Learning Targets:

- Write functions that describe the relationship between two quantities.
- Explore the composition of two functions through a real-world scenario.

SUGGESTED LEARNING STRATEGIES: Create Representations, Identify a Subtask, Group Presentation, Graphic Organizer, Debriefing, Self Revision/Peer Revision

Recall that Jim has a lawn service called Green's Grass Guaranteed. On every mowing job, Jim charges a fixed $30 fee to cover equipment and travel expenses plus a $20 per hour labor charge.

1. On a recent mowing job, Jim worked for 6 hours. What was the total charge for this job?

2. **Model with mathematics.** If Jim works for t hours, what will he charge for a mowing job? Write your answer as a cost function where $c(t)$ is Jim's charge for t hours of work.

It takes Jim 4 hours to mow 1 acre. Jim prepares a cost estimate for each customer based on the size (number of acres) of the property.

3. The APCON company is one of Jim's customers. APCON has 2 acres that need mowing. How many hours does that job take?

4. Another customer has a acres of property. Write the equation of a function in terms of a for the number of hours t it will take Jim to mow the property.

5. How much will Jim charge APCON to mow its property? Justify your answer.

My Notes

My Notes

MATH TIP

In a linear function $f(x) = mx + b$, the y-intercept is b. The variable m is the rate of change in the values of the function—the change of units of $f(x)$ per change of unit of x. When the function is graphed, the rate of change is interpreted as the slope. So $y = mx + b$ is called the slope-intercept form of a linear equation.

The functions in Items 2 and 4 relate three quantities that vary, based on the needs of Jim's customers:

- The size in acres a of the property
- The time in hours t needed to perform the work
- The cost in dollars c of doing the work.

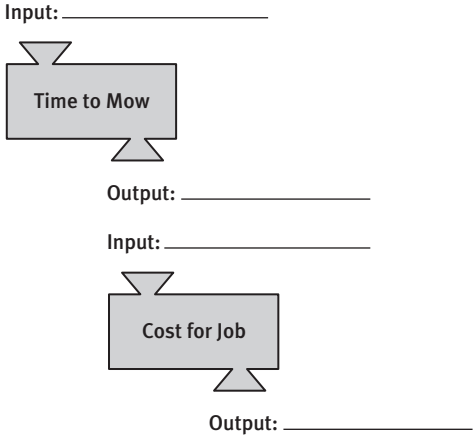
6. Attend to precision. Complete the table below by writing the rate of change with units and finding the slope of the graph of the function.

Function	Rate of Change (with units)	Slope
$c(t) =$		
$t(a) =$		

7. Complete the table below by naming the measurement units for the domain and range of each function.

Function Notation	Description of Function	Domain (units)	Range (units)
$c(t)$	cost for job		
$t(a)$	time to mow		

8. Calculating the cost to mow a lawn is a two-step process. Complete the graphic organizer below by describing the input and output, including units, for each part of the process.

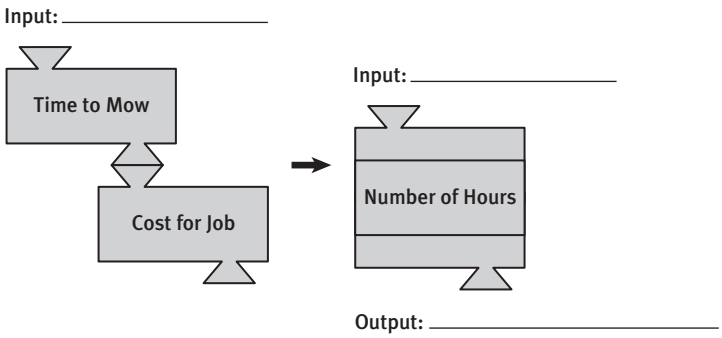


My Notes

The graphic organizer shows an operation on two functions, called a ***composition***. The function that results from using the output of the first function as the input for the second function is a ***composite function***.

In this context, the composite function is formed by the time-to-mow function and the cost-for-job function. Its domain is the input for the time function, and its range is the output from the cost function.

9. Make sense of problems. The cost to mow is a composite function. Describe its input and output as you did in Item 8.



When a composite function is formed, the function is often named to show the functions used to create it. The cost-to-mow function, $c(t(a))$, is composed of the cost-for-job and the time-to-mow functions.

The $c(t(a))$ notation implies that a was assigned a value $t(a)$ by the time-to-mow function. Then the resulting $t(a)$ value was assigned a value $c(t(a))$ by the cost-to-mow function.

10. Complete the table by writing a description for the composite function $c(t(a))$. Then name the measurement units of the domain and range.

Function Notation	Description of Function	
$c(t(a))$		
	Domain (units)	**Range (units)**

MATH TERMS

A **composition** is an operation on two functions that forms a new function. To form the new function, the rule for the first function is used as the input for the second function.

A **composite function** is the function that results from the composition of two functions. The range of the first function becomes the domain for the second function.

My Notes

Jim wants to write one cost function for mowing a acres of property. To write the cost c as a function of a acres of property, he substitutes $t(a)$ into the cost function and simplifies.

$c(t)$	$= c(t(a))$	Substitute $t(a)$ for t in the cost function.
$c(t(a))$	$= c(4a)$	$t(a) = 4a$, so write the function in terms of a.
	$= 30 + 20(4a)$	Substitute $4a$ for t in the original $c(t)$ function.
$c(t(a))$	$= 30 + 80a$	

11. Attend to precision. Write a sentence to explain what the expression $c(t(2))$ represents. Include appropriate units in your explanation.

12. Construct viable arguments. Why might Jim want a single function to determine the cost of a job when he knows the total number of acres?

13. Explain what the expression $c(t(50))$ represents. Include appropriate units in your explanation.

14. Explain what information the equation $c(t(a)) = 50$ represents. Include appropriate units in your explanation.

My Notes

Check Your Understanding

15. Given the functions $a(b) = b + 8$ and $b(c) = 5c$, write the equation for the composite function $a(b(c))$ and evaluate it for $c = -2$.

16. The first function used to form a composite function has a domain of all real numbers and a range of all real numbers greater than 0. What is the domain of the second function in the composite function? Explain.

17. The notation $f(g(x))$ represents a composite function. Explain what this notation indicates about the composite function.

LESSON 5-2 PRACTICE

Model with mathematics. Hannah's Housekeeping charges a $20 flat fee plus $12 an hour to clean a house.

18. Write a function $c(h)$ for the cost to clean a house for h hours.

19. What are the units of the domain and range of this function?

20. What is the slope of this function? Interpret the slope as a rate of change.

Hannah's Housekeeping can clean one room every half hour.

21. Write a function $h(r)$ for the hours needed to clean r rooms.

22. Write a function $c(h(r))$ to represent the cost of cleaning r rooms.

23. What is the value and meaning of $c(h(12))$?

24. **Look for and make use of structure.** Explain how a composition of functions forms a new function from the old (original) functions.