation and is not meant to suggest that such separate representations are necessary to use PKP. They're useful for complex cases but not necessary in general.

A skeleton melody line is formed of a sequence of chromatic scale symbols, within bar lines, with ups and downs indicated by arrows — for example **|5↗7↘5↘3|↘1,|** — omitting details of rhythm and timing, but adding commas to show phrasing. The interval changes in the chromatic scale, measured in half tones, accurately represent the way people with musical ears hear and remember melody lines. Rhythm and timing, assumed to be known by ear or from written music, are included while playing.

A mixed note/interval representation may seem desirable for melody lines that repeat the same interval sequence starting from different notes. For example, **|5↗7↘5↘3|↘1,|6↘4↘2|↘p7,|** repeats the interval sequence **4h↘3h↘4h** starting from notes **7** and **6**. The interval-based representation could be integrated into a note-based skeleton melody line but there are good reasons for not doing so: mixed notations are inherently more complex overall; understanding equivalent short segments from note representations is easy, as just described; and harmonically equivalent note sequences may have different interval sequences (e.g., **7↘5↘3↗1** is harmonically equivalent to **7↘5↘3↘1** but has a different interval sequence).

Chord Roots

Full chords represented by chord symbols are results in PKP, not starting points. Identifying implied chords requires a notation for chord roots. The origin of the chromatic scale makes its use for chord roots obvious: replace the numbers by RN (Roman Numeral) symbols: **I-pII-II-pIII-III-IV-pV-V-pVI-VI-pVII-VII**. Keep in mind that tonic root “**I**” is not the same as Ionian anchor “**1**” in a mode table. Assigning these root symbols to harmony shapes is simple. The implied symbols are often far from simple in music notation but that's not a problem for PKP.

SOME OBSERVATIONS

It's difficult to overstate the benefits that follow from understanding modes in terms of mode signatures that represent how they actually are on the keyboard. The strong chromaticism of the family modes and their differing numbers of notes tend to make key signatures no more than references for many accidentals, and to make chord symbols complex. For example, the closest classical modes to blues family mode are Mixolydian and Dorian, which differ from the family scale by two notes, but key signatures for blues pieces tend to be all over the map, leaving scale notes to be determined by more than two accidentals.

Most chord symbols are based on reference chords from the 7-note scales of classical modes. Departures from the 7-note scales of these chords are identified by often-complex suffixes consisting of one or more plain, sharped, or flatted degree numbers (e.g., **b5**, **#5**, **b9**, **9**, **#9**, **11**, **#11**, **b13** and **13**). This is fundamentally complex because it's several levels of representation distant from the keyboard.

Degree numbers are relative to chord roots, degree numbers for reference chords from 7-note scales must be interpreted in terms of actual scales with different numbers of notes, chord roots constantly change, notation for chord roots has to be translated into piano keys.

PKP representations of the same chords are generally simple because they're determined by building blocks from an actual mode. The complexity of chord-symbol suffixes is misleading because the suffixes generally turn out to identify notes of the building blocks of chords.

Very little notation is needed to use these ideas for actual pieces of music. Annotations are required on the written music to show anchor symbols above the staff next to chord symbols, and chromatic scale symbols on the staff next to melody notes. The only additional notation required for each piece is a 12-column mode table identifying melody and harmony modes by entries that mark chromatic scale positions. The separate summaries of melody plus harmony in two text lines, and the separate Lego-like pictures of the flow of harmony building blocks, are intended to represent mental models implied by the notation that guide playing. Writing them down can be useful for complex cases but is not required in general, beyond getting familiar with the notation.

CHAPTER 3: SOME SIMPLE EXAMPLES

The examples in this chapter are basically simple but exploring them plumbs surprising depths of musical sophistication. Anyone who has followed this so far should be able to understand and play the examples. The objective is not to show what a piece “actually is,” or how the composer or arranger might have viewed it, but to arrive at a simple way of understanding it in purely keyboard terms, and annotating the understanding on the written music to guide playing it. The result is what the piece “is” to the player.

It’s useful to think of music as divided into domains identified as **basic classical** (inherently not chromatic), **chromatic classical** and **non-classical** (both inherently chromatic). The term “classical” refers to the modes of key signatures, not divisions of music into categories such as classical, jazz and pop. The domains determine how chromaticism enters the picture. In the chromatic classical domain, chromaticism enters via successions of parallel classical modes (and possibly corresponding tonic changes that are the opposite side of the same coin). In the non-classical domain, chromaticism enters via scales that are more general than classical modes. Some chromatic pieces may be interpreted as being in either the chromatic classical or the non-classical domain, with the choice depending on the helpfulness of the interpretation.

For strongly chromatic pieces, the core harmony is often so tritone-intensive that core fifos may be omitted; their inclusion in some examples is sufficient to give a sense of how to include them.

I think of PKP as freeing the understanding of music from the straightjacket of music notation. This is a hard sell to musicians already expert in playing from music notation, because of the ten thousand or so hours at the keyboard required to get there, and because of the shared language provided by music notation for all musicians. The long hours are needed to become an expert pianist at some level, but not everyone has this ambition. It’s possible to enjoy playing the piano for pleasure, without being expert at it. For people so inclined, PKP provides access to pieces that would otherwise be inaccessible. For the musically ambitious, PKP can provide helpful insight into the deep structure of piano music, independently of practicing. Nothing here requires abandoning music notation, only looking at it from a different, complementary angle.

The examples are, in order;

Happy Birthday

Backwater Blues

Summertime

I Got Rhythm

Over the Rainbow

Traumerai

The anchor line identifies the building blocks in the foregoing table. When working from written music with chord symbols, such an anchor line is developed by analysis, working backwards from the end, not by sight reading the music going forward. The home tonic is understood from the melody line. The tritone anchors are determined relative to the home tonic from a table of tritone chords in Appendix C. The tonic and the tritone anchors establish a scale. Core fifos from the scale are faired in between the tritones as needed. However, it's also possible, as illustrated here, to develop core harmony independently of chord symbols. In either case, the full chords are interpreted results.

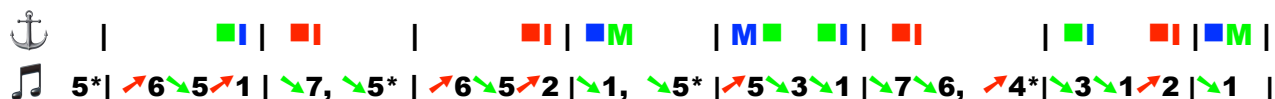
The mode tritone is placed in positions relative to the melody line that anticipate resolution in the mode. This is important because the melody line by itself is ambiguous until major tonality is established by the first appearance of note **3**. The postponement of resolution to the tonic is signified in the first and third melody phrases by the tritone harmonizing the final melody note of the phrase. Immediate resolution at the end of the second and fourth phrases is signified by the tritone harmonizing the pre-resolution melody note. The multiple appearances of the mode tritone “seed” the core, which is completed by fairing in fifos from the building-block menu. Core fifo anchors of suitable functionality are faired into the line of tritone anchors (**I** before **I**, **M** or **M** after **I**). The choice of **M** at points of melodic resolution avoids putting a dissonant half tone below the tonic, assumed to be played in the next octave up. The lightweight notation enables substitutions to be made easily, if desired (e.g., **A** or **A** for **I**).

Most transitions in this core harmony are **slides** or **morphs** (a slide moves a building block to a different keyboard position while holding its size, and a morph changes its size while holding one end fixed). The exception is the **wobbly slide I-M** which changes both position and size in one step. The unusual term represents the hand movements exactly — slide the hand while moving the fingers (“wobbling” them) for the size change. The wobbly slide **I-M** may be understood as a contraction of **I-M-M** (a tritone slides down a half tone and then morphs into a fourth). Thinking this way can be useful because it suggests a 2-step anchor line that may identify mode changes (e.g., **I-M** stays in the Ionian mode, and **I-M-M** goes to the parallel Mixolydian mode).

Different added root lines for the same core yield voicings of different chord progressions. The number of different possible chord progressions with this same core tends to be startling to the uninitiated. Chord roots are only references for specifying notes, and not actually fundamental to the sound of a chord progression (more on this later). In contrast, the core building blocks are fundamental to the sound.

From Octave Shapes to Chords

Octave shapes formed from anchored building blocks provide a simple way of adding depth to thin core harmony without adding new notes. Recall that octave shapes are formed by adding building blocks above or below the core of sizes identified by **■**, **■** or **■** attached to anchor symbols as prefixes or suffixes. The anchor line is simple and the octave shapes are either centered on it or go up from it. The shape **M■** in bar 5 is **■M** from bar 4 transposed up a fifth to follow the melody line (“voice leading”).



Melody notes (✕) are included in the following picture of this harmony to show context. The home tonic and the mode tritone determine the scale that supplies the fifos. Chord roots identified by dashes within the shapes, are visibly provided by context. Different choices of roots yield different chords but these choices are simple and obvious. This is voice-leading harmony overall, not just in bar 4. The voice leading is too close in bars 1, 2, 4 and 5 because the top notes are melody notes, but this is easily adjusted from context, if desired. The vertical bars in these shapes suggest shrinking the upper building blocks by 1 or 2 scale steps, putting the upper note of the shapes a major third above the anchor line. These shapes, in this context, voice the chords on the right as results. Different roots yield different chords (e.g., the first two shapes could voice chords **IVM7** and **IVM7b5**, among many other possibilities — see Appendix C).

		5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
		<u>S</u>	x	x	x	x	@	P	A	D	M	I	L	<u>S</u>	x	x	x	x	@	P	A	D	M	I	L	<u>S</u>	
bar 1		✕	
		✕	
		✕	
	■	x	.	-	.	.	I	✕	IIIm7
2	■	x	I	.	-	.		.	✕	V9
3		✕	
		✕	
	■	x	I	.	-	.	.	✕	V7
	■ M	.	.	x	.	.	-	.	.	.	M	.	.	.	x	.	.	✕	I6
4		✕	
	M ■	M	.	.	.	-	.	x	✕	.	VIIm9
		
	■	x	.	-	.	.	I	✕	IIIm7
5	■	x	I	.	-	.		.	✕	V9
		✕	
6	■	x	.	-	.	.	I	✕	✕	.	.	IIIm7
		✕	
	■	x	I	.	-	.	.	✕	V7
8	■ M	.	.	x	.	.	-	.	.	.	M	.	.	.	x	.	.	✕	I6

Important takeaways from this example are these: octave shapes provide a simple basis for voicing chord progressions, in context, without reference to chord symbols (except for identifying tritones from tritone chords); and chord roots are not inherently fundamental to the shape or sound of a chord progression.

NON-CLASSICAL DOMAIN: BACKWATER BLUES

Home Tonic - F (2-flats key signature: Mixolydian of F)

This is a simple, 3-chord, 12-bar blues in F, which I learned some years ago in a blues piano workshop at the then Jazz School in Berkeley, as representative of “probably half the blues pieces played by pop and jazz musicians.” It’s a simple piece but exploring it plumbs musical depths.

Anchor		M		D		M		M	
Musical Note		↗5↘4↘p3↘1		↗5↘4↘p3↘1 1— —					
Anchor		M		D		M		M	
Musical Note		↗5↘4↘p3↘1		↗5↘4↘p3↘1 1— —		↗5↗p7↘5			
Anchor		I		D				M	
Musical Note		↗p7*,		↘5 ↗1↘p7↘5↘p5↘4↘p3↘1 1— —					

The main scales plus selected harmony building blocks are summarized below. As for classical modes, the building blocks arranged in order going down the keyboard suggest possible core harmony sequences. There are more building blocks but these are the only ones used by this core harmony.

	F	G	A	Bb	C	D	Eb	F					
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
harmony	.	.	.	x	x	x	.	.	.	x	x	x	.
melody	@	.	.	D	.	x	L	\$.	.	x	.	@
blues family	@	.	x	x	x	x	x	\$.	x	x	.	@
selected building blocks	L	@
	I	x	.
	M	x	.
	.	.	.	D	x

DMI

//DL (6-note minor blues)

//DM.L

L - blues family

I - substitute for L

M

D

Substituting **I** for **L** follows from conventional thinking about chord roots. Dominant-7 chords on roots **I**, **IV** and **V** are conventionally regarded as standard blues chords: the tritones **M**, **D** and **I** are from these chords. The problem with this is tritones are more fundamental than chord roots to the blues. For example, a variety of blues chords with different roots emerge from tritone substitutions for dominant-7 chords (same tritone, root a tritone away, all non-tritone notes different). Chords **I7**, **IV7** and **V7** have tritone substitutes **pV7**, **VII7** and **pII7**. Using these substitutions muddies the chord waters, but core harmony remains clear. Tritone substitution is not limited to dominant-7 chords (or even to chords — recall **alt** modes). The **V** chord containing **L** is **VM7(II)**, of which chord **pIIM7(II)** is a tritone substitute. These chords and others like them may appear in blues pieces — for example, the footnoted chord progression on the first page of Chapter 1 — defeating all but experts by their complexity in music notation.

Given the general case, it’s best to start off thinking of **//DM.L** as the fundamental blues scale, and of the substitution of **I** for **L** as a means of simplifying chord progressions of simple blues pieces. In

either case, the function of the single appearance of the tritone in this harmony is signaling a “turnaround” — the beginning of the last 4 bars of a 12-bar blues — as an aid to improvisors.

Harmony based on un-inverted and inverted tritone cores is shown next. It’s easy to see how these simple voicing lines follow from altering tritone-based octave shapes. The simple voicing lines are sufficient because the tritones provide sufficient harmonic variety. Omitted chord roots are identified by dashes.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
@	.	A	D	M	I	L	\$.	x	x	.	@	
@	.	.	.	M	x	.	.	I7
@	.	.	D	.	-	.	.	.	x	.	.	.	IV7
@	.	.	.	M	x	.	.	I7
@	.	.	.	M	x	.	.	I7
@	.	.	D	.	-	.	.	.	x	.	.	.	IV7
@	.	.	D	.	-	.	.	.	x	.	.	.	IV7
@	.	.	.	M	x	.	.	I7
@	.	.	.	M	x	.	.	I7
.	.	x	.	.	I	.	-	.	.	.	+	.	V7
@	.	.	D	.	-	.	.	.	x	.	.	.	IV7
@	.	.	.	M	x	.	.	I7
@	.	.	.	M	x	.	.	I7

5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
\$.	x	x	.	@	.	A	D	M	I	L	\$	
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7
.	.	x	D	.	-	.	\$	IV9
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7
.	.	x	D	.	-	.	\$	IV9
.	.	x	D	.	-	.	\$	IV9
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7
.	.	.	.	+	I	.	.	\$	V7
.	.	x	D	.	-	.	\$	IV9
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7
.	.	.	x	.	-	.	.	.	M	.	.	\$	I7

All-tritone core harmony is a relatively common feature of blues pieces. An all-tritone core is not only simpler than a mixed tritone/fifo core, it’s also musically more definite. Assuming the family blues mode with no substitution of **L**, the fundamental tritone sequence of this piece is as shown next.

(M-D-M-M)-(D-D-M-M-(L-D-M-M))

Chord substitution is a favorite trick of jazz musicians, often having the contrary effects of simplifying the keyboard flow of the harmony while complicating its symbolic representation in chord notation. In PKP terms, chord substitution for this piece boils down to holding the core and changing other notes. For example, raising the bass note a whole tone for the last four bars on the left above, voices the chord sequence **V7-IV7b13-I9-I9**. Tritone substitute chords discussed earlier are another example. Make enough substitutions and symbolic chord progressions can quickly become difficult to comprehend for all but experts (again, see the footnoted chord progression on page 1 of Chapter 1).

MIXED DOMAINS : “SUMMERTIME”

Home Tonic - D (1 flat key signature: Aeolian of D)

I learned this well known Gershwin piece in the form presented here some years ago in a piano comping course given by Susan Muscarella at the then Jazz School in Berkeley. In the summary below, the anchor line is from a chord progression that will be presented later as a result. The only reference I have for this version of the piece is my course notes. Like *Backwater Blues*, it's a simple piece, the exploring of which plumbs musical depths.

The 6-note melody line is pentatonic minor except for a single half tone at end of bar 8 that could imply many minor scales, but the classical assumption would be Aeolian or Dorian. The harmony is strongly chromatic but is more than ornamental because, with the exception of bar 5, the tritone core identifies a succession of sub-modes of the minor family mode identified by **//ADI+** (recall that the plus superscript means fill in the top fourth). The sub-modes are shown above the anchor line, providing an example of how modal harmony may be notated. This is a minor context in which the **//AI** mode is the harmonic minor.

The notation shows a melody line with 20 notes and two systems of harmony. The first system has three modes: **//DI**, **//DM**, and **//AI**. The second system has two modes: **//A** and **//AI**. The modes are indicated by anchor symbols and letters above the melody line.

The modes are summarized next. The arrows at the right indicate some possibilities for inter-mode segues that are exploited in the above anchor line.

	D	E	F	G	A	Bb	C	D					
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
6 melody notes	@	.	x	x	.	x	.	\$.	.	x	.	@
family //AD.I+	@	.	x	x	.	x	.	\$	x	x	x	x	@
melodic minor //D.I	@	.	x	x	.	x	.	\$.	x	.	x	@
harmonic minor //A..I	@	.	x	x	.	x	.	\$	x	————	x	@	
Aeolian mode //A	@	.	x	x	.	x	.	\$	x	.	x	.	@
ornamental mode //DM	@	.	x	x	x	x	.	\$.	x	x	.	@

↓

↑

some nice segues

The skeleton melody line and the anchor line shown above are sufficient to play the melody with 3-note octave shapes for the harmony. The result has depth because of the three notes, and variety because of the tritones.

Adjusting the harmony to provide more shape variety is easy, as illustrated next for bars 1-8 plus the resolution bar 16. The numeric suffixes or prefixes on anchor symbols indicate offsets of voicing notes above or below the core, measured in half tones. These would be added to the above anchor

symbols to indicate desired voicings. These shapes voice the chords on the right, if not always in place then in the flow (omitted chord roots are identified by dashes). All the voicings except for the II chord are rootless, demonstrating once again that chord roots are not inherently fundamental to the sound of a chord progression.

bar	shape	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	a voicing of
		@	.	A	D	.	I	.	\$	x	x	x	x	@	.	A	D	
2	D2	.	.	.	D	.	-	.	.	.	x	x	.	IV-13
3	I4	I	.	-	.	.	.	x	.	.	.	x	.	V-7#5
4	D5	.	.	.	D	.	-	.	.	.	x	x	.	IV-13
5	M5	-	.	.	.	M	x	.	-	.	.	x	I-7#9 ornamental
6	D5	-	.	.	D	x	.	.	-	.	x	I-m6(9)
7	A4	.	.	A	x	.	.	.	x	.	.	II-m7b5
8	AI	.	.	A	.	I	.	-	.	.	x	V-7b9
16	4\$	-	.	.	.	x	.	.	.	\$	-	.	I-m triad resolution

The notation for voicing notes has the useful property of telling the size of a shape directly: it's the size if the building block plus the size of voicing interval. For example, a tritone with a fourth on top (often called "all fourths" because a tritone is an augmented fourth) has a size of 6+5=11 half tones, a half tone less than an octave. This makes finding the shape on the keyboard easy — find the treble note a half tone below the bass note an octave up, and complete the shape by adding an internal note a tritone above the bass note. All-fourths shapes generally imply complex chord symbols because this is not a basic chord shape from a classical mode.

As summarized next, there's a strong contrast between the simplicity of these shapes and the complexity of the chord symbols. The adjustment of basic chord symbols to fit scales, or places in scales, they don't quite fit is like "banging square pegs into round holes" — the results tend to be messy.

chord scale notes (bars 1-8)

root of *I-m6(9)*, "5" of *IV-13*, "7" of *II-m7*
 root of *II-m7b5*, "9" of *I-m6(9)*, "13" of *IV-13*
 "7" of *IV-7*, "#5" of *V*, "#9" of *I-7#9*
 "b5" of *II*, "b9" of *V-7b9*
 "b5" of *IV-7b5* (not in the above but could be)

chromatic scale notes

1
2
p3
p6
7

Home Tonic - Bb (2-flats key signature: Ionian of Bb)

 | | I I | | I I | M D A | M I | M I |

 | — 5 ↗ 6 | ↗ 1 ↗ 2 | 2 ↘ 1 | ↘ 6 ↘ 5 | 5 ↗ 6 | ↗ 1 ↗ 2 ↗ 4 ↘ 2 | ↗ 3* ↘ 2 ↗ 3 ↘ 2 | ↘ 1 — |

 || A | A || A | P || \$ | L || I | I ||

 | ↗ 3 ↗ 3 | ↗ 3 p5 | ↘ 3 3 | 3 ↘ 6 | ↗ 2 2 | 2 ↗ 3 | ↘ 2 ↘ 2 | ↘ 5 |

		Bb	C	D	Eb	F	G	A	Bb					
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	P	A	D	M	I	L	\$	x	x	x	x	@
bar 1-8	//I	@	.	x	.	x	I	.	\$.	x	.	x	@
bars 9-10	altA	.	x	A	.	x	.	x	.	x	x	.	x	.
bars 11-12	altP	.	P	x	.	x	.	x	\$.	x	.	x	.
bars 13-14	//L	@	.	x	.	x	.	L	\$.	x	.	x	@
bar 15-16	//I	@	.	x	.	x	I	.	\$.	x	.	x	@

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	\$	x	x	x	x	@	
I	.	.	-	.	.	I	x	<i>II-m7</i>
I	I	.	-	.	.	.	x	.	<i>V-7</i>
I	I	.	-	.	.	x	.	.	<i>V-m7</i>
M	-	.	.	.	M	x	.	<i>I-7</i>
D	.	.	.	D	.	.	-	.	.	x	.	.	.	<i>IV-7</i>
A	.	.	A	x	.	-	.	.	<i>pVII-7</i>
M	-	.	.	.	M	x	.	.	.	<i>I-6</i>
I	I	.	-	.	.	.	x	.	<i>V-7</i>

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occur. Each 2-bar segment is nominally in the Ionian mode of a different secondary tonic, but the modes are daisy-chained together from one to the next via shared notes instead of resolving to the secondary tonics. In terms of the 3-note shapes shown, the tritones are as ornamental as they are in the previous bars because all the non-tritone notes are from the home Ionian mode. There's one departure in the full chords — the VIIIm7 chord in bar 9 includes note **p5**.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	A	D	M	I	L	\$	x	x	x	x	@	
9	A	.	.	A	x	.	x	.	.	<i>VII-m7</i>
10	A	.	.	A	.	-	.	.	.	x	.	x	.	.	<i>III-7</i>
11	A	.	.	A	.	-	.	.	x	.	.	.	x	.	<i>III-m7</i>
12	P	.	P	x	.	-	.	x	.	<i>VI-9</i>
13	\$	x	\$.	x	.	.	.	<i>VI-m7</i>
14	L	x	.	-	.	.	.	L	.	.	x	.	.	.	<i>II-7</i>
15	I	x	.	-	.	.	.	I	.	.	.	x	.	.	<i>II-m7</i>
	I	x	I	.	-	.	x	.	.	<i>V-9</i>

This sequence doesn't explore the scale changes to the degree it could, but the scales are available to be explored in improvisations.

ORNAMENTED BASIC CLASSICAL : “OVER THE RAINBOW”

Home Tonic - Eb (3 flats key signature: Ionian of Eb)

This is an exercise in creating strongly chromatic harmony from scratch, illustrating the extent to which chromatic harmony can be purely ornamental. The first eight bars of the piece are sufficient for the purpose, which are so well known to everyone that there’s no need for a reference. The melody line is straight Ionian, trending downwards in zig-zags over an octave range.

As shown next, the downward trending melody line is given a downward trending tritone anchor line consisting of the alphabet in reverse order starting on **L** and wrapping around: **L-I-M-D-A-P-L-I**. The tritones are spread out over the melody line and positioned for consonance with it.

Anchor | **L** | **I** | **M** | **D** | **A** | **P** | **L** | **I** |

Musical notation: 1 1 7 5 6 7 1 1 6 5 6 4 3 1 2 3 4 2 7 1 2 3 1

The simple tritone “seed” on the left below provides a framework for adding core fifos and some other variations on the right. The double tritones provide some elegant symmetric relationships between successive shapes (e.g., **PM-A** shrinks the outer notes of the double tritone inwards a half tone; and **A-AI-M** moves a fifth aligned with the bottom note of **AI** to a fifth aligned with the top note).

	Eb	F	G	Ab	Bb	C	D	Eb
1	1	p2	2	p3	3	4	p5	5
2	6	p7	7	1	2	3	4	5
3	6	p7	7	1	2	3	4	5
4	6	p7	7	1	2	3	4	5
5	6	p7	7	1	2	3	4	5
6	6	p7	7	1	2	3	4	5
7	6	p7	7	1	2	3	4	5
8	6	p7	7	1	2	3	4	5

Left side (Seed):

1 **L** **L** x

2 **I** **I** x

3 **M** **M** x

4 **D** **D** x

5 **A** **A** x

6 **P** **P** x

7 **L** **L** x

8 **I** **I** x

Right side (Variations):

1 **M** **M** x

2 **L** **L** x

3 **I** **I** x

4 **M** **M** x

5 **M** **M** x

6 **D** **D** x

7 **PM** **PM** x

8 **A** **A** x

9 **AI** **AI** x

10 **M** **M** x

11 **MP** **MP** x

12 **L** **L** x

13 **I** **I** x

14 **M** **M** x

Here’s a summary of the melody and final core harmony.

Anchor | **M** **L** | **I** | **M** **M** | **M** **D** | **PM** | **A** **AI** | **M** | **MP** | **L** | **I** | **M** |

Musical notation: 1 1 7 5 6 7 1 1 6 5 6 4 3 1 2 3 4 2 7 1 2 3 1

MIXED : “TRAUMERAI” (SCHUMANN)

Home Tonic - F (1 flat key signature: Ionian of F)

The reference for this beautiful classical piece is the *Classical Fake Book*, 2nd Edition, Hal Leonard (2013). It provides chord symbols that yield the anchor line shown (the chord symbols are not shown because there's nothing new in them). It opens and closes with four bars in the Ionian mode of the home tonic. In between, it can be understood as a succession of parallel classical modes, or as a single, extended non-classical mode — the 9-note major family scale **//AMI+**. The latter is simpler because it provides all the melody and most of the harmony, and minimizes the number of mode changes to think about. Ornamental elements of the harmony (highlighted in yellow) are the common **P** and **L** functional substitutions for tritones **A** and **I**.

The image displays a musical score for 'Traumerai' by Schumann, focusing on the home tonic of F (Ionian mode). The notation is presented in five staves, each beginning with a treble clef and a common time signature. The first staff shows a sequence of notes: 5, 1, 7, 1, 3, 5, 1, 1, 7, 6, 5, 1, 2, 3, 4, 6, 1, 2, 3, 5, 2, 5. Above the notes are chord symbols: \$, M, I, M, \$, A. The second staff continues the sequence: 1, 7, 1, 3, 5, 3, 3, 2, 1, 7, 1, 3, 6, 1, 7, p7, 6, 7, 5, 5. Above the notes are chord symbols: \$, A, PM, L, I, \$. The third staff shows: 1, 7, 1, 3, 5, p7, p7, 6, 5, 4, 6, 2, 3, 4, 3, 2, 6, 1. Above the notes are chord symbols: \$, I, M, PM, M, P, M. The fourth staff shows: 4, 3, 4, 6, 1, 4, 4, 3, 2, 1, 3, 6, 7, 1, 7, 6, 3, 3, 2, 5. Above the notes are chord symbols: I, M, AI, I, M, I. The fifth staff shows: 1, 7, 1, 3, 5, 3, 3, 2, 1, 6, 5, 1, 2, 3, 4, 6, 2, 3, 4, 6, 6, 7, 1. Above the notes are chord symbols: \$, L, I, I, \$. The notation is repeated for bars 1-8.

repeat bars 1-8

The melody scales and building blocks from them are summarized next. The scales provide more building blocks than this but the others aren't used. Given this picture, reading the above anchor line is straightforward. As always, the single building blocks of the core may be played as octave shapes that add depth without adding new notes.

	F	G	A	Bb	C	D	E	F					
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
//I	@	.	x	.	x	x	.	\$.	x	.	x	@
//AMI+	@	.	x	.	x	x	.	\$	x	+	x	x	@
\$=@	\$	@
I	I	x
I	I	x
I	I	x
M	M	x
M	M	+
A	.	.	A	x
A=\$.	.	A	\$

Ionian mode (first & last bars)

major family scale (middle bars)

building blocks from the family scale

Ionian mode (first & last bars)

major family scale (middle bars)

building blocks from the family scale

Tonic Changes

There are no tonic changes in the melody line. The bar-and-a-half melody segment in bars 13-14 is that of bars 1-2 transposed up a fourth, misleadingly suggesting an Ionian tonic change up a fourth. However, the continuation of this change would require the appearance of note **p7**, not **7**. The segment is an excursion in the Ionian mode of the home tonic, not a tonic change.

CHAPTER 4 : THE BUILDING-BLOCK WORLD

This chapter goes into the details of using building blocks to identify tonic scales and to form shapes that voice chords. There are no new concepts here, only development of ones already presented, so it's possible to skim the chapter to see what's in it, as a preparation for knowing where to look for details when needed, and then to go on to the more advanced examples in the next chapter.

TONIC SCALES FROM BUILDING-BLOCK CONTENT

The following short dictionary mode signatures for tonic scales summarizes in half a page all the scales and more in scale dictionaries such as *The Source*. The mode signatures on the left identify scales by tritone content.

<u>signature</u>	<u>1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1</u>	<u>scale type</u>
PADMIL	x P A D M I L x x x x x x	chromatic (12 notes)
//P.DM.L	x P . D M . L x . x x . x	diminished (8-notes, min-maj)
//AD.IL	x . A D . I L . x x . x x	diminished (8 notes, min)
PA.MI	. P A . M I . x x . x x .	diminished (8 notes, maj)
ADMI	. . A D M I ——— x x x x .	no name (8 notes, min-maj)
//A.M.L	x . A . M . L . x . x . x	whole tone (6 notes, maj)
P.D.I	. P . D . I . x . x . x .	whole tone (6 notes, min)
//DM.L	@ . x D M x L \$. x x . @	min-maj blues family (9 notes)
//DMI	@ . x D M I . \$. x x x @	variation
//ADM	@ . A D M x . \$ x x x . @	variation
//P.DM	@ P x D M x . \$. x x . @	variation
//ADD.I	@ . A D . I . \$ x x + x @	minor family (9 notes): aka //AD.I⁺
//A.MMI	@ . A . M I . \$ x + x x @	major family (9 notes): aka //A.MI⁺
//DM	@ . A D M I . \$. x x . @	pentatonic union (8 notes), basic blues scale
//D.I	@ . A D . I . \$. x . x @	melodic minor (7 notes): a master mode
//AD.I	@ . A + . I . \$ x ——— x @	harmonic minor (7 notes)
//A.MI	@ . A . + I . \$ x ——— x @	harmonic major (7 notes)
//A.MI	@ . A . + I . \$ x + . x @	“bebop” major (8 notes)
//ADMI	@ . A + + I . \$ x ——— x @	harmonic min-maj (8 notes): a master mode
//I	@ . A . M I . \$. x . x @	Ionian (7 notes): a master mode
//M	@ . A . M ——— \$. x ——— @	pentatonic major
//D	@ ——— D . I . \$ ——— x . @	pentatonic minor

The scales above the double line are atonal, with mirror symmetry (same interval sequence going up and down). The ones without the // prefix are not true parallel modes because they don't contain the home tonic. The scales below it are asymmetric and tonal. These scales are completed by populating

the anchor sets in the lower fifth of the scale frame with fifo-only anchors. The letters **ADMI** are fundamental to these anchor sets, with different omissions of letters or different identifications of them as fifo-only anchors determining different scales. Minor-third intervals of scales are shown as solid lines to make them stand out to the eye. These are by no means the only possible scales but the dictionary is easily extended if anyone sees a need.

The focus of PKP is tonal music but atonal scales are included because shared letters of their defining words make them visible structural parents of scales lower down in the dictionary; and also because they may be used ornamentally in tonal music. Atonal scales have no minor, major or minor-major tonality by themselves but representing them by words that place them in the context of the home tonic gives them the relative tonality shown on the right (the presence of **D**, **M** or **DM** identifies minor, major or minor-major tonality).

There are no 5-letter words because the implied scales would be too close to the chromatic scale to be usefully distinguished from it. As scale identifiers, single-letter words are reserved for scales with seven or more notes, which includes classical modes but not 6-note blues scales containing single tritones that are understood as sub-scales of higher level blues scales.

The zone within the dictionary ranging from 4-letter words down to 2-letter words that identify parallel modes is complex in music notation. Scales must be understood as enumerated notes relative to one of more than twelve possible different key signatures. The role of classical modes as the defining scales of key signatures makes music from scales that differ markedly from them misleadingly complex. The dictionary makes this zone simple by pushing note symbols and interval inversions down to a lower conceptual level. The three scales in the middle formed around three asymmetric triple tritones provide important families of tonal scales.

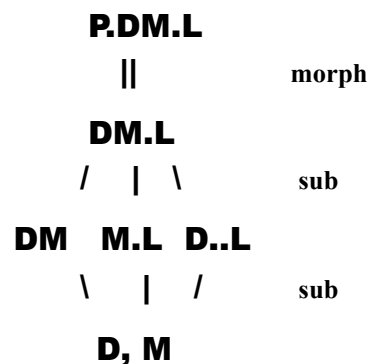
The table covers all possible mode changes. As explained in Chapter 2, tonic changes are understood to be implied by mode changes, when a melody line requires them.

SCALE FAMILIES

Scale families provide a simple, unified view of a very large number of scales of music notation of different kinds in different key signatures that are, in aggregate, overwhelmingly complex for all but experts.

Blues Family

The blues hierarchy is shown next.



At the top is the diminished scale **P.DM.L** that is a kind of “parent.” This scale morphs into the **DM.L** scale (the **P** tritone morphs into a fifo while holding its upper note, the pitch center, fixed). The

morphing breaks the symmetry of the diminished scale, yielding an asymmetric tonal scale that contains the other scales as sub-scales that follow from pruning anchors.

The words **M.L** and **D..L** determine parallel “melodic” and “harmonic” modes that share with classical modes the properties of seven notes and no adjacent half tones.

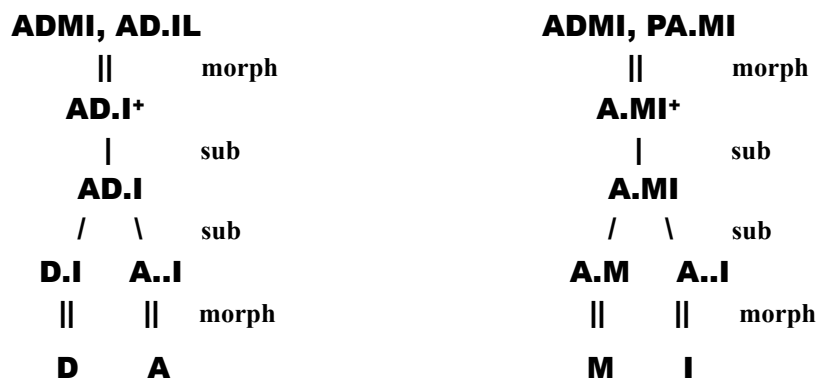
Conspicuously missing are tritones **A** and **I** from the classical modes conventionally regarded as the default minor and major modes of key signatures. Fifo anchors **A** and **I** are in the scales but the associated tritones would be ornamental extras.

Blues has high tritone content but it also has high fifo content (five possible fifo anchors from the **DM.L** scale). A strong blues sound follows from core harmony that has correspondingly high tritone content (for example, the footnoted chord progression of *Goodbye Porkpie Hat* in the opening chapter). Weakening the core tritone content in favor of in-scale fifos weakens the blues sound.

I learned the 9-note scale in conventional terms some years ago in a blues piano workshop at the then Jazz School in Berkeley, but this intuitively simple way of understanding and remembering it is my own invention and appears to be novel. Pianists and music teachers I have asked don’t seem to know about it. Books I have consulted don’t say anything about it.

Minor and Major Families

The union of the parallel Aeolian and Ionian modes forms a 10-note scale of mixed minor-major tonality determined by the word **ADMI** that is a master scale of family scales of major and minor tonality. As shown next, the minor and major hierarchies are slightly more complex than the **//DM.L** blues hierarchy because **ADMI** is not the only possible parent atonal scale, and the classical modes at the bottom are morphs of the modes above them, not sub-scales.

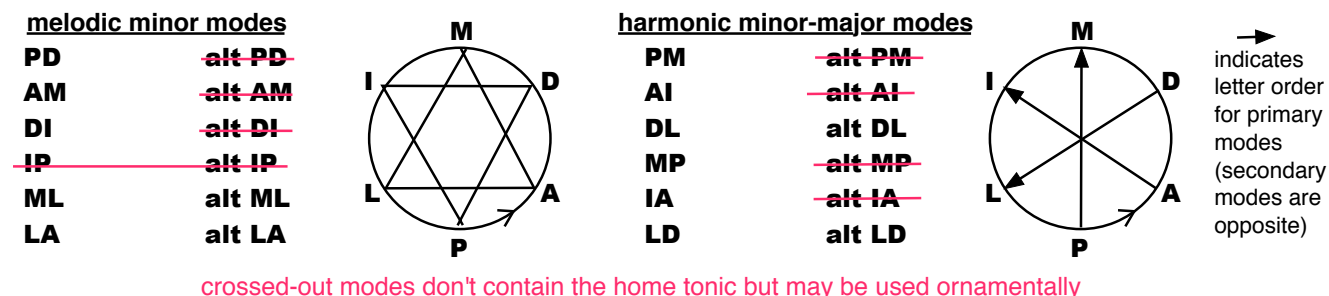


The **ADMI** scale reduces to 9-note minor or major family scales by morphing the **M** or **D** tritone into a fifo with the same top note. The morphed scales are identified by the augmented words **AD.I⁺** or **A.MI⁺** where the plus superscript indicates the top note of the missing tritone is retained. The result is both interesting and simple: an all-half-tone top end for both scales, with bottom ends of different tonalities.

The double tritones **D.I** and **A..I** on the left, and **A.M** and **A..I** on the right, determine parallel “melodic” and “harmonic” modes that share with classical modes the properties of seven notes and no adjacent half tones.

PARALLEL “MELODIC” AND “HARMONIC” MODES

The systematic development of the concept of parallel modes in music notation is restricted to single- and double-tritone modes identified by the terms “melodic” and “harmonic” that share with classical modes the properties of seven notes and no adjacent half tones.



The melodic and harmonic modes are different in kind because each of the words is unique for the former and half of the words are repetitions of the other half in the opposite order for the latter. This is because the double tritones of the latter are circularly symmetric (all inter-note intervals are minor thirds in any inversion). The harmonic modes can be dauntingly complex in conventional terms (Appendix D) because the irregularity of the master mode makes rotated transpositions doubly irregular. The irregularity can be finessed by making the master mode mixed minor-major, leaving the choice of its tonality in a particular parallel mode to context.

The three words **A.M**, **D.I** and **M.L** that fall out of the earlier scale hierarchies determine four parallel modes of the melodic minor mode identified by **D.I**. The three words determine four modes because any word containing **L** determines a primary parallel mode and an **alt** mode with all non-tritone notes different.

The two words **A..I** and **D..L** that also fall out of the foregoing scale hierarchies determine eight possible parallel modes of a harmonic minor-major master mode.

Details follow for information, but most of the modes fall directly out of family scales in the context of particular pieces of music.

Parallel “Melodic” Modes

These modes are summarized below in the same format as for parallel classical modes to enable easy comparison. As for classical modes, there are seven parallel modes (checked, with the master mode double checked). The unchecked modes are pseudo modes that don't contain the home tonic. Neither **IP** mode contains the home tonic but the major mode is designated **//IP** because of its place in the mode table. The twelve modes are so close to whole-tone scales that they are easy to know on the keyboard. A whole-tone scale is formed of five stacked whole tones, and these scales contain four stacked whole tones highlighted in yellow for the six primary modes. The notes highlighted in blue that provide references for the whole tone stacks are the tonics going down by fifths (**6-2-5-1-4-p7**) of the relative master mode.

As for classical modes, the **alt** modes are tritone substitutes with all non-tritones notes different, and the ones containing **L** are parallel modes because the **L** tritone contains the home tonic (the difference here is there are two of these modes). Some useful relationships to classical modes are shown on the right, but keep in mind that the best way of understanding how the modes emerge in

music is as sub-modes of higher level family modes in the scale hierarchy. It's also useful to remember that these scales are morphed from the whole tone scales identified by **A.M.L** or **P.D.I** by splitting one scale note into two notes a half tone above and below it (the lower note is the one highlighted in blue above).

		1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	tonality notes highlighted
		@ P A D M I L \$ x x x x @	
✓	//LA	x . x . x . x . x . x . x . x	major (lydian sharp 5)
✓	alt-LA	x . x x . x x . x . x . x	minor
	//IP	. x x . x x . x . x . x .	major (ionian sharp 1)
	alt-IP	. x . x . x . x x . x x .	minor
✓	//ML	x . x . x . x . x . x . x	major (lydian flat 7)
✓	alt-ML	x x . x x . x . x . x . x	minor-major
✓✓	//DI	@ . x x . x . x . x . x x	minor (ionian flat 3)
	alt-DI	. x . x . x x . x x . x .	minor
✓	//AM	x . x . x . x . x . x . x	major (mixo flat 6)
	alt-AM	. x x . x . x . x . x x .	major
✓	//PD	x x . x . x . x . x . x . x	minor (dorian flat 2)
	alt-PD	. x . x x . x x . x . x .	

Parallel “Harmonic” Modes

Making the minor-major mode the master reduces fourteen parallel modes that are often intricate and difficult to comprehend or remember (Appendix D), into the eight simple parallel modes check marked below (double checked for the master mode). The four unchecked modes are tritone partners that don't include the home tonic. The secondary tonics of the primary modes are highlighted in blue; the ones highlighted in grey are offset a tritone from these. The pattern is very simple: **alt** modes switch the empty and full minor thirds; the empty minor third is always bookended by half tones. Two-letter words with the same letters that identify different primary modes have the letters in opposite order.

		@ P A D M I L \$ x x x x @
		1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
✓	//DL	x . x D ——— L x . x x x x
✓	alt-DL	x x . D x x L . x x ——— x
✓	//LD	x ——— D x . L x x x . x x
✓	alt-LD	x x x D . x L ——— x x . x
✓✓	//AI	x . A x x I . x x ——— x x
	alt-AI	. x A ——— I x . x x x x .

✓

alt-IA

✓

//PM

x **P** — **M** **x** . **x** **x** **x** **x** . **x**

alt-PM

• **P** **x** **x** **M** **x** • **x** — **x** **x** •

//MP

x **P** . **x** **M** — **x** **x** . **x** **x** **x**

alt-MP

—P x . M x x x . x x—

se

//DM.L

//DL

e.g., “Romanian” (Appendix D)

CHORDS FROM BUILDING BLOCKS

These flows

be
th
fa
ha
or
sh

se

chord

shape

1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3 3 4 p5 5 p6 6

@ . A . M I . \$. x . x @ . A . M I . \$. x

I-M7

434

O . . M . . O . . x

II-m7

... 0 ... I ... 0 ... x ...

III-*m*

.....M.....O.....X.....O.....

IV-M

434

.....I.....O.....X.....O.....

V.7

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99

VII

V1-m

VC

building blocks of chords I, II and V here, but not of the others. The core building block of a chord in PKP is the most unique building block of the chord in the tonic scale. This is always a tritone for chords containing a tritone. Otherwise it's one of the two fifos of the scale anchored a half tone apart (e.g., fifths anchored by **I** and **M** for the Ionian mode). Think of the bottom or top segments shown in grey as the visible parts of enrichment fifos, of sizes to be determined, “snapped onto” the core building blocks underneath them at scale points (the **o-o** fifos are all fifths for basic seventh chords going up from these roots, but these are not the only possibilities).

The shapes in the second column represent interval stacks created by the combinations of building blocks. The numbers are counts of half tones: **3** represents a minor third, **4** represents a major third. This is a variation of a notation called “figured bass notation” (Appendix C), in which different numbers represent counts of scale steps, not half tones, between chord notes. In this variation, the building blocks are partial stacks: **33** is a tritone, **43** and **34** are fifths. The shapes are exact for any position on the keyboard. Pinpointing the position is conventionally accomplished by specifying chord roots (e.g., the **IVM7** chord could be written **IV434**). In PKP, the pinpointing is accomplished by core anchors. A core anchor with a single outside voicing note above or below it provides a 3-note voicing of a chord that's often sufficient in context. The voicing note for these chords and this mode is always a major or minor third above or below the building block, with the choice determined by the scale.

Chords From Core Building Blocks

Putting the chord roots in down-by-fifths order brings forward to the eye the simple way in which chord progressions for the Ionian mode may be formed around two **I-I-M** core anchor lines joined by morphing the final fifth **M** of the first line into the fourth **M**. The transtions are all morphs except for one wobbly slide. Three-note voicings are completed by adding major or minor thirds determined by the scale above or below the core, and notated by adding a numeric prefix or suffix to the anchor symbol, specifying the size of the thirds.

The chords are core building blocks (anchored) with overlapped enrichment fifos (**o-o**). The 3-note voicings omit the inner note of the enrichment fifos, which in this case is either the pitch center of the octave or a note a whole tone above it, both of which tend to be implied by context in passages of music from the scale. The intervals above and below the core are not themselves building blocks, but the projecting ends of overlapped fifos.

shape	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	chord
I 4	I	x	IV-M7
I 3	I	x	VII-m7b5
M 3	M	x	III-m7
M 3	M	x	VI-m7
3 I	I	x	II-m7
3 I	I	x	V-7
4 M	M	x	I-M7

Transforming this so the chords go up from the roots yields the difficult-to-play result shown next. This is difficult to play because the hand must be lifted and moved by large jumps, while also adjusting

the fingers for “wobbles” between minor and major thirds. Such shape progressions may also be understood as sliding a fixed “scale shape,” with 2 scale steps between each note, to different positions in the scale. This is conceptually simple but doesn’t represent the wobbles, which are specific to particular modes and must be learned from practicing. Understanding such progressions in terms of actual keyboard shapes enables knowing the shapes for any mode and root sequence independently of practicing them.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6		
<u>shape</u>	@	.	A	.	M	I	.	\$.	x	.	x	@	.	A	.	M	I	.	\$.	x		<u>chord</u>
I4	I	.	.	O	.	.	x					O		IV-M7
I4	x				O	.	I		.	.	O	.	VII-m7b5
M3	M	.	.	O	.	.	.	x				O	III-m7
M3	x	.				O	.	M		.	O	.	.	VI-m7
3I	.	.	O	.	I	.	.	O	.	.	.	x	II-m7
4I	O	x				O	.	I		V-7
4M	O	.	.	.	M	.	.	O	.	.	.	x	I-M7

There’s obvious benefit in learning the easier-to-play morphed version first and then switching to this jumpy version, if desired, by inverting selected building blocks (which is always easy when building blocks are known by their anchors).

Triad Chords

Seventh chords and their inversions, sixth chords, are the workhorse chords of 4-part harmony. Three-note voicings of them just shown are still seventh or sixth chords. Triad chords are different in kind because they’re fundamentally three notes. Triad chords are formed by splitting fifths (but not fourths) asymmetrically to form stacks of opposite-sized thirds (major or or minor), or splitting tritones symmetrically to form stacks of minor thirds.

A very simple notation for triad chords is provided by adding a superscript to the anchor of fifth or tritone that identifies a symmetric or asymmetric split: ▲ or ▼ for asymmetric splits (▲ indicates the major third is on the bottom and ▼ indicates it’s on top); and • for symmetric splits. For example, **A▲** is a major triad (chord symbol **II**, notes **2-p5-6**), **A▼** is a minor triad (chord symbol **II^m**, notes **2-4-6**), and **A•** is a diminished chord (chord symbol **II^{dim}**, notes **2-4-p6**). Triad chords may be voiced in a more spread-out fashion by holding the inner note and inverting the building block: e.g., the major triad **@▼** (notes **1-p3-5**) may be voiced as the fourth **\$** with a major third underneath: **4\$** (notes **p3-5-1**).

Augmented fifths (8 half tones) may, like tritones, be split symmetrically. The result is a stack of two major thirds (4 half tones each) that forms an augmented major triad. If an augmented fifth on any anchor **X** is represented by **X+**, then **X+•** represents a stack of two major thirds. The chord so formed is an augmented major triad.

The fact that stacks of minor and major thirds form 3-note chords suggests that stacks of the building blocks may provide 3-note shapes that voice chords with more notes.

Stacked Building Blocks

Stacked building blocks that voice chords with four or more notes are remarkably simple objects on the keyboard. They may be represented in anchor notation in a simple way that matches their actual

simplicity. One of the building blocks is represented by a color-coded anchor symbol (X, X or X). The other is represented by a symbolic prefix or suffix (■, ■ or ■) that represents an attached building block of the color-coded size. For example, X■ is a stack of two fourths that spans a minor seventh, X■ is a stack of a tritone and a fourth that spans a major seventh (often called an “all fourths” shape, considering a tritone as an augmented fourth), and X■, X■ or X■ are octave shapes. X■ is larger than an octave. The symbols in all cases may include inversions of the core building blocks. and may be in either order.

This notation is a more visual substitute for stacks of building blocks than the notation presented earlier that uses numeric suffixes or prefixes for voicing notes (for example, X■ and X■ mean the same as X5 and X5). The numbers remain necessary for non-building-block voicing intervals..

Open Voicings

Returning to the original 4-note chords, the 4-note open voicings of them in (a), next, follow from inverting the enrichment fifos upward into the next octave and identifying the inversions by anchors. The shapes are represented by symbols of the form **XY** in which the backslash emphasizes that the building blocks are stacked, not overlapped. The shapes could be identified by the notation shown on the left in parentheses, but parsing the interval stacks is cumbersome and error prone.

Examples (b) and (c) are simple variations that provide voicings of a variety of complex-looking chord symbols. This vividly illustrates the banging-square-pegs-into-round-holes nature of chord symbols. Simple variations in the building blocks yield complex variations in the chord symbols because of having to adjust them to “holes” in scales they don’t quite fit.

Example (c) is particularly simple: move one note down a scale step and then move all notes down a scale step. The shapes in the last two steps are “all fourths” (counting the tritone as an augmented fourth). These are far from the only possible variations but they give a good sense of the often deceptively complex nature of chord progressions.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
shape		@	.	A	.	M	I	.	S	.	x	.	x	@	.	A	.	M	I	.	S	.	x	.	x	@
(a)	I\A	(I27)	I	@	.	A	x	.	.	IIIm7
	I\A	(I37)	I	x	.	A	x	.	.	V7
	M\@	(M17)	.	.	.	M	x	@	S	.	.	.	IM7
(b)	I\M	(I45)	I	@	.	.	.	M	x	.	.	IIIm9, IVM7
	I\M	(I55)	I	x	.	.	.	M	x	.	.	V7(13), VIIIm7b5
	M\A	(M35)	.	.	.	M	x	.	A	S	.	.	.	IM9, IIIIm7
(c)	I\M	(I45)	I	@	.	.	.	M	x	.	.	IIIm9, IVM7
	I\M	(I55)	I	x	.	.	.	M	x	.	.	V7(13), VIIIm7b5
	M\A	(M55)	.	.	.	M	x	.	A	S	.	.	.	IM6(9), IIIIm7(11)

The double-anchor notation covers the 3-note shapes described earlier: simply omit the top note of the upper building block. That said, it’s simpler to work with core building blocks and offsets from them if 3-note shapes are sufficient.

FINAL OBSERVATIONS

Chord symbols in tonal music often way over-specify notes because many of their notes are provided by context (mode scale, melody line, neighboring chords). A chord root is no more than a reference for an accurate specification of a set of notes. Inversions are often notated as radically different chords on different roots. Many different roots may provide chord symbols that do the job, perhaps with the addition of suffixes indicating extended or altered notes (often this is like “banging square pegs into round holes” — the result is messy). The mutual substitutability of many scale fifos offers many mutually consonant or harmonically equivalent chords.

Tritones are the only fixed elements provided by chord symbols (same size in either inversion, not substitutable in the ordinary sense of mutual consonance or harmonic equivalence), and so can be relied upon to provide a fixed element of any chord containing them. They identify parallel modes from which building blocks may be chosen to form shapes that provide suitable voicings of, or substitutes for, written chords. Knowing the scales makes it easy to fill in core fifos that morph to or from tritones, and to add enrichment fifos determined by bass or treble lines below or above the core.

Early in my musical adventure, I learned about simple “open” voicings of complex chords in a “piano comping” course given by jazz pianist Susan Muscarella at the Jazz School in Berkeley (now the Jazz Institute). The voicings were based on chord progressions that were complex to begin with, making figuring out the voicings in terms of reorganized degree numbers a complex process with difficult-to-remember results.

At the time, jazz pianist Taylor Eigsti was an artist in residence at the Jazz school who, I heard, was teaching a chord-symbol-free way of creating voicings based on representing chords as “scale shapes” determined by counts of scale steps between adjacent chord notes. This resonated with my developing ideas about building blocks, so I contacted him to learn about his method. He told me that he recommends, to beginning jazz piano students, a practicing regime of moving scale shapes of chords to different positions in and between classical modes, without reference to chord symbols. The objective is developing the instinctive moves required of jazz pianists. Although the scale shapes are fixed, the keyboard shapes are “wobbly,” requiring adjustment of the fingers to play. This combined with lifting all the fingers off the keyboard and moving them by often largish intervals to other keyboard positions makes this style of playing difficult. The method has no notation and is learned from exercises.

I decided to find for myself a simple, common way of notating such different ways of playing chord progressions. PKP is what I found.

CHAPTER 5: ADVANCED EXAMPLES

This chapter explores a smorgasbord of example pieces that I found difficult when I first encountered them in music notation, and that seem to be generally understood as difficult because of strong departures from classical modes, often combined with difficult or multiple written key signatures. The idea of a smorgasbord is to offer something for everyone.

A takeaway from this chapter is confirmation that tritones are structurally fundamental to music understood in interval terms, and that a DNA-like alphabet is sufficient to cover any tonic scale or chord progression that may be played on the piano, independently of the key signature(s) in which a piece may be written. This is different in kind from “the score is sacred” view of many classical musicians because it permits variations of harmony based on common cores and implied modes.

In a famous session of PBS’s *Piano Jazz*, Bill Evans, in conversation with Marian Macpartland, said words to the effect that he advocated taking a piece apart to understand its architecture before putting it back together in an improvisation. I thought this was a great concept but wondered how “architecture” could be conceptualized. I suggest that the concept of musical domains introduced in Chapter 3 provides a way. The domains identified there, namely **basic classical** (not chromatic), **chromatic classical** (chromaticism introduced via parallel classical modes) and **non-classical** (uses non-classical modes that are inherently chromatic), are explored in some depth in this chapter.

I have heard it said, and believe it to be true, that composition is frozen improvisation. Therefore it, like improvisation, requires a sense of architecture. I suggest that exactly such a sense of architecture is revealed by the examples in this chapter. The fact such a sense can be extracted from such a wide variety of pieces without any information other than that provided by the piece itself suggests that it was put there by a composer or arranger, based on an intuitive understanding that’s no doubt different in kind from PKP, but that can be represented simply by PKP.



The examples are intended to be understood by playing them, not just looking at pages, and so should be approached in front of a piano on which they can be tried. Start with the melody line and the tritone core to establish basic flow. Play the tritone core as octave shapes first. Then play the actual shapes shown, including interpolated fifa shapes. Then try variations.



The examples are as follows, in order of presentation:



Goodbye Pork Pie Hat
Prelude to an Afternoon of a Faun
Giant Steps
Round Midnight
Body and Soul
Laura
When Sunny Gets Blue
Every Time We Say Goodbye
All of Me
No Greater Love
All the Things You Are
Chelsea Bridge
Lush Life
The Peacocks (the only one in annotated music notation)

Home Tonic - Eb (3 flats key signature: Ionian of Eb)

The key signature of 3 flats and the home tonic of Eb jointly identify the Ionian mode **III** as the reference mode for the accidentals that determine blues notes. This is an example of the often misleading nature of key signatures. A 5-flats or 6-flats key signature indicating Dorian or Aeolian modes of Eb would be closer to the blues.

 | 4M | L4 | | ML | | L4 | | 4M | |
 5 | ↗1 ↗p3 ↘1 p3 | 4 | ↘p3, ↘1 p7 | ↗1 p3 ↘1 p7 ↗1, ↘5 | ↗1 p3 ↘1 p3 | 5 | ↘p3, ↘1 p7 | ↗1 p3 ↘1 p7 ↗1, 1 |

 | L4 | | A4 | | I | | P | | L | | L4 | |
 | 5 | ↘p7 ↘1 ↗p3 ↘4 p3, ↘p7 p3 | ↗5 | 1 | ↘1 p3, ↗p7 ↘5 p3 ↘p7 p6, | ↘p3 p5 | 1.p2 | 7 | ↘p5 ↘2 | ↗p3 ↘1, ↗p5 ↘4 |

 | 3M 3D | | 3I | | L4 | | ML | | P | | ML | |
 | ↘p3 ↘1, | ↗4 | ↘p3 ↘1 p7, ↗p5 ↘4 p3 ↘1, | ↗p3 ↘1, | ↗5 | ↘4 p3, ↘1 p7 ↘5 |

The harmony line determines the harmonic flow shown next, which is almost all 3-note rootless voicings of written chords (the omitted roots are identified by dashes). The idea is to hold a picture like this in the mind's eye while playing, not necessarily to write it down. The free use of tritone substitute chords (e.g., **I7/pV7**, **II7/pVI7**, **pIIIM7#II/VM7#II**) makes for a complex written chord progression. These chords have all non-tritone notes different (including roots a tritone apart) but the same harmonic function; in other words, they combine different fifos with the same tritone.

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them. The stack of two fourths does the job. A more accurate voicing of the *sus* chords would be a stack of fourths up a whole tone from this (■\$) but the stack shown is sufficient to get started.

The bass line provided by the chord roots is missing but could easily be added back underneath if desired. That said, this harmony evokes the haunting, rich sound of the piece in a very satisfying way — small note changes in a smoothly flowing keyboard pattern make big sound changes.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	A	D	M	I	L	\$	x	x	x	x	@	
1	4M	x	.	.	.	M	x	.	.	I7#9
	L4	x	L	.	-	.	x	.	.	.	pVI9(13)
2	I	x	-	.	.	.	I	x	.	.	pIIIM9
	ML	x	.	.	.	M	.	L	.	.	.	x	.	.	pV7#11
3	I	x	.	-	.	.	I	x	.	.	II9sus
	L4	x	L	.	-	.	x	.	.	.	pVI9(13)
4	I	x	.	-	.	.	I	x	.	.	II7sus
	4M	x	.	.	.	M	x	.	.	I7
5	I	x	I	x	.	.	IVm11
	L4	@	L	.	-	.	x	.	.	.	pVI7(13)
6	A4	.	.	A	x	.	.	.	x	IIIm7b5
	I	x	I	.	-	.	x	.	.	.	V7#5#9
7	P	.	P	x	.	-	.	x	VI13#11
	L	x	.	-	.	.	L	x	.	.	II7(13)
8	L4	x	L	.	-	.	x	.	.	.	pVI7
	I	x	-	.	.	.	I	x	.	.	pIIIM7
9	4M	x	.	.	.	M	.	-	.	.	.	x	.	.	pV7(13)
	3D	x	.	.	D	.	-	x	.	.	IV7
10	I4	x	I	.	-	.	x	.	.	.	V7
	L4	x	.	-	.	.	L	.	.	.	x	.	.	.	II7
11	M L	x	.	.	.	M	.	L	.	.	.	x	.	.	I7#9 + pVI7
12	P	.	P	x	pIIIM7#11
	ML	x	.	.	.	M	.	L	.	.	.	x	.	.	pV7#11

variation

back to the blues

turnaround

2-chord mashup


smooth segue to bar 1

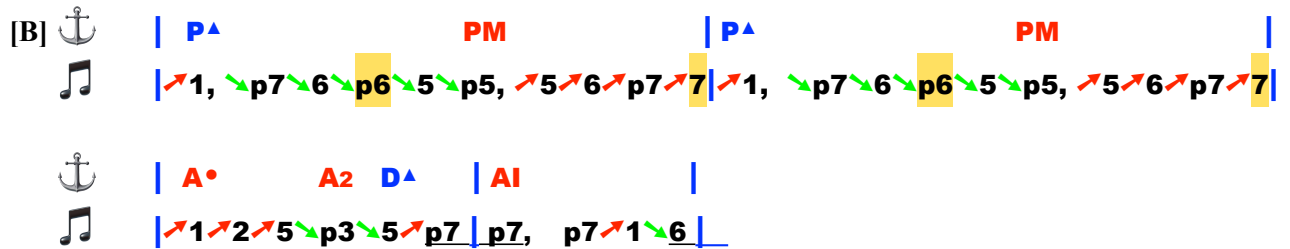
NON-CLASSICAL DOMAIN : “PRELUDE TO AN AFTERNOON OF A FAUN”

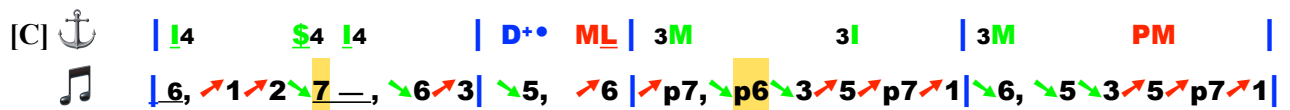
Home Tonic - C# (4 sharps key signature: Aeolian of C#)


This and the previous piece have something unexpected in common, namely the **//DM.L** blues scale. I was motivated to investigate this piece by things said about its violation of music convention in the article *Beauty in the Void*, Alex Ross, The New Yorker, Oct. 29, 2018. I was curious about the non-classical modes it might use. My source for the written music is the *Classical Fake Book*, 2nd Edition, Hal Leonard (2013), page 222. The key signature is 4 sharps and the home tonic is C#, identified by the final note of the melody line.

The only unusual notational feature in the skeleton melody line below is the double arrow at the end of [A] indicating a jump to the pitch center two octaves up. Yellow highlighting identifies ornamental passing notes (ornamental because they fill in between scale notes and are never harmonized).

[A] 

[B] 

[C] 

[D] 

Transposing all the melody notes outside the home octave into corresponding positions in it (below) reveals the blues scale and the pentatonic minor sub-scale.

	C#	D#	E	F	F#	G	G#	A#	B	C#			
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
//DM.L	x	.	x	x	x	x	x	\$.	x	x	.	x
ornamental									+		+		
//D	@	.	.	x	.	x	.	\$.	.	x	.	@
	sections [A], [B], [C]												
	section [D]												

The blues scale provides a structural handle on the piece that's missing otherwise. That music notation provides no structural handle is verified by the amazement expressed in the referenced article at Debussy's departures from “music convention.”



The annotated harmony shown next in Lego form satisfies the written chords mostly in place, and always in the flow. The harmony building blocks from the family scale that appear in the previous example also appear here. Blues tritones **M** and **L** have a strong presence. Blues tritone **D** is missing from the harmony but is present in the melody line (e.g., bars 3-4). Tritones **P**, **A** and **I** are ornamental passing tritones analogous to ornamental passing notes in melody.



These passages use many triad chords, which are 3-note shapes consisting of symmetrically split tritones (e.g., dim chord **A•**) or asymmetrically split fifths (e.g., major triad **P▲**). Fitting the triads into the flow sometimes inverts them or converts them into seventh chords.

		C#	D#	F	F#	G#	A#	B	C#					
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	P	A	D	M	I	L	\$	x	x	x	x	@
[A]	1-4	-	-
[B]	1	P▲	.	P	.	.	x	.	.	x	.	.	.	D
		PM	.	P	.	.	M	.	.	~	.	.	x	Bdim/D=Ddim7
	2	P▲	.	P	.	.	x	.	.	x	.	.	.	as above
		PM	.	P	.	.	M	.	.	~	.	.	x	as above
	3	A•	.	.	A	.	.	x	.	.	x	.	.	D#dim
		A2	.	.	A	.	.	~	.	.	x	.	x	B7
		D▲	.	.	.	D	.	.	.	\$.	.	x	F
	4	3I• = AI	.	.	x	.	.	I	.	.	x	.	x	F#dim
[C]	1	I4	x	.	~	.	.	I	.	.	.	x	.	F#/D#
		\$4	.	.	x	\$.	.	x	G#
		I4	x	I	.	.	.	x	.	F#/D#
	2	D+•	.	.	.	D	.	.	.	~	.	.	x	C+ (stacked major thirds)
		ML	x	.	.	.	M	.	.	L	.	.	x	B7b5
	3	3M	.	x	.	.	M	x	.	A#
		3I	.	.	x	.	.	I	x	B6
	4	3M	.	x	.	.	M	x	.	A#
		3M• = PM	.	P	.	.	M	.	.	x	.	.	x	Fdim
[D]	1-2	-	-
	3	@▼	@	.	.	x	.	.	.	x	.	.	.	E6

Home Tonic - F# (empty key signature plus accidentals)

The melody line and incomplete core harmony are shown next for home tonic F#=**1** (highlighting is of segments from different modes explained following this summary). Resolutions to home tonic **1** and to secondary tonics A#=**3**, D=**p6** and Eb=**6** establish the primary melody tonality as major to the ear. The core harmony includes all of the tritones but only the fifos that harmonize these four melody notes. The remaining fifos follow from obvious morphings. Learning the piece from this summary and then adding the morphed fifos by eye and ear is easier than trying to learn everything at once because the alignments between melody and full core harmony are unconventional.

 | **L** | **A** | **P** | | **L** | **A** | **M** | **M** | |
 | 1 \rightarrow p6 \rightarrow 4 \rightarrow p2 \rightarrow 3 | 3 , | \rightarrow 4 \rightarrow p3 \rightarrow p6 \rightarrow 3 \rightarrow p2 \rightarrow 6 \rightarrow 1 | 1 , | |

 | **A** | **P** | | **L** | **L** | | **M** | **M** | | **A** | **A** | **M** | |
 | \rightarrow p2 \rightarrow 7 \rightarrow 3 | 3 , | \rightarrow 4 \rightarrow p3 \rightarrow p6 | p6 , | \rightarrow 6 6 \rightarrow 1 | 1 , | \rightarrow p2 p2 \rightarrow 3 | 3 , | \rightarrow 1 1 | |

As shown in the following table, the melody scale considered by itself is an altered harmonic-minor-major mode of tonic **6**: the unaltered mode is determined by double tritone **A..I**; the alteration substitutes tritone **D** for tritone **A**. This only substitutes the anchor because the other note is already in the scale. This is the clearest way of identifying the alteration. Identifying it by an altered degree number would be confusing because the 8-note scale has an extra degree number in its bottom fifth. The melody scale is actually an assembly of fragments of the Ionian modes but this is the overall result. Playing the melody line knowing this scale is easy by itself. Playing the full core harmony is easy by itself. The only difficulty is the sometimes confusing alignment between the two.

	F#	G		A#	B		Eb		F#				
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
melody	x	x	.	D	x	I	————		x	x	.	x	x
//M	x	.	x	.	M	x	.	x	.	x	x	.	x
alt-L	x	x	.	x	.	x	L	.	x	.	x	.	x
alt-A	.	x	A	.	x	.	x	.	x	x	.	x	.

altered-harmonic-minor-major@6

Mixolydian@1

Locrian@1

tritone partner of Aeolian@1

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of the melody line cross over between these modes. The highlighted melody segments in bars 1-2 and 5-6 are **4h**↗**3h**↗**4h** sequences from **alt-L** and **alt-A** that are also from the overall melody mode.

The core harmony is best learned first without the light-shaded fifos, which can be added later by simple morphing. Melodic resolutions to the home and secondary tonics are marked “>” on the left. The morphs are all downward, with one exception (**A-A** in bars 14-15 because the context leading up to it is different from before). This core harmony fits the written chords as shown on the right. Dashes indicate omitted roots. Octave shapes that go well with the flow of the melody are indicated on the left.

		F#	G#	A#	B	C#	D#	E	F#					
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	P	A	D	M	I	L	\$	x	x	x	x	@
bar														
1	-													
	L													
2	-													
	A													
> 3	P													
4	-													
	L													
5	-													
	A													
6	-													
	M													
> 7	M													
8	-													
	A													
> 9	P													
10	-													
	L													
> 11	L													
12	-													
	M													
> 13	M													
14	-													
	A													
> 15	A													
16	-													
	M													

IVM7	(from bar 16)
pVI7	
pIIIM7	
III7	
VIM7	
bIIIm7	
pVI7	
pIIIM7	
III7	
VIM7	
I7	
IVM7	
VIIIm7	
III7	
VIM7	
bIIIm7	
pVI7	
pIIIM7	
Vm7	
I7	
IVM7	
VIIIm7	
III7	
VIM7	
Vm7	
I7	(to bar 1)

IVM7 (from bar 16)

pVI7

pIIM7

III7

VIM7

bIIIm7

pVI7

pIIM7

III7

VIM7

I7

IVM7

VIIIm7

III7

VIM7

bIIIm7

pVI7

pIIM7

Vm7

I7

IVM7

VIIIm7

III7

VIM7

Vm7

I7 (to bar 1)

Improvisations could be based on the single melody mode. Or they could be based on the two main harmony modes //M and **alt-L**. The **alt-A** mode may be considered ornamental because it has only one note not in the other two modes (7). Among other possibilities, this suggests blues improvisations.

NON-CLASSICAL DOMAIN : “ROUND MIDNIGHT” (MONK)

Home Tonic - Eb (6 flats key signature: Aeolian of Eb)

The sources are *The Ultimate Jazz Fakebook*, Hal Leonard, 1988, p. 322 for the main part of the piece (shown first) and *Standards Real Book*, Sher Music, 2000, p. 369 for the optional introduction (shown next). The main part of the melody line is in the 9-note minor family scale **AD.I*** relative to which the highlighted notes **p5**, **3** and **p2** are ornamental. Much of the harmony is from this scale, with a few highlighted ornamental elements that fit the flow (e.g., **L-I**, **I-M**).

Anchor | **D** | **I** | **D** | **I** **M** |

Music: — ↗5 ↗1 ↗2 ↗5 p3, — ↘5 | ↗1 ↘7 ↗1 ↗5 ↘4, | — ↗1 p3 ↗5 p7 ↘6, — ↘1 | p5 ↘4 4 ↘3, |

⊕ 1

Anchor | **A** | **I** | **L** | **I** |

Music: — ↗4 p6 ↗1 p3 ↘2, — ↘p6 | ↘5, ↘1 ↘7 ↘p7 ↘6 ↗4, ↘1 ↗2 | ↗p3* ↘2 ↘1 ↗2 ↘1 | ↘7 ↘5, — :||

2

Anchor | **L** | **D** | **L** | **I** | **D** | **L** | **I** |

Music: | ↗p3, ↘2 ↘1 ↗2, ↘1 ↘7 | ↗1 — 1 ↗2 | ↗p3* ↘2* ↘1 ↗2 ↘1 | ↘7 ↘5, — ↘1 ↗2 | ↗p3 ↘2 ↘1 ↗2 ↘1 | ↘7 ↘5, — ↗5 |

Anchor | **A** | **L** | **I** | **M** **A** | **L** | **I** |

Music: | ↗p6 p6 ↘5 5 | ↘4 ↘3* ↘p3 — ↗5 | ↗1* ↘p7* | ↘p6, ↘1 ↘7, ↗5 ↘4 | p2 ↘p7 ||

⊕

Anchor | **L** | **I** | **D** |

Music: | ↗p3 ↘2 ↘1 p3 ↘1 p7 | ↗1 — |

Here follows the optional, 8-bar introduction. The three 2-bar segments are the same interval sequence going down by whole tones, ending up a half tone above the home tonic. The final two bars settle on the home tonic in a highly idiosyncratic way not fully shown here (almost anything will do in these bars).

Anchor | **L** | **D** |

Music: | — ↗7 ↗1, ↗2 ↗3 p5 ↗7, ↘p5 ↘3 ↗7, ↘p5 ↘3 | ↗2, ↘1 ↘5 ↘p3 ↗4, |

Anchor | **M** | **P** |

Music: | — ↗6 p7, ↗1 ↗2 ↗3 ↗6, ↘3 ↘2 ↗6, ↘3 ↘2 | ↗1, ↘p7 ↘4 ↘p2 ↗p3, |

Anchor | **I** | **L** |

Music: | — ↗5 p6, ↗p7 ↗1 ↗2 ↗5, ↘2 ↘1 ↗5, ↘2 ↘1 | ↗p7, ↘p6 ↘p3 ↘7 ↗p2, |

Anchor | **ML** | **I** |

Music: | — — — — | — — — — |

The main melody scale is as follows. The very different key signatures for two pieces with the same Eb tonic that are similar in kind, namely this piece (6 flats) and *Goodbye Pork Pie Hat* seen earlier (3 flats) illustrate the possible arbitrariness of key signatures.


	Eb	F	Gb		Ab		Bb	Cb		Db		Eb	
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
ADMI	@	.	x	x	x	x	.	\$	x	x	x	x	@
AD.I+	@	.	x	x	.	x	.	\$	x	x	x	x	@

CHROMATIC CLASSICAL DOMAIN : “BODY AND SOUL”

Home Tonic Db (5 flats key signature: Ionian of Db)

This piece is strongly chromatic in a way that’s particularly complex in music notation. My source for the written music is *The Ultimate Jazz Fakebook*, Hal-Leonard (1988), p67. Three successive sections have three different key signatures: **A** 5 flats (Db-Ionian); **B** 2 sharps (D-Ionian); **C** 1 flat (D-Aeolian, but actually Dorian due to a natural in the written melody line). Ionian and Dorian of tonic D are nearby parallel modes (only 2 notes different). Ionian of Db is a distant from these, with many notes different (5 notes different from **B**). The obvious home tonic is Db because **A** starts and ends the piece. The large distance between **A** and **B** is more than compensated by the simplicity of every note of **B** being a half tone up from every note of **A**.

A  | **A** **P** | **M** | **M** **D** | **LD** |

 | **A** | **I A** | **M** | **IA** | **I** |


B  | | **I** | **M** | |


C  | **M** | **A DL** | **M M** | **D A P PD** |


The melody scales are classical modes. The strongly chromatic harmony is not, but is consistent with the melody. When I first encountered this piece, I struggled with trying to understand how this harmony implied the melody scales and finally realized I had it backwards. Here follow the melody scales.

		Db	Eb	F	Gb	Ab	Bb	C	Db					
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	P	A	D	M	I	L	\$	x	x	x	x	@
A	//P.DM.L	x	x	.	x	x	.	x	x	.	x	x	.	x
	//A.MI	@	.	x	.	x	x	.	@	x	.	x	x	@
	//I	@	.	x	.	x	x	.	\$.	x	.	x	@
B	//A.MI	@	.	x	.	x	x	.	@	x	.	x	x	@
	altL@p2	x	x	.	x	.	x	x	.	x	.	x	.	x
C	//P.DM.L	x	x	.	x	x	.	x	x	.	x	x	.	x
	altM@p2	.	x	.	x	x	.	x	.	x	.	x	x	.

harmony (first 4 bars)

harmony (last 4 bars)

melody (all)

harmony

melody: Ionian of tonic p2



harmony, ending returns to **A**



melody: Dorian of tonic p2



NON-CLASSICAL DOMAIN: “LAURA”



Home Tonic - C (empty key signature: Ionian of C)

My source for this piece is *The Jazz Book*, John Brimell, CPP/Belwin, 1989, p24. The source book is sub-titled *Today's Easy Adult Piano* but this piece is “easy” only in the sense that the density of notes on the page is low. It's musically much deeper than it looks on the music page, and somewhat difficult to figure out because the melody lines of some of the sections are sparse, and therefore compatible with multiple interpretations. The interpretation presented here is simple in the sense that every section except the turnaround section (d) is understood as originating in the 8-note, harmonic-minor-major scale determined by the double tritones **DL-PM-AI-AI-AI** and the corresponding tonic sequence **5-4-p3-1**, both of which are highlighted in blue below. Except for (d), the successive tonics are determined by tritone slides instead of by melody lines coming to rest on tonic notes. The melody lines end on notes shared with next mode, thus daisy-chaining the modes together without actually resolving in the current mode. Scale fifos are omitted but are easy to fair into the flow. The piece is easy to play, once memorized, without knowing this interpretation, but knowing it means knowing the otherwise-often-mysterious origin of all the notes — a solid basis for improvisation.



(a)  | | **LD** |
 | 7 7 | 7 ↘ p7 ↗ 7 ↘ p7 ↗ p5 ↗ 5 | ↗ 6 ...



(b)  | | | **PM** | | |
 6 ↘ 3 | 3 | ↗ 6 6 | 6 ↘ p6 ↗ 6 ↘ p6 ↗ 3 | 4 | ↗ 5 | ...

(c)  **D** | **AI** | **I** | | | | **DL** |
 5 — | — ↘ p3 ↗ 4 | — ↘ p3 ↗ 4 | p3 ↗ 4 | ↗ 5* ↘ 2 — | — ↗ p3 ↗ 5 ↗ p7 | ↗ p3* ↘ p6 — |

(d)  | **LA** | | **L** | | **AI** |
 | — ↗ 6 | ↗ 2 — | — ↗ 3 ↘ 1 ↗ 2 — |

go to (a) to repeat or to (e) to finish

(e)  | | **AI** | | |
 | — ↗ 4 ↗ 5 | — ↘ 3 ↗ 4 ↗ p6 ↗ 7 | ↗ 2 ↘ 1 ↗ 2 | ...

(f)  | | **DL AI** | | **AI** | | |
 2 ↘ 7 ↗ 1 | ↗ p3 p3 | — ↗ 3 ↘ p3 ↘ 2, ↘ 6 ↗ 7 | ↗ 1 — | — |

The shared harmonic-minor-major scale is very simple within the octave of its tonic: a circularly symmetric double tritone provides a stack of 3 minor thirds going up from a whole tone above the bottom of the octave (6 notes); the bottom minor third is filled with half tones (2 notes, highlighted in yellow) and the top minor third is empty; the frame of the scale octave adds 2 notes for a total of 8. The corresponding parallel modes of the home tonic are determined by the position of the empty minor

third a half tone below tonics highlighted in blue. Not every section visits all the scale notes. However the resolution section (e) does, thus establishing a basis for the rest. A useful feature of this interpretation is it accommodates the adjacent half tones of the filled-in minor thirds as scale notes, thus providing a clean representation of every section, free of passing notes.

	C	D	E	F	G	A	B	C					
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
(a) //DL	x	.	x	x	—	x	x	.	x	x	x	x	x
(b) //PM	x	x	—	x	x	.	x	x	x	x	x	.	x
(c) //alt-IA	—	x	x	.	x	x	x	x	.	x	x	—	
(d) //M	x	.	x	.	x	.	.	x	.	x	.	.	x
(e) //ADMI	x	.	x	x	x	x	.	x	x	—	x	x	
(f) //DL	x	.	x	x	—	x	x	.	x	x	x	x	

7*, ↘p7↗7↘p7↘p5↗5↗6

6↘3, ↗6*↗p6↗6↘p6↘3↗4↗5,

5↘p3↗4, ↘p3↗4↘p3↗4↗5*↘2,

↗p3↗5↗p7↗p3*↘p6,

↗6↗2, ↗3↘1↗2,

↗4↗5, 3↗4↗p6↗7↗2↘1↗2,

↘7↗1 ↗p3*, ↗3↘p3↘2

↘6↗7↗1

7*, ↘p7↗7↘p7↘p5↗5↗6
6↘3, ↗6*↘p6↗6↘p6↘3↗4↗5,
5↘p3↗4, ↘p3↗4↘p3↗4↗5*↘2,
↗p3↗5↗p7↗p3*↘p6,
↗6↗2, ↗3↘1↗2,
↗4↗5, 3↗4↗p6↗7↗2↘1↗2,
↘7↗1↗p3*, ↗3↘p3↘2
↘6↗7↗1

The melody notes of (c) are 6 notes of **//A**, excluding only the home tonic, which is provided by context. However, thinking this way would make (c) a special case, which it isn't in the context of this interpretation.

The harmony provides the double tritones that determine all the melody scales but that of (d), which has no tritones. The 4 different melody notes of (d) originate in the pentatonic major mode of the home tonic, which provides a simple segue to (a) for a repeat or to (e) to end the piece.

The other tritones in the harmony are ornamental in the context of the local melody scale.

NON-CLASSICAL DOMAIN: WHEN SUNNY GETS BLUE

Home Tonic - G (1 flat key signature: Dorian of G)

My source for this piece is sheet music by Jack Segal and Marvin Fisher from Hal-Leonard (1956), publication number HL00351105. The melody line from this source is a **//DM.L** blues in G, with a 4-bar section in the bridge that visits **//L**. The harmony shown below is a modification of the written harmony, which is a mix of fifos and tritone chords that sound fine but are confusingly irregular. Ornamental touches are highlighted in yellow — the ones in the melody line are from the written music and the ones in the harmony are part of the reharmonization.

The notation is organized into three systems, each with a chord line (anchored with an anchor symbol) and a melodic line (starting with a treble clef and a double bar line).
System 1: Chords are M, D, L, D, M, D L, I. The melodic line includes notes like 2, 1, 2, 4, 4, p3, p7, p5, 4, p3, p3, 2, 1, p7, 6, 5, 1, 6, and a repeat sign.
System 2: Chords are M, D, A, P, L, I. The melodic line includes notes like 5, p7, 1, p7, p2, p7, 1, p7, 4, p7, 1, p7, p2, p7, 1, p7.
System 3: Chords are M, D, L M D, M L. The melodic line includes notes like 5, p7, 1, p7, 4, p3, p7, 2, 4, 2, 4.
A second system follows, with chords \$, \$, L, L, \$, \$, L, L and a melodic line ending with ...//L.
A third system has chords I, M, M, A, A, L, M, D and a melodic line with notes like p7, 1, p7, 1, 2, p3, 3, 1*, 1, 6*, 6, 4, 5, 6, p7, 1, 4.
Below this is a section labeled 'repeat bars 1-6, followed by this ending, repeated twice'. The chords are D, D, A D, A D. The melodic line includes notes like 2, 4, 5, 4, p6, 4, p2, 1, 1.

Play the harmony in the first instance as octave shapes by doubling the treble line of the core an octave down. The 5-tritone runs down by half tones in bars 4-5 and 6-7 are very easy to play and sound good with the melody line. They include ornamental tritones **I**, **A** and **P** that are easily morphed into in-scale fifos, if desired.

The Lego-like harmony is shown next. Keep in mind that this is only for illustration because it's implied by the anchor line. The harmony of the first four bars of the bridge is in the **//L** mode (Lydian). The final four bars of the bridge return to the original blues scale. The transition between them in bars 13-14 is marked by the distinctive sound of a fourth-fourth slide down a half tone.



		G	A	Bb	C	D	E	F	G					
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	.	A	D	M	I	L	\$.	x	x	.	@
1	M	.	.	.	M	x	.	.
	D	.	.	.	D	x	.	.
2	L	@	L
	D	.	.	.	D	x	.	.
3	M	.	.	.	M	x	.	.
	D	.	.	.	D	x	.	.
4	L	L	@
	I	I	x
5	M	.	.	.	M	x	.	.
	D	.	.	.	D	x	.	.
	A	.	.	A	x	.	.	.
6	P	\$	P
	L	L	@
	I	I	x
7	M	.	.	.	M	x	.	.
	D	.	.	.	D	x	.	.
8	L	L	@
	M	.	.	.	M	x
	D	.	.	.	D	x
9	M	.	.	.	M	x
	L	L	@



		G	A	Bb	C	D	E	F	G					
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	.	A	D	M	I	L	\$.	x	x	.	@
10	\$	\$	x
	\$	\$	@
11	L	L	@
	L	L	x
12	\$	\$	x
	\$	\$	x
	\$	\$	@
13	L	L	@
	L	L	x
14	I	I	x
	M	M	x
15	M	M	x
	A	A	x
16	A	A	\$
	L	L	@
17	M	M	x
	D	D	x



NON-CLASSICAL DOMAIN: EVERY TIME WE SAY GOODBYE



Home Tonic - Eb (3-flats key signature: Ionian of Eb)

This piece is hauntingly lovely, with ambiguous changes that go well with the words about love and loss. The melody line comes to rest at the end on the home tonic **1** in the unmistakable Ionian-mode sequence **5-4-3-2-1-7-1** and much of the melody line is in the major family scale //A.MI* of which Ionian is a sub-scale. Minor (or minor-ish) segments in melody and harmony are highlighted in yellow. Except for one, all-minor, 4-bar section, minor touches are brief and sometimes in opposition (minor in melody and major in harmony, or vice versa). There's sufficient ambiguity in much of this piece that there's little point in overthinking scale implications, which is why there are no annotated mode signatures or identified secondary tonics. The ambiguity is a deliberate feature of the piece by a creative composer.



(a)  | M | I | I M | PM I I |
 | 3 3 3 — | — 3 3 3 | 3 — — 3 | ↗ 4 — ↘ 2 ↘ 1 ↘ 7 |



(b)  | M PM | I I | M M | D A |
 | — ↗ 5 5 5 | — 5 5 5 | 5 — — ↘ 3 ↗ 4 ↗ 5 | ↗ p6 — ↘ 4 ↘ 3 ↘ 2 |

(c)  | D D | A I | D M | M |
 | — ↗ 1 1 1 | 1 ↘ 7 — ↘ 5 | ↗ p7 — ↘ 3 ↗ 4 ↗ 5 | ↗ 6 — |

(d)  | D A | A A | L L | A I |
 | — ↘ p6 p6 p6 | p6 ↘ 5 — ↘ 1 ↗ 2 | ↗ p3 p3 ↘ 1 ↗ p3 ↘ 2 | — — |

(e) repeat (a)-(b)

(f)  | \$ DL A | I | M M | D A |
 | — ↗ 1 1 ↗ 2 ↘ 1 | 1 ↘ 7 — ↘ 5 ↗ 6 | ↗ p7 ↘ 3 ↗ 6 ↘ 2 | ↘ 1 ↘ 7 ↗ 1 ↗ 5 ↘ 4 |

(g)  | M M | L I | M | DL AI |
 | ↘ 3 3 3 — | — ↘ 2 ↘ 1 ↘ 7 | ↗ 1 | — — |

Cross references with the notes of the key-signature scale were provided earlier.

The relative timing of the melody line and the rich, mixed harmony is organized in an irregular way that I have always found impossible to remember in chord terms. The chords sometimes harmonize melody notes and sometimes provide transitions between them, in patterns that are easy to get wrong. It's much easier to get the core harmony right because the flow cues what comes next to the eye and ear.

In phrase (f) shown next, the two melody bars with the famous words “how strange the change from major to minor” stay resolutely in major for the melody, while the highlighted harmony, in these bars only, goes from major to minor. The harmony of the first two of these bars is intricate in chord terms but simple on the keyboard. The final voicing for the second bar implies the double tritone **AI** that’s a half tone down from **DL** in the first bar. In effect, the core sequence is **DL-AI** down a half tone. Octave voicings for the remaining bars are sufficient.

			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
			@		A		M	I		\$	x	+	x	x	@
(f)	\$	<i>I</i>	\$	@
	DL	<i>bIII-dim7</i>	.	.	.	D	.	.	L	.	.	x	.	.	@
	A	<i>II-m7b5</i>	.	.	A	x
	I	<i>V-7</i>	I	.	-	.	.	.	x	.
	M	<i>I-7</i>	-	.	.	.	M	x	.	.
	M	<i>IV-M7</i>	M	-	.	.	.	x	.	.	major
	D	<i>IV-m7</i>	.	.	.	D	.	-	.	.	.	x	.	.	to minor
	A	<i>pVII-7</i>	.	.	A	x	.	-	.

The sound of the piece is relatively insensitive to the exact placement of the harmony relative to the melody. When the melody is on a downbeat, playing the harmony on the following upbeat — and vice versa — often sounds fine. For this reason, the harmony placements shown don’t necessarily align exactly with the chord placements in the written music. Many of the harmony shapes are rootless voicings of the written chords.

NON-CLASSICAL DOMAIN : “ALL OF ME”

Home Tonic - C (empty key signature: Ionian of C)

I learned this version of *All of Me* in C (along with *No Greater Love* in Bb coming up) in Susan Muscarella’s piano comping course some years ago at the then Jazz School in Berkeley (now the Jazz Institute), as an example of using 4-note “open” voicings of extended and altered seventh chords to accompany soloists. The chord progression is strongly chromatic and therefore complex in chord symbol terms. The open voicings rearrange the chord notes and sometimes omit the roots. The standard way of explaining such voicings is by rearranged degree numbers of chord scales, in which the same notes from adjacent chords have different degree numbers relative to different roots that are sometimes omitted. Such representations are indirect relative to the keyboard, difficult to comprehend as a whole, or remember, and difficult to annotate on the written music because they require too many symbols. This complexity goes away when the shapes are understood in terms of core building blocks.

The skeleton melody line and harmony anchor line are shown next. Trying to understand this as a succession of classical modes bogs down in complexity. The simplest way of remembering it is this: melody and harmony are both from the 9-note, major family mode **A.MI+** (//I plus notes **p6** and **p7**), with the passing ornamentation shown highlighted in yellow. The two appearances of minor note **p3** in the melody line are passing notes because omitting them has no substantive effect on the sound. The appearances of **P** and **L** in the harmony are ornamental substitutes for **A** and **I** because substituting the latter for them sounds fine.

Anchor | M | I | M | | A | M | | P | | P | M | P | \$ |
 Music | 1 ↘ 5 ↘ 3 — | 3 , ↗ 1 ↗ 2 ↘ 1 | ↘ 7 ↘ p6 ↘ 3 — | 3 , | ↗ 6 ↘ 5 ↘ 3 — | 3 , | ↘ p3 ↗ 3 ↗ p7 ↘ 6 | ↘ 5 ↘ 4 | 4 , |

Anchor | A | | A | | | A | A | | \$ | L | | I | | I | |
 Music | ↘ 3 ↘ p3 ↘ 2 — | 2 , ↗ 3 ↗ p6 ↗ 7 | ↗ 2 — ↘ 1 — | 1 , | ↘ 7 ↘ p7 ↘ 6 — | 6 , 6 ↗ 2 ↘ 7 | ↘ 6 — | ↗ 7 , |

repeat first 8 bars

Anchor | L | | L | | A | | P | P | | I | | I | I | \$ | P | \$ | I |
 Music | ↗ 2 — ↘ 1 ↘ 7 | ↗ 2 , ↘ 1 | ↘ 7 — ↘ 3 ↗ 5 | ↗ 7 , ↘ 6 | ↗ 1 — ↘ 6 ↗ 1 | ↗ 3 3 | ↘ 1 — | — — |

Scales

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	\$	x	x	x	x	@	
//A.MI+	@	.	x	.	x	x	.	@	x	x	x	x	@	melody & harmony (first 16 bars)
//I	@	.	x	.	x	x	.	@	.	x	.	x	@	melody & harmony (last 8 bars)
P, L		+					+							ornamental subs for A, I (all harmony)

The next picture shows what this looks like in terms of successive building blocks on the keyboard. This intended to represent a picture in the mind’s eye implied by the above, not something to be written

down. Play this harmony in the first instance as octave shapes formed by doubling the treble line of the core an octave down. The only exception is bars 3-4 in which the tritones establish outer notes a minor seventh apart that may be held for both, only moving the middle note.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	A.MI⁺	@	-	A	-	M	I	-	\$	x	+	x	x	@		A.MI⁺	@	-	A	-	M	I	-	\$	x	+	x	x	@	
1, 17	M	-	.	.	.	M	.	.	.	x	.	.	-		9	A	.	.	A	.	-	.	.	.	x	
	I	I	.	-	.	.	x	.		10	A	.	.	A	.	-	.	.	.	x	
2, 18	M	-	.	.	.	M	.	.	.	x	.	.	-		11	A	.	.	A	.	-	.	.	.	x	
3, 19	A	-	.	.	.	x	.	.	.	A	12	A	.	.	A	.	-	.	.	.	x	
4, 20	M	M	x	.	.		13	\$.	.	-	x	@
5, 21	P	\$.	-	.	P	14	L	.	.	-	L	@
6, 22	P	\$.	-	.	P	15	I	I	@
	M	-	\$.	.	.	@	16	I	I	.	-	.	.	x

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	A.MI⁺	@	-	A	-	M	I	-	\$	x	+	x	x	@	
25	L	L	.	.	-	.	x	.	
26	L	L	.	.	-	.	x	.	
27	A	-	.	.	.	x	.	.	.	A	
28	P	x	.	-	.	P	
	P	x	.	-	.	P	
29	I	.	.	-	.	.	.	I	x	
30	I	I	.	-	.	.	x	.	
	I	I	.	-	.	.	x	.	
31	\$.	.	-	\$.	.	.	@	
	P	x	.	-	.	P	
32	\$.	.	-	\$.	.	.	@	
	I	I	.	-	.	.	x	.	

Open Voicings

Four-note “open” voicings developed from the same core are shown next. This is intended to represent a picture in the mind’s eye implied by the notation in the below left column (which would be annotated above the staff). The **X\Y** notation identifies non-overlapping building blocks **X** on the bottom and **Y** on top (the backslash distinguishes this from overlapped building blocks represented by **XY**). The shapes are voicings of the chords shown on the right. Chords on roots I, II and V are from the Ionian mode. The other chords (highlighted in yellow) provide visibly simple segues between the Ionian voicings. The chord symbols are all satisfied serially in the flow. This is remarkably simple, conceptually, compared to the standard way of understanding the same thing in terms of degree numbers of chord scales. The four notes are difficult to play with the left hand, but easy-to-play 3-note voicings of the same chords follow from omitting the top notes; the top notes can be added by the left

hand under the melody line.

Bars 1-8

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	A.MI+	@	-	A	-	M	I	-	\$	x	+	x	x	@	-	A	-	M	I	-	\$	x	+	x	x	@	
1	M\A	M	x	.	.	-	.	A	\$	I-6(9)
	I\M	I	.	-	.	.	.	x	.	.	.	M	x	V-9(13)
2	M\A	M	x	.	.	-	.	A	\$	I-6(9)
3	L\A	-	.	L	.	.	.	x	.	.	.	A	x	III-9
4	M\A	M	x	.	.	.	A	x	III-9b5
5	P\L	x	.	-	.	.	P	.	.	-	.	L	x	.	VI-9(13)
6	P\I	x	.	-	.	.	P	.	.	.	I	.	.	.	x	.	.	.	VI-7b13
7	\$\M	\$.	.	.	@	-	.	M	x	II-m9
	P\I	x	.	-	.	.	P	.	.	.	I	.	.	.	x	.	.	.	VI-7b13
8	\$\M	\$.	.	.	@	-	.	M	x	II-m9

Bars 9-16

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	A.MI+	@	-	A	-	M	I	-	\$	x	+	x	x	@	-	A	-	M	I	-	\$	x	+	x	x	@	
9	A\I	x	A	.	M	x	III-7
10	A\I	x	A	.	.	I	x	.	.	III-7b9
11	\$\M	\$.	-	.	@	.	.	M	x	VI-m7
	A\I	x	A	.	.	I	x	.	.	III-7b9
12	\$\M	\$.	-	.	@	.	.	M	x	VI-m7
13	\$\M	\$.	.	.	@	-	.	M	x	II-9sus
14	L\I	L	@	-	.	M	x	II-9
15	I\I	I	@	-	.	M	x	II-m9
16	I\I	I	.	-	.	.	x	.	.	.	M	x	V-9
	I\I	I	.	-	.	.	x	.	.	.	D	x	V-7b9b13

Bar 17-24 - repeat bars 1-8

Bars 25-32

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
A.MI⁺	@	-	A	-	M	I	-	\$	x	+	x	x	@	-	A	-	M	I	-	\$	x	+	x	x	@	
25 L\M	L	.	.	-	.	x	M	x	.	.	.	VI-6(9)
26 L\M	L	.	.	-	.	.	@	.	.	.	M	x	.	.	.	VI-m6(9)
27 A\M	A	.	.	.	M	x	.	.	.	III-m9
28 P\L	P	x	.	.	VI-9(13)
P\I	P	x	.	.	VI-7b9b13
29 A\M	I	x	.	-	.	M	x	.	.	.	II-m9
30 I\M	I	.	-	.	.	.	x	.	.	.	M	x	.	.	.	V-13
I\D	x	.	.	.	D	x	.	.	V-b9(13)
31 M\A	M	x	.	.	-	.	A	x	.	.	.	I-6(9)
P\I	P	.	.	.	I	x	.	.	VI-7b9b13
32 \$\M	@	.	-	.	M	x	.	.	.	II-m9(11)
I\D	I	.	-	.	.	.	x	.	.	.	D	x	.	.	V-7b9b13

Understanding how the shapes satisfy the chord symbols requires knowing the relationship between the numeric suffixes of the chord symbols and the chromatic scale. I explain this only to make clear that the shapes satisfy the written chords, not to suggest figuring the shapes out this way. In bars 3-4, chord suffix “9” is chromatic scale note **p5** and chord suffix “b5” is chromatic scale note **p7**. The 3-note voicings in bars 3-4 don’t include **p5** but it’s included in bar 5, thus satisfying the bar-4 chord in the flow. The bar-4 shape jumps up a minor third in bar 5, with a wobble that puts a fourth on top instead of a major third. The wobble provides degree “13” of the chord. It also provides degree “9” of the bar-4 chord, thus satisfying two chord symbols at one stroke.

Home Tonic - Bb (2 flats key signature: Ionian of Bb)

	7↗1↗2						
	M5	D5	A4	P4) ... //A.MI+
	↘1, ↘7↘6↘3 ↗5,	↘p5↘4↘1 ↗3	3	3↗7↘p7)
	I5	M5	I5	I5)
	↘6	6↗3↘p3 ↘2	2,	↘7↗1↗2)
<u>repeat bars 1-4</u>							
	L4	I4	I5	M5	D5) ... //I
	↘6,	6↘3↘2	↗1 — ↗2 —	↘1,	—)
<hr/>							
	A..I	\$3	I..A	\$3) ... //AI
	↗3↗p6↗7↗2	↘1,	↘p6↗7↗2↗4 ↘3,)
	I..A	\$4	L4	I3) ... //AI-//I
	↗3↗p6↗7↗2	↘1↘7↘6↘3	↘2,	—	— ↗7↗1↗2)
<hr/>							
<u>repeat bars 1-4</u>							
	L4	I4	I5	M5	I4	I5) ... //I
	↘6*↗3↘2	↘1—↗2—	↘1—,	—	—)


	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
// A.MI ⁺ (major family)	@	.	x	.	x	x	.	@	x	x	x	x	@
// AI (major context)	@	.	x	.	x	x	.	@	x	.	.	x	@
// I (Ionian major)	@	.	x	.	x	x	.	@	.	x	.	x	@
P,D,L		+		+			+						


ornamental subs for **A,I**


CHROMATIC CLASSICAL DOMAIN : “ALL THE THINGS YOU ARE”



Home Tonic - Ab (4 flats key signature: Ionian of Ab)


I learned this strikingly beautiful piece early in my musical adventure, and found it easy to play but difficult to understand. The source is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). The skeleton melody line and associated core harmony for bars 1-26 where all the changes occur are shown below. Notes highlighted in blue identify the home and secondary tonics of modes shown at right.


(a)  | \$ | I | I | M | M |
 ♪ | 1 | ↗4↘1 | ↗7* | 7↗3 ↘7 | ↘6* | ... //I : Ionian

(b)  | D | D | | |
 ♪ | 6↗p3↘6 | ↘p6 | — , | ... altD@p6 : Phrygian

(c)  | A | \$ | L | L | M | P | PM |
 ♪ | ↘5 | ↗1↘5 | ↘p5* | p5↗7↘p5 | ↘3* | 3↗p5↗5↘p5↘3 | ... //L@5 : Ionian

(d)  | | L | M | M | | D | D |
 ♪ | ↘p3, ↘p5↗7↘p5 | p5↘3* | — ↘5↗p6↗3 | ↘p3 | — |
 | M | | | |
 ♪ | — ↘p5↗7↘p3 | p3↘p2* | ... altM@p3 : Phrygian

(e)  | P | | @ | A |
 ♪ | — ↘2↗p3↗p2 | ↘1* | — | ... transition

(f)  | | I | ...
 ♪ | 1 | ↗4↘1 | ↗7* //I : Ionian

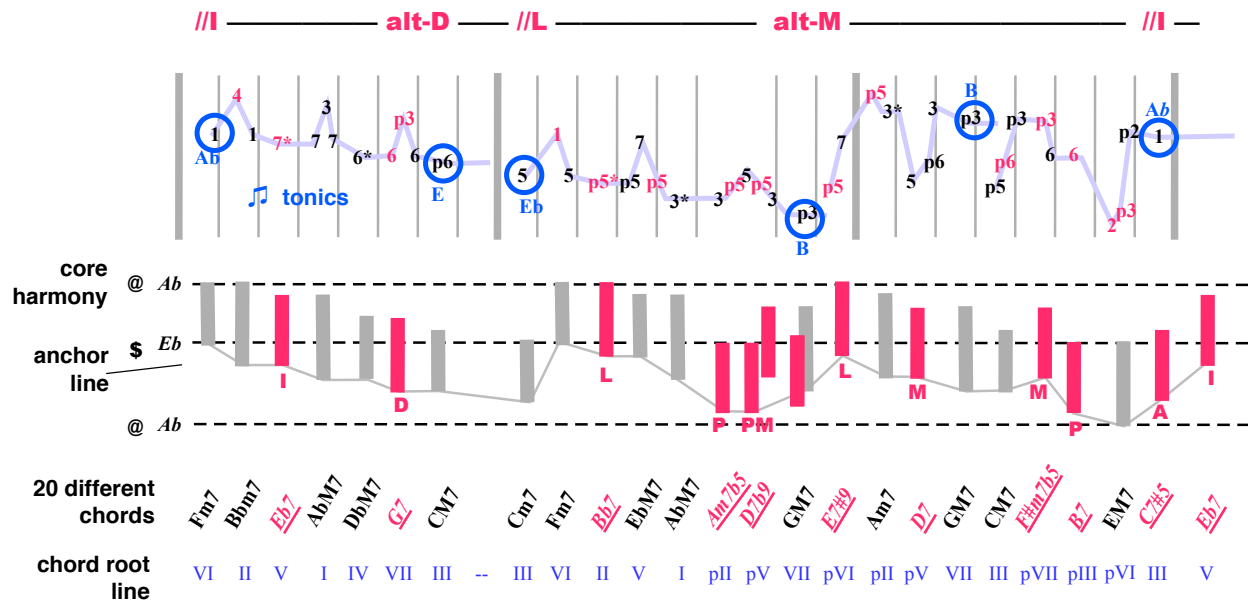
The melody notes transposed into the home octave provide the home-tonic modes.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	\$	x	x	x	x	@	
(a) //I	x	.	x	.	x	x	.	x	.	x	.	x	x	Ionian of tonic 1
(b) altD@p6	.	x	.	x	x	.	x	.	x	x	.	x	.	Phrygian of tonic p6
(c) //L@5	x	.	x	.	x	.	x	x	.	x	.	x	x	Ionian of tonic 5
(d) altM@p3	.	x	.	x	x	.	x	.	x	.	x	x	.	Phrygian of tonic p3
(e)	x	x	x	x	x	transition
(f) //I	x	.	x	.	x	x	.	x	.	x	.	x	x	Ionian of tonic 1

The melody line alternates between relative Ionian and Phrygian modes of the tonic sequence 1↘p6↗5↘p3↗1 highlighted in blue (in letter notes this is Ab-E-Eb-B-Ab). The tritone change I-L and the tonic change 1↗5 identify an Ionian-Ionian change. The tritone change D-M and the tonic change

p6~p3 identify a Phrygian-Phrygian change. The parallel and alt modes of the home tonic are relative modes of these (the alt modes follow from the absence of the home tonic from the two Phrygian modes). The strong presence of the Phrygian mode may seem strange but is not unusual in strongly chromatic music.

The following overview sketch contrasts the conceptual and actual simplicity of the piece with the complexity of the written music. The chord progression begins with a simple Ionian root line that starts to be misleading at bar 7 and continues to be so until bar 13. Being misleading stops after that but complexity continues. The transition back to the home tonic is particularly complex.



NON-CLASSICAL DOMAIN : “CHELSEA BRIDGE”

Home Tonic - C#/Db (4 sharps for Aeolian of C#, 5 flats for Ionian of Db)

The source for this hauntingly beautiful Strayhorn piece is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). There are two key signatures but only one home tonic, the black key is identified by both Db and C#. The notational complexity that follows from this has been discussed elsewhere in these pages. This piece multiplies the complexity by returning to the original Ionian mode at the end via naturals relative to the 4-sharps key signature. Between the parallel classical modes are chromatic passages that multiply the notational complexity once more. The result is a confusing muddle of many more than twelve note symbols relative to the two written key signatures. The piece is musically sophisticated by any measure, but this notational complexity is way out of proportion to the sophistication.

I first learned this piece by rote with great difficulty from the written music, but thought that anything that sounds this good must have simple musical logic behind it. It does, as shown below: the basic structure is major-minor-major where the major and minor sections open with **///I** and **///A** and then branch out into non-classical modes of the same tonality. The concluding major section is **///I**. Yellow highlighting identifies ornamental elements relative to identified scales. Parallel modes of the melodic minor determine melody and harmony scales in bars 2-5. All the scales are shown on the next page.

Anchor	M D A P	... ornamental
Musical	: — 6↗7, ↗1↗2, ↗3↗4↗5	... ///I
<hr/>		
Anchor	L.A (@*) M.L (L ⁺) L.A (@*) M.L (L ⁺) P	... determines /// modes
Musical	↗p6—, ↗6↘p5 p5, ↗5↗p6 p6, ↗6↘p5—, ↗5↘p2 p2	... moves between modes
<hr/>		
Anchor	I I M M D A	... determines ///I
Musical	↗3 ↘1—, ↗3↗5—, ↗3↘1— : — 5↗p6	... moves in ///I
<hr/>		
Anchor	A A) ... ///A
Musical	↗p7* p7↗1↗p3↗5↗p7↗2, ↗p3 ↗2↘p7↗5↘p3↗1 ↗7↗p5↗2)
<hr/>		
Anchor	P I I) ... alt-IP
Musical	↗p2, — ↗p3 ↗4 ↗5↗p6↗p3— ↗p2↗7 ↗p7 ↗p2—, ↗7↗p7)
Anchor	M	
Musical	↗6, ↗p5↗6↗p2, ↗6 ↗p2	... ornamental
<hr/>		
Anchor	M.L D A P	... ornamental
Musical	↗p5, ↗6↗7, ↗1↗2, ↗3↗4↗5	... ///I

In bars 2-5, the harmony double tritones may be implied by playing their outer notes as augmented fifths moving down and up by whole tones, as shown next. Playing these augmented fifths with the

sparse melody line is very easy, and sounds good.

		p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		x	x	@	P	A	D	M	I	L	\$	x	x	x	x	@
bars 2, 4	<u>L.A</u>	.	.	x	.	x	.	.	.	x	.	x
	@+	.	.	x	x
bars 3, 5	<u>M.L</u>	x	.	x	.	.	.	x	.	x
	L+	x	x

Here are the melody scales. In bars 2-9, the various parallel modes, all with major tonality, are determined jointly by the melody and harmony. In bars 2-5, the melody line sequences **p6-6-p5-5** and **p5-5-p6-6** move back and forth between the **LA** and **ML** modes. The slide of an augmented fifth up and down by a whole tone captures the signature note difference. In bars 6-9, the sparse melody line exercises the major triad of the **//I** mode identified by the harmony. In bars 13-15, the **IP** minor mode follows the **//A** minor mode like the several major modes follow the opening **//I** major mode.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	\$	x	x	x	x	@	
bar 1	@	.	x	.	x	x	.	x	.	x	.	x	@	//I
bars 2, 4	x	.	x	.	x	.	x	.	x	.	x	.	x	//LA) aug fifths highlighted
bars 3, 5	x	.	x	.	x	.	x	.	x	.	x	.	x	//ML)
bars 6-9	@	.	x	.	x	x	.	x	.	x	.	x	@	//I
bars 10-12	@	.	x	x	.	x	.	x	x	.	x	.	x	//A
bars 13-15	.	x	.	x	.	x	.	x	x	.	x	.	x	altIP (Phrygian flat one)
bars 16	.	x	x	.	.	x	.	.	.	ornamental transition
bar 17	@	.	x	.	x	x	.	x	.	x	.	x	@	//I





The melody and harmony have an elegant simplicity in these terms that's not difficult to understand, remember or play.





NON-CLASSICAL DOMAIN : “LUSH LIFE”













Home Tonic - Db (5 flats key signature: Ionian of Db)

This beautiful Strayhorn piece is melodically and harmonically rich, and challenging to play from the written music because there are often two or more chords per bar, many of them chromatic relative to the written key signature of five flats (major tonic Db). The piece is so chromatic that there is more than one way of understanding it. It can be seen as daisy-chained, short segments of successive parallel classical modes, but this quickly bogs down in complexity.

The view of it here is in terms of two non-classical parallel modes of the home tonic for both melody and harmony: one is the 10-note **//ADMI** minor-major mode in the verse [A] and chorus [C]; the other is the **alt-IP** mode (Ionian sharp one) in the bridge [B].

[A]  | @ A | M A | M A | M I A D |
 5 :: ↗1*↗2* | ↗3*↘2* | ↗3*↘2* | ↗3*↗4*↗5*↗p6* |
 1 _____ 2 _____
 | I I | A | I M | M D |
 ↗p7*—↘p6↗p7↘p6|↗5*↗p7↘p3*|↘2*, — ↘5 :|| ↘2*, — ↗7 |

[B]  | I P | A P | DL | D P | D P | DL |
 |↘7*↗p2*|↗3*↗5*|↗7↘p2↘6↗7*|7 — 7|↘7*↗p2*|↗3*↗5*|↗7↘1↘6↗7*|
 | I | M | MP | DL | AI |
 |↘6↗7↘5↗6↗7↘3|3 — | — ↗5↘4↘3|↘2| — 2*↗3↘1|↗5 — | — |

[C](a)  | M I | M I | D A | M L I |
 | 5↗6↗p7↗p3↘p6|↘5↗6↗7↘p3↘p7|↘6↘5↗1↗3↗4↗5 | ↘p7 — ↘p6 |
 (b)  | repeat bars 1-2 of (a) | M D | P A |
 | " | ↘6↘5↗1↗3↗5↗p6|↘7 — ↘1 |
 (c)  | A L | DL | M | A P L I |
 | ↗2↗3↗4* — ↘p3 | ↘2↗3↗4*↘p3 | ↘p6↗p7↗7* — ↘p7 | ↘p6↗p7↗7*↘6 |
 (d)  | repeat bars 1-2 of (a) | DI AI | P |
 | " | ↘6↘5↗1↗3↗4↗5 | ↗6 — ↘3 |
 (e)  | A | L I | DL | P |
 | ↗5↘4↗5*↘4 | ↘3↘2↗3↘7 | ↗2↘1↗2 — 2↘1 | 1↘p7↗6 — ↘3 |
 (f)  | A | L I | A D M M | A D M |
 | ↗5↘4↗5*↘4 | ↘3*↘2↗3↘5 | ↗p7↗7↗1↗p2 | ↗2↗p3↗3 |

These modes lead naturally to short, easy-to-remember segments in classical and other modes that follow from the flow. For example, **alt-IP** morphs into **///I** halfway through [B] by altering one note. From time to time, the harmony of the different sections substitutes **P** and **L** for **A** and **I**, but thinking of these as ornamental relative to the **//ADMI** keeps things conceptually simple. The grey shading in [C] highlights differences.

There are no voicing extensions because the intent is to play octave shapes for single tritones and for fifos, which adds depth without adding new notes (the piece is so melodically and harmonically rich that the voicings don't have to add variety). Harmony fifos are mostly omitted for simplicity, except for characteristic sequences of them in [A] and the ending bars of [C].

The double tritones **PM**, **AI** and **DL** that appear in many places in the harmony are diminished seventh chords, commonly used ornamentally in any context in which they sound good. The double tritones have circular symmetry (all intervals are minor thirds), so only three of them with different notes exist. They can be played as four notes but they can also be voiced with three notes (a tritone with a minor third on top, or the opposite) or even with two notes (outer notes a major sixth apart), when the context implies the other note(s). A sequence of two or three of them, often used as an ornamental transition between modes, can be played as a sequence of major sixths anchored by any three of six tritone anchors (e.g., **P-A-D** going up and **L-I-M** going down, to name just two possibilities — others require inversions of individual tritones).

The chromatic melody-line run in the final two bars is a signature of the piece.

MIXED DOMAIN : “THE PEACOCKS”

Home Tonic - F (5 flats key signature: Phrygian of F)

This wraps up the chapter with an example in annotated music notation (in handwritten annotations, circling tritone anchors, or their chords of origin, or both is helpful). This strongly chromatic piece is one of the most haunting jazz pieces I have heard or learned to play. It sounds so “right” as written that jazz improvisations rarely stray far from it.

The home tonic is determined to be F by the final notes of bars 9 and 17. In bars 1-9, the combination of the 5 flats key signature, the home tonic and the lack of accidentals in the melody line identifies the melody scale as Phrygian (mode signature //P). The melody line in bar 8 runs up through all the notes of this scale starting a fourth up (fifth down) from the tonic. The contrasting harmony is chromatic. The over-lines in bars 2 and 4 indicate held treble notes.

@ = F

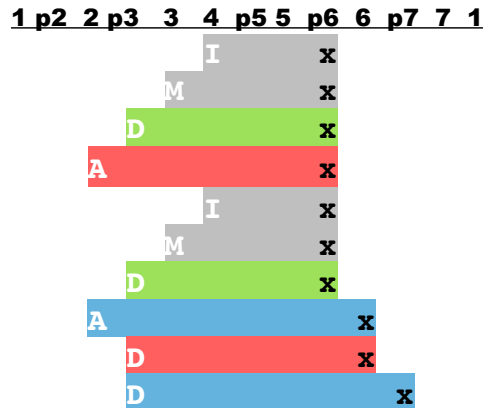
Anchor: //ADMI Scale: //P = 1-p2-p3-4-5-p6-p7-1

5 8 10 12 14 16 17

Chords and Annotations:

- Bar 1: Bbm5, BbmM, Bbm7, Bbm6, Bbm5, BbmM, Gbm7
- Bar 2: Fm7, Bb13, Ebm7
- Bar 3: C#9, F7#9, Bbm5, (F7alt)
- Bar 4: Cm7b5, F7, Bbm5
- Bar 5: Cm7b5, F7, Bbm5, BbmM
- Bar 6: Ebm7b5, Ab7, Eb7b5
- Bar 7: Eb7b5
- Bar 8: C#9, F7alt

Harmony in bars 2-9 contrasts strongly with the melody. The core harmony in these bars, the first part of which is shown next, is visibly from the atonal **ADMI** scale (which is minor-major relative to the home tonic).



The melody of the second section consists mainly of individually simple sequences from the full chromatic scale that are prompted by the harmony.

bar 11:	p5↘p3↘2↗4 3↘p2↗1↗p3	↘3h↘h↗3h	net ↘3h (minor third)
bar 13:	p6↘4↘3↗5 p5↘p3↘2↗4		
bar 15:	7↘p6↘5↗p7 6↘p5↘4↗p6		
bar 16:	3↘5↗p6 4↘p6↗6 p5↘6↗p7 5↘p7↗7	↘9h↗h	net ↘8h (aug. fifth)
bar 17:	p2↘3↗4 2↘4↗p5 p3↘p5↗5 3↘5↗p6 4↘p6↗6		

CHAPTER 6: OBSERVATIONS & CONCLUSIONS

Music notation is here to stay and must be lived with, but this doesn't preclude alternate views of piano music from a different perspective. I looked for an alternate view of harmony and found one based on intervals.

The central role of tritones in PKP tends to seem inappropriate to people savvy in music notation, for several reasons. In general, it goes against received musical wisdom. More specifically, tritones cannot even be written down in music notation until a key signature is established, so imagining them as identifiers of scales is difficult. Anything involving multiple tritones is complex in music notation because it implicitly brings in other key signatures — the more tritones, the more complexity. The very concept of fifos is unconventional and even more so is the concept that they can be treated as morphed tritones, understood from context. The payoff is insight into deep structure that is difficult to see in music notation. Appreciating these things requires stepping outside the box of music notation, at least momentarily.

Thinking in notes and thinking in intervals *is* different in kind. Many experts who have learned the hard way to think in notes apparently find thinking in intervals too alien to contemplate. Comments from them have run the gamut from “obviously wrong” to “wrong headed” to “naively simplistic” to “overwhelmingly complex.” One expert said I had “found a tritone hammer and saw everything as a nail” as if the very existence of tritones as useful size tools was unthinkable. The book *Modalogy*, devotes 50 or so complex pages to parallel modes without ever mentioning the possibility that tritones might simplify things by providing defining notes.

Because I started out seeing intervals and notes as dual views of the same thing, I came to find it natural to switch back and forth. I have made my best effort in this document to convey the simplicity of this. I believe, based on my personal experience as an adult beginner, that any beginner could easily learn to think this way.

I know from personal experience that PKP helps with learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. It turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

The jumpy chord root lines in many of the examples demonstrate that referring chords to constantly moving roots creates misleading complexity. This complexity is both conceptual (the underlying, smoothly flowing core harmony is obscured) and actual (playing the jumpy harmony is technically difficult because all the fingers must be lifted and moved by large intervals). It's easier to learn the core harmony and introduce the jumps later, if desired, via inversions, which are always easy on the keyboard, given an anchor line.

GENERAL BENEFITS

PKP annotations above the staff tell you many things. Combined with context provided by the melody line, they tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not right at any point by identifying tonic scales in play in a very immediate way. They tell you about transitions between major and minor tonality and between different tonics. They provide a basis for enrichment and improvisation.

Scale, modes and chords are seen as part of a continuum expressed in the same terms. This seems to me to be a very powerful benefit, compared to seeing everything beyond classical modes as

special cases identified by accidentals (which includes chromatic chords with suffixes that imply accidentals). The representations of scales and chords in terms of tritone clusters are unfamiliar to expert pianists but the interval stacks that they imply on the keyboard are completely familiar.

With PKP, playing music is guided by note-based symbology on the music page (one dual view) plus PKP annotations above the staff (the other dual view) that suggest core harmony and make changing tonic scales and tonics directly visible to the eye in terms of changing dictionary words.

A scale dictionary of less than a page covers much ground. The PKP dictionary provides a novel, high-level, conceptual view of tonic scales that's easier to hold in the mind's eye than the very much larger number of spelled out tonic scales in music notation. The dictionary covers $12 \times 39 = 468$ scales of 12 different tonics in music notation (more than is covered by scale dictionaries such as *The Source*). Words of 1-4 letters from the 6-letter alphabet identify, by tritone content, 39 tonic scales/modes with 7 or more notes, 1-4 tritones, and no intervals larger than a minor third. The count of 39 scales/modes includes 11 single scales covered by single words and $4 \times 7 = 28$ parallel modes covered by transpositions of 1-2 letter master words (only the master mode is identified in the dictionary). A selected home tonic anchors all of this to the keyboard.

The words are analogous to the words of biological DNA in the sense that they identify deep structure without spelling out the details. Words of biological DNA are “expressed” as proteins, the building blocks of life. **PADMIL** words are “expressed” as building blocks of scales and harmony. Biological DNA is “extracted” from biological samples and interpreted by sophisticated machines called “sequencers.” **PADMIL** scales and building blocks are “extracted” and “sequenced” from musical lines by sophisticated machines called human eyes. Biological DNA is “inherited” from parents. **PADMIL** words are “inherited” from general knowledge about them encoded in the dictionary.

Symmetry breaking in the scale dictionary and symmetry breaking in core harmony are two sides of the same coin. In the dictionary, it yields scales with fewer tritones. In harmony, it releases the tension of dissonance to produce consonance. Seeing the simplicity of this requires forming mental images of it relative to the schematic tonic octave that are independent of specific assignments of the octave to the keyboard.

Understanding harmony is possible independently of practicing. Keyboard symmetries centering around tritones enable high-level representations of harmony that are simple in their own terms and deep in harmonic meaning in any terms.

Music is freed from its key-signature straightjacket. Harmony scales are understood at a glance from dictionary words aggregated by eye, instead of having to be figured out or remembered from key signatures, accidentals and chromatic chord symbols. Tonic changes are no more difficult than any other changes.

Anchor lines that define harmonic cores replace chord root lines as the fundamental musical lines of harmony. Chord roots are visibly secondary elements in the flow of the music because inversions move them into the body of the chord, where they become no more than another chord note that may be omitted if implied by context.

Sight reading anchor lines provides deep context. Anchor lines tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not by identifying tonic scales in play. They tell you about transitions between major and minor tonality and between different tonics.

Multiple explicit or implicit key signatures become a non-issue. This is true whether the key

signatures imply tonic changes or only scale changes for the same tonic. Tonic changes amount to no more than moving (conceptually) a tonic pointer within the home octave and reinterpreting the alphabet letters relative to it, without changing any notation.

Chord symbols become a non-issue. Chord complexities that result from banging square pegs (chord symbols) into round holes (places in scales where they don't quite fit) are misleading. Chords described by chord symbols are, on the keyboard, almost universally composed of combinations of PKP's two kinds of building blocks, of which only tritones are always core. A melody line plus an anchor line above the staff imply a voicing of an unidentified chord progression. Specific chords are identified by assigning roots.

Learning blues prepares you for anything music notation can throw at you. Blues is an example of "folk music" that evolved independently of music notation and therefore tends to be regarded as naive when measured against its depth and complexity. This condescending view is unwarranted. Blues is harmonically sophisticated in a highly organized way that makes it, to my mind, a better starting point than classical modes of key signatures for understanding many forms of music.

Sharps and flats are not needed. Building blocks are the same number of half tones no matter how they are notated. This enables PKP to dispense with sharps and flats without any loss of musical accuracy. Expert pianists tend to think they are still needed to identify context because they have learned to use them for that, but note positions relative to tonics are sufficient.

The simple flow of harmony in harmonic cores, in which building blocks slide or morph into other building blocks, is universal. Nothing constrains harmony to be played this way but the flow is so natural and easy to play that much is to be gained by learning pieces this way and then transitioning to other playing styles if desired.

Tritone-intensive harmonic cores of much music provide so much harmonic variety that voicing them requires only adding depth. Depth is easily added by doubling the treble line of the harmonic core an octave down, or the bass line an octave up, and then making adjustments to move points on the line closer to the core, if desired.

No more piano wizardry is required to have fun playing these kinds of music than the ability to sight-read melody lines in music notation. Anyone with eyes to see intervals on the keyboard can play harmonic cores from anchor lines annotated above the staff.

The simple way harmony is represented provides great freedom in choosing how to play it. Playing styles may range between the extremes of smoothly flowing harmony within the octave of the home tonic of piece to jumpy harmony in which all harmony notes go up from a chord root line. Improvisation follows naturally and is fun, even if not expertly done.

The simplification provided by the alphabet is different in kind from simplifying chord symbols. Any sophistication in the harmony is in the tonic scales of origin of the chords.

Mode signatures provide an above-the-staff markup notation for the concept of "modal jazz" which is otherwise a concept without a notation.

PKP annotations above the staff are not always necessary. With experience, straightforward chord progressions may be sight-read in PKP terms.

SOME REFERENCES

1. Barta, *The Source: The Dictionary of Contemporary and Traditional Scales*, Hal Leonard (1995), for helping me to be sure I was not missing important scales.
2. Mehegan, *Jazz Improvisation 1: Tonal and Rhythmic Principles*, Watson-Guptyl (1984), for Roman-numeral chord notation, and for teaching me (unintentionally) that it is not a solution for complex chromatic chord progressions, but a problem if pushed beyond its basic function of specifying chord root lines.
3. Eskelin, *Lies My Music Teacher Told Me*, Stage Three Publishing (1994) for insight into the nature of scales and musical “perfection,” and for encouraging me to think outside the box.
4. Dmitri Tymoczko, *A Geometry of Music* (2011) for stimulating discussions of how to think about music from different angles.
5. Mark Levine, *The Jazz Theory Book*, Sher Music Co. (1995) for providing examples of well known jazz scales and harmonic forms in conventional notation, against which to verify PKP coverage.
6. George Russell, *The Lydian Chromatic Concept of Tonal Organization*, <http://www.georgerussell.com/lc.html>, for making me aware that PKP covers the concept, because nothing is changed by replacing the Ionian mode by the Lydian mode as the default reference major mode for any piece of music.
7. Edward Frenkel, *Love and Math: The Heart of Hidden Reality*, Perseus (2013) (on Kindle), for many insights into the usefulness of symmetry.
8. Jeff Brent with Schell Barkley, *Modality — Scales, Modes & Chords: the Primordial Building Blocks of Music*, Hal Leonard (2011), for the most comprehensive treatment I have found of this subject. The result is painstakingly accurate but overwhelmingly complex because it sticks to representations based on sharps and flats that follow from the misalignment of pitches of notes of overlapping octaves.
9. Ross W. Duffin, *How Equal Temperament Tuning Ruined Harmony (and Why You Should Care)*, W.W. Norton (2007), for an understanding of the piano’s deviation from musical perfection.
10. Stephon Alexandar, *The Jazz of Physics: The Secret Link Between Music and The Structure of the Universe*, Basic Books (2016). This amazing book resonates strongly with my own ideas. The author is a jazz musician and physicist who speaks of doing physics as informing the playing of jazz and playing jazz as informing the doing of physics. Both require rising above formalisms to form conceptual representations, with symmetries highlighted as particularly important.

ACKNOWLEDGMENTS

I did not take this musical journey alone. I received comments and help from many people over the decade or so the ideas were germinating and consolidating.

Music theorist Paul Steinbeck encouraged me to continue writing at a time when I was becoming discouraged about finding a way of bringing my ideas before the music community. Jazz pianist, teacher and composer Taylor Eigsti has been an inspiration to me. Although I have never been a piano student of his, I have learned much from him in sporadic discussions in person and by email. I am deeply grateful for his willingness to take time away from a busy schedule to engage in these discussions. A short series of piano lessons from SF jazz pianist Michael Parsons helped me to see more clearly the relationship between my ideas and standard jazz-piano practice.

Thanks to SMT (Society for Music Theory) members Charise Hastings, Peter Shultz and Neil Newton for insightful email comments and encouragement following announcements of my website on an SMT mailing list. Thanks to music professor Robert Rawlins for email encouragement, and for helpful examples and comments. Aaron Blumenfeld and Susan Muscarella gave helpful courses at the Jazz School in Berkeley (now the Jazz Institute) that provided many examples to chew on (including, from Susan Muscarella's course, some interesting variations on *Summertime* in D minor that helped me understand ornamental scales). Amateur pianist and Jazz School Board Chair Susan Brand, and concert pianist and music entrepreneur Robert Taub, provided early encouragement.

My piano teacher in San Francisco for several years, Ken Fishler, provided inspiration and how-to information on chord voicings, while patiently tolerating and responding to my question-everything approach. My first piano teacher in Ottawa, Canada, Sally Robinson, started me off right as an adult beginner by helping me to learn favorite harmonically sophisticated pieces by following her fingers on the keyboard without understanding anything about what I was doing except that it sounded right — being able to play these pieces, even if clumsily and by rote, enabled understanding to seep in gradually. I think that without this particular way of starting out — fingers on the keyboard first, written music later — I would not have started thinking about keyboard harmony in the way I did. It forced to my attention the large gap between simplicity on the keyboard and complexity of the full notation that represents it.

I wore out the patience of many music professionals by bombarding them with unsolicited email requests for comments on my ideas. I would like to thank four in particular, who politely responded to numerous emails in spite of being uncomfortable with my ideas: Harry Likas, Dmitri Tymoczko, Jeff Brent and Daniel Glover. Their criticisms helped me change my explanations without changing my mind.

Thanks to friends Marva Black, Mike Budde, Peter Marchant and Selinda Spugies for helpful insights on aspects of this material that helped me to understand how to get the ideas across better. Thanks to singer extraordinaire and friend Lorna Kollmeyer for helping me figure out how to organize piano accompaniment in guitar-strumming style.

My wife Sheila's accurate musical ear helps me know what does and does not sound good on the piano, in addition to making my life generally interesting. My grandsons Joshua and Ethan Feiber provided encouragement and comments; Joshua set up the website www.pianotheoryman.com as birthday gift; Ethan, who learned very young to play the piano impressively well by ear, has experimented with the notation and found it helpful for approaching written music.

SOME COMMENTS FROM READERS

The following comments provide a kind of history of the development of PKP (the dates on the left identify when the commenters read different drafts of this material). The unconventionality of putting tritones on center stage has tended to get in the way of expert musicians accepting the ideas at all, let alone seeing them as simple, which has not been helped by the fact that it took me a long time to find the simple way of explaining the ideas that appears in this document. I continue to think the ideas are important for learning the piano in a less restrictive manner than is conventional but have not yet been able to find an audience. At the very least, the more recent comments show that the PKP method is more than just a fantasy of an over-enthusiastic amateur.

(2016) Taylor Eigsti more recently than below. "... a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

(2011) Paul Steinbeck. Assistant Professor of Music Theory; Washington University, St. Louis

"The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music."

(2009) Robert Rawlins. University Music Department Chair (Rowan University); jazz musician; teacher; author of several books on jazz

"I became aware of Raymond Buhr's novel method for analyzing and voicing chromatic chord progressions in 2008 through a draft of a paper he wrote on the subject. I have kept up to date on developments of the method and we have had many email exchanges discussing issues of interpretation and application. I am a member of his intended target audience—a jazz musician who is not a pianist who needs to work out harmonic patterns on the piano from time to time. I am also a music teacher who has actually tried out aspects of his method on students. I can vouch from personal experience for his method's helpfulness in dealing with complex chromatic chord progressions."

(2009) Susan Brand. Board Chair, The Jazz School in Berkeley; amateur pianist

"When Raymond Buhr consulted me about his theory of chromatic chord progressions, I was immediately struck by his ability to analyze and attempt to simplify this complex subject. Mr. Buhr brings a unique perspective and a great deal of enthusiasm, depth of understanding and originality of viewpoint to the subject. Over the years I have watched the continuous work that he has put into editing and rethinking his work. He has had ongoing consultations with many knowledgeable musicians/teachers/editors and all have contributed to the development of the method described in this book. His ideas offer a way of understanding musical theory that will add greatly to the field and will be extremely helpful to musicians and music educators."

(2008) Taylor Eigsti. Jazz pianist, composer, teacher, former Artist in Residence at The Jazz School in Berkeley

"Through the brilliant lens of an engineer, Raymond Buhr has laid out an analysis of harmony that is a unique and complex look at the right-brain from the left-brain's perspective."

APPENDIX A: UNCONVENTIONAL ELEMENTS

TERMINOLOGY & NOTATION

- **anchor:** identifies a building block by the position of its bottom end relative to the home tonic
- **anchor set:** set of tritone and fifa anchors that define a scale by construction
- **anchor line:** anchor sequence written above the staff (outlined by circles for tritones & boxes for fifas)
- **alphabet: PADMIL** identifies anchors by the first letters of the names of classical modes
- **building block:** tritones or fifas (fifths or fourths) anchored at alphabet positions (size distinctions determined by color coding).
- **chromatic scale: 1-p2-2-p3-3-4-p5-5-p6-6-p7-7-1**
- **context:** provided by mode signatures for melody and harmony plus flow of both
- **core:** sequence of building blocks of harmony identified by an anchor line
- **family:** a set of sub-scales of a family scale defined by a single mode signature
- **fifa:** fifth or fourth that are opposite inversions (add up to an octave)
- **flow:** formed by morphs and slides of building blocks
- **frame:** defined by the tonic @ and pitch center \$ of a tonic octave (plus sometimes by tonality)
- **mode signature:** word identifying a tritone cluster and implying an anchor set via the scale dictionary
- **pattern:** organized arrangement of intervals on the keyboard or over time
- **pitch center:** note identified by a fifth/fourth octave split, symbolized by \$
- **morph:** small change in the size of a building block while holding one end fixed
- **outside:** not in a given tonic scale (as distinct from “chromatic” meaning not in a key-signature scale)
- **shape:** combination of building blocks
- **slide:** size-preserving movement of a building block
 - **wobbly slide:** combined morph and slide
- **phlat:** prefix **p** identifying chromatic-scale notes in the whole tone gaps of the major scale of a tonic
- **tonic pointer:** suffix of form @**t** attached to an anchor symbol to indicate a secondary tonic
- **word:** set of alphabet letters with optional dots indicating skipped letters

ABOUT THE ALPHABET

An effort must be made to avoid confusing letters of the **PADMIL** alphabet with other uses of the same letters in music theory and notation (**A**, **D** and **I** are particularly troublesome). In principle, any six letters would do for the alphabet, but the connection made to classical modes by this alphabet is too useful to discard. I thought of substituting the Greek letters epsilon (€) and delta (Δ) for **A** and **D** and the English letter **Y** for **I**, to yield **PEΔMYL**, pronounced almost the same and therefore easy to remember as having the same meaning (epsilon represents “Ae” of Aeolian). However, the mix of different alphabets introduces a different kind of confusion. I also thought of **PQRMYL** as an arbitrary way of getting rid of **A**, **D** and **I**. Or even **UVWXYZ**, which has the undesirable side effect of also getting rid of useful **P**, **M** and **L**. I have not been able to think of anything better than **PADMIL**.

APPENDIX B: ABOUT SCALES

KEY-SIGNATURE SCALES

The following summary of the standard interpretation of the major and natural minor tonic scales of key signatures (Ionian and Aeolian modes) is a helpful reference. It shows the sharped notes of sharp scales in red and the flatted notes of flat scales in blue. Although there are only five black piano keys, scales with six flats or sharps exist because of the scale-spelling rule that the same letter note cannot appear on both sides of a half-tone interval. For example, note B is *Cb* in a 6-flats scale.

Major (Ionian) and relative minor (Aeolian) scales are shown on the same line because they have the same key signature. Parallel minor scales (diagonal lines) have the same tonic, which does not necessarily mean the same note symbol because the same piano key may be represented by different note symbols (e.g., the parallel *Db* major and *C#* sharp minor scales, which have the same black-key tonic on the piano, have different note symbols for it).

	<u>Ionian</u>	<u>rel. Aeolian</u>	<u>key sig.</u>	<u>Ionian scale</u>	
	C	A	empty	C-D-E-F-G-A-B-C	
	F	D	1 <i>b</i>	F-G-A- <i>Bb</i> -C-D-E-F	
	<i>Bb</i>	G	2 <i>b</i>	<i>Bb</i> -C-D- <i>Eb</i> -F-G-A- <i>Bb</i>	
	<i>Eb</i>	C	3 <i>b</i>	<i>Eb</i> -F-G- <i>Ab</i> - <i>Bb</i> -C-D- <i>Eb</i>	
	<i>Ab</i>	F	4 <i>b</i>	<i>Ab</i> - <i>Bb</i> -C- <i>Db</i> - <i>Eb</i> -F-G- <i>Ab</i>	
	<i>Db</i>	<i>Bb</i>	5 <i>b</i>	<i>Db</i> - <i>Eb</i> -F- <i>Gb</i> - <i>Ab</i> - <i>Bb</i> -C- <i>Db</i>	
	<i>Gb</i>	<i>Eb</i>	6 <i>b</i>	<i>Gb</i> - <i>Ab</i> - <i>Bb</i> - <i>Cb</i> - <i>Db</i> - <i>Eb</i> -F- <i>Gb</i>	
	<hr/>				
	F#	D#	6 #	<i>F#</i> - <i>G#</i> - <i>A#</i> -B-C- <i>D#</i> - <i>E#</i> - <i>F#</i>	
	B	G#	5 #	B- <i>C#</i> - <i>D#</i> -E- <i>F#</i> - <i>G#</i> - <i>A#</i> -B	
	E	C#	4 #	E- <i>F#</i> - <i>G#</i> -A-B- <i>C#</i> - <i>D#</i> -E	
	A	F#	3 #	A-B- <i>C#</i> -D-E- <i>F#</i> - <i>G#</i> -A	
	D	B	2 #	D-E- <i>F#</i> -G-A-B- <i>C#</i> -D	
	G	E	1 #	G-A-B-C-D-E- <i>F#</i> -G	
	C	A	empty	C-D-E-F-G-A-B-C	
	<hr/>				
					switch from flat to sharp keys

Ionian tonics going down by fifths

parallel (same tonic) Ionian & Aeolian modes

--- same tonic symbols

— different tonic symbols

Avoiding multiple written key signatures by using accidentals to identify implicit key changes replaces one kind of complexity by another because accidentals have many more uses than this (e.g., ornamental passing notes, or identifying scales that originate in no key signature). Sorting out the possibilities requires considerable head-scratching at a very low conceptual level.

Key-signature-based music notation has stood the test of time and is here to stay because of the huge legacy of music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation.

CROSS REFERENCES

Cross-referencing music notation and PKP notation requires the following table. This is the simplest case. More generally, naturals may enter the note-symbol picture to cancel sharps or flats of key signatures, and white piano keys may be represented by sharps or flats (B=C*b*, C=B*#*, F=E*#*, E=F*b*) to satisfy the scale-spelling rule that the same letter note must not appear twice. Experts develop rules of

thumb about sharps and flats as indicators of what's going on, and see PKP as discarding these clues — not so, the clues are present in PKP in a different way.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7
@	P	A	D	M	I	L	\$	x	x	x	x
C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B
C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C
D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db
D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D
E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb
F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E
F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F
G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb
G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G
A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab
A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A
B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb

SCALES AS INTERVAL STACKS

The representation of scales as interval stacks is an adaption of a notation called “Figured Bass Notation” (Wikipedia) for identifying harmony by annotating bass notes on a staff with number stacks that represent counts of scale steps going up from the bass notes. The adaption replaces counts of scale steps with counts of half tones, and lists the stacks horizontally instead of vertically. For representing scales, the numbers are restricted to **1** (half tone), **2** (whole tone) and **3** (minor third). Any scale may be spelled out as a stack of these numbers that adds up to twelve. For example, the Ionian mode is identified by **| 2212 | 221 |** and the Aeolian mode by **| 2122 | 122 |**, where the vertical lines indicate the scale frame. The Ionian stack also identifies inter-note interval sequence of the singer’s solfege scale **do-re-mi-fa-so-la-ti-do**. The interval stacks are the same for parallel or relative modes, which aids clear thinking about the nature of modes and the relationships between them. For example, the interval stack of the Aeolian mode is visibly a rotation of that of the Ionian mode.

In cases where the numbers may be confused with other numbers (e.g., degree numbers), **1**, **2** and **3** may be replaced by **h**, **w** and **w+**.

APPENDIX C: ABOUT CHORDS

Chords identified by chord symbols with four or more notes boil down to combinations of the two kinds of building blocks. Here follow some examples. Two fifos (fifths or fourths) form major-7, minor-7 or major-6 chords. One fifo and one tritone form dominant-7, half-diminished-7 (a.k.a. minor-7- \flat 5) or minor-6 chords. Three fifos or two fifos and a tritone form 9th, 11th and 13th extensions of these chords. When tonic scales depart from key signatures, the same kinds of building blocks are available from the scales but now more than one tritone is available. For example, two tritones form diminished-7 or dominant-7- \flat 5 chords. Two tritones and one fifo (that may share a note with a tritone) form variations of other chord types such as dominant-7 \flat 9. Omitting inner notes of combinations may yield different intervals, or thinned voicings of chords. For example, an augmented fifth (8 half tones) is combination of two tritones a whole tone apart with their inner notes omitted; and a 3-note voicing of a 4-note seventh chord is composed of the outer notes plus the most important inner note.

ANCHOR LETTERS FROM TRITONE CHORDS

The following table of tritone chords summarizes the simplest examples of chords with tritones in different positions.

offset of tritone bass note above root R	suffixes on root symbol R										
	7	7#9(13)	9 ♭ 13 (or #5)	7 ♭ 9	7 ♭ 5 (or #4 or #11)	dim7 (or o7)	m7 ♭ 5	m6	M7(11)	♭ 9	M7#11 (or ♭ 5)
fourth									X		
major third	X	X	X	X	X						
minor third		X				X		X			
whole tone			X								
half tone				X						X	
0					X	X	X				X

Only chord variations that alter tritone content are included. For example, R7, R9, R7 \sharp 9, R13 and R9(13) are all variations of R7 with the same tritone content and so are all represented in the table by R7 (variations are left to context). Diminished seventh chords (Rdim7) have no counterpart in classical modes. Sus chords are not shown because they have no tritones. They are typically V-7 \sharp 3 chords. A Vsus-V-I progression is a substitute for a II-V-I progression in which one note is altered between the first two steps.

Inverting the tritone of a chord into the home octave (if it isn't already there) determines its PKP anchor. For example, The tritone of a V-7 chord is partially outside the home octave but its inversion is within it. The bass note of its inversion is a fourth above the home tonic which makes its anchor **I**.

Double tritones are the normal upper limit for tritone content of chords encountered above the staff in fake books, lead sheets and sheet music.

MISLEADINGLY COMPLEX CHORD SYMBOLS

A startling variety of chords are voiced by simple shapes formed of a tritone combined with different fifos, as illustrated next for the Ionian mode (some are rootless, all are in the mode).

1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
@	P	A	D	M	I	L	\$	x	x	x	x	@
.	x	x	.
.	.	x	x
I		II					V					
V7, II m 7(13), IM9(11),												
.	x	x	.
.	x	x	.
					IV		V					
V9(13), IV M 7b5												
.	x	x	.
x	x
		II					V					
V7(11), II m 7(13)												
.	x	x	.
x
I							V					
V13(11), IM7b5												
.	x	x	.
.	.	x
		II			IV					VII		
VII m 7b5, IV M 6b5, II m 6												

Non-classical modes provide many new chords. Here are some examples.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	A	D	M	I	L	\$	x	x	x	x	@
minor family	@	.	x	x	.	x	.	\$	x	x	x	x	@
AD	.	.	A	D	x	x	.	.	.
D.I	.	.	.	D	.	I	x	x	.
A..I	.	.	A	.	.	I	.	.	x	.	.	x	.
A°	.	.	A	.	.	x	.	.	x
3D+	x	.	.	D	x	.
4\$.	.	.	x	.	.	.	\$	@
no simple conventional symbol IV7b5, VII7b5 II _{dim} 7, IV _{dim} 7, pVI _{dim} 7, VII _{dim} 7 rootless 7b9 rel. to roots a 1/2 tone down II _{dim} ImM7, pIIIM ⁺ Im (minor triad @▼, spread out)													
major family	@	.	x	.	x	x	.	\$	x	x	x	x	@
MI	M	I	x	x	.
A.M	.	.	A	.	M	x	.	x	.
A..I	.	.	A	.	.	I	.	.	.	x	.	x	.
3\$	x	.	.	\$	@
no simple conventional symbol III7b5, pVII7b5 as above I (major triad, spread out)													
blues family	@	.	x	x	x	x	x	\$.	x	x	.	@
DM	.	.	.	D	M	x	x	.	.
M.L	M	.	L	.	.	.	x	x	.
D..L	.	.	.	D	.	.	L	.	.	.	x	x	.
no simple conventional symbol pV7b5, I7b5 analogous to A..I													

FIGURED BASS NOTATION; EXTENDED CHORDS

Figured bass notation provides a simple representation of chords from scales with 7 notes and no adjacent half tones. It represents chords by stacks of numbers going up from a bass note, in which each number is a count of the scale steps to the next note up. The stacks are annotated on a staff as literal stacks of numbers (vertical lists). I write them here as horizontal lists separated by plus signs. For classical modes, a major or minor third is always two scale steps and whole tone is always one scale step (e.g., a seventh chord is **2+2+2** and a sixth chord is **2+2+1**).

Figured bass notation can be helpful for understanding extended chords of classical modes that follow from splitting chord steps of size **2** into **1+1** sequences and then moving the added note up an octave. The **+4** on top of two of the extended chords on the right is a consequence of avoiding **1+1+1** sequences in the in-place forms. The corresponding chord symbols have degree-number suffixes 6, 9, 11 or 13 added to basic symbols to identify the extensions.

<u>chord type</u>	<u>(2) = split</u>	<u>in place</u>	<u>extended</u>
seventh	2+2+2	2+2+2	-
ninth	(2)+2+2	1+1+2+2	2+2+2+2
eleventh (seventh+11th)	2+(2)+2	2+1+1+2	2+2+2+2+4
thirteenth (ninth+13th)	(2)+2+(2)	1+1+2+1+1	2+2+2+2+2+4

APPENDIX D: ABOUT PARALLEL MODES

MODES FROM MODALOGY

The non-classical parallel modes as presented in the book *Modality* are summarized next, except with notes in numeric-chromatic-scale notation, instead of in the RN (Roman Numeral) notation with sharps and flats used in *Modality*. Corresponding PKP mode signatures are shown on the left.

The purpose is twofold. One is to verify the PKP view of modes relative to *Modality*. The other is to illustrate the complexity of the conventional representations. For example, the tritone anchored by **D** that is a component of many of the minor modes is understood as **p3-6** in the symbolic chromatic scale, and that's it. Inversions in different places in harmony are left to context. In *Modality*, tritone anchor **p3** is \flat III or \sharp II and tritone anchor **6** is VI or \flat VII, and that's only the anchors.

The alphabet soup of names and the intricate details of the scales makes them very difficult to distinguish functionally, or to remember as enumerated scales. Seeing the possibility that tritones might be fundamental scale-defining objects for these modes is effectively impossible in these terms. This impossibility is demonstrated by a discussion in *Modality* of defining and non-defining notes of the many and various modes that never mentions tritones.

The simple, unique mode signatures of PKP are novel.

Parallel Modes of the Melodic Minor | whWW | wWh |

alt-LA	1-2-p3-4-p5-p6-p7-1	Aeolian Diminished (Aeolian- ♭ 5)
LA	1-2-3-p5-p6-6-7-1	Lydian Augmented (Lydian-#5)
alt-ML	1-p2-p3-3-p5-p6-p7-1	Jazz Altered (Locrian- ♭ 4)
ML	1-2-3-p5-5-6-p7-1	Lydian Dominant (Lydian- ♭ 7)
DI	1-2-p3-4-5-6-7-1	Melodic, or Jazz, Minor (Ionian- ♭ 3)
AM	1-2-3-4-5-p6-p7-1	Jazz Mixolydian (Mixolydian- ♭ 6)
PD	1-p2-p3-4-5-6-p7-1	Jazz Phrygian (Phrygian-#6)

Parallel Modes of the Harmonic Minor | whWW | hW+h |

DL	1-2-p3-p5-5-6-p7-1	Romanian, Dorian #4, Mishebarakh
alt-DL	1-p2-p3-3-p5-p6-6-1	Leading Tone Minor Diminished, Super Locrian ♭7
LD	1-p3-3-p5-5-6-7-1	Lydian Blues Major, Lydian #2
alt-LD	1-p2-p3-4-p5-6-p7-1	Jazz Phrygian Diminished
AI	1-2-p3-4-5-p6-7-1	Harmonic Minor, Aeolian ♮ 7, Jazz Minor ♭ 6, Mohammedan
IA	1-2-3-4-p6-6-7-1	Ionian Augmented
PM	1-p2-3-4-5-p6-p7-1	Phrygian Dominant

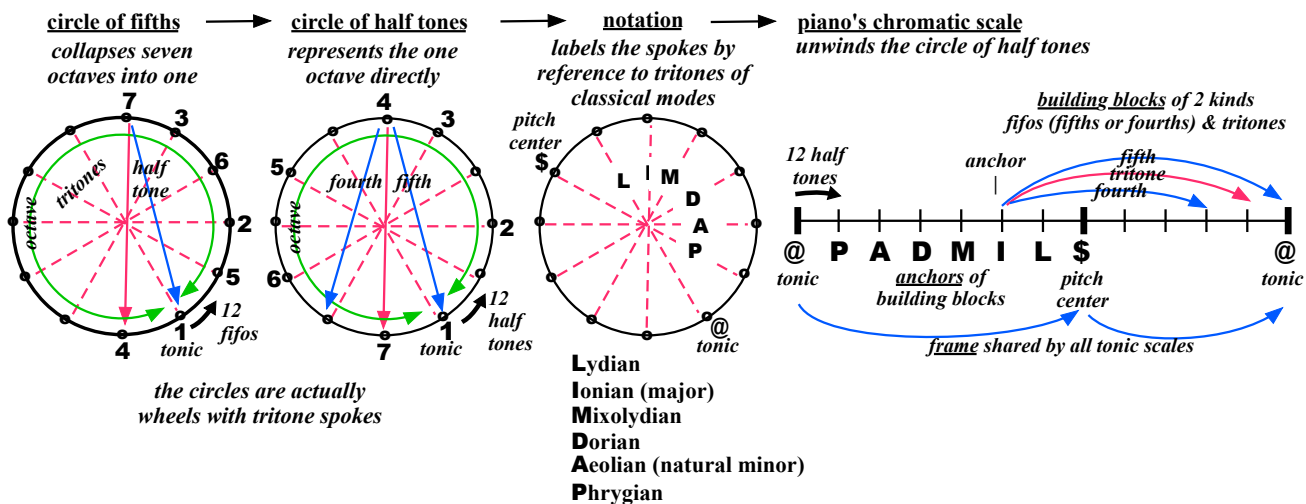
Parallel Modes of the Harmonic Major | wwHw | hW+h |

DL	1-p3-3-p5-p6-6-7-1	Lydian Blues Augmented, Lydian Augmented #2
alt-DL	1-2-p3-4-p5-6-p7-1	Jazz Minor #4, Lydian Diminished
LD	1-2-p3-p5-5-6-7-1	Lydian Melodic Minor, Lydian ♭ 3
alt-LD	1-p2-p3-4-p5-p6-6-1	Leading Tone Major Diminished, Locrian ♭7, Locrian Diminished-7
AI	1-2-3-4-5-p6-7-1	Harmonic Major
PM	1-p2-3-4-5-6-p7-1	Jazz Phrygian Dominant, Mixolydian ♭ 2
MP	1-p2-p3-3-5-p6-p7-1	Altered Phrygian Dominant, Phrygian ♭ 4, Superlocrian ♮ 5, Superphrygian

APPENDIX E: ABOUT SYMMETRY & SYMMETRY-BREAKING

The elements of PKP are summarized here in a way that provides a link between the concept of symmetry breaking in these pages and in the book *The Jazz of Physics*.

The circle of fifths shown below left presents the basic elements of tonic scales and chord progressions from them in one simple picture. The points going counterclockwise around the circle are 12 notes a fifth apart (7 half tones) across 7 keyboard octaves (12 fifths x 7 half tones = 7 octaves x 12 half tones). The circle is manifestly a wheel with tritone spokes. The notes of the major scale are the numbered points around the half circle delimited by the highlighted tritone spoke (6 half tones).



The circle visibly establishes two kinds of Lego™-like **building blocks** as fundamental elements of music, namely fifths or fourths referred to jointly as **fifos** for simplicity, and tritones. The circle as labeled identifies fifths going up the keyboard through 7 octaves or fourths going down the keyboard through 5 octaves. For the major scale, it identifies a basic chord root line going down by five fifths (3½ octaves) or five fourths (2½ octaves) to the tonic (equivalent to progressively zig-zagging down a fifth and up a fourth within one octave).

The pictorial geometry of the circle of fifths and the musical geometry of its interpretation are different in a way that is counterintuitive (different numbers of octaves going around the circle for fifths and fourths, very different musical sizes of lines of almost the same pictorial length that cut across the circle). This becomes confusing for more general chord progressions of this and other scales.

The circle of half tones rearranges the spokes of the circle to put the labeled points around it in scale order. The result is an intuitively natural **@1** between pictorial geometry and musical geometry. Around the circle remains an octave and across any spoke remains a tritone. The half tones across the circle now go around it and the fifos around the circle now go across it.

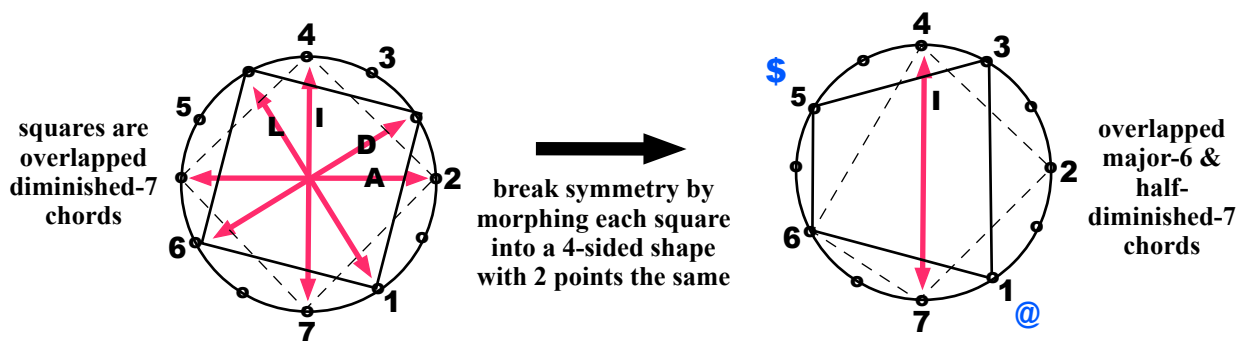
The end points of all the spokes of either circle visibly identify all the points around it, suggesting a notation that labels spokes (building blocks) instead of points (notes). This simple observation is the twist that leads to PKP's unique combination of simplicity and depth. The spoke labels, called **anchors**, directly identify tritones but also identify fifos. In the chromatic scale that unwinds from the circle, each anchor letter identifies a tritone and two fifos morphed from it. The letters identify relative positions of the anchors in the chromatic scale, and outlining them differently in annotations above the

staff distinguishes the different kinds of building blocks (circles for tritones and squares for fifos, e.g., ① and □). Tritone anchors are unambiguous independently of context and fifo anchors are ambiguous but the ambiguity is generally resolved by context. No anchors are needed in the top pitch half of the octave because inversions cover them.

The letters of the **PADMIL** alphabet mark the relative positions of the anchors of the single tritones of classical modes, but this is only to provide a correspondence between PKP notation and music notation. It does not bind the letters to the classical modes, or to tritones. The letters are in a special boldfaced font to distinguish them from other uses of some of them in music notation/theory (there is no actual possibility of confusion because the different notations are never mixed together).

SYMMETRY BREAKING IN THE CIRCLE OF FIFTHS

According to *The Jazz of Physics*, symmetry-breaking is a deep feature of how both music and the universe work. Here's a view of breaking the symmetry of a diminished scale to yield a major scale that corresponds to an example on page 66 of the book (in the book, the dotted square and its transformation are not shown and no mention is made of tritones). This kind of thing is fascinating to anyone with a mathematical bent but probably not anyone else.



The symmetry-breaking is shown next in progressive steps that morph tritones into fifos.

