THE EMPEROR HAS NO CLOTHES:
MUSIC AS IT ACTUALLY IS ON THE PIANO KEYBOARD

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I approached the piano as an adult beginner interested in learning to play jazz, which eventually led me to wonder how jazz pianists can improvise in terms of a music notation that seemed to me to be misleadingly complex for the piano. In the words of the naive child in the fairy tale, "the emperor has no clothes," meaning the piano, seen directly without the obscuring "clothes" of music notation, is much simpler than music notation for it. Music notation is misleadingly simple for pieces in easy key signatures that don't depart from the key signature, and then gradually increases in complexity as pieces become more "interesting," to the point of becoming suddenly unapproachable by all but experts.[1] This is sad be because the complexity increase is purely notational. The piano is not just for experts, but this limits access to it by amateurs.

When I took up the piano as an adult beginner, I became, in effect, the naive child in the fairy tale who thought he saw things more simply. I had learned the interval-based "solfege" scale **`do-re-mi-fa-so-la-ti-do`** in primary school, and played trumpet in school bands as a youth — and so knew enough to be puzzled by the simplicity of the solfege notation relative to the complexity of formal music notation, but never had the time to give it much thought. After a period of piano lessons as an adult beginner, the increasing notational complexity of pieces I was encountering revived my puzzlement. When I retired soon after, I had the time to indulge my curiosity. This document is the result. It's a record of my personal journey to a "no clothes" view of piano music. It's limited in scope by the pieces I studied, mostly short pieces from "jazz and standards" or "classical" fake books. Outside its scope are virtuoso pieces from scores black with notes that only experts can attempt. That said, the pieces I studied are as harmonically sophisticated as is possible on the piano.

I "reverse engineered" pieces of music I was learning, to understand them in terms of **keyboard shapes**, which the piano is uniquely equipped to present to the eye in terms of intervals measured in half tones, the pitch intervals between adjacent piano keys. I learned that chord symbols, which should help with the complexity problem, are actually part of it.[2] Making them part of the solution requires seeing them as shapes formed from combining or splitting a small number of Lego-like **building blocks** that are the same shape anywhere on the keyboard. The result is a "no-clothes" notation I call **PKP**, standing for **P**icturing **K**eyboard **P**atterns. The basic elements of PKP are hidden in plain sight on the keyboard and in standard practice for voicing chord progressions of music notation. PKP bundles these elements into a lightweight notation that's both deep, and simple enough to be annotated on the

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[1] An example of misleading complexity is provided by one of the simplest and most distinctive changes in music to the ear, namely a change from major to minor tonality of the same tonic. The example is for the tonic provided by the black piano key immediately above C, variously known as C# or Db. The change is from 5-flats of Db major to 4-sharps of C# minor (Appendix B provides a summary of key-signature scales). The change naturalizes 5 notes and then sharps 4 notes — 9 symbol changes to move 3 notes down a half tone! The different symbols for the major tonic Db and the minor tonic C# seem, misleadingly, to imply slightly different pitches for the home tonic. What they actually imply is slightly different pitches for the notes C and D when used as references for sharps or flats. These pitch differences don't exist on the piano.

[2] The following chord progression for the haunting E*b* blues *Goodbye Porkpie Hat* written by Mingus as a tribute to Lester Young is a "poster child" for misleading chord complexity. The progression is E*b*7#9—B9(13)—EM9—A7#11—D*b*9sus—B9(13)—D*b*7sus—E*b*7—A*b*m11—B7(13)—Fm7*b*5—B*b*7#5#9—C13#11—F7(13)—B7—EM7—A7(13)—A*b*7—B*b*7—D*b*7—E*b*7♯9—B7—EM7#11—A7#11 (Appendix C provides a summary of chord symbols). There are 24 chords, an average of 2 per bar for 12 bars, only 5 of which (highlighted in blue) don't contain tritones. The tritone content is the basis of deep structure of startling simplicity that is the subject of this document (Chapter 4 presents this piece as an example).

written music. The notation can also be written down separately as a playable shorthand. PKP is not a replacement for music notation but a lightweight complement to it. Its unique combination of simplicity and depth are either a serendipitous side effect of the organization of the piano keyboard or a fundamental property of music that's obscured by music notation. Either way, it provides contextual cues to help in learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. This turns the conventional relationship between practicing and understanding on its head: understanding guides practicing instead of only emerging from it.

PKP IN A NUTSHELL

A 6-letter, DNA-like **alphabet** identifies the positions and sizes of building blocks. Letters identify single building blocks and words identify shapes that form scales and chords. The shapes are determined by a two-part, **universal home octave** that can be positioned anywhere on the keyboard by assigning a piano key to the **home tonic**. Everything else is position-independent. The two parts are the alphabet, just identified, and a chromatic scale aligned with it that's a simple generalization of the solfege **do-re-mi** scale, except with different symbols. Pieces of tonal music are understood in terms of **parallel modes** of the home tonic. This is in contrast to the key signatures of music notation that identify relative modes (same notes) and put parallel modes (same tonic) in different key signatures. This difference is the source of PKPs unique combination of simplicity and depth.

PKP and music notation provide **dual views** of music based on intervals vs. notes. Duality is a well known way of dealing with complexity in math and physics, so its helpfulness for dealing with the complexity of music notation should not come as a surprise. The reason it's helpful is it introduces simple **symmetric** shapes that provide **constructors** for scales and chords. The most surprising aspect of PKP is the central role played by symmetries based on tritones . Tritones are building blocks of size half a keyboard octave (6 half tones). No one savvy in music notation would suspect them to be fundamental because anything involving more than one tritone is complex in music notation. The other building blocks are fifths and fourths a half tone larger or smaller than tritones. Fifths and fourths are the same kind of building block (consonant, invert into each other) warranting a new term for the kind, namely **fifo** (standing for fifth or fourth). The small difference in size between tritones and fifos (a half tone) and the large difference in sound (dissonance vs. consonance) makes these building blocks fundamental to both the structure and the sound of music.

The gist is this: **Tritones are simple. Fifos are understood as morphed tritones (the reason for the new term). Shapes formed by combining or splitting tritones and fifos are the fundamental shapes of music, namely scales and chords. Two kinds of context are provided, namely mode context determined by mode signatures expressed in alphabet words, and flow context determined by the flow of building blocks when the music is played. This joint context guides the completion of core harmony by selected fifos that fit both the flow and the mode (always fifos because all the tritones are already included in the mode signature and the harmony).** There are nuances but this is the substance. Intervals larger or smaller than building blocks mostly emerge as outer or inner intervals of shapes, but their occasional appearance as building-block-like objects is covered in simple way, without adding to the alphabet, by representing them as expanded or shrunk building blocks. Melody notes or harmony building blocks outside the scale of a definitive mode signature may appear ornamentally.

The practical difference is seeing harmony in terms of the smooth flow of building blocks anchored

by alphabet letters within the home octave, instead of the often jumpy root line of chords that must be understood relative to constantly changing roots. Successive building blocks slide and morph into each other by small intervals. The flow is simple because the building blocks are almost the same size, but the sound can be sophisticated because small differences in size create large sound differences: a change in size of half tone may turn dissonance (a tritone) into consonance (a fifth or fourth), or vice versa. There is no conflict between these dual views, which are complementary, and easily transformed into each other. The thing about the interval-based view is its independence of keyboard position, in other words of the actual notes being played. The solfege **`do-re-mi`** scale has this independence and PKP's harmony notation builds on it. **The result is a new way of understanding and creating tonal music.**

Why go to all this trouble when the same chord voicings can be figured out in music notation? One answer is, chord symbols over-specify notes in tonal music. Tonal music provides context that chord symbols ignore. **Core harmony** is provided by the flow of core building blocks plus completion building blocks determined by context. Inversions or substitutions of building blocks provide substitute chords. Chords represented by chord symbols are interpreted results. Turning this around, think of chord symbols not as specifications but as menus of possible notes..

I had many questions about music notation but one stands out: Given that scales are determined by key signatures, how can altering a particular note by a half tone not only tell the ear that the scale has changed but also what the new scale is? PKP provide an answer that goes to the heart of the enterprise, explained in Chapter 2.

IF THESE IDEAS ARE SO GOOD, WHY ARE THEY UNKNOWN?

I have been asked: If the ideas are so good, why has no enterprising young pianist discovered them? The answer has several parts. So much time is required to master the piano as an instrument, and music notation as the representation of music for it, that any young person who aspires to mastery has no time for anything else. The deceptive simplicity of "starter" pieces in simple keys gives no warning of later complexity, enabling music notation to get a toehold in the mind as the only way of thinking about music, encouraged by piano teachers who have come to this way of thinking through long hours of practicing.

The thinking "outside the box" that produced PKP brought in many elements from outside music that I happened to know about in different ways. Training in math and physics provided the concepts of duality and symmetry. Research experience in software design — in my pre-retirement job as a professor of systems and computer engineering — provided experience in inventing simple notations to understand complex processes. Added to this are personal temperament and training that leads me to question conventional wisdom, a very strong sense of the existence of a useful dual view of piano music, and time made available by my then-recent retirement

In the process of developing these ideas, I searched for signs of them in the literature, and reached out to experts for the same thing, and found nothing equivalent. Closest were ideas about symmetry in the book *The Jazz of Physics*, and about chords as "scale shapes" learned from discussions with jazz pianist Taylor Eigsti.

My admitted lack of musical credentials is reasonable cause for skepticism that I have anything useful to say, so the opinions of PKP of a couple of experts are worth quoting before proceeding. Musical theorist Paul Steinbeck: "The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation.

This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music." Jazz pianist/composer/teacher Taylor Eigsti: "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

GUIDE TO READERS

The depth of the insights provided by PKP led me to think they might be of interest to others, such as newcomers to the piano who have begun to be daunted by the complexity of music notation, pop and jazz musicians who are not pianists but want to explore harmony on the piano, "wannabe" expert pianists who might appreciate a helpful notation that exposes fundamentals, and anyone with a stake in the piano and curiosity about these issues. Expert pianists are not in this list because they have already figured out personal ways of dealing with the complexity, to the extent that they probably don't even see it anymore; however they may be curious about a notation that makes these claims because it could be useful for teaching people who don't have the ambition or time to become expert.

Chapter 2 develops the concepts and notation. In it, appearances of new terms are boldfaced, and conventional terms from music theory are in quotation marks. Chapter 3 develops a mode hierarchy that provides the framework for understanding music in its terms, illustrated at every step by example pieces presented in the notation. Chapter 4 provides a smorgasbord of advanced pieces that I found difficult on first encounter (the first example is the piece of footnote 2 on the opening page). Chapter 5 provides observations and conclusions. References, acknowledgements and comments from some readers of earlier drafts follow. Appendices are about the relations between conventional representations and PKP, for terminology (A), scales (B), chords (C), parallel bridging modes (D), and hidden symmetries (E).

There's very little music notation in the form of notes on a staff in this document because the purpose is to see the emperor without the clothes, not to describe the clothes. PKP is intended to be used to annotate written music, but this is is helpful only if the annotations are understood in their own terms, independently of music notation. This is possible because the concepts are simple and are directly related to how the ears hear music. Interpretations in terms of music notation are results not a starting points.

The main elements of PKP are **melody lines** in chromatic scale notation (annotated on the staff or written separately), **harmony lines** in alphabet notation (annotated above the staff or written separately), and simple **mode tables** that provide a cross reference between the two. These are supplemented by Lego-like pictures of shapes formed from building blocks that can be directly understood in terms of music notation because the building blocks can be so understood.

No question, this is a different way of thinking that takes time and effort to internalize and to reconcile with conventional ways. It makes the piano more approachable for anyone who loves music in its full harmonic richness but finds music notation for it too complex. Don't be misled, by the many pictures of Lego-like stacks of building blocks in these pages, into thinking drawing such stacks is a necessary part of using PKP. It isn't: the stacks are only to illustrate how the notation maps to actual building blocks on the keyboard, seen in the mind's eye as determined by the home octave.

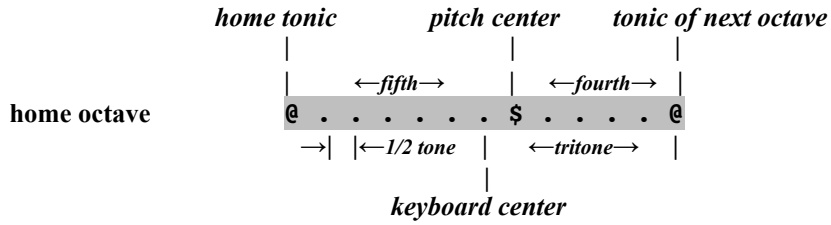
CHAPTER 2: CONCEPTS & NOTATION

PKP concepts and notation enable probing deep and complex waters in music notation without becoming overwhelmed by details. The first ten or so pages explain the fundamentals. The subsequent pages develop progressively more sophisticated example pieces that are easy to understand: *Happy Birthday to You* illustrates one classical mode (same key signature) used for everything. *Over the Rainbow* illustrates single-classical-mode melody with ornamental chromatic harmony that departs from the melody mode, bringing in all the tritones of the chromatic scale. *Here's That Rainy Day* inserts short chromatic melody/harmony segments into a single classical mode. *I Got Rhythm* illustrates cascaded, classical mode/tonic changes known in jazz as "Rhythm Changes" because of their use in this piece. *Giant Steps* pushes the complexity limits of these cascaded changes to an extreme that essentially breaks this way of looking at them for anyone but experts, but exposes a new kind of underlying simplicity. These examples build towards a simple, coherent view of classical and non-classical modes described in the next chapter.

STARTING FROM COLD

The central concept of PKP is a position-independent **universal home octave** for any piece of music. This conceptual octave has two parts: a 12-half-tone chromatic scale, and a DNA-like **alphabet** for representing **building blocks** from the scale.

Developing a simple notation for this concept requires starting cold from the simple but accurate conceptual representation of a home octave on the piano shown next, namely a line divided into 12 equal parts identifying 12 half tones.



The bottom **@** is the home tonic and the top **@** is at once the top note of the home octave and the tonic of the next octave up, which looks the same on the keyboard and is harmonically equivalent. Thus one conceptual home octave represents a stack of actual home octaves in a position-independent way. Bear with me while I now explain a few things about this simple picture that are fundamental to understanding PKP.

A half tone is the musical interval played by adjacent piano keys. Successive half tones that are the same linear size in the picture represent successively larger pitch intervals, which increase uniformly within the octave in such a way that the top pitch is double the bottom one. This makes the octave the most consonant interval in music, and therefore the most fundamental. The term "octave" refers to the number of intervals (8) in basic 7-note scales within it, which contain 5 whole tones and and 2 half tones arranged in such a way that the total scale interval is either a half tone or whole tone below the top of the octave. Either way, 8 intervals of the 12-half-tone octave provide 7 scale notes.

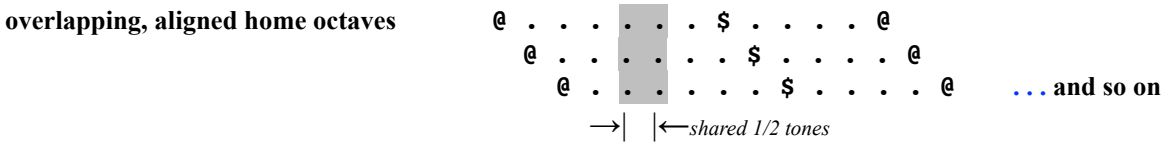
The picture splits an octave into two closely related kinds of **building blocks**. One kind is composed of fifths and their inversions, fourths. These are the most consonant intervals next to octaves, warranting the new term **fifo** (standing for fifth or fourth) as a building-block identifier. The other kind

is composed of **tritones** and their inversions, which are also tritones. These are the most dissonant intervals next to half tones. Without any additional information, these two types of intervals are, plausibly, fundamental to piano music — and not only to it but also to all music, given that the piano is accepted worldwide as a general purpose musical instrument. How could they not be fundamental? The octave is the most fundamental interval of music and these are the most fundamental splits of it.

The **fifo split** introduces the pitch center **$**, which forms a consonant **scale frame @-$-@** shared by most tonic scales. Scales without pitch centers exist but are simple derivatives of ones with pitch centers, warranting referring to the latter as **primary** (e.g., for those familiar with classical modes, the Lydian mode of a key signature is a primary mode and the Locrian mode is a derived mode of it). The **tritone split** introduces the geometric center of the 12-half-tone octave, which is the keyboard center of the piano octave.

The **building blocks** identified by these splits differ by only a half tone in keyboard size but are hugely different in musical sound, namely consonance vs. dissonance. The interplay between consonance and dissonance is fundamental to much harmony and these building blocks mediate the interplay, when you look closely at chord progressions and their voicings. The only music without dissonance — which means without half tones or tritones — originates in only two kinds of scales of which I am aware, namely 5-note scales called "pentatonic" (the scales of key signatures without their half tones and tritones), and a rare, 6-note, atonal scale called "augmented" that appears in Coltrane's famously difficult — but actually simple in PKP terms — jazz classic *Giant Steps*. covered at end of this chapter.

The piano's alignment of half tones of overlapping home octaves offset by half tones enables building blocks to be understood as universal components of keyboard shapes anywhere on the keyboard. This simple picture is the cornerstone of PKP's combination of simplicity and depth because it provides the basis for a position-independent representation of building blocks that's independent of both music notation and chord notation that goes with it.



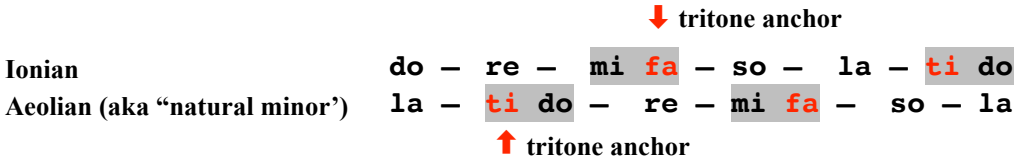
The sharing of half tones is a result of the piano not providing variable pitch piano keys to play overlapping octaves "perfectly," meaning with the same uniform increase of pitch sizes of half tones in each octave. The difference is minimized across all piano octaves by "equal temperament tuning" that makes scales within different octaves sound uniformly the same to the ear, except for overall pitch differences. The difference is not important to the ear for the half tones themselves, because they're strongly dissonant when their notes are sounded together, and human ears are insensitive to small errors in the pitch sizes of dissonant intervals. Larger intervals made up of these half tones must sound right in context to most ears, otherwise the piano would not be acceptable as a general purpose music instrument. Music notation doesn't actually help with finding "perfect" pitches independently of the piano because they must be learned by ear in any case (e.g., from solfege scales). This suggests that the piano's chromatic scale is as musically accurate as the scales of music notation, in the sense of identifying intervals measured in half tones that can be either played on the piano as given or learned

by ear independently of the piano.[3]

Building up from SolfegeScales

The singers' solfege scale **do-re-mi-fa-so-la-ti-do** is a natural starting point for developing PKP, independently of its original purpose of learning intervals by ear. It's a position-independent, primary major scale, based on intervals measured in half tones relative to a given starting pitch **do**. The scale has two half tones **mi-fa** and **ti-do** and otherwise is all whole tones. It's a primary scale because it includes the scale frame **do-so-do** Its tonality is major because **do-mi** is a major third. The scale is the master mode of a family of solfege modes determined by rotating it to start from different scale positions. For example, the rotation **la-ti-do-re-mi-fa-so-la** is a primary minor scale. It's primary because it includes the scale frame **la-mi-la**. Its tonality is minor because **la-do** is a minor third. For singers, the starting pitches of these scales can be anything and the remaining pitches follow by ear. For pianists, the scales follow from known keyboard intervals going up from designated piano keys.

These modes may be aligned on a shared tonic tonic **do** as shown next (whole tones are spaces with dashes, half tones are spaces without dashes). The names on the left are of the classical modes that provide the scales of the default major and minor modes of key signatures. The alignment makes the modes **parallel**, which means that in music notation they would have different key signatures, which is a complex way of representing the simple change from major to minor of the same tonic (recall footnote 1 of Chapter 1). The note symbols of these modes may be replaced by the symbols of a shared, 12-half-tone chromatic scale described later, but the concepts are clear without the distraction of unfamiliar scale symbols.



Four takeaways from this picture are independent of solfege notation and lead directly to PKP notation. 1) **Symmetric constructor shapes** (highlighted) determine the scales. The constructors consist of the mode tritone with two adjacent half tones, outside it for Ionian (**xx-xx**) and inside it for Aeolian (**xx-xx**). Red identifies tritone notes, which are always 6 half tones apart, enabling these constructors to be understood purely in terms of intervals measured in half tones, without any note symbols. 2) **Symmetry breaking** by whole tones creates the scales (only whole tones because the scale's two half tones are already in the constructors). 3) **Tritone anchors** — the nearest note of a tritone above the home tonic identified by a special anchor symbol that says "tritone" — are sufficient identifiers of tritones. Because tritones are the same size in either inversion, knowing a tritone anchor means knowing the tritone as a **building block** of fixed size that goes up from the anchor in the home octave but could go down from it in a piece of music. 4) A **tritone anchor and a tonic** identify a primary classical mode in interval terms as definitely as key signature and tonic do in note terms.

THE ALPHABET

The anchor alphabet is novel and so could be anything. I initially used the six first letters of the names of the classical modes that determine the scales of key signatures, listed in the unconventional

[3] See the books *How Equal Temperament Tuning Ruined Music* and *Lies My Music Teacher Told Me* for more on these matters.

order of their single tritones going up the keyboard, namely "P" for Phrygian, "A" for Aeolian, "D" for Dorian, "M" for Mixolydian, "I" for Ionian and "L" for Lydian/Locrian (one letter for both because they contain the same tritone). This unconventional order identifies tritones of parallel modes going up the keyboard by half tones (the conventional order is for relative modes that all contain the same tritone). The alphabet letters by themselves identify the tritones, not the modes — modes are identified by **signatures** formed of letters or words from the alphabet.

However, I had to partly abandon this obvious alphabet because three letters have other meanings in music notation: "A" and "D" are letter notes and "I" is a symbolic chord root. Some substitutions fix this: "O" replaces "A" of Aeolian, "R" replaces "D" of Dorian and "Y" replaces "I" of Ionian (think of "Ionian" as "eye-onian" and let "Y" stand for "eye"). The other letters still serve, so the alphabet becomes **PORMYL** pronounced "pormil" and shown in a distinctive font (Arial black) to identify the letters as anchor symbols (circling them in handwritten annotations accomplishes the same purpose). The anchor letters refer to classical modes but the anchored tritones exist independently of any mode. The other modes will emerge as we go along.

The anchors of the Aeolian and Ionian tritones identified by arrows in the earlier picture in solfege terms are the letters **O** for Aeolian and **Y** for Ionian. The combination of the alphabet and the scale frame forms an **alphabet octave** that may be written as **@PORMYL$xxxx@**. The upper fourth has no anchors because the tritones anchored in the lower fifth supply the notes. The pitch center **$** and the two tonic notes **@** are tritone completion notes of **P** and **L** but never tritone anchors. The only tritone anchors are the six alphabet letters.

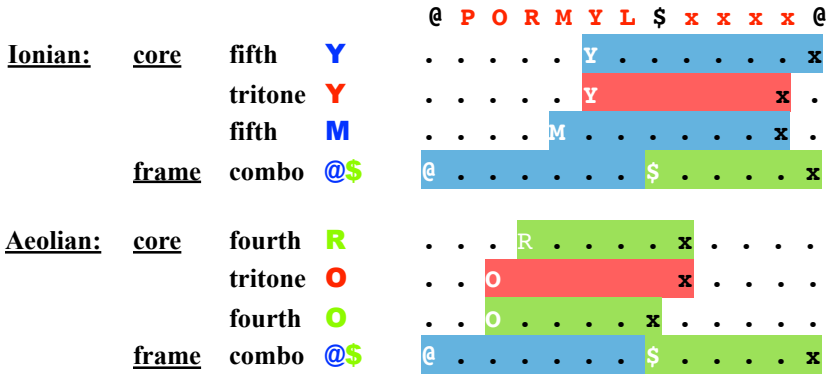
The parallel Ionian and Aeolian modes identified earlier in foregoing diagram may now be summarized without reference to note symbols of any kind, as follows. The header line of the table is in a different font (fixed width Courier) for formatting reasons.

		@ P O R M Y L $ x x x x @
Ionian:	tritone **Y**	. . . . . x . . . . . x .
	constructor	. . . . x x . . . . . x x
	mode	o . o . x x . o . o . x x
Aeolian:	tritone **O**	. . x . . . . . x . . . .
	constructor	. . x x . . . x x . . . .
	mode	o . x x . o . x x . o . o

The alphabet covers both kinds of building blocks, namely tritones and fifos, with the differences determined by color coding: red for tritones, blue for fifths, green for fourths. The main building blocks of these modes are shown next, in a notation that enables visualizing the building blocks in Lego-like terms. Anchors are shown in white text in the building blocks to make anchor lines stand out (e.g., **Y-Y-M-@**, **Y-Y-M-$**). Looking ahead, such anchor lines provide harmonic sequences, the formation of which is guided by both mode context and flow context. Hold that thought.

The frame identified by the 2-letter word **@$** is an example of a Lego-like "snapping together" of overlapped building blocks at half-tone points. The overlap here is by the half tone at the top of the fifth anchored by **@** and the bottom of the fourth anchored by **$**, requiring one color to overlap the other (green overlaps blue to identify the fourth as primary). The combination is an **octave stack** formed of

opposite inversions of building blocks. Other octave stacks and other overlaps of building blocks will come into play as we go along.



Inversions of building blocks are identified by underlined anchor letters. Inversions are the same color for tritones (O is an inversion of O) and opposite colors for fifos (O is an inversion of O, and O of O); the underlining is of the inversion, not the the original. The fifos of the scale frame are the only building blocks with two possible anchors, one for the fifo going up and the other for the one going down (e.g., @ and $ are the same fifth).

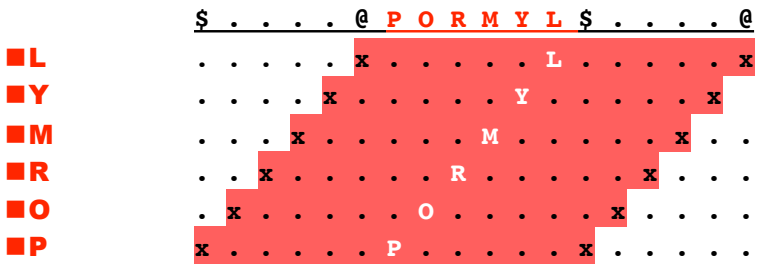
Octave stacks formed of opposite inversions of building blocks are useful objects for several reasons: they are among the simplest shapes on the keyboard, provide opposite inversions of building blocks for free, and are almost chords, and so provide starting points for forming chords. This warrants a special notation for them: an anchor symbol with a box suffix or prefix of a color that indicates the size of the octave completion interval. For example, the @$ octave stack may be notated as ■$ or @■, depending on which fifo is regarded as primary. This goes for any octave stack. For example, the tritone stack OO is ■O or O■, and the fifo stack OO is O■ or ■O.

It's worth pausing here to return to the importance of tritones and fifos as fundamental building blocks of music. They split the most fundamental interval of music, the octave symmetrically (tritones) and asymmetrically (fifos) — it's difficult to imagine that anything so structurally fundamental would not also be musically fundamental. The morphings of these building blocks into each other form a closed set that provides transitions between dissonance (tritones) and consonance (fifos) that are fundamental to the sound and emotional impact of music. The workhorse chords of harmony, namely triads, sixth chords and seventh chords are either split building blocks or overlapped combinations of pairs of building blocks. The "guide intervals" of standard chord voicing practice are these building blocks, except notated several levels removed from the keyboard (they're identified by pairs of degree numbers of chord scales relative to constantly changing chord roots).

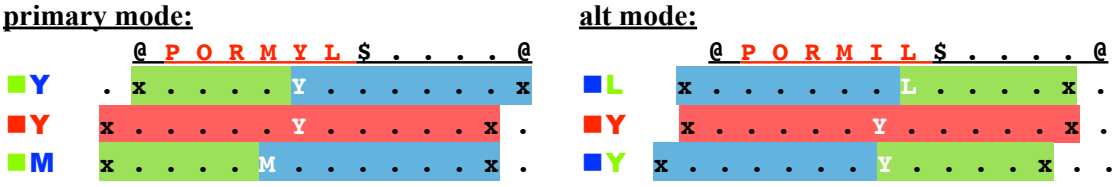
SIMPLE PROPERTIES OBSCURED BY MUSIC NOTATION

The primitive data elements of PKP are not the building blocks themselves, but anchor-centered octave stacks formed of a building block and its opposite inversion. The simple properties obscured by music notation follow from the many fifo stacks that may be morphed from tritone stacks, which provide many different choices for fifos to fit both the mode context and the flow context.

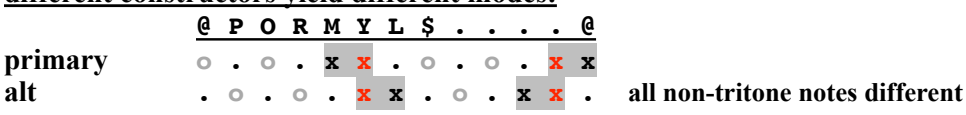
The six, anchor-centered, symmetric tritone stacks identified by the alphabet are the starting points. The alphabet octave in the top line is extended to include inversions going down into the next octave (or equivalently, into the home octave from the next octave up).



Tritone stacks morphing into fifo stacks identify fifos as morphed tritones. As illustrated next for the Ionian tritone, a tritone stack morphs directly into 4 fifo stacks, each of which provides 2 fifos (1 fifth, 1 fourth). As above, the notation on the left represents octave completion intervals by box prefixes that identify the color of the inversion (red completes red, green completes blue, blue completes green).



As shown next, the different morphings yield different symmetric constructors **xx–xx** and **xx–xx** that determine different modes. The primary mode is a parallel mode of the home tonic because it includes it. The alt mode doesn't include the home tonic and so is not a parallel mode of it, but is a primary parallel mode of a tonic transposed by tritone (i.e., inverted) — same tritone, all non-tritone notes different. During a mode change, the constructors determine inversions of building blocks of new modes. Once a new mode is established, the inversions are harmonically equivalent within it.



All tritones except **L** yield a single parallel mode of the home tonic. The **L** tritone is unique because it includes the tonic, so its primary and alt modes are both parallel modes (called respectively, Lydian and Locrian). This accounts for the 7 parallel modes of the home tonic — 6 primary modes determined by 6 tritones and one alt mode determined by one of the tritones, namely **L**.

These morphings answer a question in opening chapter: How can altering one particular note by a half tone not only tell the ear that the scale has changed but also identify the new scale? Given the establishment of a classical mode by one of these core sequences, morphing the tritone in the opposite direction from the one that established the mode will sound wrong to the ear, suggesting the opposite core. This means changing one core note of a core fifo by a half tone changes the mode. Knowing the simple relationship between the cores enables rejection of fifo possibilities that don't fit a particular mode. For example, the establishment of an Ionian mode by the core sequence **Y-Y-M** excludes the

fourths of its alt mode determined by the core sequence **L-Y-Y**.

Classical modes contain a multiplicity of fifos, which is at once responsible for both musical richness and notational complexity, if all are specified by pairs of note symbols in music notation. Understanding fifos as morphed tritones does an end run around the notational complexity because it enables the choice of specific fifos to be postponed, avoiding over-specification of notes up front that are actually determined by context provided by the flow and mode of the music.

The simple, transparent nature of this way of understanding available building blocks is in sharp contrast to the complexity of the same thing in music notation. It makes possible the transformation of core building blocks into chords by adding core building blocks from context. It makes possible the substitution by eye of consonant fifos of chords for one another to fit context — think of this as chord substitution from the bottom up. It aids in the understanding of mode/tonic changes. Example pieces coming up explore the possibilities.

Other Intervals in Alphabet Terms

Building blocks are not, of course, the only intervals of music. Intervals smaller than building blocks are major thirds, minor thirds, whole tones and half tones; intervals larger than building blocks are augmented fifths, major sixths, minor sevenths and major sevenths. The larger and smaller intervals spreading out in opposite directions from the building blocks are inversions of each other, so knowing one means knowing the other. These intervals appear in music mostly as inherent parts of shapes formed from building blocks determined by alphabet letters or words. The infrequent special cases when one of them acts as a separate building-block-like object are easily handled by annotations on letters. Details are explained as needed for example pieces, and summarized in Appendix A.

A UNIVERSAL CHROMATIC SCALE

The chromatic scale is not needed to understand the concepts, and introducing it earlier would only have been distracting. However, it's needed now to complete the picture. It consists of seven numbers identifying the fixed positions of the piano keys of the master Ionian mode within any home octave, and five prefixed numbers identifying the fixed positions in its 5 whole-tone gaps. The only reason for the choice of the Ionian mode as the basis for the scale is to provide a strong connection to music notation, in which this mode is conventionally understood as the master mode of a key signature.

conceptual home octave	**@ P O R M Y L $ x x x x @**
master major scale (Ionian mode)	**1 . 2 . 3 4 . 5 . 6 . 7 1**
piano keys in its whole-tone gaps	**p2 p3 p5 p6 p7**
universal chromatic scale	**1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1**

The prefix "**p**" stands for "phlat" and means "next piano key down." The prefix is not a conventional flat because it applies only to five specific notes that have whole tones below them in the Ionian mode. The numbers are not degree numbers that count scale notes, but fixed note positions within the home octave. There are no flats or sharps that can be attached to any piano key to raise or lower its pitch. The chromatic scale itself does the raising or lowering (e.g., **4-p5** and **p5-5** raise the pitch of of **4** and **p5** by a half tone). The key-signature rule that the same note symbol (e.g., **5**) cannot appear in successive scale symbols does not apply.

The simplicity of having a universal chromatic scale for any home octave contrasts strongly with music notation's different chromatic scales identified by sharps, flats and naturals relative to different

key signatures.

The chromatic scale notation is not novel. It's adapted from a well known Roman-Numeral (RN) based scheme for identifying **chord roots** relative to a tonic, described, for example, in Mehegan's jazz piano instruction book. The possible chord roots in the chromatic scale are all the notes of the chromatic scale with plain numbers replaced by RN symbols. The only novel feature is the use of the "phlat" prefix **p** that avoids Mehegan's confusing use of conventional flat symbols that are not actual flats in music notation. His notation accurately represents the black keys of the C octave but is confusing for other other octaves with black keys in different scale positions.

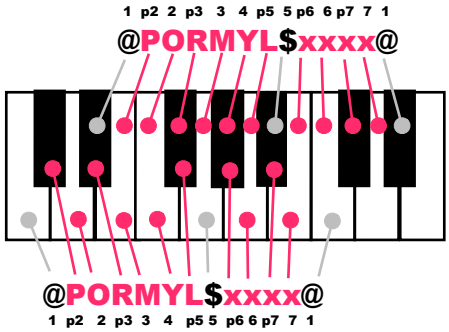
Here follows the adapted chord notation aligned with the alphabet. The RN chord root notation is used only to illustrate the interpretation of PKP shapes as chords and does not go beyond the roots. For example, the chord **Eb*7b9*** is **I*7b9*** relative to tonic Eb and **pIII*7b9*** relative to tonic C. The PKP shapes relative to these roots and tonics determine the same chords in a different, simpler way.

chromatic scale	**1**	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**
chord roots in PKP	**I**	**pII**	**II**	**pIII**	**III**	**IV**	**pV**	**V**	**pVI**	**VI**	**pVII**	**VII**	**I**

The home tonic is the reference for understanding changes to secondary tonics identified by **tonic pointers** of the form **@t**, where **t** is a chromatic scale symbol identifying a tonic (the home tonic pointer is implicitly **@1**). Tonic pointers enable parallel-mode changes to be interpreted as tonic changes.

ON THE KEYBOARD

The **universal home octave** is a combination of the alphabet octave and the chromatic scale, typically appearing as a pair of header lines on mode tables in which entries identify note positions. It maps to the keyboard as follows for two possible home tonics.



The mix of black and white piano keys is visibly very different for different home octaves but the difference is manageable because of the intuitively simple nature of the notations. The chromatic scale mirrors the look of the C octave on the keyboard, enabling other octaves with different mixes of black and white piano keys to be understood in the same terms.

The only caveat is the necessity of keeping the scale frame of the home tonic fixed in the mind, to avoid confusion with recently played pieces with different home tonics that may still be in mind. This can be helped by putting removable stick-on labels on the tonic and pitch-center keys, and by running through scales, arpeggios and chord sequences for a new tonic to get the new representations into the mind and fingers before approaching an actual piece.

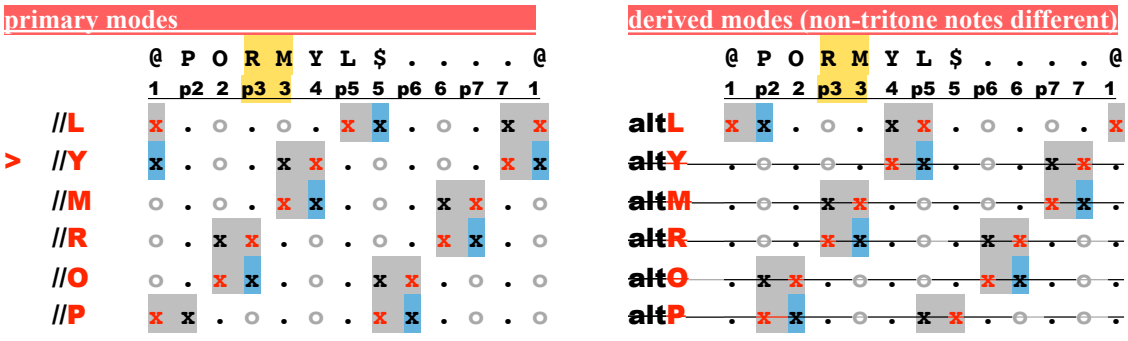
The difficulty of doing this is much less than the difficulty of dealing with the many and various different chromatic scales of music notation for different home tonics. The advantage of doing it is thinking of pieces of music with different key signatures in common terms. Music notation still has to be dealt with, but my experience has been that it's relatively easy to think in these terms when the PKP notation is annotated on the written music. With a bit of experience, the PKP notation can be used by itself to provide a shorthand description of the melody and harmony of an entire piece.

PARALLEL CLASSICAL MODES

Here follows a simple table that covers all possible parallel classical modes and corresponding tonic changes, which means all possible key signatures. The modes on the left are both **parallel** (identified by the prefix "//") and **primary** (contain both the tonic and the pitch center, which is say the scale frame). Different tonalities are determined by the yellow-highlighted segments of the universal home octave (the **POR** modes are minor and the **MYL** modes are major). Successive mode changes are determined by the tritones moving in half-tone steps, with the grey-highlighted symmetric constructor shapes **xx—xx** and **xx—xx** alternating at each step. The successive modes differ by one note, provided by the mode tritone.

The home-octave modes are relative modes of the transposed master mode (same notes in different orders). The master tonics go down by fifths in the sequence **5-1-4-p7-p3-p6** (blue highlighting), enabling any mode change to be understood as a tonic change to the transposed master mode or a relative mode of it.

The derived modes on the right, identified by "**alt**" prefixes, are **tritone substitutes** of the primary modes — same tritones, all non-tritone notes different. The modes with strikethroughs are not parallel modes because they don't include the home tonic, but still have important roles to play in chromatic music. In the master tonic sequence **p2-p5-7-3-6-2** going down by fifths (blue highlighting), each tonic is a tritone away from the corresponding primary tonic.

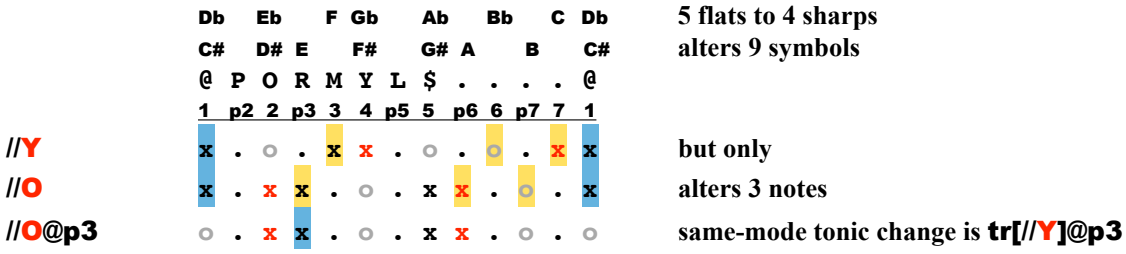


It's worth pausing here for a moment to reflect on the remarkable simplicity of this compared to understanding the same thing in terms of enumerated notes relative to all possible key signatures in music notation.

Two Sides of the Same Coin

Mode/tonic changes for classical modes are "two sides of the same coin." Footnote 1 of Chapter 1 provided an example of music notation's complex representation of mode changes that are simple on the keyboard. We now have the notation to illustrate the simplicity, not only of the mode changes but also of the tonic changes that are the "other side of the coin".

The mode change from major (Ionian **//Y**) to parallel minor (Aeolian **//O**) is simple by any measure. The corresponding same-mode tonic change is Ionian to Ionian up a minor third. This is implicit in the notation at bottom left but may be made explicit by the notation at bottom right, which means "transpose **//Y** to tonic **@p3**."



Beyond Classical Modes

Looking ahead, the tritone basis of PKP is fundamental to going beyond classical modes in a simple but deep way. The details are developed in the next chapter, but keep the following elements in mind. The tritone basis begins with the single tritones of classical modes and continues with multi-tritone combinations (doubles, triples, quadruples). Such tritone combinations provide symmetric **constructors** of modes, analogous to the ones that determine classical modes in the foregoing table.

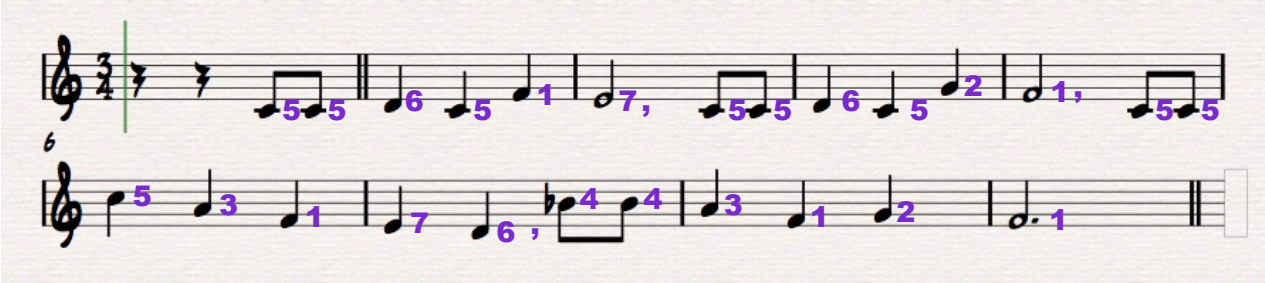
Double tritones take us into the domain of modes called *melodic minor*, *harmonic minor* and *harmonic major*, parallel modes of which provide a bridge between classical modes and non-classical modes. On one hand, they share the classical-mode properties of 7 notes and no adjacent half tones; on other hand, they share double tritones with many non-classical modes, of which they are sub-modes. Suites of parallel modes of these modes as master modes have been formally developed in music notation (see, for example, the book *Modalogy*). These modes are deeply complex in music notation, but simple in PKP terms.

Triple and quadruple tritones take us into the domain of parallel modes that are different in kind from classical modes (different numbers of notes, adjacent half tones), which include *generalized major and minor modes*, *blues modes* and *atonal modes*. The bridge modes just identified fall out of these modes as sub-modes.

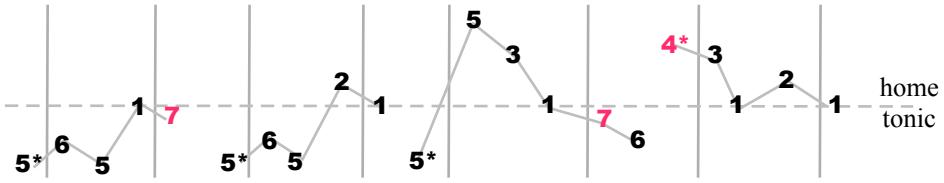
MELODY LINES

"Melody" and "harmony" are interchangeable concepts. Any single line of a piano piece (sometimes a pair of them) may be treated as melody, automatically relegating other lines to harmony. A melody in tonal music is normally a single line that determines a "song" that can be sung, whistled or hummed, independently of the details of harmony, rhythm and timing — the only specified timing is one note after the other. The distinction between melody and harmony becomes blurred for melody lines accompanied by, for example, walking bass lines that can themselves be song-like.

Shown next is the piece *Happy Birthday to You* in F-major, with chromatic scale symbols annotated on the staff next to melody notes and commas indicating phrasing. This is visibly the Ionian mode, but knowing any mode generally requires entering the note positions in a mode table. The melody annotations pave the way for adding harmony in alphabet notation next to chord symbols above the staff. Understanding a piece begins with its melody line played forward but understood backwards, because resolutions to tonics occur at the ends of pieces (here) or sections. Sometimes the end of a piece or section sets up for resolution at the beginning of the repeated phrase.



Independent representations are useful for getting an overview of pieces with long or complicated melody lines. Sketching the line on graph paper and labeling the points with chromatic scale symbols is one way of getting an independent view (asterisks indicate repeated notes). But this takes up too much vertical space on the page for longer or more complicated pieces.



A vertically compact summary follows from collapsing the line of chromatic scale symbols into a single, text line. The line of symbols by itself is ambiguous — is the next note up or down? — but the ambiguity is easily removed without adding any new symbols by highlighting pivot points between melody arcs that trend up from low points (grey) or down from high points (yellow) — think of yellow as mountain peaks illuminated by sunlight and grey as valleys in shadow. Notes within between pivot points of an arc are understood to go to the nearest note with that symbol. Zig-zags within arcs don't need highlighting because the trend remains in the direction of the arc. In rare cases, arrow prefixes (↑↓ not illustrated here) indicate moving the nominally-next-note up or down an octave.

5* | 6 5 1 | 7, **5*| 6 5 2 | 1,** **5* | 5 3 1 | 7 6,** **4* | 3 1 2 | 1**

Bar lines plus commas indicating phrasing are often sufficient reminders of timing, but timing may also be explicitly indicated by a line of downbeat markers (**^**) under the melody line. These markers identify timing independently of anything else, which means using a different timing requires only ignoring or changing the downbeat markers (or avoiding using them in the first place). Timing choices for notes between identified downbeats are left open — the only constraint is squeezing them in. The choice between a rest and an extended note for a downbeat marker between melody notes is left open.

5* | 6 5 1 | 7, **5*| 6 5 2 | 1,** **5* | 5 3 1 | 7 6,** **4* | 3 1 2 | 1**
^ ^ ^ ^ ^ **^ ^ ^ ^ ^ ^** **^ ^ ^ ^ ^ ^** **^ ^ ^ ^ ^**

Improving Readability

Readability may be improved, if needed, by offsetting pivot notes up or down from the text line.

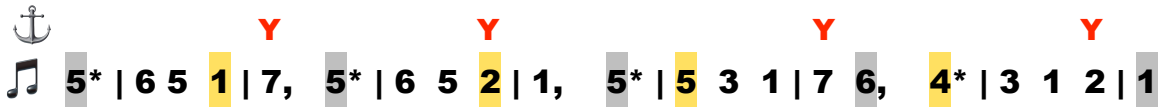
5* | 6 5 1 | 7, **5* | 6 5 2 | 1,** **5* | 5 3 1 | 7 6,** **4* | 3 1 2 | 1**

EXAMPLE: *HAPPY BIRTHDAY*

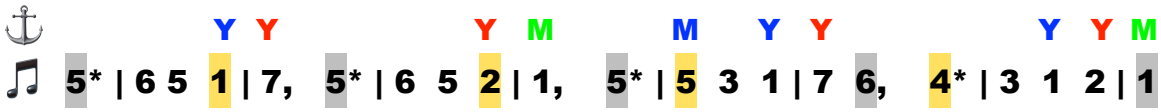
This example of one classical mode (Ionian) for everything follows from the earlier melody line. The relevance of this example goes beyond this particular piece or classical modes in general because the same concepts and notations apply across the board. The skeleton melody line seen earlier is identified by 🎵 and the new harmony line by ⚓ (these identifications are helpful for lengthier, more complicated pieces than this, but it's best to begin as one intends to continue). This harmony could follow from annotating anchors of building blocks next to chord symbols above the staff, but is developed independently of the written music here, as an exercise in the bottom-up development of harmony. The melody is short and simple, so pivot notes are not offset.

Core Harmony

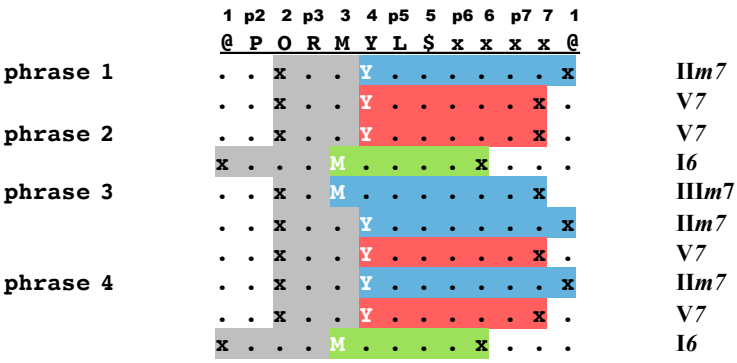
The flow of harmony is developed in steps, with tritones added first. This example doesn't start from chords, but the logic of the tritone placement can be understood without reference to them. The tritones identify resolution in the mode and so are placed just before ends of melody phrases that resolve to the tonic, or at the ends of melody phrases that don't resolve, anticipating future resolution (melody phrases are delimited by commas). The fixed-size tritones establish a flow pattern — very simple here — that, with the known mode, constrains the choice of fifos. The anchor-line tritones could be from either major **//Y** or minor **altY**, but the former is the only choice that fits this melody line.



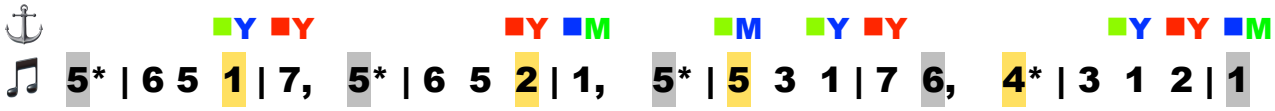
The next step is adding fifos from the mode to fit the flow. A chord progression would tell where to add them, but the logic of the additions is clear here without that. The **morph** **Y-Y** sets up for resolution (a morph holds one end of a building block fixed while moving the other a small amount). The **slide** **M-Y** exercises the scale (a slide moves a building block while preserving its size). The **wobbly slide** **Y-M** performs resolution (a wobbly slide moves a building block while also adjusting its size). This unusual term represents the hand movements exactly: slide the hand while wobbling the fingers for the size change. The use of a **M** instead of **M** for resolution avoids a possible half-tone dissonance that would be introduced by playing the tonic melody note at the top of the same octave.



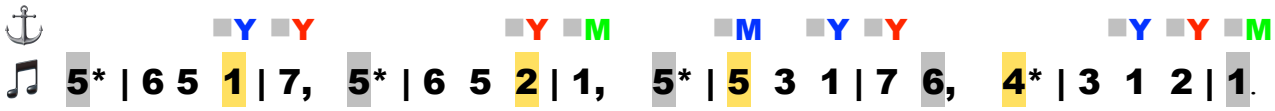
Here follows a Lego-like view of the implied harmony on the keyboard. This is shown horizontally instead of vertically because this is how shapes on the keyboard are viewed when sitting in front of it. If the harmony is assumed to originate in basic seventh or sixth chords from the Ionian mode, the bass line is determined, because the only choices of intervals down to it from the anchor line are major or minor thirds, only one of which is possible at any point. The resulting 3-note voicings are compatible with the conventional chord progression shown on the right. This simple flow captures the sound of the chord progression because the small position and size changes make big sound changes.



The creation of such voicings is helped by starting from octave stacks, which are almost chords. This notates the octave stacks but the stacks are so simple that notation is not normally required — simply complete the octave going up or down (down here, but up is also possible).

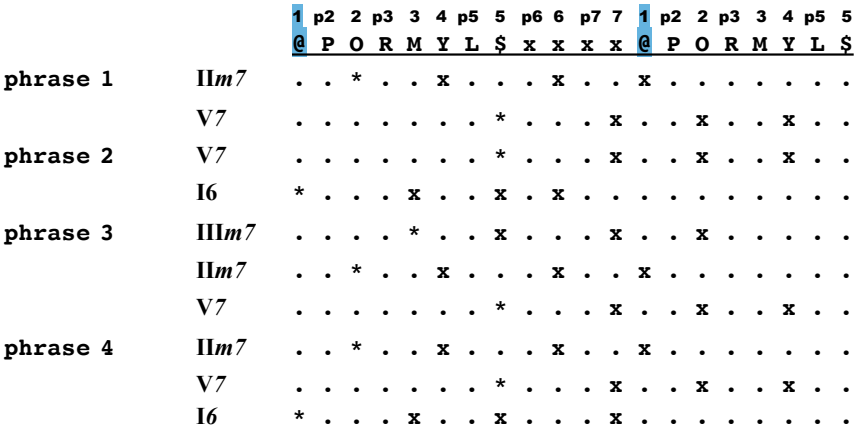


Shrinking the octave stacks up the from the bottom in generally obvious ways yields the 3-note voicings of the chords shown above, which may be notated in the harmony line as follows. The grey boxes identify smaller voicing intervals to be determined by context. The grey boxes are useful for indicating whether the voicing interval is below or above the building block. More notation than this is seldom required.



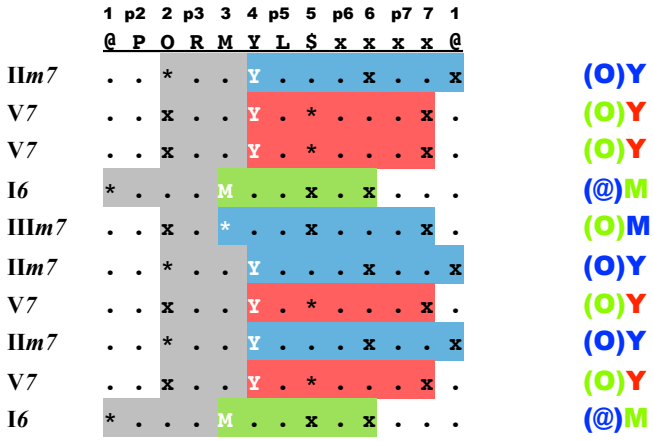
Starting from Chord Symbols

I have attempted above to demonstrate that developing harmony from the bottom up in terms of building blocks is relatively simple. Now let's do the same thing starting from chord symbols. Chord roots are marked * and chord notes x. The scale shapes of seventh chords are all the same, namely four notes two scale steps apart going up from roots, sometimes into the next octave. Sixth chords lower the top note of a seventh chord a whole tone.

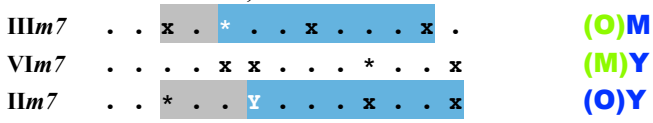


Playing these chords starting from cold is technically difficult because the whole hand must move by often large jumps, while adjusting the fingers to form the appropriate inter-note intervals among different mixes of black and white piano keys for different tonics. The trick is wrapping the chord notes around within the home octave and identifying building blocks anchored in the home octave in the result. Once these are "in the fingers" from practicing, playing the inversions that yield the chords as written, or in voice leading form, or in any other form or variation, is easy.

The following picture adds something new to the familiar core harmony, namely two-letter words that represent combinations of pairs of building blocks that form the chords (e.g., **OY** represents a pair of overlapped fifths offset by a minor third). The parentheses identify the chord-completion fifos added to the core building block (always fifos because all the tritones are in the core). This is often too much notation because the bass line of the added fifos is implied by the mode (it's either the tonic or a whole tone above it). The same goes for the treble line (it's either the pitch enter or a whole tone above it). The core harmony may also be voiced above the core by inverting all the completion fifos. The basic sound of the chord progression is provided by the core. The added fifos provide variety and depth of sound. The core is audibly more important to the sound than the chord roots.

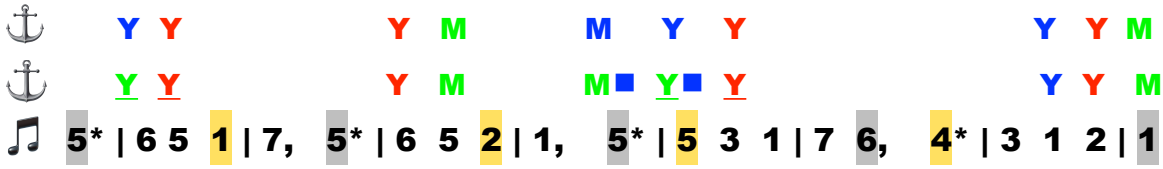


The chord **III*m7*** is often the start of a root sequence **III-VI-II-V-I** that goes down by fifths. The **VI** chord is omitted here because it's not needed, but its notes are in the **III-II** sequence.

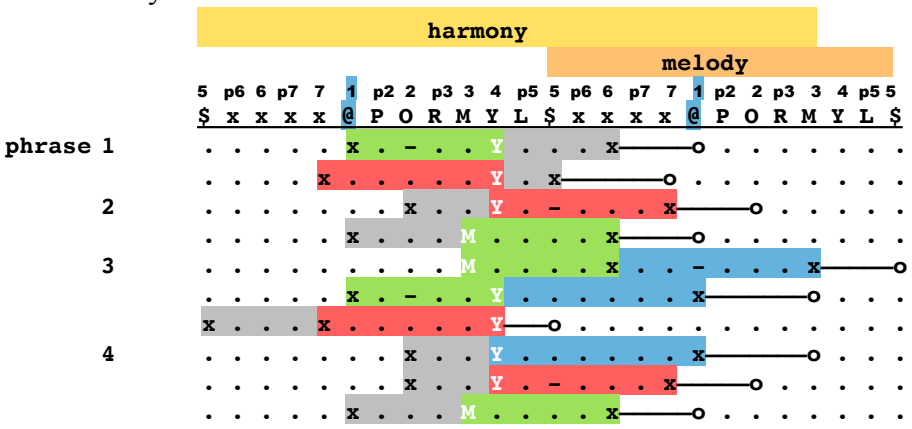


Core Harmony as a Basis for Any Harmony

As an example, voice-leading harmony is shown next (original on top). Some building blocks now go down from the anchor line and some go up. The purpose is to put the anchor line closer to the melody line. The first two shapes in phrase 3 are stretched into octave shapes to get as close as possible to the higher melody line here.



Here is the result on keyboard, shown for the harmonized melody notes (identified by **o**). The voicing intervals shown in grey for the inverted building blocks are the internal notes of the original un-inverted ones. The harmony and melody overlap but each harmony shape as close as possible below its harmonized melody note.



More About Chords

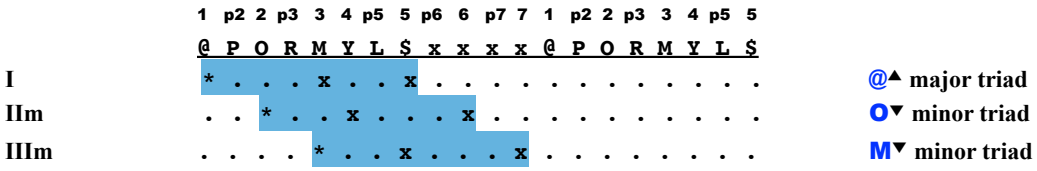
Chords are results not starting points in PKP, except for identification of tritones. Chord symbols of tonal music over-specify notes, so many chord progressions can be implied by core harmony within the home octave. As illustrated by these examples, combinations of building blocks yield smaller and larger intervals as inner or outer intervals, without any need to notate them specifically.

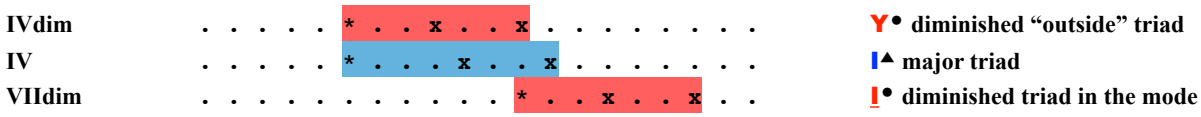
Different inversions of the same combination of building blocks may have different chord symbols, increasing notational complexity without any increase in substance: for example, the inversion **I6/VI** is **VI*m7*** (the slash suffix indicates an inversion going up from a note identified by a root symbol); the inversion **II*m7*/IV** is **IV*M6***; the inversion **V*7*/II** is not normally given a separate chord symbol because it doesn't have the shape of a basic seventh or sixth chord (but it may be given an altered chord symbol).

Seventh and sixth chords can be understood as combinations of triad chords that share two notes: **I*M7*** is a combination of a **major triad** with root **I** and a minor triad with root **III**; **II*m7*** is a combination of a **minor triad** with root **II** and a major triad with root **IV**; **V*7*** is a combination of a major triad with root **V** and a **diminished triad** with root **IV**. Turning this around, a seventh or sixth chord may be implied by a sequence of triad chords.

Triads are split building blocks, requiring some new notation. A superscript notation preserves the visibility of the building blocks. Major or minor triads are asymmetric splits of fifths into a minor third and a major third in either order, symbolized by superscript "▲" indicating the larger interval is on the bottom, or superscript "▼" indicating the larger interval is on top. Diminished triads are symmetric splits of tritones, symbolized by superscript "●". Fourths enter the triad picture via inversions of fifths.

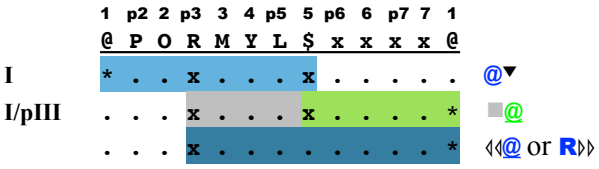
Here follow some examples for the Ionian mode.





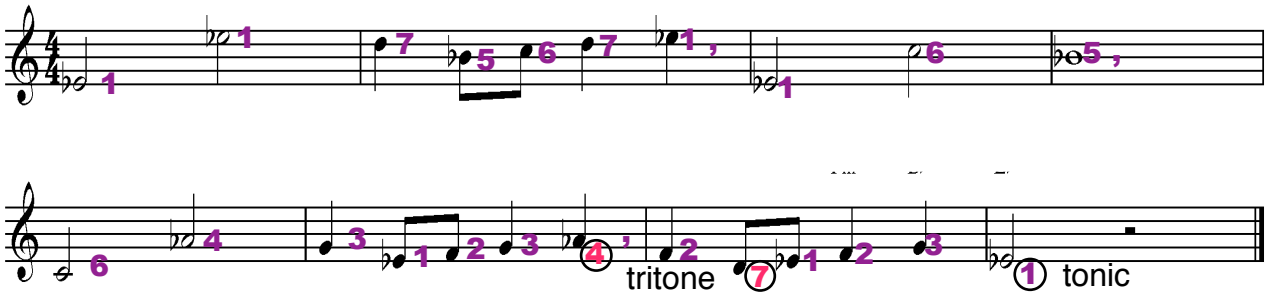
A mix of triad chords, sixth chords and seventh chords tends to be complex because it increases the number of chord types to be parsed. Triads are unlike chords formed of combinations of building blocks, in that inversions are not of the same form as the original (an inversion of a split building block is not a split building block). The different forms of inversions add to the complexity.

It's sometimes useful to represent inversions of minor or major triads by outer intervals (not so for diminished triads because this would obscure the mode-identifying tritone). For example, the outer interval of the first inversion of minor triad @▼ is the major sixth ⧏⧏@ going down or R⧐⧐ going up (a fifth expanded by two half tones, with an inner note that may often be left to context).

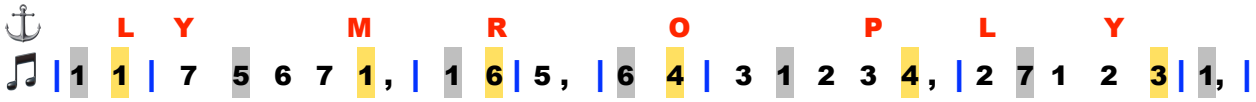


EXAMPLE: *OVER THE RAINBOW*

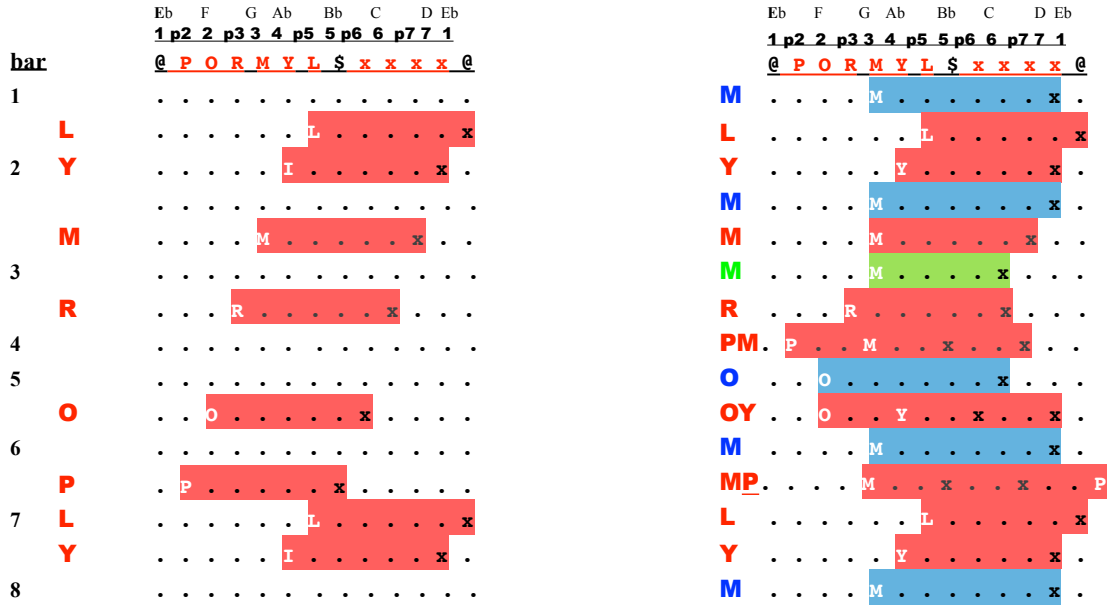
This example develops strongly chromatic ornamental harmony from scratch for the opening melody phrase of the well known piece *Over the Rainbow*. This illustrates that written harmony may be just harmony, without any other purpose than to sound good. It need not originate in the melody scale and may not have any intended meaning as a harmony scale. It just *is*. The first eight bars of this piece are sufficient for the purpose. The melody line is straight Ionian, trending downwards in zig-zags over an octave range to home tonic Eb (the implied Ionian key signature is 3 flats).



Here follows the skeleton melody line plus a skeleton harmony line consisting of all the tritone anchors in reverse order starting on **L** and wrapping around: **L-Y-M-R-O-P-L-Y**. This line is an example of constant-keyboard-shape harmony: the same keyboard shape (a tritone) moves to different positions in the chromatic scale.



As shown next, the tritones provide a framework for adding core fifos and some other variations.



The double tritones provide elegant symmetric relationships between successive shapes (e.g., the sequence **PM-O** shrinks the outer notes of the double tritone inwards a half tone; and **O-OY-M** moves a fifth aligned with the bottom note of **OY** to a fifth aligned with the top note).

Here follows a summary of the melody plus new harmony. Play the single building blocks as octave shapes that go with the flow. The harmony could but isn't intended to imply parallel mode changes, or the tonic changes that are the other side of the same coin. It's only ornamentation.

Over the Rainbow (orginal home tonic = Eb)

This works because the five tritones not in the Ionian mode morph into mode fifos or from them, demonstrating the flexibility of the many fifos being understood as morphed from the many fewer tritones.

This example of ad hoc chromatic harmony sets the stage for a more architectural view of chromatic harmony.

TOWARDS AN ARCHITECTURAL VIEW

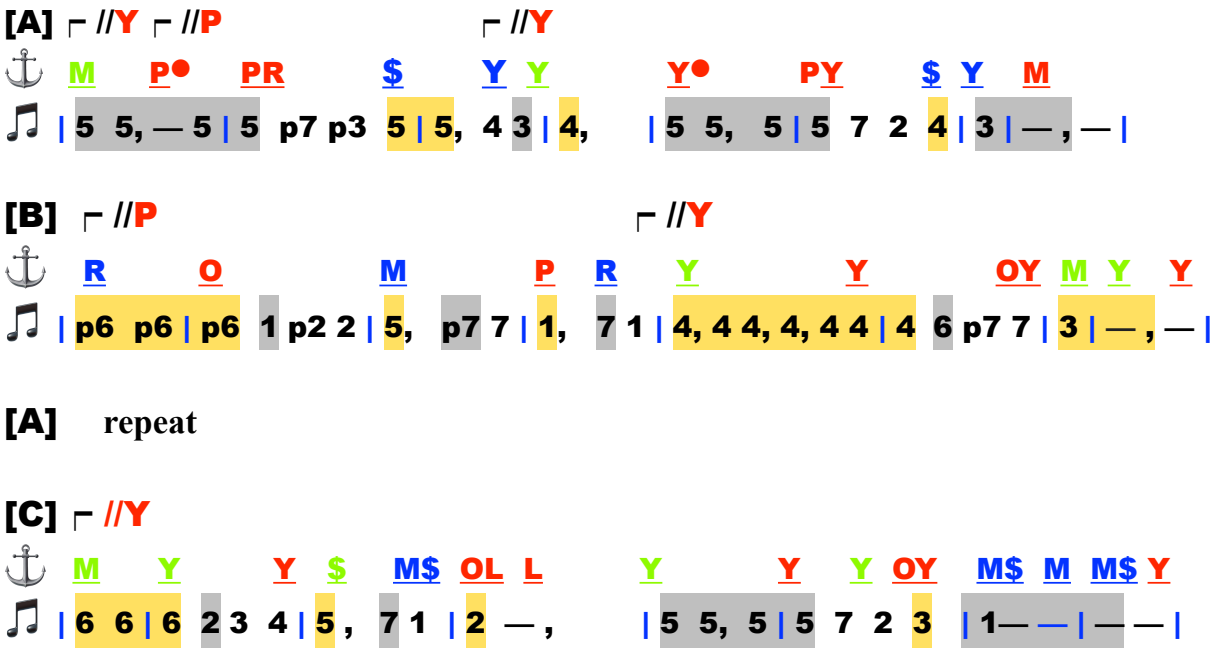
In a famous session of NPR's *Piano Jazz*, Bill Evans, in conversation with Marian Macpartland, said words to the effect that he advocated taking a piece apart to understand its "architecture" before putting it back together in an improvisation. I thought this was a great concept but wondered how "architecture" could be conceptualized.

My view of architecture is in terms of different **domains of chromaticism**. "Chromaticism" means departures from a written key signature, which means, for classical modes, departures from a single classical mode that go to one or more other classical modes in succession. The domains are **basic classical** (chromaticism is absent or ornamental), **chromatic classical** (chromaticism is introduced via parallel classical modes or associated tonic changes that are "two sides of the same coin") and **non-classical** (chromaticism uses hierarchy of a non-classical parallel modes introduced in the next chapter). Our immediate concern is with the chromatic classical domain but seeing it up front as one of three domains is helpful.

EXAMPLE: ***HERE'S THAT RAINY DAY***

This evocative piece in home tonic G is from *The Ultimate Jazz Fakebook*. Working through the details is an instructive exercise in figuring out chromatic music that sounds coherent but looks notationally incoherent in music notation. The piece is squarely in the chromatic classical domain, in which a sequence of parallel modes determines melody and harmony. In the following summary, annotations marked " ⌐ " identify the governing mode as mainly Ionian (major, //Y), except for two short segments in Phrygian (minor, //P). The melody lines of the short segments are ambiguous but the harmony makes Phrygian an obvious governing minor mode. Ornamental passing notes and harmony shapes in both these modes give a misleading sense of notational incoherence, when expressed in music notation, but the mode context provides coherence in PKP notation. The minor segments give this mostly major piece a sad-happy vibe.

Play the melody line and the tritones to get a sense of the sound of the piece. Fill in the fifos later to smooth the flow and add depth.



The melody modes identified above are as follows, with passing notes identified by plus signs.. The home octave is inverted because the melody lines mostly start below the tonic.

	D					G							D
	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5
	$	x	x	x	x	@	P	O	R	M	Y	L	$
Phrygian //P	x	x	.	x	.	x	x	+	x	.	x	.	x
Ionian //Y	x	.	x	+	x	x	.	x	.	x	x	.	x

Deriving the Harmony

Getting a grasp on the harmony shown next is easy if you keep the governing modes identified on the right below in mind. Remember that these and other Lego-like pictures are presented not as pictures

to be drawn but as illustrations of how to work things out at at the keyboard.

Deriving the harmony from the written chords starts with identifying tritone chords relative to the home tonic (red text). Their tritones are provided by a table in Appendix C. Roots are identified by asterisks, for reference only, because most may be omitted. The tritones establish fixed points in the flow that provide references for filling in overlapping fifos to voice the non-tritone chords.

[A]

			D					G							D
			5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5
			$	x	x	x	x	@	P	O	R	M	Y	L	$
bar 1	I-*M9*	M	.	.	.	.	x	*	.	–	.	M	.	.	.
2	pIII-*7*	P●	x	.	.	x	.	.	P	.	*	.	.	.	.
	VI-*7b5*	PR	x	.	*	.	.	.	P	.	R	.	.	.	.
3	pVI-*M7*	$	.	*	.	.	.	x	.	.	–	.	.	.	$
4	pVI-*6*	Y	.	*	.	x	.	–	.	.	–	.	Y	.	.
5	II-*m7*	Y	.	.	–	.	.	x	.	*	.	.	Y	.	.
6	V-*7*	Y●	*	.	.	.	x	.	.	x	.	.	Y	.	.
	V-*7b5*	PY	*	–	.	.	x	.	P	.	.	.	Y	.	*
7	I-*M7*	$	.	.	.	.	x	*	.	–	.	M	.	.	.
8	V-*m7*	Y	*	.	.	x	.	.	.	–	.	.	Y	.	*
	I-*7*	M	.	.	.	x	.	*	.	.	.	M	.	.	.

[B]

			D					G							D
			5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5
			$	x	x	x	x	@	P	O	R	M	Y	L	$
bar 1	IV-*m7*	R	.	x	.	.	.	–	.	.	R	.	*	.	.
2	pVII-*9*	O	.	x	.	*	.	–	.	O	.	.	–	.	.
3	IV-*M7*	M	.	.	x	.	.	–	.	.	.	M	*	.	.
	VI-*9*	P	x	.	*	.	–	.	P	.	.	.	.	.	.
4	pVI-*M7*	R	.	*	.	.	.	–	.	.	R	.	.	.	*
5	II-*m7*	Y	.	.	–	.	.	x	.	*	.	.	Y	.	.
6	V-*9*	Y	*	.	–	.	x	.	.	–	.	.	Y	.	.
	V-*7b9*	OI	*	x	.	.	x	.	.	O	.	.	Y	.	*
7	I-*M7*	M	–	.	.	.	x	*	.	.	.	M	.	.	.
8	II-*m7*	Y	.	.	.	.	.	x	.	*	.	.	Y	.	.
	V-*7*	Y	*	.	.	.	x	.	.	.	.	.	Y	.	*

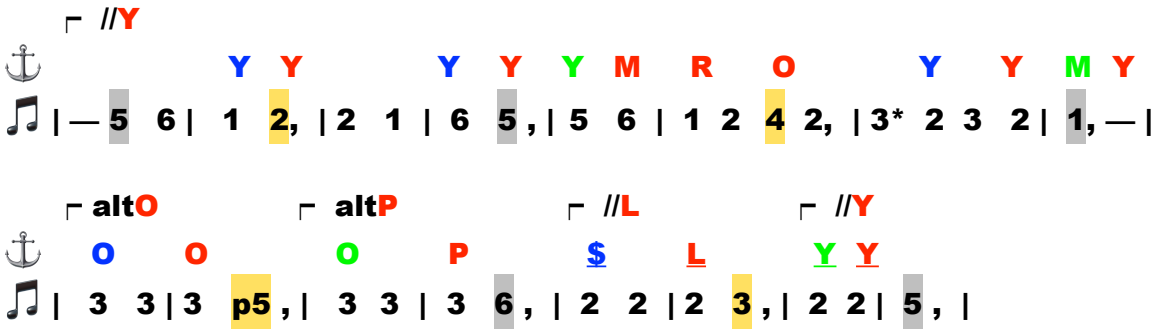
[C]

			D					G							D
			5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5
			$	x	x	x	x	@	P	O	R	M	Y	L	$
bar 1	IV-*M7*	M	.	–	.	.	x	.	.	.	.	M	*	.	.
2	II-*m7*	Y	.	.	–	.	.	x	.	*	.	.	Y	.	.
	V-*7*	Y	*	.	.	.	x	.	.	x	.	.	Y	.	*
3	III-*m7*	$	.	.	.	.	.	.	.	x	.	*	.	.	$
	VI-*m7*	M$	.	.	*	.	.	x	.	.	.	M	.	.	$
4	II-*7b5*	OL	.	x	.	.	.	x	.	*	.	.	.	L	.
	II-*7*	L	.	.	–	.	.	x	.	*	.	.	.	L	.
5	II-*m7*	Y	.	.	–	.	.	x	.	*	.	.	Y	.	.
6	V-*7*	Y	*	.	.	.	x	.	.	–	.	.	Y	.	*
	II-*m7*	Y	.	.	.	.	.	x	.	*	.	.	Y	.	.
	V-*7b9*	OY	*	x	.	.	x	.	.	O	.	.	Y	.	*
7	I	M$	.	.	x	.	.	*	.	.	.	M	.	.	$
	VI-*m7*	M	.	.	x	.	.	–	.	.	.	M	.	.	.
8	I	M$	.	.	*	.	.	*	.	.	.	M	.	.	$
	V-*9*	Y	*	.	–	.	x	.	.	–	.	.	Y	.	*

EXAMPLE: *RHYTHM CHANGES*

The Gershwin piece *I Got Rhythm* (*The Standards Real Book*, Sher Music (2000), p191) is the origin of widely copied chord changes called "Rhythm Changes" by jazz musicians. The changes move through successive parallel modes of the home tonic, first by ornamental harmony that suggest parallel mode changes without executing them, and then by actual changes. Dealing with these changes can be challenging in music notation, depending on the key signatures involved. In the following summary, parallel mode changes occur at points marked " ⌐ ."

I Got Rhythm (original in Bb)

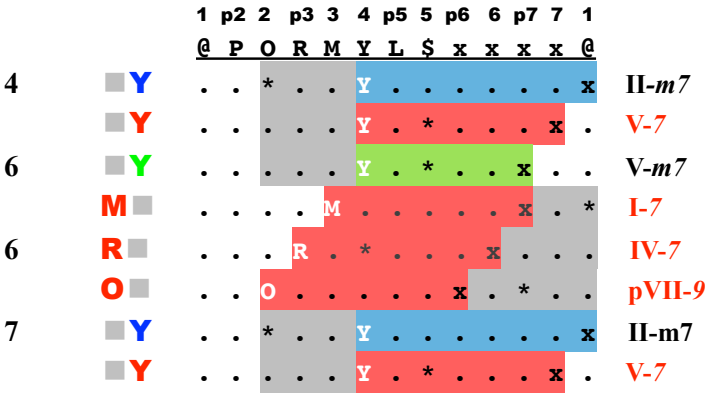


The **M-R-O** tritone sequence in bars 1-8 is purely ornamental because the melody line stays in the Ionian mode of the home tonic. The melody line in bar 10 goes to chromatic note **p5** that's visibly and audibly not a passing note, identifying a mode change determined by the **O** tritone. The mode is not **//O** because this is a minor mode and the melody line in bars 10-11 is major: the mode **altO** (same tritone, all non-tritone notes different). The same goes for tritone **P** in bars 12-13: the mode is **altP**. The mode in bars 14-15 is **//L** and bar 16 returns to **//Y**. These are very simple mode changes: in bars 10-16, one note changes from one mode to the next, supplied by the tritone. The only big change (4 notes) is **//Y-altO** in bars 8-10.

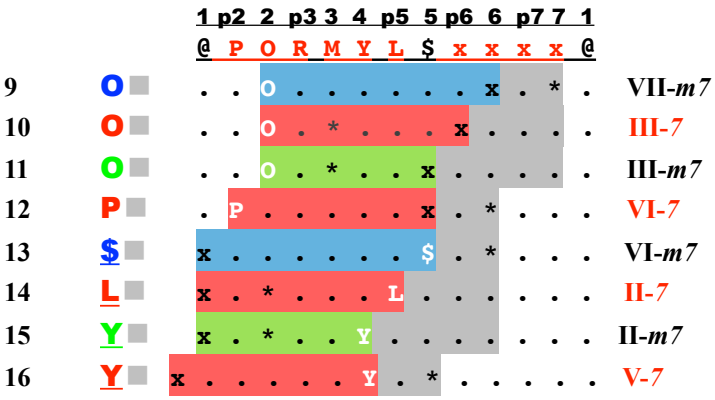
The parallel mode changes are shown next (altered notes highlighted). The other side of the coin of parallel mode changes is Ionian tonic changes shown on the right (up a major sixth and then down by fifths). The parallel modes provide the notes.

		Bb	C	D	Eb	F	G	A	Bb	<—written key
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	Y	L	$	x	x	x	x	@	
bar 1-8	//Y	x	.	x	.	x	x	.	x	.	x	.	x	x	
bars 9-10	altO	.	x	x	.	x	.	x	.	x	x	.	x	.	**tr[//Y]@6**
bars 11-12	altP	.	x	x	.	x	.	x	x	.	x	.	x	.	**tr[//Y]@2**
bars 13-14	//L	x	.	x	.	x	.	x	x	.	x	.	x	x	**tr[//Y]@5**
bar 15-16	//Y	x	.	x	.	x	x	.	x	.	x	.	x	x	

The harmony of bars 4-7 is shown next. The grey box prefixes and suffixes on the left indicate voicing intervals of unspecified size less than a fourth that are left to context. This notation could be shown in the anchor line using grey box prefixes and suffixes.



The harmonic sequence for bars 9-16 where the mode changes occur is shown next.

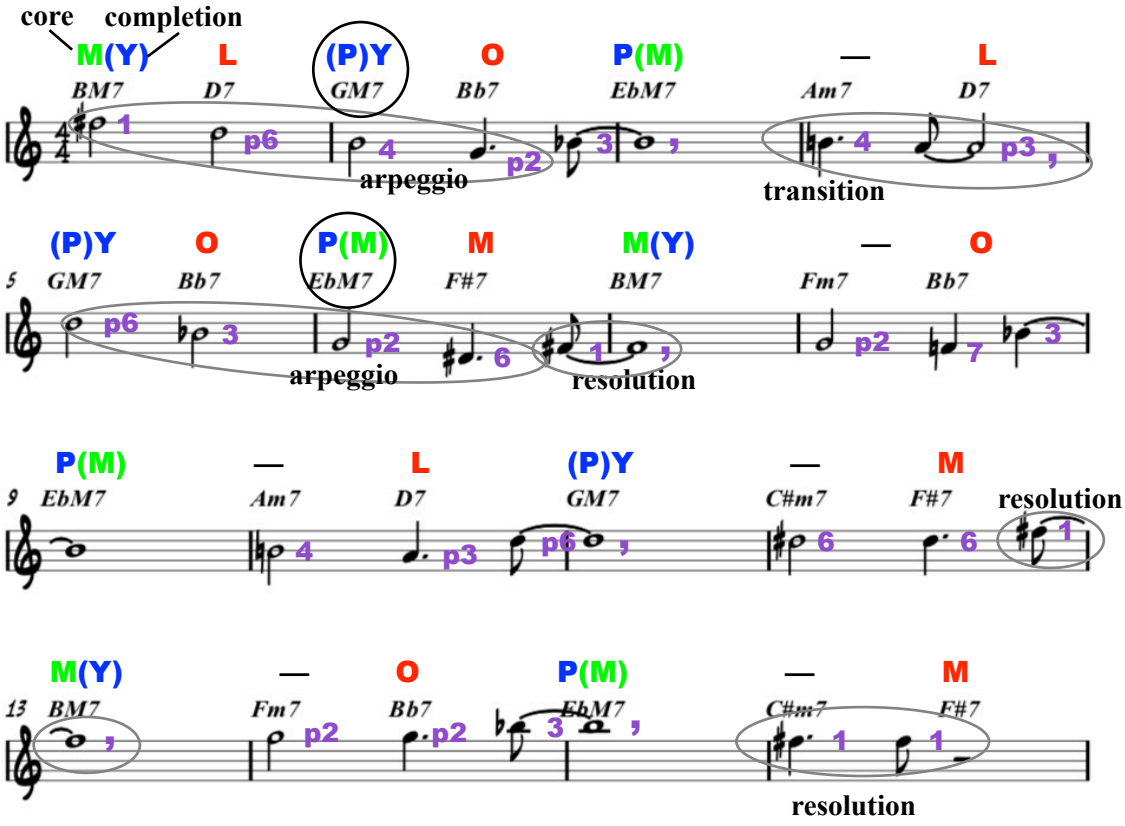


The **VII-*m7*** chord in bar 9 includes note **p5**, of the **altO** mode. After that, the only notes that go outside the Ionian home tonic mode are one note from each of the three non-Ionian tritones, which is no different from the first eight bars. The parallel modes provide more differences than this that are not exploited here but could be.

EXAMPLE: *GIANT STEPS*

Coltrane's famously difficult jazz classic *Giant Steps* "pushes the envelope" of the chromatic classical domain. In music notation, it presents as an example of successive classical modes. It's famously difficult because it moves between Ionian tonics G (1 sharp), B (4 sharps) and Eb (3 flats) that differ from each other by 4 notes (the "giant steps"), and the changes are rapid. It's simpler than it looks for several reasons. The core harmony is composed of 3 tritones in rotation, with the same fifo always morphed from each. The melody line is a symmetric mashup of the 3 fifos, with a couple of passing notes added that break the symmetry to give an 8-note melody scale. The home tonic of the melody scale is F# (5 sharps), a half tone below Ionian tonic G.

The chart shown below is an annotated Sibelius copy of a fake-book chart (*The Real Book*, 6th Edition, Hal-Leonard). The chord sequence determines the succession of Ionian modes just described. Relative to the home tonic F#, the Ionian tritones of these modes are anchored by **L**, **O** and **M**. The core tritone-fifo sequences of the classical modes are **O-P**, **L-Y** and **M-M** in these terms. These core sequences normally imply parallel classical modes but the modes are actually not part of this picture because notes of the melody line are cherry picked from them to provide a rather simple melody scale formed only of the notes of the 3 fifos plus a couple of passing notes.



The melody scale is determined by the outlined arpeggios of the **G*M7*** and **Eb*M7*** chords in the opening bars. The chords are **(P)Y** and **P(M)** in building-block terms, where the parentheses identify

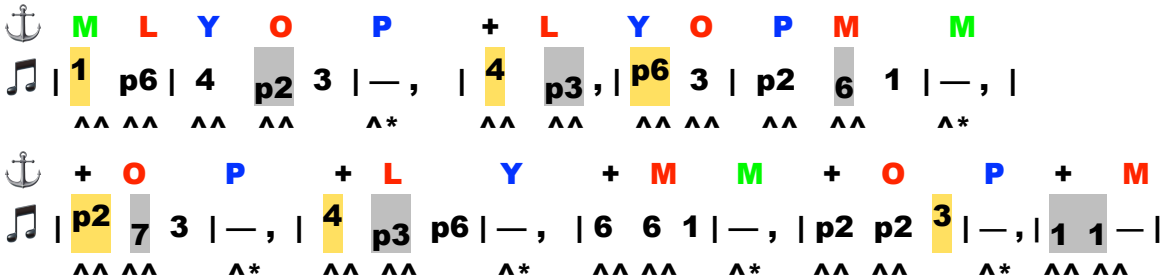
completion fifos for the core **Y** and **P** fifos. Normally, core-completion fifos are left implicit in core anchor lines, but making them explicit here is important because the notes of the completion fifos determine the **PMY** melody scale, which is a 6-note, atonal, augmented scale morphed from the tritones of the 6-note, atonal, whole-tone scale **OML**.

The details are summarized next in tabular form (as before, * identifies chord roots). The **PMI** scale is called "augmented" because its 5th note is a half tone above the pitch center. The tritones determine the harmony and melody but are absent from the **PMY** scale (only one note from each is in the scale). Actually, the scale is a mashup of all three major seventh chords but the BM7 arpeggio is not in the melody line and would add no new notes if it was. Melody notes **p3** and **7** are ornamental passing notes relative to this scale. Identifying them as such is useful because, as actual scale notes they would form extra tritones **R** and **Y** with scale notes **6** and **4**. Tritone **R** is equivalent to **O** and tritone **Y** to **M** here, in the sense that they morph into the same fifos, but explicitly adding them would add symbolic clutter without adding compensating insight.

		F# G B Eb F# <—tonics
		1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
		@ P O R M Y L $ . . . . @
D*7*	L	x . . . . . x . * . . . x
G*M7*	(P)Y	. * . . . x . . x . . . x — highlighted core **Y**
B*b7*	O	. . x . * . . . x . . . .
Eb*M7*	P(M)	. x . . x . . . x * . . . — highlighted core **P**
I*7*	M	* . . . x . . . . . x . .
B*M7*	M(Y)	. . . . x * . . . x . . x — highlighted core **M**
atonal scale	PMY	x x . . x x . . x x . . x
melody scale		x x . + x x . . x x . + x

Here is a skeleton summary of the piece, showing only the core building blocks above the melody line (the plus signs are placeholders for fifos to be added from context).

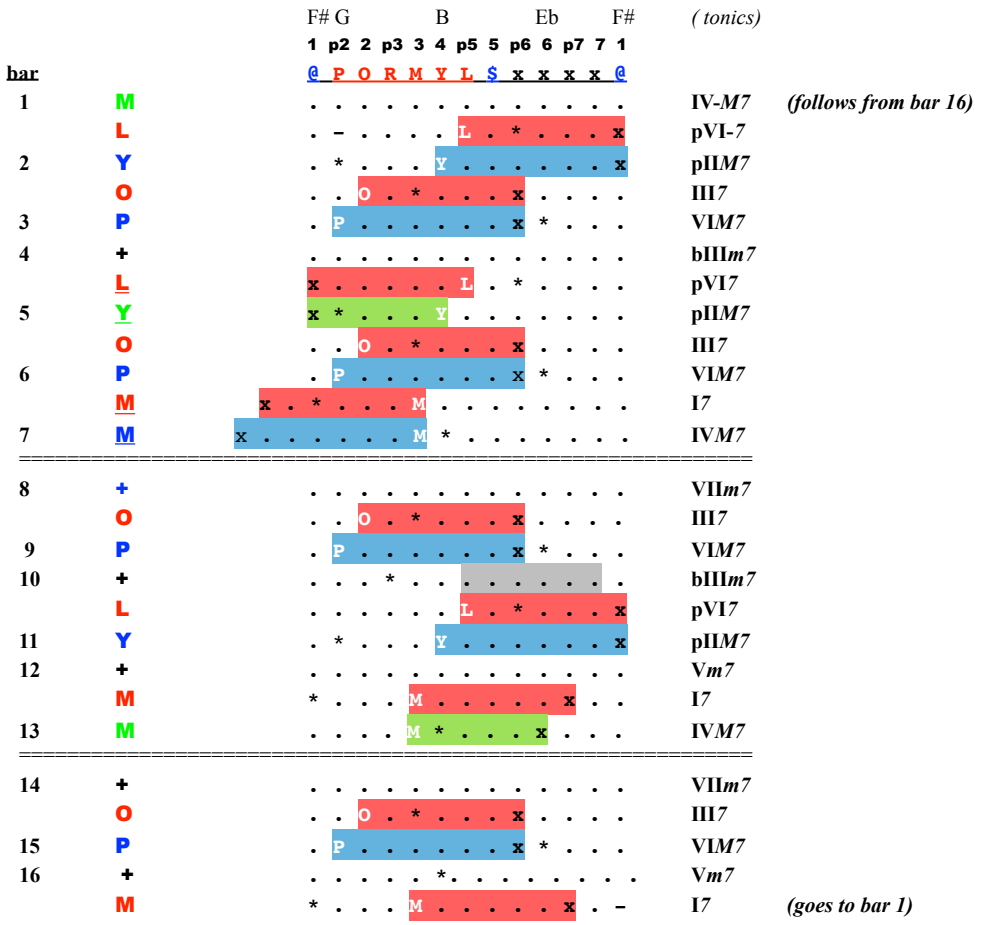
Giant Steps (original home tonic = F#)



The only trace left of three rapidly changing Ionian modes from distant key signatures is the **OML** harmony tritones morphing into the **PMY** fifos — each tritone always provides the same next fifo in the core harmonic sequence. The only thing to remember is this piece doesn't actually exercise the

modes identified by the tritones. This skeleton gives a good sense of the sound of the piece and is easy to play. Once this is "in the fingers" adding the missing details is easy.

Here follows a Lego-like view of the core harmony that includes some inversions to fit the flow. This simple harmony fits the written chords shown on the right (roots are indicated by * in the table). The blank spaces are easily filled in from the flow.



Observations

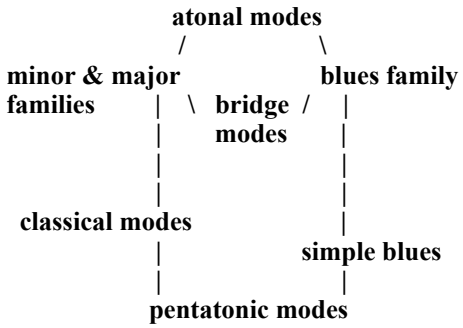
I spent countless hours trying to figure out this piece over the course of development of these ideas, and annoyed experts I consulted with half-baked ideas about its organization. An example of a half-baked idea was imagining that the essence of the piece is scales defined by double tritones from the whole tone **OML** scale (next chapter). No matter how much I massaged this view, it became complicated and sounded wrong. Experts concluded my "engineer's view" of piano music is inherently complex. It isn't — I just didn't fully understand it at the time. It took me a long time to understand the underlying symmetries that make it simple.

This interpretation is easy to understand and play and provides a dramatically simpler basis for variations than the rapidly changing, distant classical modes of the original.

CHAPTER 3: A HIERARCHY OF MODES

This is the "other half of architecture" beyond the chromatic classical domain. The many different approaches to chromaticism beyond this domain may be organized into the following hierarchy of modes determined by words from the alphabet. These modes are chromatic to begin with and so reduce the need for mode/tonic changes to add chromaticism.

The reason for presenting classical modes before presenting this hierarchy is they provide the alphabet, which provides a link to music notation, thus grounding this unconventional view in something familiar to anyone who studies the piano. This hierarchy is comprehensive enough to cover all the modes and beyond in scale dictionaries such as *The Source* What's more it goes beyond scale dictionaries by covering the modes for all possible keyboard positions of the home tonic. The hierarchy is comprehensive but not exhaustive; there's room for identifying more modes, if anyone sees the need.



The hierarchy provides the framework for developing and understanding chromaticism in a systematic way, bottom up, starting from simple pentatonic modes. Classical modes (e.g., minor Aeolian and major Ionian) were defined independently of this hierarchy in the previous chapter because they're the scales of key signatures but, in this view of them, they're developments of pentatonic modes. Classical modes and simple blues are shown at different levels because they're different in kind. Modes higher up on the two sides of the hierarchy are **mashups** of modes lower down, identified by words with more letters. Steps from pentatonic modes to basic blues and from pentatonic modes to classical modes yield scales that are different in kind, and the steps from there to family modes widens the difference. The family modes are similar in kind. Bridge modes (e.g., melodic minor, harmonic minor, harmonic major) provide a bridge between the two sides of the hierarchy. These modes emerge as sub-modes of those higher up in the hierarchy, but are important modes in their own right. They share with with classical modes the properties of seven notes and no adjacent half tones (they differ from them by one note); they share double tritones with modes higher up.

At the top of the hierarchy are atonal modes (e.g., diminished, whole tone) compatible with multiple tonics, that can often be understood as parents of modes lower down (or as departures that go outside these modes).

There are no 5-letter mode signatures because the implied scales would be too close to the chromatic scale to be usefully distinguished from it. The zone within the mode hierarchy that identifies parallel modes by 2-letter to 4-letter words is complex in music notation. Scales must be understood as enumerated notes relative to one of more than twelve possible different key signatures. The role of classical modes as the defining scales of key signatures makes music from scales that differ markedly from them misleadingly complex. The hierarchy makes this zone simple by pushing note symbols and

interval inversions down to a lower conceptual level.

The chromaticism of such changes is identified in music notation by accidentals in melody lines and chromatic chords in harmony, without any explicit indication of their origins, which may be classical modes, non-classical modes, a mix of the two, or ornamentation that has no scale implications. Explicit key-signature changes indicate the origin is in classical modes, provided there are no accidentals in the melody and no chords that go outside the key signatures, but much strongly chromatic music is written with a single key signature.

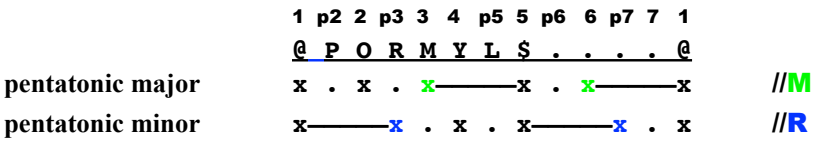
The examples that illustrate the concepts in this chapter are as follows.

Backwater Blues (blues)
Summertime (minor family)
Traumerai (major family)
Laura (sequential families)

PENTATONIC MODES

Pentatonic modes are fundamental to "folk music" in many cultures worldwide because they have no dissonance (no half tones, no tritones), which makes music from them singable, hummable or whistleable by ordinary "folk" with musical ears but no musical training.

Parallel pentatonic major and pentatonic minor modes are shown next. The scale frame provides three notes, one more note results from specifying major or minor tonality, and a final note follows from splitting a remaining major third into whole tones — the only way of splitting it that doesn't introduce half tones. This yields scales with inter-note intervals of whole tones and minor thirds, which are fundamental by any measure. The minor thirds are made to stand out visually in this notation by joining their notes by horizontal lines that say "no notes here." The modes may be understood as having the fifo mode signatures shown on the right.



The modes have no half tones but an instrument with half tones is required to play the modes for different tonics a half tone apart. The piano is such an instrument. The most visible instances of pentatonic modes on the piano are the clusters of 5 black piano keys: the major mode starts on the bottom black key of the 3-tritone cluster, and the relative minor mode on the next black key down. A lot of fun can be had and insight gained by experimenting with these black-key modes.

From Pentatonic Modes to Classical Modes

The six primary parallel classical modes follow directly from pentatonic modes as shown next (the 7th parallel mode, **altL**, follows from **//L**). The minor third intervals of the pentatonic modes are split into a half tone and a whole tone in all possible ways. The primary modes bring in all the notes of the piano's chromatic scale.

Examples of pieces using classical modes were provided in Chapter 2, so none are needed here.

	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
	@ P O R M Y L $ . . . . @	
pentatonic major	x . x . x———x . x———x	
//L	x . x . x——x–x . x——x–x	Lydian
//Y	x . x . x–x——x . x——x–x	Ionian
//M	x . x . x–x——x . x–x——x	Mixolydian
pentatonic minor	x———x . x . x———x . x	
//R	x——x–x . x . x——x–x . x	Dorian
//O	x——x–x . x . x–x——x . x	Aeolian
//P	x–x——x . x . x–x——x . x	Phrygian

From Pentatonic Modes to Blues

I once heard a jazz musician say in an interview that learning the blues before learning music notation paved the way for understanding everything that music notation later threw at him. I thought this sounded right. Starting with pentatonic modes makes sense because whole stretches of many simple blues pieces are in these modes. The unique, sad-happy sound of the blues comes into play by "bending" major-mode notes into minor (equating minor with sad and major with happy). The 6-note blues modes taught to beginners as "the" blues modes are pentatonic modes with one added "bent" note that gives the major mode a minor twist (**p3**), and makes the minor mode even more "bent" (**p5**). The latter is the famous flatted 5th, where "5th" refers to the 5th note of a primary classical mode, in other words the pitch center — calling it the flatted 5th in the context of blues scales with different numbers of notes is confusing.

	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
	@ P O R M Y L $ . . . . @	
//RM	x . x + x———x . x———x	6-note "bent" pentatonic major
//RL	x———x . x + x———x . x	6-note "bent" pentatonic minor
//RML	x . x x x x x x . x x . x	9-note family mode: blues "mashup"
//RM	x . x x x x . x . x x . x	8-note sub-mode: pentatonic "mashup"

These views of blues have an elegant simplicity. The family mode is not my invention, only this representation of it is. I first learned of it in conventional terms in a blues piano workshop at the then Jazz School in Berkeley. The family mode captures characteristic features of blues, namely mixed minor-major tonality, adjacent half tones, more scale notes than classical modes, and the addition of three "bent" notes of the master major classical mode (Ionian), namely flatted 3rd, 5th and 7th. It's possible to extend this into a 10-note mode by including the **Y** tritone, but this not only gets too close for comfort to the chromatic scale, it also loses something distinctive about the 9-note blues, namely a whole tone gap below the upper tonic and the top scale note. I find it more helpful to view the **Y** tritone as ornamental in the blues. Incidentally, the key signatures of the **//R** and **//M** classical modes are good choices for blues pieces because only two accidentals are required for the family mode.

The distinctive *sound* of the blues relative to the parallel classical modes is conveyed, for example, by the **M•-R•** transition between diminished triads anchored by major and minor scale notes within the blues mode. A signature blues sequence "crushes" **3** into **p3** or vice versa, expressing the fact that they're in the same mode. They're in different modes in classical major-to-minor modulations.

blues mixed major-minor

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	Y	L	$	.	.	.	.	@
M●	.	.	.	x	.	.	o	.	.	x	.	.	
R●	.	.	.	x	.	.	o	.	.	x	.	.	.

classical major-to-minor

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	Y	L	$	.	.	.	.	@
M▼	.	.	.	.	x	.	.	o	.	.	.	x	.
R▲	.	.	.	x	.	.	.	o	.	.	x	.	.

EXAMPLE: ***BACKWATER BLUES***

This is a simple, 3-chord, 12-bar blues, which I learned in the blues piano workshop mentioned earlier, as representative of "probably half the blues pieces played by pop and jazz musicians." The home tonic of the source is F. It's a simple piece but exploring it plumbs musical depths. The simple melody line establishes the home tonic by resolving to it in several places. Commas at the ends of phrases only identify the last note of the phrase, leaving open whether it's sustained or not. Timing is left open. Swing-feel 4/4 time would be appropriate and could be notated by an extra downbeat line. The harmony is all tritone-based, which is not uncommon in blues. The tritones are from dominant seventh chords **I***7* (**M**), **IV***7* (**R**) and **V***7* (**Y**). Offsetting the ups and downs in the melody line isn't needed here because everything is mostly obvious downs.

Backwater Blues (original home tonic = F)

⚓ M R M M
♫ | 5 4 p3 1, | 5 4 p3 1, |—— |—— |

⚓ R R M M
♫ | 5 4 p3 1, | 5 4 p3 1, |——| — 5 p7 5 |

⚓ Y R M M
♫ | p7 p7, 5 | 1 p7 5 p5 4 p3 1, |—— |—— |

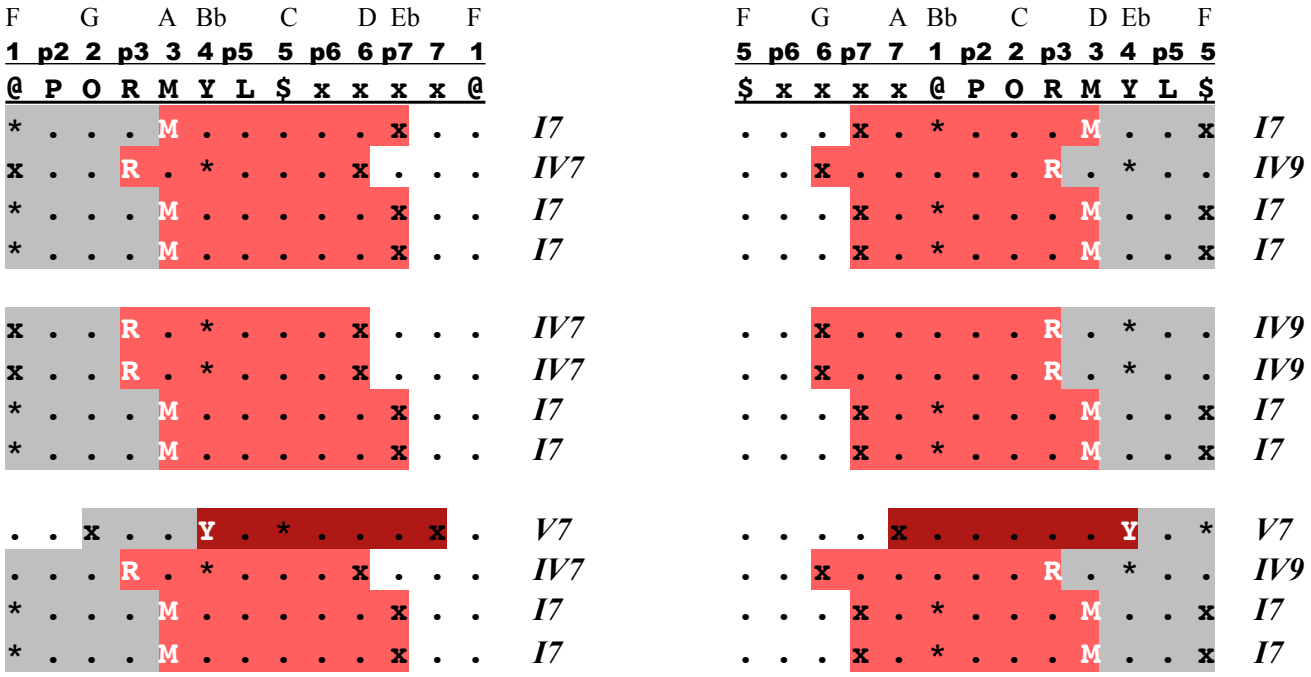
The first bar illustrates a feature of many blues pieces that helps in establishing both the fact of a blues and identifying the home tonic, namely opposite tonality of melody (minor) and harmony (major). As shown next, this bar implies, all by itself, the shared blues scale **//RM**.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	Y	L	$	x	x	x	x	@	
major harmony in bar 1	.	.	.	.	x	.	.	.	.	.	x	.	.	
minor melody in bar 1	x	.	.	x	.	x	.	x	.	.	.	.	x	
implied shared blues mode	x	.	+	x	x	x	.	x	.	x	x	.	x	**//RM**

The melody in bar 10 adds tritone **L**, rounding out the shared mode to **//RM.L**. The actual melody scale in all but bar 10 is pentatonic minor, and in bar 10 is the 6-note blues extension of it identified by **//RL**, but all of the melody and harmony originates in the family scale, with one exception — one appearance of **Y** in the harmony as a substitute for **L**. The tritone is a "turnaround" marker that identifies the beginning of the last four bars of a 12-bar blues. Blues tritone **L** would seem to be a natural turnaround marker, but is substituted by **Y** because the **V***7* chord containing it makes for a simple chord progression —all chords the same kind. The **V** chord containing **L** is different in kind, namely **V***M7(11)*.

Harmony based on un-inverted and inverted tritone cores is shown next. It's easy to see how these simple voicing lines follow from altering tritone-based octave shapes. Grey box prefixes and suffixes could be used to suggest these shapes in anchor lines but this tends to be overkill because the choices

are so simple and visibly obvious on the keyboard. As before, chord roots in the Lego-like shapes are marked *.



Simple 3-chord blues pieces such as this often use dominant-7 chords **I***7*, **IV***7* and **V***7*. The **V***7* chord containing tritone **Y** is such a familiar feature of music that it tends to be borrowed for blues to round out a trio of chords of the same kind. This is so common that many musicians understand basic blues to be defined by this chord trio. This is not general because blues pieces in general use chords of too many different kinds for chord content to be a useful characterizing feature. Tritone content is more helpful and the **//RM.L** signature is particularly helpful because it captures fundamental features of the blues in a compact and intuitive way.

A Sampling of Blues Chords

Apart from simple 3-chord blues such as this, blues chords are all over the map. This is because the classical modes that determine the symbols for seventh chords are different in kind from blues modes (different numbers of notes, no adjacent half tones, only single tritones). Blues chords in general are basic chords altered to fit a blues mode (think "banging square pegs into round holes"). A sampling of such chords is provided next. The sharp and flat suffixes are degree numbers in the 7-note scales that define the basic chords, and are quite confusing relative to blues scales with more or fewer notes. The chord symbols are intended only to illustrate the complexity — knowing them is not required to read on. The takeaway here is that combinations of building blocks *are* the chords in context. An amazing amount of chord complexity can be introduced by holding the tritone and changing the completion fifo (or vice-versa). Finding a chord symbol to identify such combinations can sometimes be difficult.

Complex chords such as the ones on the right often boil down in context to a tritone plus a voicing interval that implies a completion fifo, or two overlapped tritones (a double tritone). Roots on the right

are all over the map, giving lie to the idea that blues is characterized by roots **I-IV-V**. The roots are indicated by * entries in the Lego-like representations of the chords. Color-coding of fifo completions in parentheses on the left is mirrored by color-coding of corresponding **x** entries.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
	@	P	O	R	M	Y	L	$	x	x	x	x	@	P	O	R	M	Y	L	$	examples
M•	.	.	.	.	*	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.	III*dim*
(@)M	x	.	.	.	M	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.	I*7*, III*m7b5#5*, III*m7b5b13*
M(R)	.	.	.	.	M	.	.	–	.	.	x	.	*	.	.	x	.	.	.	.	I*7#9*
R•	.	.	.	*	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.		pIII*dim*
R(I)	.	.	.	R	.	*	.	.	.	x	.	.	x	.	.	.	.	.	.	.	IV*7*
R(O)	.	.	.	R	.	*	.	.	.	x	.	.	.	.	x	.	.	.	.	.	IV*13*
(O)M	.	.	*	.	.	.	M	.	.	x	.	.	x	.	.	.	.	.	.	.	II*7*
(O)I	.	.	x	.	.	Y	.	*	.	.	.	x	.	.	.	.	.	.	.	.	V*7*
I($)	.	.	.	.	.	Y	.	*	.	.	.	x	.	.	x	.	.	.	.	.	V*7*
L($)	.	.	.	.	.	.	L	*	.	.	.	x	x	.	x	.	.	.	.	.	V*M7(11)*
L•	.	.	.	.	.	.	*	.	.	x	.	.	x	.	.	.	.	.	.	.	pV*dim*
L($)	.	.	.	.	.	.	L	*	.	.	.	.	x	.	x	.	.	.	.	.	V*M7#3*
(P)L	.	x	.	.	.	.	L	.	–	.	.	.	x	.	.	.	.	.	.	.	**tritone sub for V*M7#3***
ML	.	.	.	.	M	.	*	.	.	.	.	x	.	*	.	.	.	.	.	.	I*7b5*, pV*7b5*
RM	.	.	.	R	*	.	.	.	.	.	x	x	.	*	.	.	.	.	.	.	I*7#9(13)*, III*7b9b5*
RL	.	.	.	R	.	*	L	.	.	.	x	.	*	.	.	.	.	.	.	.	I*dim7*, IV*7b9*

See Appendix C for more on chords.

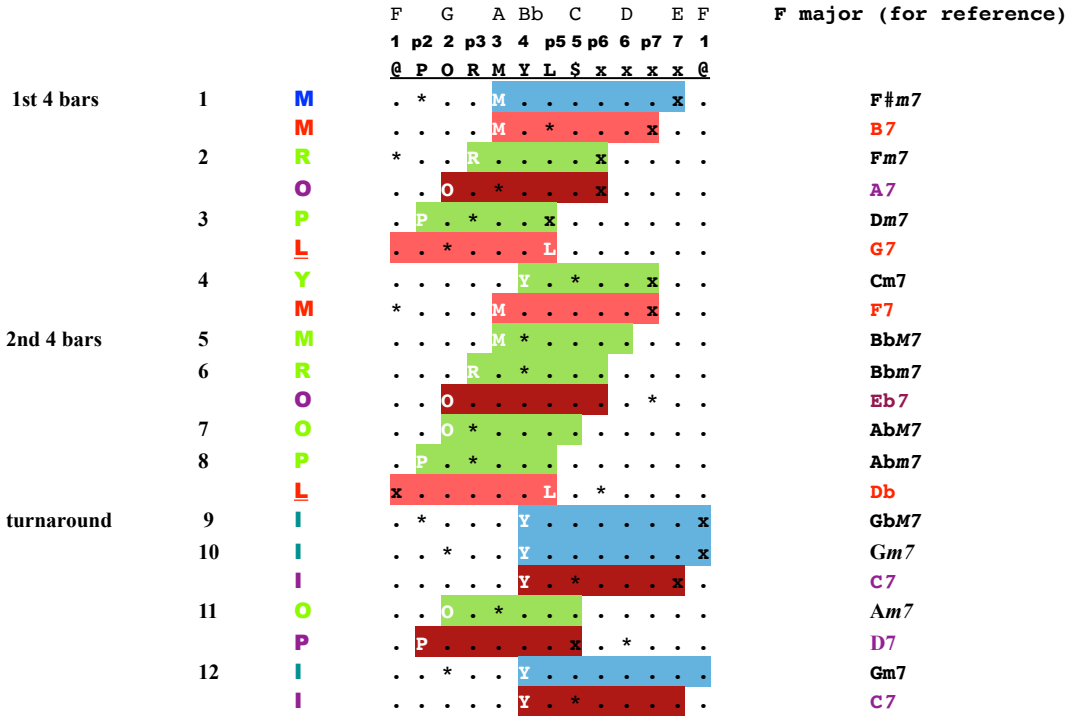
Some Variations of 3-Chord Blues

The following table shows variations of 3-chord blues that progressively depart from the **I-IV-V** format but stick to only basic seventh chords.

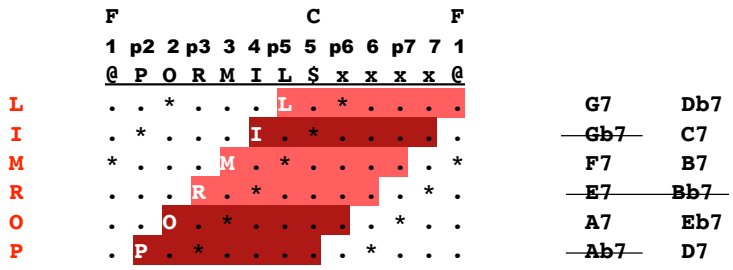
															turnaround			
1	M	M	M	M	R	R	M	M	Y	Y	M	M						
2	M	M	M	M	R	R	M	M	Y	R	M	Y						
3	M	R	M	M	R	R	M	M	L	Y	M	Y						
4	M	R	M	M	R	R	M	P	L	Y	M	Y						
5	M	R	M	M	R	R	M	P	+	Y	M	+	Y					
6	M	R	M	M	R	O	M	P	L	Y	M	+	Y					
7	M	R	M	+ M	R	O	M	+ P	+	Y	+ P	+	Y					
8	M	R	M	+ M	R	O	+	P	+	Y	+ P	+	Y					
9	M	R	M	+ M	R	+ R	M R	O P	+	Y	+ P	+	Y					
10	+	+ O	+ L	+ M	R	R	+	+ L	+ Y	+ Y	+ P	+ P						
11	+	+ +	+ +	+ M	+	+	+	+	+	Y	+ +	+ P						
12	+	+	+ +	+ M	+ +	+	+	+	+	Y	+ +	+						
13	+	+	+ +	+ M	+	+ O	+	+ L	+	+ Y	+ P	+ P						
14	+	+ O	+ L	+ M	+	+ O	+	+ L	+	Y	+ P	+ Y						
15	+	+ O	+ L	+ M	+	+ R	+	+ L	+	Y R	+ P	+ Y						
16	+ M	+ O	+ L	+ M	+	+ O	+	+ L	+	+ Y	+ P	+ Y						
17	+	+ M	+ +	+ +	+ +	+ R	+	+ P	+	+ +	+ +	+						

This is an adaption of a table of chord progressions for tonic F handed out in the blues piano workshop mentioned earlier. It further gives the lie to idea that blues is determined by **I-IV-V** roots. The table vividly illustrates the simplicity of developing voicings of notationally complex chord progressions from tritone content. Tritone chords are represented by their anchors and fifo anchors are represented by placeholder plus signs, leaving fifos to be faired in from context. The two highlighted anchor lines are representative of the two parts of the table. The top part shows progressively different variations of *Backwater Blues* (the highlighting indicates combining segments from different lines). This shades into *bebop blues* chord progressions in the bottom part, of which the highlighted bebop blues line is a representative example.

Here follows the development of the highlighted bebop blues line, which is simplicity itself in building-block terms. Fifos are faired in between fixed tritones by morphing in a visibly obvious way.



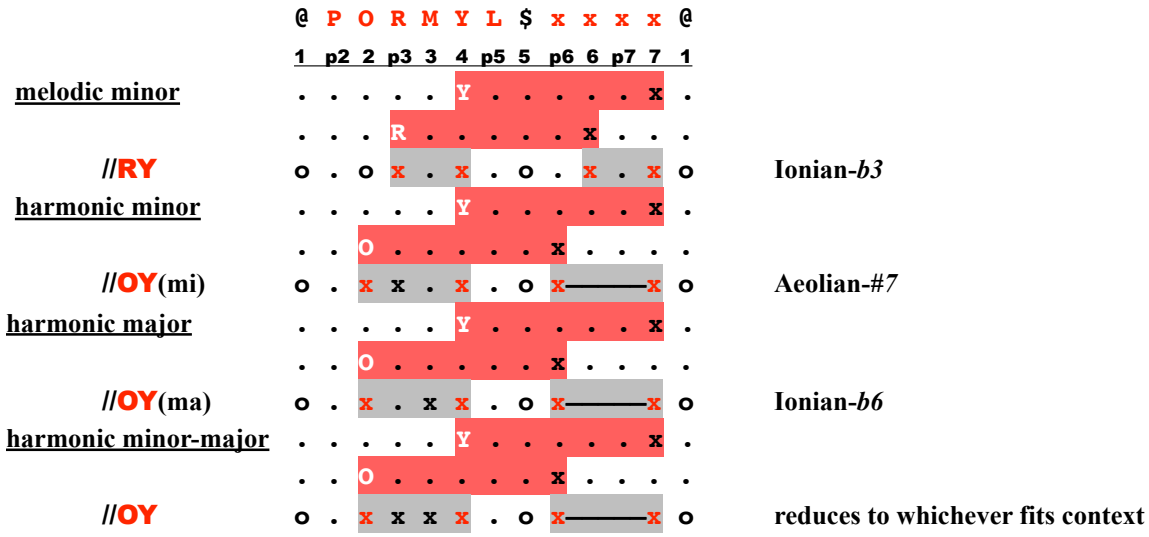
As shown next, pairs of tritone substitute chords are used extensively (same tritone, root offset by a tritone, all non-tritone notes different) for blues of a given tonic **@1** (shown as F at the top for concreteness).



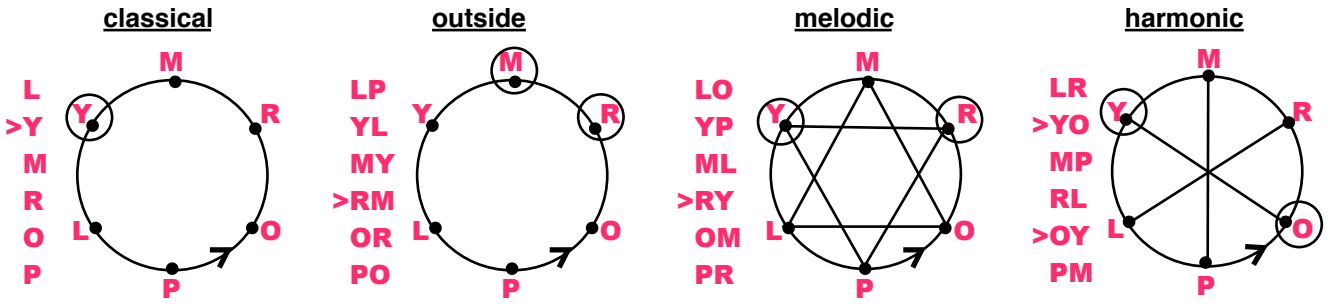
Complex as such bebop blues chord progressions are, the chords themselves are plain seventh chords, which is not representative of blues in general. In fact, the bebop progressions don't sound like blues by themselves: they need a melody line to provide a blues sound.

BRIDGE MODES

We have now progressed up the two sides of the mode hierarchy to the **bridge modes**, so called because they're sub-modes of higher family modes on both sides. As shown next, the master modes are enriched classical modes with seven notes, no adjacent half tones, and one note lowered or raised a half tone. They're also more than this because they're the first step in understanding non-classical modes in terms of multiple tritones, which is important because it's simple. For the **OY** modes, the parenthetical additions of "mi" for minor and "ma" for major are needed because the double tritones alone are ambiguous. This ambiguity is finessed by defining an understood 8-note master mode with both master tonalities and leaving the choice to context. This simplifies things by reducing the mode suites to two, a 7-note **melodic suite** and an 8-note **harmonic suite**. How this works will emerge as we go along.



Understanding the parallel modes is helped by viewing the **PORMYL** alphabet as a circular loop, in which the modes are determined by master words transposed by alphabet steps that wrap around to the other end of the alphabet, as follows ("**>**" identifies the master modes).



This picture starts off with classical modes as a reminder that the 2-letter words determining the modes are the single letters of the classical modes in the same order, with one letter added after it, provided by a transposed master word. The "outside" suite of modes is identified as such because it contains adjacent half tones not allowed in classical modes. The **melodic modes** are determined

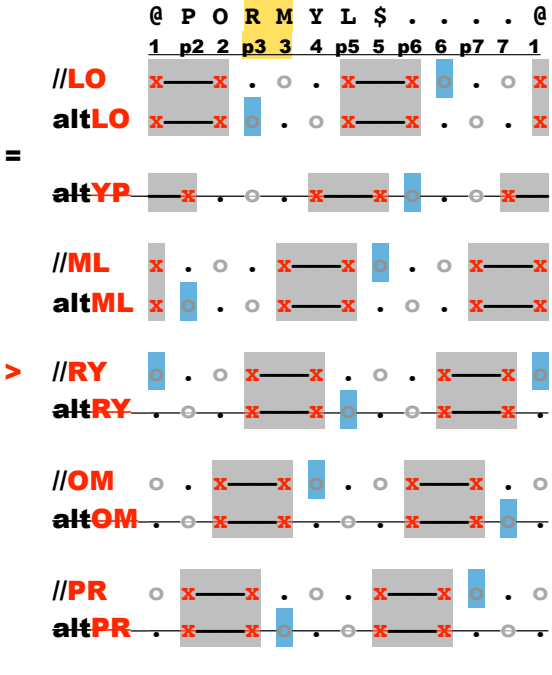
unambiguously by six unique double tritones offset by whole tones. The **harmonic modes** are determined by only three unique double tritones offset by minor thirds, anchors of which are repeated in the opposite order.

The circular loop places the anchors of the double tritones within the home octave. The parallel modes fill in the gaps in the home-octave scales with whole tones going up the keyboard. The mode signatures provide clues to the family origins of the modes. The only problem is this: unlike classical modes, in which successive primary modes in alphabet order differ by only one note, successive primary modes here are relatively distant from each other. This is a particular problem for the harmonic modes, in which the differences are intricate. This means that the following tables are best used as references that determine the possible modes, leaving the details of the modes to their families of origin.

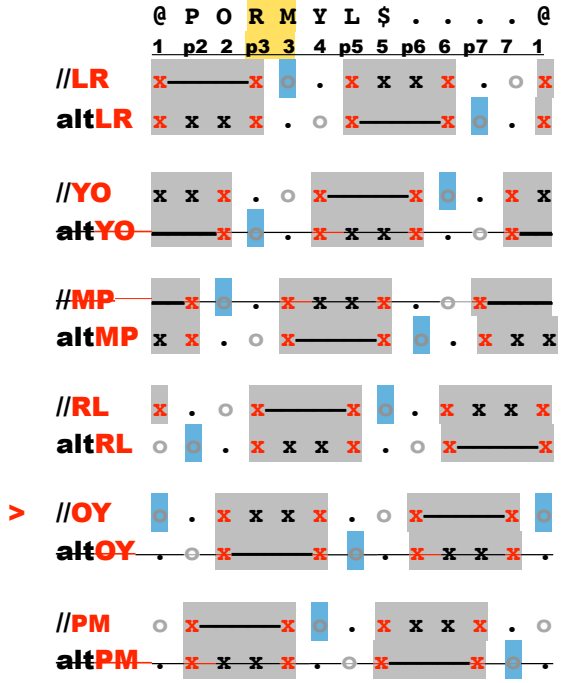
Mode Tables

Two mode tables fill in the details of the melodic and harmonic modes, for reference. The six "**//**" modes are **primary modes** that determine the rest. However, the six primary modes are no longer all parallel — only modes without strikethroughs in the tables are parallel, which includes four modes containing tritone **L**. Highlighting in the tables means the same as before: yellow determines mode tonality; grey identifies **constructor** shapes that leave only whole tones for completion; blue identifies transposed master tonics.

Melodic Modes (7 notes)



Harmonic Modes (8 notes)



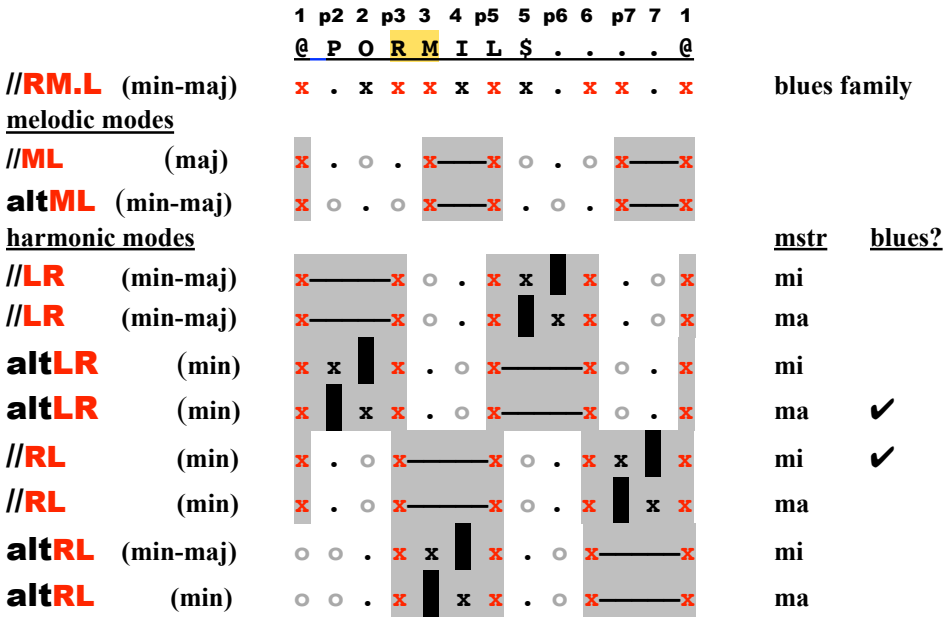
The melodic and harmonic modes are very easy to construct on the keyboard because they're organized in a visibly simple way around the whole tones and minor thirds of the constituent double

tritones. The dashes in these intervals say "no notes go here." The tritones are given. The primary modes are determined by adding one none-tritone note in just the right place, namely the blue-highlighted master tonic, which goes up by fifths or down by fourths in the tables. The 8-note harmonic modes reduce to 7 notes by omitting one non-tritone note of the filled minor third **xxxx** that doesn't fit context.

These modes in music notation are among the most complex musical objects I have encountered in my explorations. Appendix D gives a sense of this. It provides all the modes spelled out in chromatic scale symbols with an associated alphabet soup of more exotic mode names than there are modes. The multitude of names, combined with intricate scale differences that seem arbitrary from one mode to the next, make the scales difficult to distinguish or to hold in the mind in these terms.

Examples

The modes appear naturally as sub-modes of minor, major and blues families of modes, as illustrated next for the blues family, already introduced. The tonalities in parentheses following the mode signatures are of the parallel modes. The tonalities on the right are of the master mode of the 7-note harmonic modes, determined by the blacked-out notes in body of the table. The check-marked modes are exact sub-modes of the **RM.L** family mode. The others are variations that may be provided by passing notes or more general mode signatures such as **RMYL** or **P.RM.L**.



Other Suites of Parallel Modes

The 8-note "outside" mode based on the double tritone **RM** is is a blues mode, determined earlier by a **mashup** of parallel minor and major pentatonic modes (it's also a mashup of parallel Dorian and Mixolydian classical modes, suggesting possible key signatures for blues pieces). It's not a **bridge mode** because it's not an enriched classical mode (it includes adjacent half tones and has minor-major tonality). No suite of parallel modes based on a master **RM** mode exists in music practice, to my knowledge, probably because blues is sufficiently chromatic that there's there's no need. Nor do suites of parallel modes of any kind based on more than two tritones exist, probably for much the same reason. That said, such modes are easy to define if anyone sees a need.

FROM CLASSICAL MODES TO MAJOR AND MINOR FAMILIES

The minor and major families follow from a mashup of //O and //Y classical modes that's analogous to the mashup of pentatonic minor and major modes on the blues side of the hierarchy. As shown below, the mashup yields a 10-note minor-major mode identified by //ORMY that sometimes appears as a melody mode in strongly chromatic pieces (e.g., *Lush Life* in Chapter 4), but is mainly a parent of 9-note minor and major family modes that differ by one note in the lower fifth of the scale frame (grey shading). The 9-note modes follow from substituting tritone **M** by the fourth **Y** or tritone **R** by the fourth **M**, which opens a gap in the lower fifth while leaving the upper fourth untouched. The modes shown all have visibly simple, asymmetric forms. These modes are close to the chromatic scale but are unambiguously tonal because of asymmetry.

		1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
		@ P O R M Y L $ . . . . @	
//ORMY		x . x x x x . x x x x x x	10-note mashup of //O & //Y
//OR.YY	(or //OR.Y⊕)	x . x x . x . x x x + x x	9-note minor family
//O.MMY	(or //O.MY⊕)	x . x . x x . x x + x x x	9-note major family

Here follows a summary of important sub-modes and variations of them. This makes logical sense of modes that can seem ad hoc in conventional terms. As with the blues, the melodic and harmonic modes (names underlined below) are master modes of suites of parallel modes that are analogous to the suites of parallel classical modes in having 7 notes and no adjacent half tones; the difference is two tritones instead of only one, making for richer harmony. The 8-note harmonic minor-major mode is different in kind, namely a mashup of harmonic minor and harmonic major sub-modes. It's particularly useful as a composite master mode because of its simple, regular form, which reduces in a simple way to the irregular forms of the sub-modes.

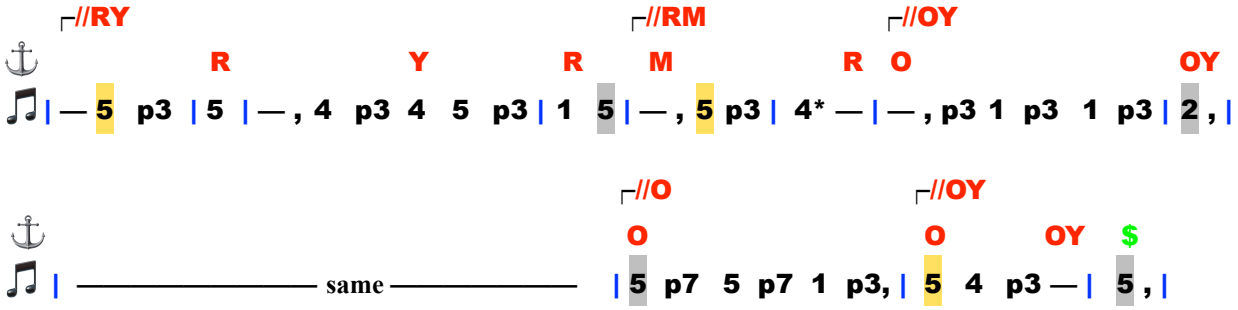
	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
	@ P O R M I L $ . . . . @	
//OR.Y	x . x x . x . x x x . x x	8-note "bebop" melodic minor
//R.Y	x . x x . x . x . x . x x	7-note melodic minor,
//O.M	x . x . x x . x x . x . x	7-note melodic major
//OR.Y	x . x x . x . x x———x x	7-note harmonic-minor
//O.MY	x . x . x x . x x———x x	7-note harmonic-major
//ORMY	x . x x x x . x x———x x	8-note harmonic minor-major

MINOR FAMILY: *SUMMERTIME*

I learned this version of this well known minor piece by Gershwin some years ago in a piano comping course given by Susan Muscarella at the then Jazz School in Berkeley (now the Jazz Institute). The home tonic of the source is D and the key signature is one flat, identifying Aeolian mode **//O**, but the piece departs significantly from this mode. In the following summary, the anchor line is from a chord progression that will be presented later as a result. The only reference I have for this version of the piece is my course notes. Like *Backwater Blues*, it's a simple piece, the exploring of which plumbs musical depths.

The 6-note melody line is pentatonic minor with one added note (**2** in bar 8) that's in most minor scales. The harmony is strongly chromatic but is more than ornamental because, with the exception of bars 5-6, the tritone harmony self identifies a succession of sub-modes of the minor family mode with signature **//OR.Y**⊕ (recall that the plus superscript means fill in the top fourth).

Summertime (original home tonic = D)



The modes are summarized next. The arrows at the right indicate some possibilities for inter-mode segues that are exploited in the above anchor line.

	D	E	F	G	A	Bb	C	D	
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	Y	L	$	x	x	x	x	@	
6 melody notes	x	.	x	x	.	x	.	x	———	x	.	x		
are from Dorian //R	x	.	x	x	.	x	.	x	.	x	x	.	x	
family //OR.Y⊕	x	.	x	x	.	x	.	x	x	x	+	x	x	
yields melodic minor //R.Y	x	.	x	x	.	x	.	x	.	x	.	x	x	↓ **Dorian-#7**
and harmonic minor //O..Y	x	.	x	x	.	x	.	x	x	———	x	x	↓ **Aeolian-#7**
and Dorian //R	x	.	x	x	.	x	.	x	.	x	x	.	x	↕
//RM is ornamental	x	.	x	x	x	x	.	x	.	x	x	.	x	↑ **8-note blues**

The skeleton melody line and the anchor line shown above are sufficient to play the melody with 3-note octave shapes for the harmony. The result has depth because of the three notes, and variety because of the tritones.

Adjusting the harmony to provide more shape variety is easy, as illustrated next for bars 1-8 plus the resolution bar 16. The numeric suffixes or prefixes on anchor symbols indicate offsets of voicing notes above or below the core, measured in half tones. These could be added to the above anchor

symbols to indicate desired voicings. These shapes voice the chords on the right, if not always in place then in the flow (omitted chord roots are identified by dashes). All the voicings except for the **II** chord are rootless, demonstrating once again that chord roots are not inherently fundamental to the sound of a chord progression.

The suffixes for on anchors show that fit the scales. For example, the opening tritone with a fourth on top (often called "all fourths" because a tritone is an augmented fourth) has a size of 6+5=11 half tones, a half tone less than an octave. This makes finding the shape on the keyboard easy — find the treble note a half tone below the bass note an octave up, and complete the shape by adding an internal note a tritone above the bass note. All-fourths shapes generally imply complex chord symbols because this is not a basic chord shape from a classical mode.

bar	core	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3	a voicing of	
		@ . O R . Y . $ x x x x @ . O R		
2	R	. . . R . * . . . x . . . . x .	IV-*13*	
3	Y	. . . . . Y . * . . . x . . . x	V-*7#5*	
4	R	. . . R . * . . . x . . . . x .	IV-*13*	
5	M	* . . . M . . . . . x . – . . x	I-*7#9*	**ornamental**
6	R	* . . R . . . . . x . . – . x .	I-*m6(9)*	
7	O	. . * . . . . . x	II-*m7b5*	
8	OY	. . O . . Y . * x . . x	V-*7b9*	
16	$	* . . x . . . $ . . . . x . . –	I-*m*	**triad resolution**

As summarized next, there's a strong contrast between the simplicity of these shapes and the complexity of the corresponding chord symbols. The adjustment of basic chord symbols to fit scales, or places in scales, they don't quite fit is like "banging square pegs into round holes" — the results tend to be messy.

chord scale notes (bars 1-8)	chromatic scale notes
root of ***I-m6(9)***, "5" of ***IV-13***, "7" of ***II-m7***	**1**
root of ***II-m7b5***, "9" of ***I-m6(9)***, "13" of ***IV-13***	**2**
"7" of ***IV-7***, "#5" of ***V***, "#9" of ***I-7#9***	**p3**
"b5" of ***II***, "b9" of ***V-7b9***	**p6**
"b5" of ***IV-7b5*** (not in the above but could be)	**7**

MAJOR FAMILY: *TRAUMERAI*

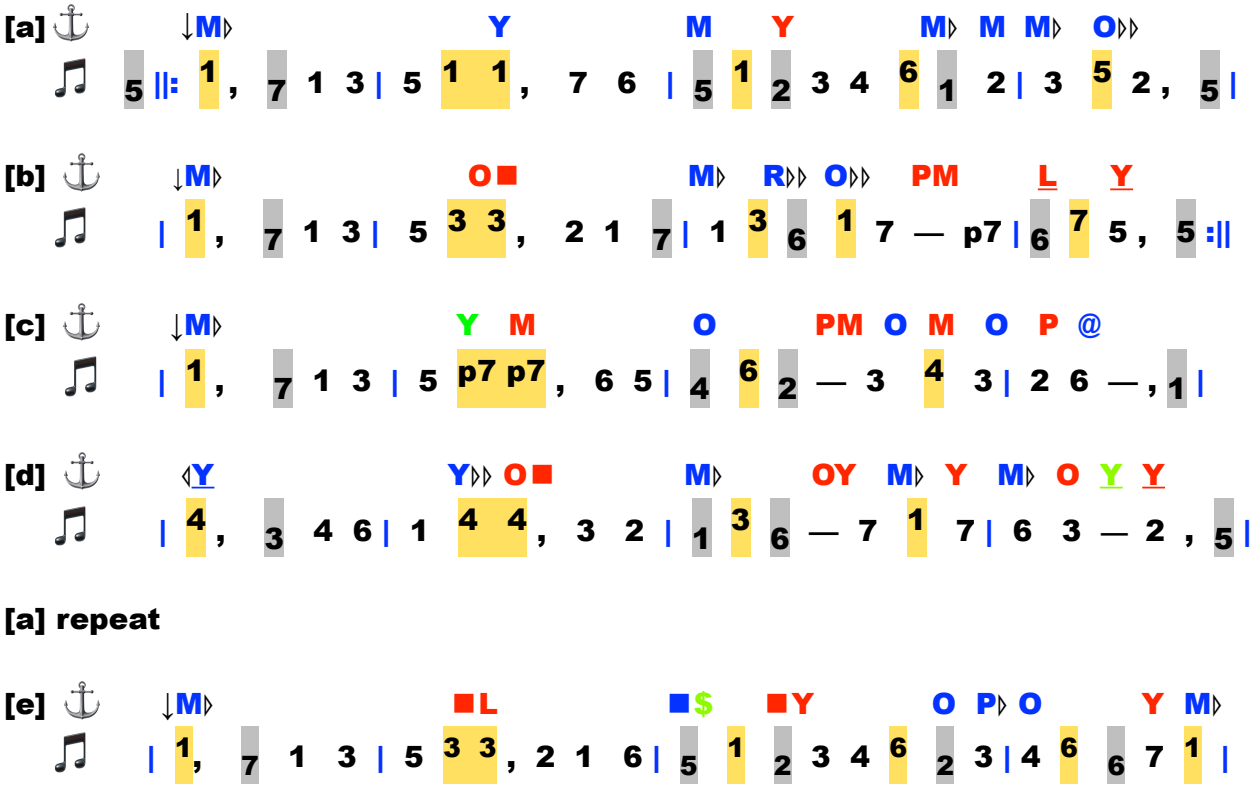
The example is Schumann's beautiful *Traumerai* (*Classical Fake Book*, 2nd Edition, Hal Leonard (2013). The home tonic of the source is F. As shown below, the melody is straight **//Y** with a mashup of it and **//M** in [b]-[c] that adds one note in a few places (**p7**). The 8-note mashup could be identified as **//MY** (no other mode in this document has this signature) but it seems simpler to think of the melody plus harmony of all sections as governed by the major family mode **//O.MY⊕**, with tritones **P** and **L** providing ornamental harmonic transitions. The melody line resolves to relative tonics **@2-@6-@2-@5-@1** of this mode that are also relative tonics of the **//Y** mode, but thinking in terms of the more general mode helps in remembering melody and harmony resolutions.

Rhythm is 4/4 with most chords on downbeats. The exceptions are diminished seventh chords (double tritones) on upbeats that provide ornamental transitions to following chords on downbeats.

The harmony in music notatation looks misleadingly simple but is actually complex in aggregate due to a mix of different chord types in different inversions, on a jumpy bass line formed of chord roots and bass notes of inversions. This interpretation replaces the jumpy bass line by a smoothly flowing anchor line that represents both building blocks and outer intervals of compound shapes (augmented fifths or major sixths) going up or down from it. It makes its own decisions about chord inversions.

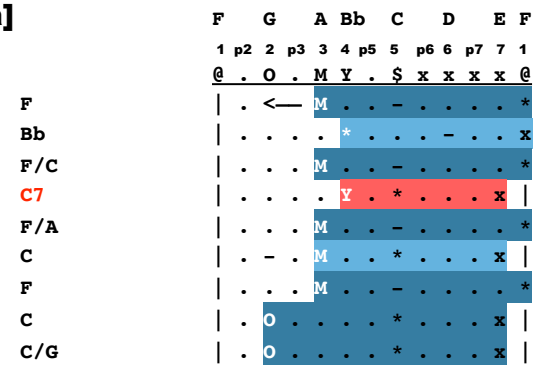
The thin core harmony captures the essential sound of the piece effectively, and implies thicker harmony that's easily filled in from context (most of the missing notes are the pitch center or the home tonic). The ups and downs of the harmony roughly follow those of the melody line, but by smaller intervals. The main challenge is remembering how the melody and harmony go together in the latter half of each section, where the resolution patterns to different relative tonics are different.

Traumerai (original home tonic = F)

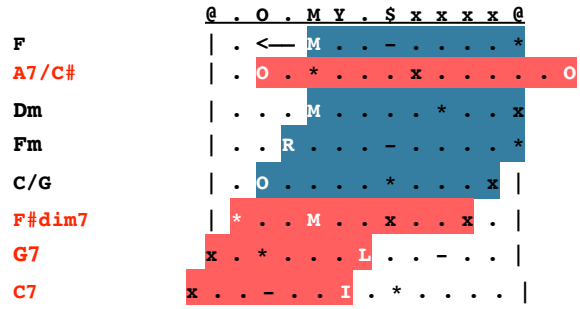


Here follow explanations of the notated harmony. The chords in the fake-book are all over the map, so I used them more as suggestions than specifications. I first learned this piece from a Schumann score and then forgot the details, but this sounds as I remembered it.

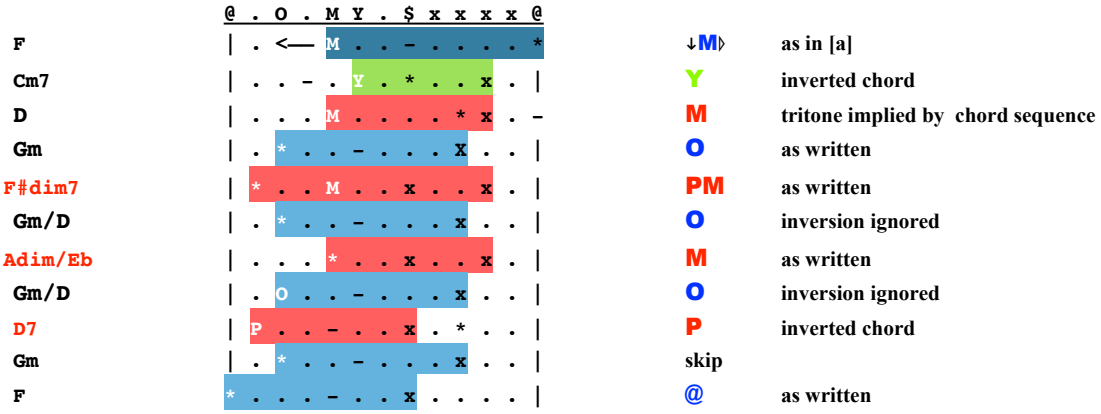
[a]



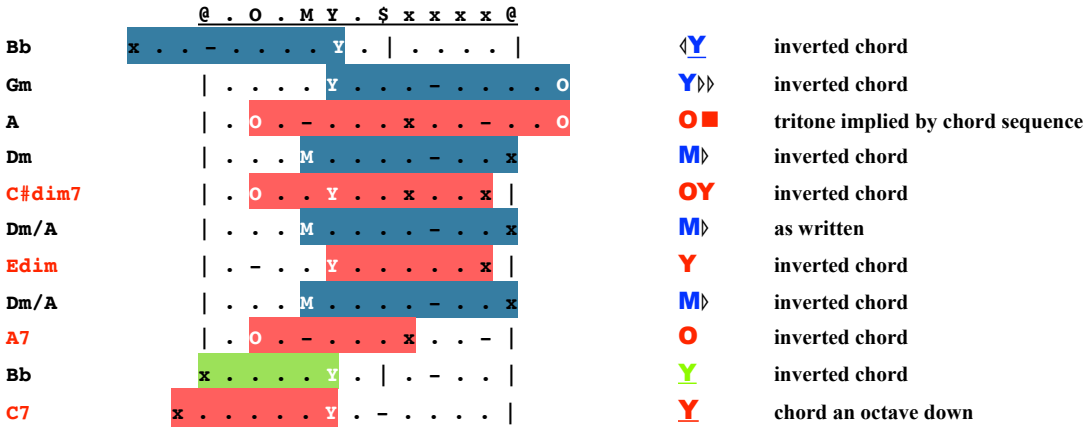
[b]



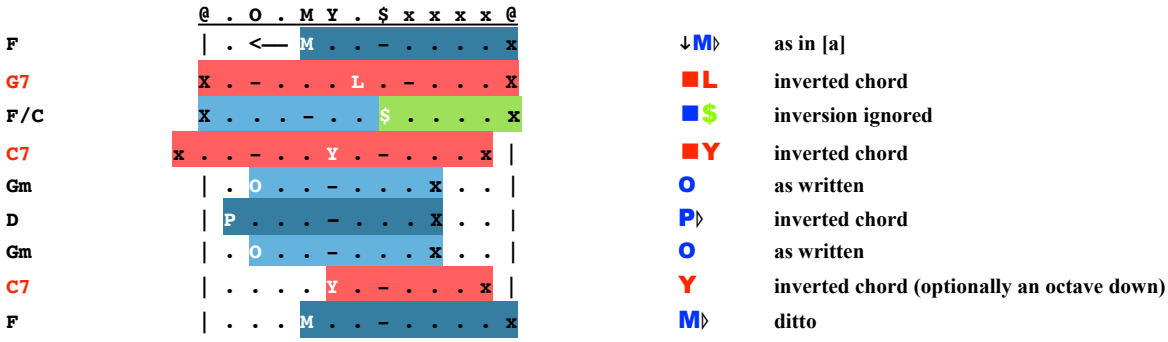
[c]



[d]



[e]



Observations

The meanings of additions or alterations identified by prefixes or suffixes on building blocks are much simpler than for additions or alterations to chord symbols because only two kinds of building blocks exist, namely tritones and fifos, and only six anchors place them directly on the keyboard.

MULTIPLE INTERPRETATIONS: *LAURA*

My source for the summary shown next is *The Jazz Book*, John Brimell, CPP/Belwin, 1989, p24. The home tonic of the source is C. The source book is sub-titled *Today's Easy Adult Piano* but this piece is "easy" only in the sense that the key signature is empty, the density of notes on the page is low, and playing it without thinking about the changes is easy. The not-so-easy part is getting a handle on the changes, due to the presence of adjacent half tones in often-sparse melody lines, and to seemingly irregular relationships between the melody lines and the richly chromatic harmony. In the following summary, fifos are omitted at points marked **+** because understanding the changes requires only the tritones, and the fifos are easily added from context determined by the tritones.

Laura (original home tonic = C)

(a) ⚓ + RL +

♫ | 7* | — , p7 7 p7 p5 5 | 6* 3 | — , |

(b) ⚓ + PM + R

♫ | 6* | — , p6 6 p6 3 4 | 5 — | — , |

(c) ⚓ OY + Y + L RL

♫ | — p3 4 | — , p3 4 | p3 4 | 5* 2 | — , p3 5 p7 | p3* p6 |

(d) ⚓ LO L + + Y OY

♫ | — , 6 | 2 — | — , 4 1 — 2 |

repeat (a)-(b) and then go to (e)-(f)

(e) ⚓ + Y● + L

♫ | 4 5 | — , 3 4 p6 7 | 2 1 2 | — , 7 1 |

(f) ⚓ LO OY Y OY + +

♫ | p3 p3 | — , 3 p3 2 6 7 | 1 — |

An interpretation in terms of mixed, parallel, classical modes: The melody lines are from successive classical modes that go from major in (a)-(b) to minor in (c) to major in (d)-(f). Ornamental notes (highlighted) "bleed" between the classical modes. The pentatonic major mode of (d) is a sub-

mode of the resolution Ionian mode of (e)-(f). The successive classical modes are determined by single tritones going down by whole tones through (a)-(b)-(c) and then up a minor third to (e)-(f). This may be interpreted as Ionian tonics going down by whole tones through (a)-(b)-(c) and then down a minor third to (e)-(f), except this is not very helpful because the modes are daisy-chained together instead of resolving. It's much simpler to think of this in terms of parallel mode changes.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	x	x	x	x	@	
(a) melody notes	.	.	.	.	.	.	x	x	.	x	x	x	.	
//L	x	.	x	.	x	.	x	x	.	x	.	x		**major**
(b) melody notes	.	.	.	.	x	x	.	x	x	x	.	.		
//M	x	.	x	.	x	x	.	x	.	x	x	.	x	**major**
(c) melody notes	.	.	x	x	.	x	.	x	x	.	x	.	.	
//O	x	.	x	x	.	x	.	x	x	.	x	.	x	**minor**
(d) melody notes	x	.	x	.	x	.	.	.	.	x	.	.	x	
//M	x	.	x	.	x	.	.	x	.	x	.	.	x	**major**
(e) melody notes	x	.	x	.	x	x	.	x	x	x	.	x	x	
/Y	x	.	x	.	x	x	.	x	.	x	.	x	x	**major**
(f) melody notes	x	.	x	x	x	.	.	.	.	x	.	x	x	
//Y	x	.	x	x	x	x	.	x	.	x	.	x	x	**ornamented major**

<u>An interpretation in terms of parallel harmonic-minor-major modes:</u> I find the local irregularity of (a)-(b)-(c) unsatisfying. The piece sounds like each melody phrase is from a fully defined parallel mode of its own. Out of curiosity, I looked for an interpretation in terms of non-classical modes and found the following one in terms of parallel modes of the harmonic-minor-major mode determined by double tritones in the harmony. There's an elegant regularity about this that fits the elegant regularity of the melody phrases to the ear.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	I	L	$	x	x	x	x	@	**<u>home tonalities</u>**
(a)	**melody notes**	.	.	.	.	.	.	x	x	.	x	x	x	.	
	//RL	x	.	x	x	———	———	x	x	.	x	x	x	x	**minor**
(b)	**melody notes**	.	.	.	.	x	x	.	x	x	x	.	.	.	
	//PM	x	x	———	———	x	x	.	x	x	x	x	.	x	**major**
(c)	**melody notes**	.	.	x	x	.	x	.	x	x	.	x	.	.	
	altYO	———	———	x	x		x	x	x	x		x	x	——	**same notes as earlier (c)**
(d)–(e)-(f) as before															

ATONAL MODES

The focus of PKP is tonal music but atonal scales summarized next are included because shared letters of their defining words make them conceptual parents of scales lower down in the hierarchy; and also because they may be used ornamentally in tonal music. Atonal scales have no minor, major or minor-major tonality by themselves but representing them by words that place them in the context of the home tonic gives them tonality relative to it. The last mode is familiar from the earlier *Giant Steps* example. It has no no tritones but is determined by tritones because it's morphed from the tritones of **O.M.L**. There are no all-atonal example pieces in this document.

Atonal modes have a symmetry that makes them compatible with multiple tonics (the shape going up from different tonics is the same). Whole tone scales have circular symmetry and diminished scales have mirror symmetry. Representing them by alphabet words selects the home tonic as a reference. Breaking their symmetry creates related home-tonic scales.

		1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
word	type name	@ P O R M Y L $ . . . . @	related to
ORMY	—	———x x x x———x x x x—	//ORMY, //OR.Y, //O.MY
P.R.Y	whole tone	. x . x . x . x . x . x . x .	//RY
O.M.L	"	x . x . x . x . x . x . x . x	//OM
P.RM.L	diminished	x x . x x . x x . x x . x	//RM.L
PO.MY	"	. x x . x x . x x . x x .	//O.MY
OR.YL	"	x . x x . x x . x x . x x	//OR.Y
PMI	augmented	x x———x x———x x———x	O.M.L

CHAPTER 4: A SMORGASBORD OF ADVANCED EXAMPLES

This chapter explores a smorgasbord of example pieces that I found difficult to understand when I first encountered them in music notation because of strong departures from classical modes, often combined with difficult or multiple written key signatures. The idea of a smorgasbord is to offer something for everyone. Any one of these pieces rewards deep study and, when the study is in these terms, what's learned is portable between different pieces with different tonics and key signatures.

A takeaway from this chapter is confirmation that tritones are structurally fundamental to music understood in interval terms, and that a DNA-like alphabet is sufficient to cover any tonic scale or chord progression that may be played on the piano, independently of the key signature(s) in which a piece may be written. This is different in kind from "the score is sacred" view of many classical musicians.

I have heard it said, and believe it to be true, that composition is frozen improvisation. Therefore it, like improvisation, requires a sense of architecture. I suggest that exactly such a sense of architecture is revealed by the examples in this chapter. The fact such a sense can be extracted from such a wide variety of pieces without any information other than that provided by the piece itself suggests that it was put there by a composer or arranger, based on an intuitive understanding that's no doubt different in kind from PKP, but that can be represented simply by PKP.

The following pieces, in no particular order, explore a wide variety of home tonics, modes and key signatures. All are in the chromatic non-classical domain. There are distinct differences in style between the different pieces that range from loose and somewhat ambiguous, to systematic and unambiguous.

Blues Family

Goodbye Pork Pie Hat
When Sunny Gets Blue
Prelude to an Afternoon of a Faun (interpreted as a blues)
Straight No Chaser
Blue Monk

Minor or Major Family

You Must Believe in Spring
Round Midnight
All of Me
All the Things You Are
No Greater Love
Body and Soul
Every Time We Say Goodbye
All the Things You Are
Chelsea Bridge
Lush Life
The Peacocks

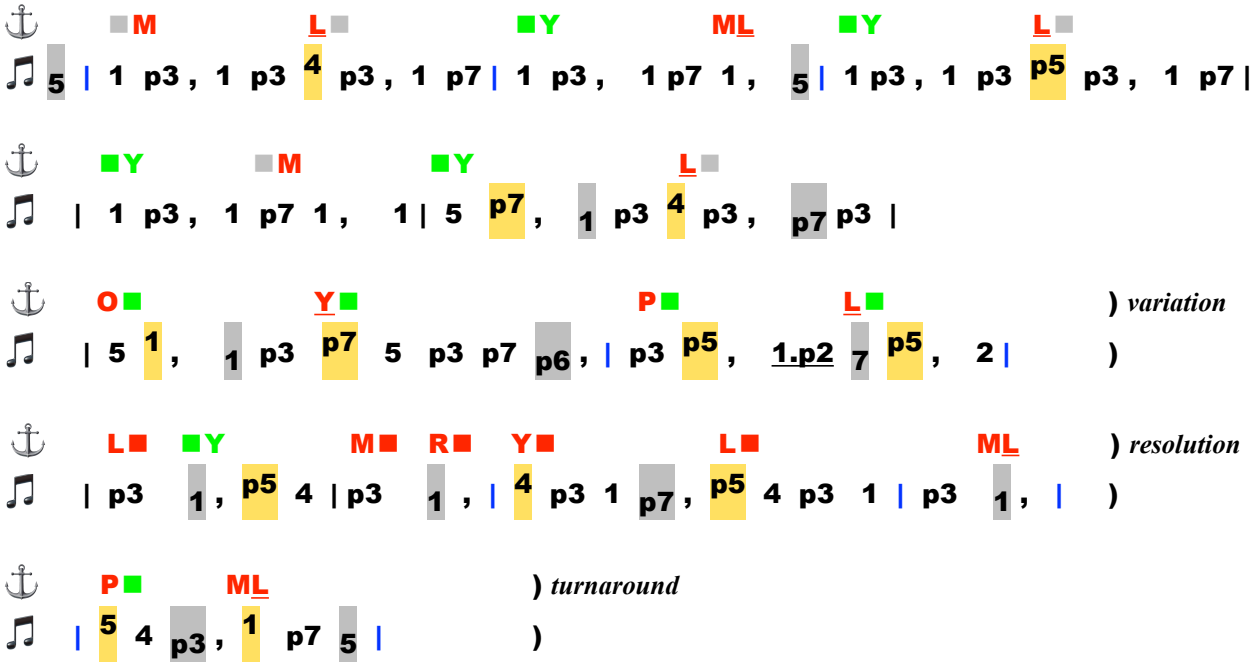
GOODBYE PORKPIE HAT

This blues in Eb, the chord progression of which is shown in footnote 2 of the opening chapter, is a poster child for chromatic music that's difficult in music notation for all but experts. My source for it is the *Mingus Fakebook*, Hal Leonard (1991). Trying to learn this piece from this source was one of the stimuli that sent me down the path to PKP. The key signature of 3 flats and the home tonic of Eb jointly identify the Ionian mode **//Y** as the reference mode for the accidentals that determine blues notes. This is an example of the often misleading nature of key signatures. A 5-flats or 6-flats key signature indicating Dorian or Aeolian modes of Eb would be closer to the **//RM.L** blues that governs the piece for this tonic.

The simple melody line is mostly in the 6-note minor blues sub-scale. Bars 6-7 are a variation that goes "outside" the blues. In bar 7, the **1.p2** notation means the first note is "crushed" into the second note on the same beat. The melody is played in swing-feel 4/4 time. No downbeat line is required because commas indicate phrases terminated by a downbeat, with the next note on upbeat.

Keep in mind here and later that this notation is only to illustrate the concepts independently of the obscuring clothes of music notation, and that, most of the time, all that's required is chromatic scale symbols annotated on the staff and augmented anchor lines annotated above the staff next to chord symbols. That said, this is a useful way of capturing the essence of tricky passages, to say nothing of it being valid for any home tonic, not just the written one.

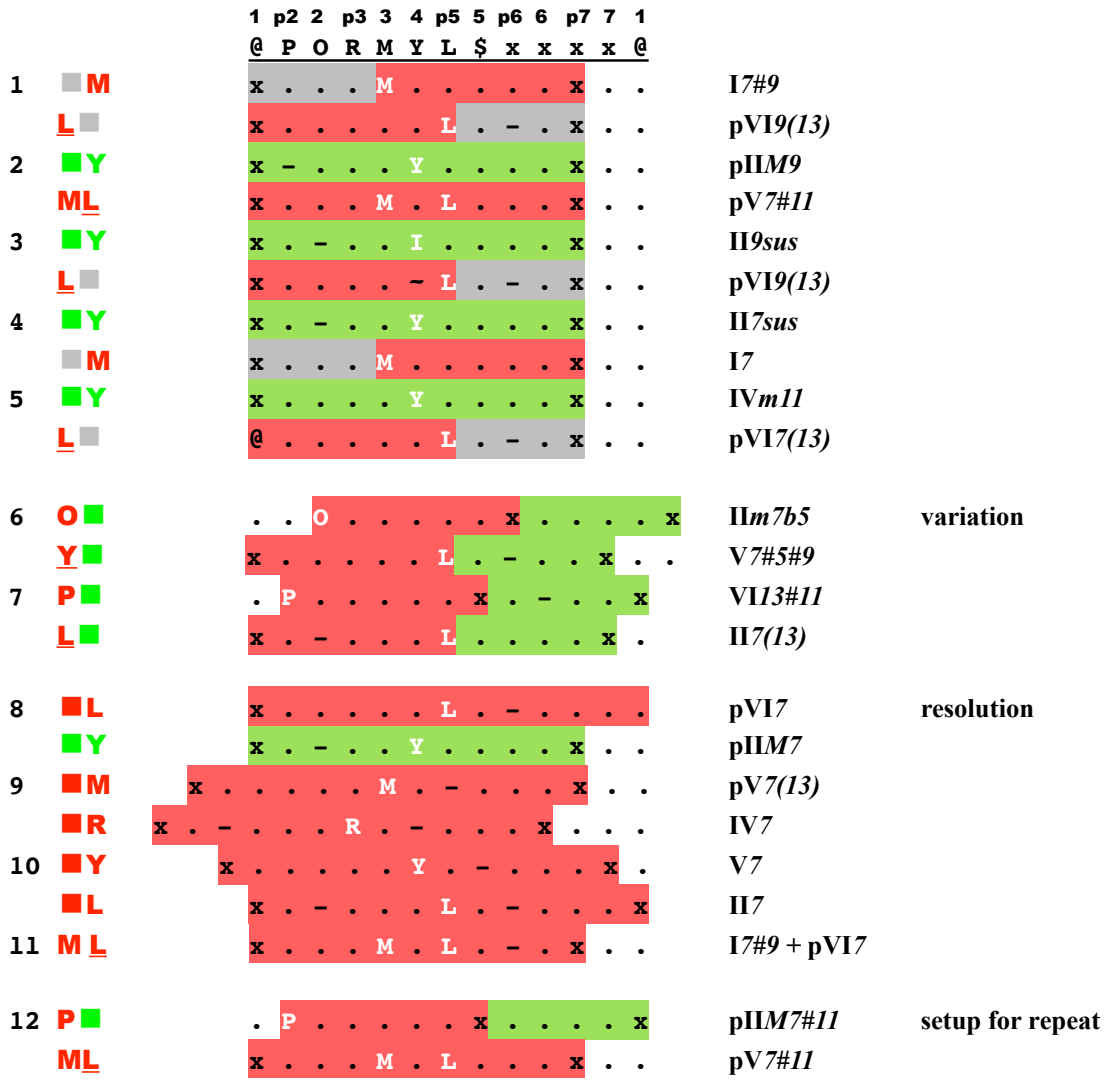
Goodbye Porkpie Hat (original home tonic = Eb)



Three styles of harmony are illustrated: in bars 1-5, the outer notes stay fixed (determined by the double tritone **ML**) while one inner note moves; in the variation in bars 6-7, the tritone shapes are "all fourths," providing an easy-to-play sequence that gives a bluesy sound to this departure from the blues; in bars 8-12, the single-tritone chords are all octave shapes, on the principle that there's so much harmonic variety in the tritone sequence that the tritone-based shapes don't have to provide it.

The Lego-like structure of this harmony is shown next, along with the written chords. Many of the alterations in the chord symbols on the right are provided by context. For example, ***#9*** of the **I*7#9*** chord in bar 1 is melody note **p3**. The free use of tritone substitute chords makes for a complex written chord progression. These chords have all non-tritone notes different (including roots a tritone apart) but the same harmonic function; in other words, they combine different fifos with the same tritone. The **pII** roots of the major seventh chords are byproducts of tritone substitutions, not indicators of visits to a mode containing this note. Several repetitions of the non-tritone shape ■I voice a variety of chords, the nuances of which can be left to context. A more accurate voicing of the *sus* chords would be provided by the shape ■$, a whole tone up from ■I, but the two shapes are mutually consonant and the the difference is relatively unimportant to the ear in this context; the ■I shape fits the flow better.

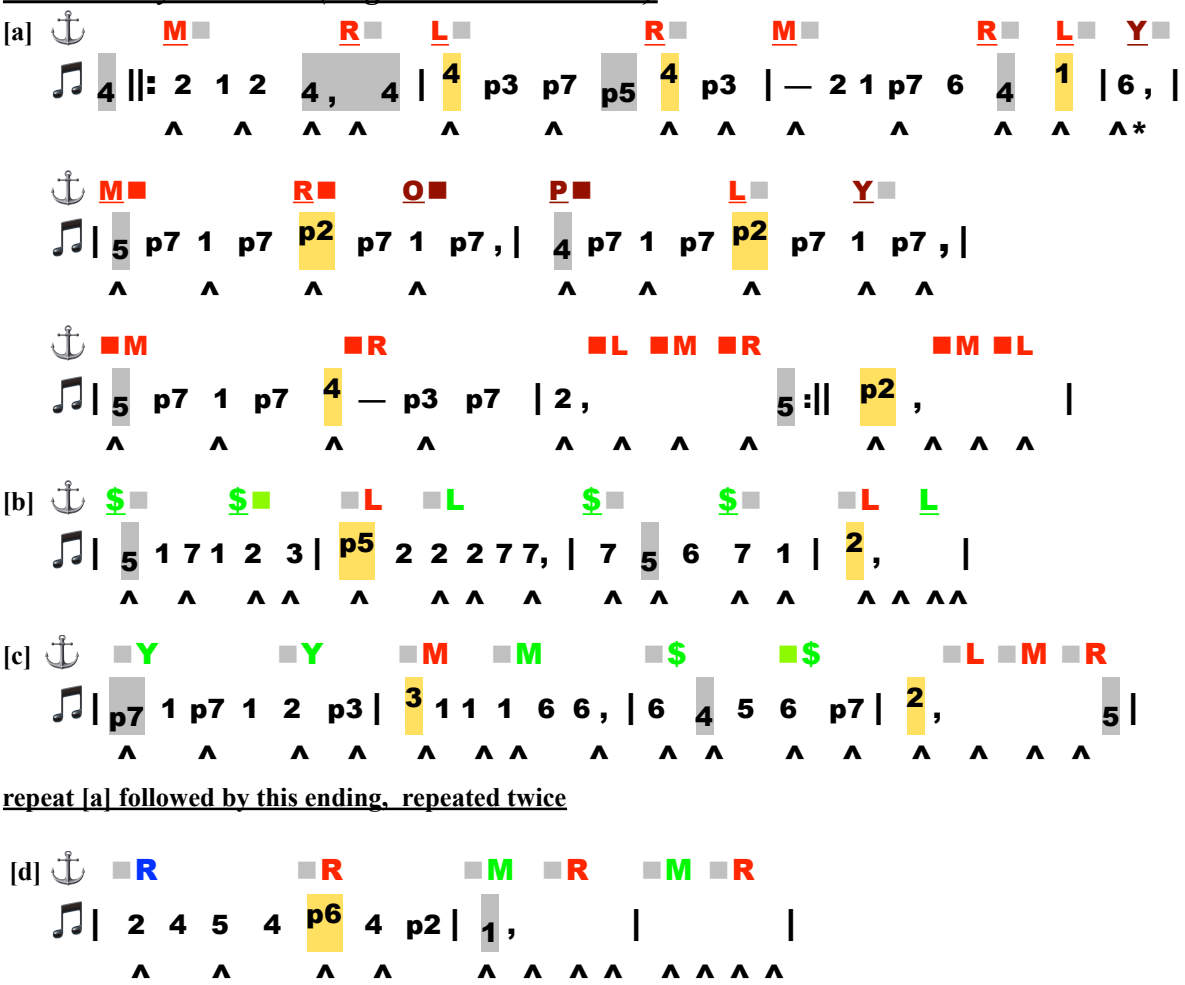
The bass line provided by the chord roots is missing but could easily be added back underneath if desired. That said, this harmony evokes the haunting, rich sound of the piece in a very satisfying way: small note changes in a smoothly flowing keyboard pattern make big sound changes.



WHEN SUNNY GETS BLUE

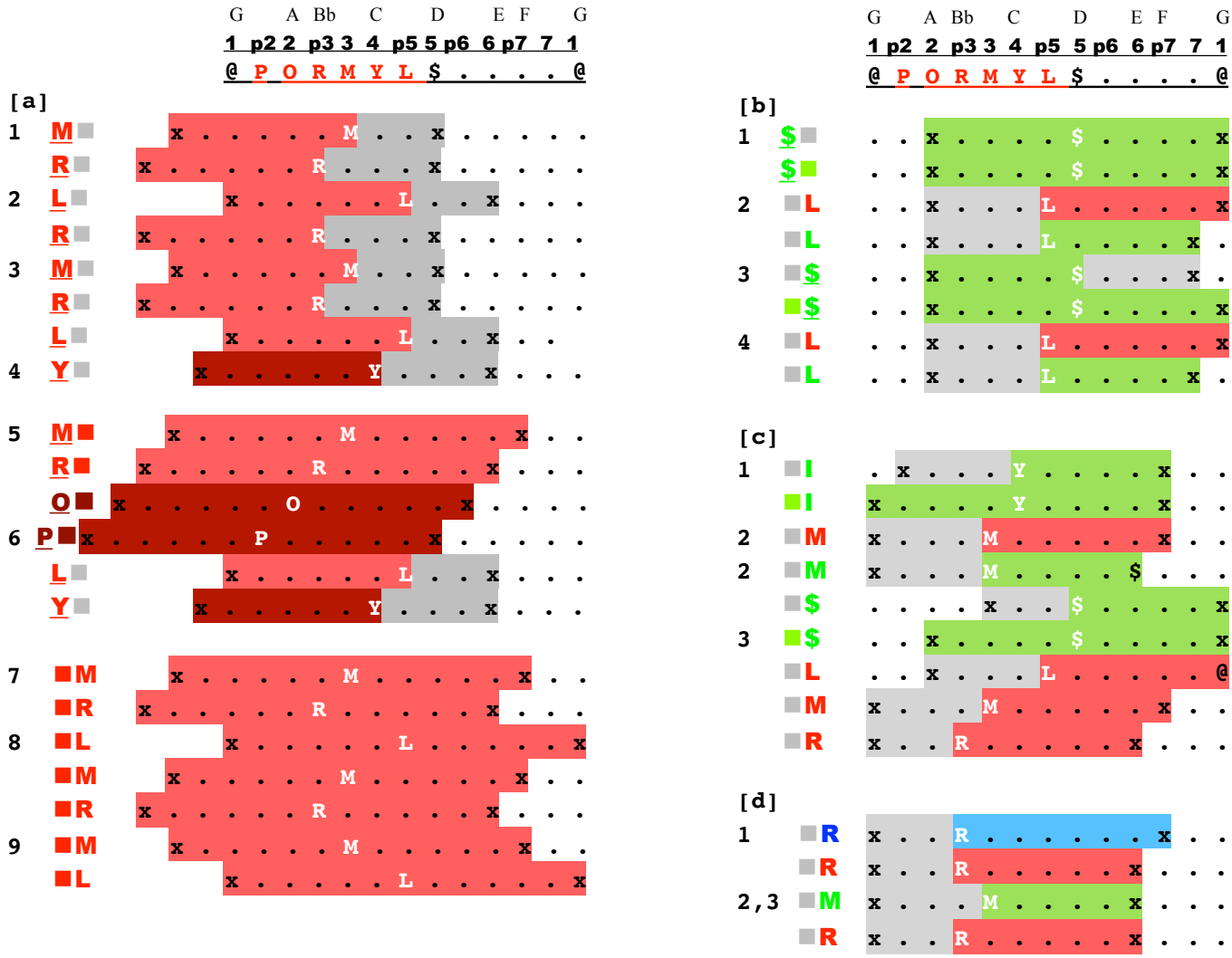
My source for this piece is sheet music by Jack Segal and Marvin Fisher from Hal-Leonard (1956), publication number HL00351105. The home tonic is G and the key signature is one flat, identifying Dorian of G as the reference classical mode. The melody line is a **//RM.L** blues, with a 4-bar section in the bridge [c] that goes to the nearby **//L** mode. The harmony shown here is a modification of the written harmony, which I found to have a confusingly irregular flow. This harmony flows smoothly to my eye and ear. Ornamental tritones in [a] (not in the blues scale) are shown in a darker shade of red.

When Sunny Gets Blue (original home tonic = G)



repeat [a] followed by this ending, repeated twice

Here's what this harmony looks like on the keyboard.

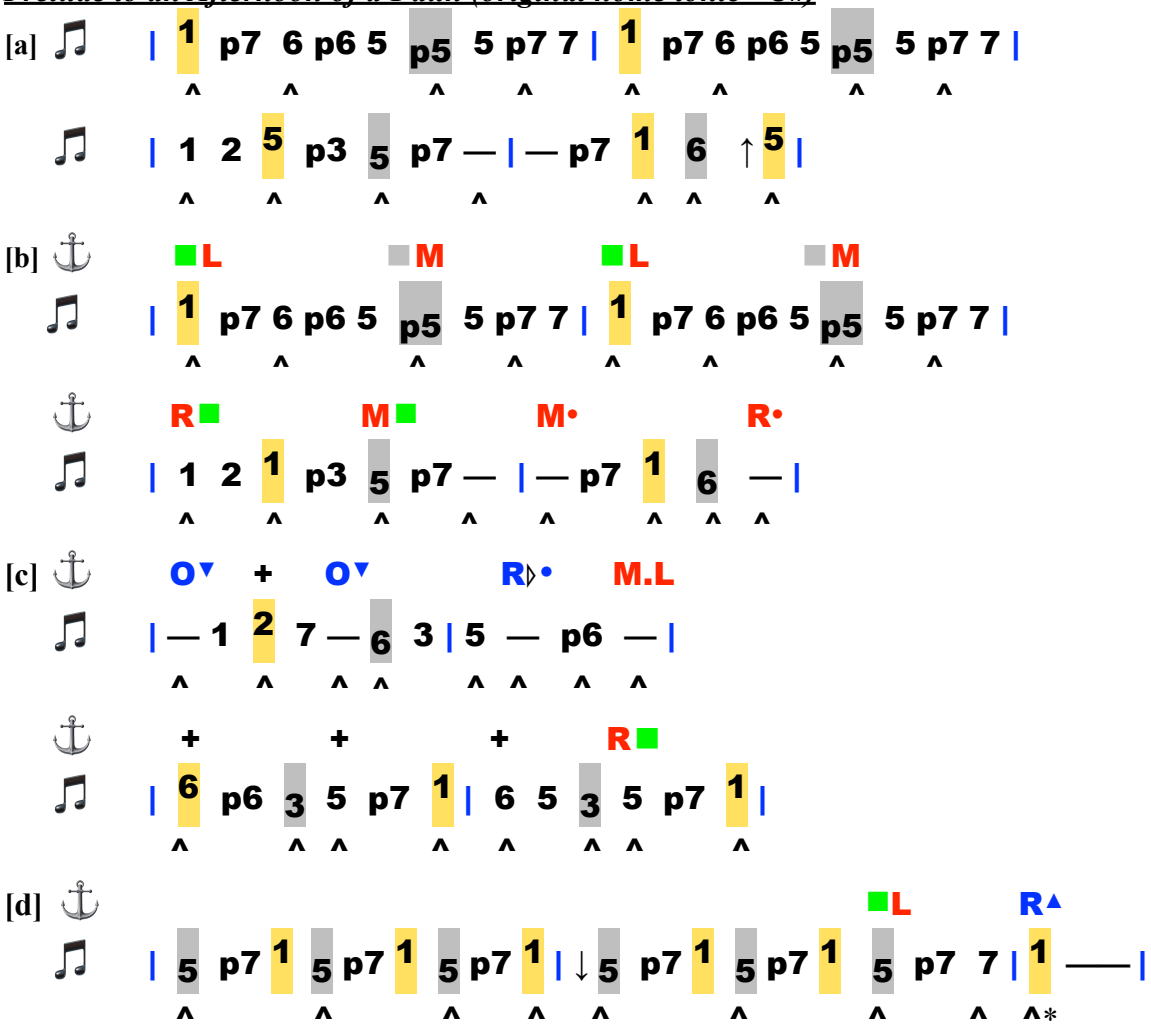


PRELUDE TO AN AFTERNOON OF A FAUN

I was motivated to investigate this hauntingly beautiful Debussy piece by things said about its creative violation of music convention in the article *Beauty in the Void,* Alex Ross, The New Yorker, Oct. 29, 2018. The violations were explained in the article in terms of unusual sequences of different keys. I wondered if there might be a way of understanding it in terms of non-classical modes. My source for the written music is the *Classical Fake Book*, 2nd Edition, Hal Leonard (2013), page 222. The piece looks innocently (and misleadingly) simple there — a melody line with a few accidentals and some mostly simple chord symbols above it for mostly triad chords. The problem is, it's written in C# minor with accidentals that bring in Db major, a combination that's inherently complex in music notation (recall footnote 1 in the opening chapter).

Here follows a summary of a variation that's faithful to the written melody line but takes some liberties with the harmony (described following this). The written time signature of 9/8 divides each bar into 3 groups of 3 eighth notes, but playing it in 4/4 time sounds very close to the original timing and leads naturally to a swing-feel blues. The "+" entries in [c] are placeholders, to be filled in, if desired, from the ensuing description.

Prelude to an Afternoon of a Faun (original home tonic =C#)



Getting a handle on the piece is helped by the following identification of the melody scales. The piece uses all the notes of the chromatic scale for both melody and harmony, but organized in a way that suggests **//RM.L** blues with ornamental passing notes **p6** and **7** (they're passing notes because omitting them, while maintaining the same timing, has no substantive effect on the sound).

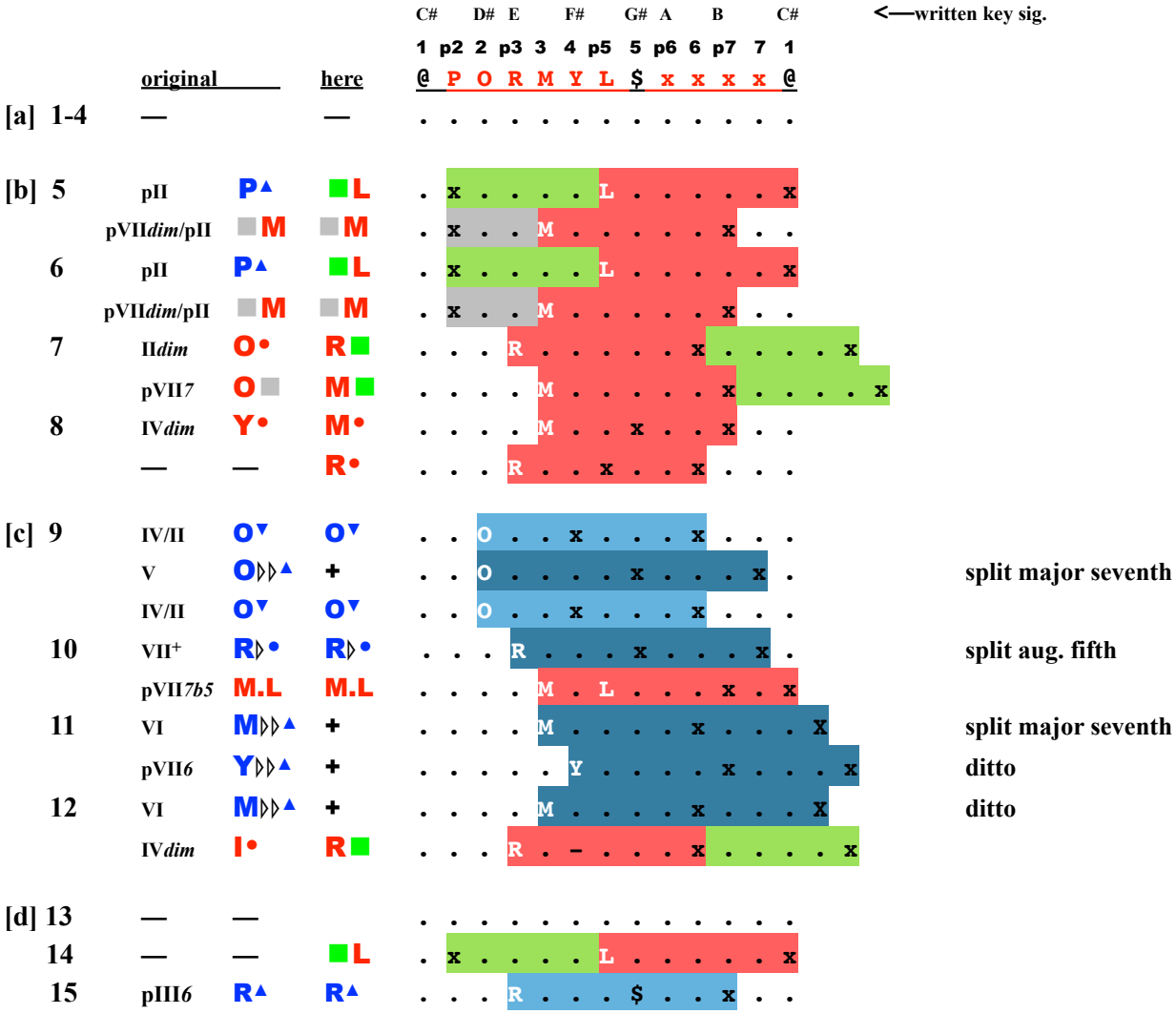
	C#	D#	E	F#	G#	A	B	C#	<—written key sig.
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	Y	L	$	x	x	x	x	@	
[a]-[c]	x	.	x	x	x	x	x	x	+	x	x	+	x	//RM.L blues melody scale with passing notes
[d]	x	.	.	x	.	x	.	x	.	.	x	+	x	//R pentatonic minor sub-mode

The original harmony is mostly from the blues scale, with the striking exception of note **p2** as the root of a major triad in the opening bars of [b]. This is striking not only because this note is not in the melody scale, but also because it's dissonant with the melody note it harmonizes (the tonic). Adding the tonic to the triad yields a major seventh chord, a simple alteration of which yields the "all fourths" shape ■**L** from the blues scale (a tritone is an augmented fourth). This basic blues shape sounds as appropriate as the original major triad and paves the way for more blues-centered harmony in the rest of the piece, in which the same shape appears for different tritones. No chord symbol is given because the convention of identifying chromatic notes by altered degree number suffixes is confusing for scales that have more notes than the classical modes that determine the basic chords.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	Y	L	$	x	x	x	x	@	
pII	P▲	.	x	.	.	.	x	.	.	x	.	.	.	.	1st written chord of [b]
pII*M7*	—	.	x	.	.	.	x	.	.	x	.	.	.	x	forms this chord with the tonic
—	■L	.	x	.	.	.	.	x	.	.	.	.	.	x	altered chord fits the //RM.L scale

A complete Lego-like picture of the harmony is shown on the next page, including the original harmony and the new blues-based harmony. A comparison for [b] leading into [c] shown below. The new harmony is entirely from the blues scale, and is evocative of the blues. The scale and the flow prompt the shapes, and the result sounds fine. The proof is in the playing.

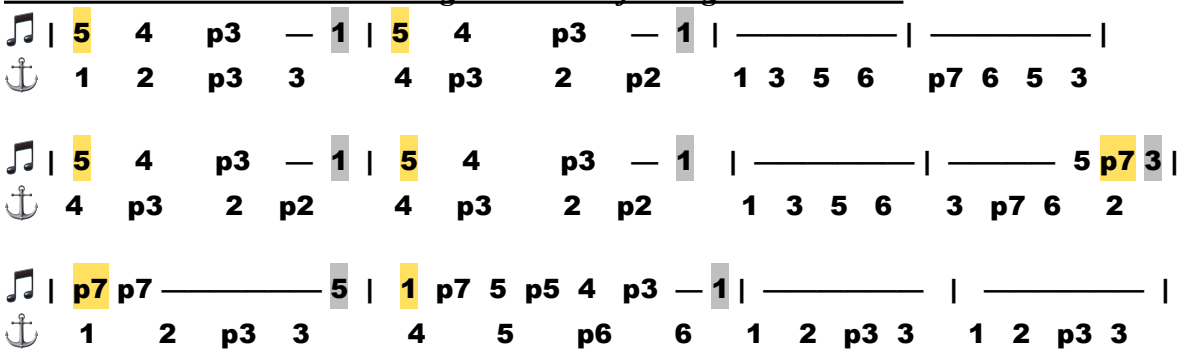
Here follows the complete harmony. Recall that ◗ or ◖ identify expansion or shrinkage of building blocks.



STRAIGHT NO CHASER

Walking bass lines provide an alternative to chordal harmony in many kinds of music, for interest and variety. They are easy to represent in the melody-line notation, easily portable in this form between different pieces with the same or different home tonics, easy to create knowing mode scales of the tonics, and easy to play. These things are illustrated by the following use of a walking bass line from Monk's *Straight No Chaser* (coming up) for *Backwater Blues* in Chapter 2. The interleaving of this bass line and the melody line of *Straight No Chaser* is a bit tricky, so this introduces the bass line in a simpler way, besides making *Backwater Blues* more fun to play. The bass line is identified as harmony by an anchor symbol on the left, but is shown under the melody line where it's actually played, because showing it above would be confusing (it's not confusing for anchor lines shown above the staff because they're not represented in note symbols). There are some transient dissonances between this melody and harmony that could be avoided by some syncopation of the melody, but such dissonances are often a feature of the blues.

Backwater Blues with the Walking Bass Line of Straight No Chaser

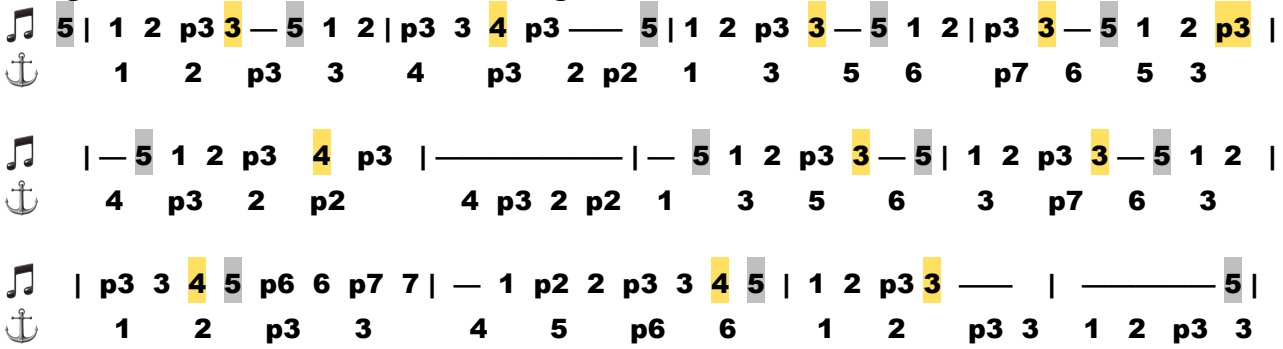


Walking bass lines are easily created (or understood) using a mode table, as illustrated next. Bass-line notes **p2** and **p6** and **7** ornamental passing notes. To be concrete, the corresponding notes of music notation are shown on top for tonic F blues.

	F		G	Ab	A	Bb	B	C		D	Eb		F	<==F home tonic	
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		
	@	P	O	R	M	Y	L	$	x	x	x	x	@		
//RML	x	.	x	x	x	x	x	x	.	x	x	.	x		
bar 1	x	.	x	x	x	.	.	.	.	.	.	.	.		up
bars 2, 5, 6	.	x	x	x	.	x	.	.	.	.	.	.	.		down
bar 3, 7	x	.	.	.	x	.	.	x	.	x	.	.	.		up
bar 4	.	.	.	.	x	.	.	x	.	x	x	.	.		down
bar 8	.	.	.	.	x	.	.	.	.	.	x	.			up
	.	.	.	x	.	.	.	.	.	x	.	.	.		down
bars 9-11	x	.	x	x	x	x	.	x	x	x	.	x	.	x x x	up
bar 12	x	.	x	x	x	.	.	.	.	.	.	.	.		up

Here follows the melody line of *Straight No Chaser* with home tonic F using this walking bass line (the original in the Monk fake-book has home tonic Bb but I learned it in F). The transition to Bb is easy, simply replace the F Dorian scale in the table below by the corresponding Bb scale. In either case, the key signature identifies the Dorian melody mode **//R** of the home tonic which is altered to **//RM** by an accidental. This is not the full blues scale because **p5** is missing.

Straight No Chaser with the Same Walking Bass Line



The melody line is in the **//RM** sub-mode of the blues family mode with two obvious passing notes (each appears only once). Recall that the mode is a mashup of parallel pentatonic modes).

F		G	Ab		Bb		C		D	Eb		F	<== Dorian of tonic F
1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
@	P	O	R	M	Y	L	$	x	x	x	x	@	
x	.	x	x	x	x	.	x	.	x	x	.	x	//RM
	+							+		+			passing notes

The intricate interleaving of the two lines makes the whole sound like more than the sum of the parts, and requires considerable practice to get right, and also considerable effort to write down correctly. The benefit of this effort is portability of concepts and of keyboard patterns to other pieces.

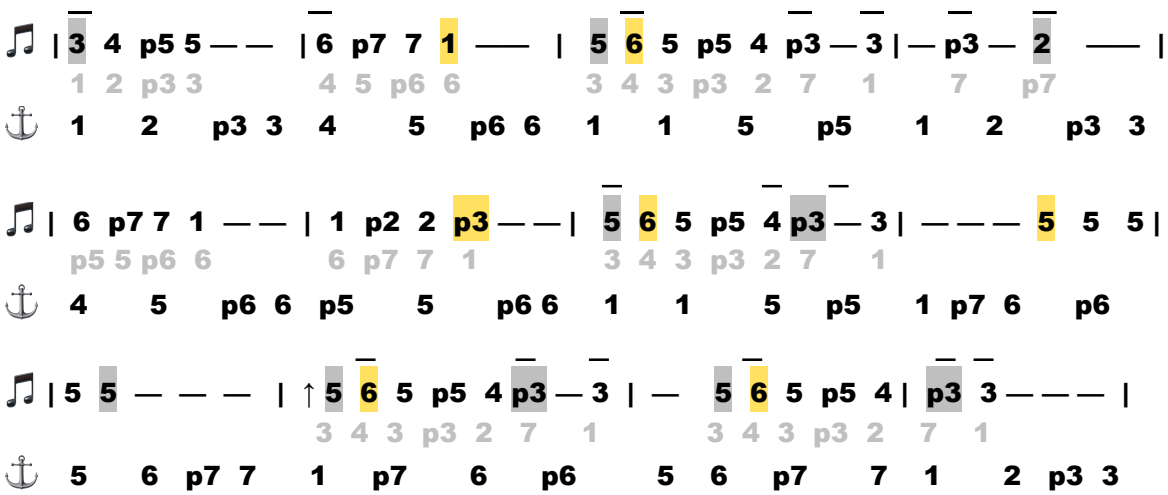
BLUE MONK

This piece is so chromatic that it seems as if several different home tonics might fit. Only one fits, namely Bb of the written 2-flats key signature, established by the first notes of a double melody line consisting of two lines offset by thirds (major or minor). This choice of tonic makes clear that the appearance of all notes of the chromatic scale is a byproduct of three passing notes.

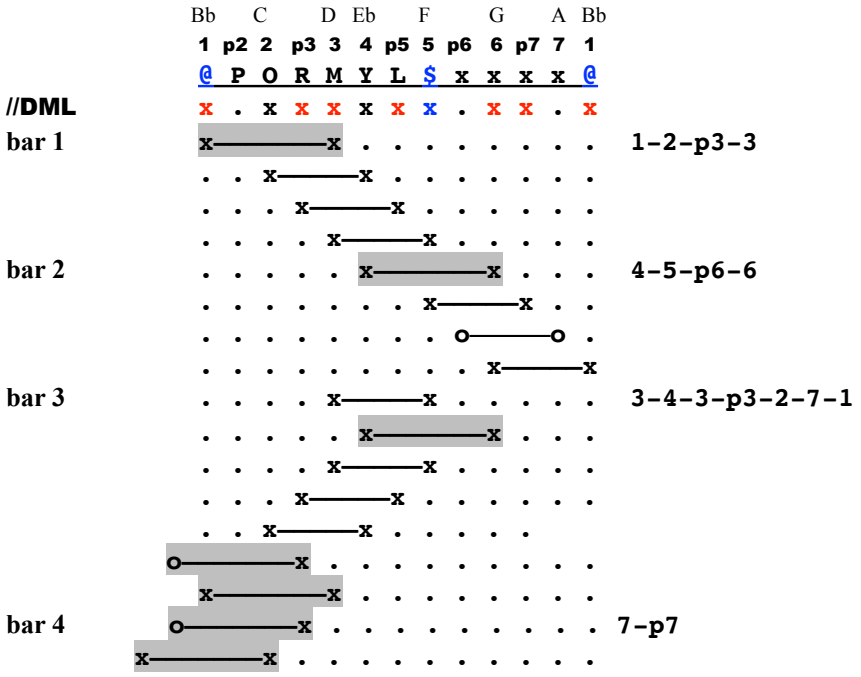
```
Bb    C     D  Eb    F     G     A  Bb    <== key signature notes
1 p2  2  p3 3  4 p5  5  p6 6 p7  7  1
@ P   O  R  M  Y  L  $  x  x  x  x  @
x .   x  x  x  x  x  x  .  x  x  .  x    //RM.L
  +                    +     +       passing notes
```

As shown next, the double melody line formed of main line with a secondary line a third (major or minor) down from it. The thirds are determined by a simple rule: switch between a major and a minor third when one note in either line moves by a whole tone. This is to avoid the "outside" notes **p2** or **p6** or **7** wherever possible; the rule is broken only when "outside" notes are unavoidable (thus they become passing notes). The positions of the resulting major thirds are marked below by over lines. For concreteness, the secondary line is shown as a sequence of lighter note symbols, but it's simpler to think of it as determined by construction from the rule. The opening sequence of a major third and three minor thirds establishes the melody scale.

Blue Monk (original home tonic = Bb)



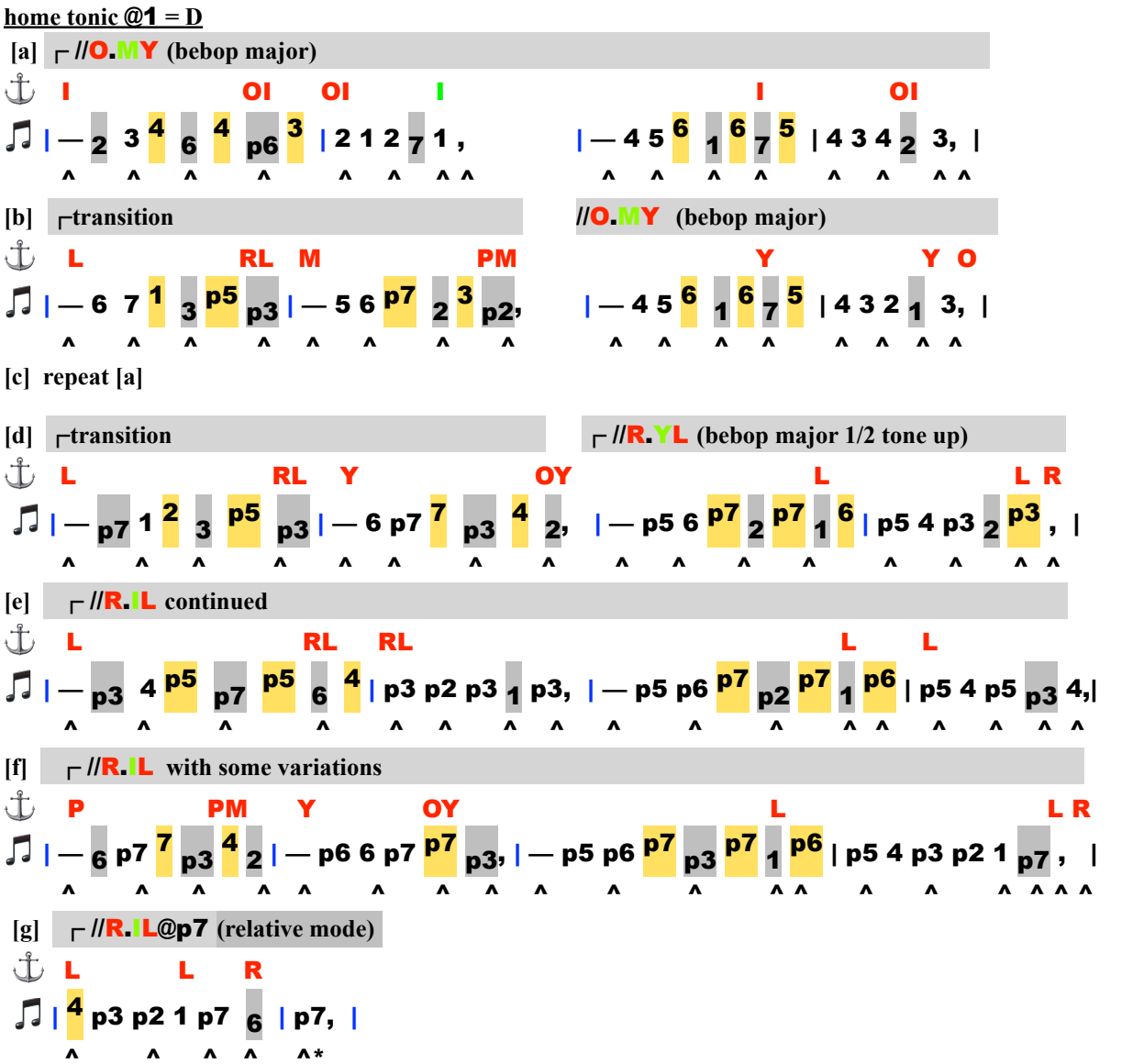
Construction of melody thirds from the rule is illustrated next for bars 1-4 (**x** marks blues-scale notes, **o** marks "outside" passing notes, grey shading highlights major thirds)



The sequence starts with a major third to avoid "outside" note **p2**. It continues with a minor third for the same reason. And so on. The few appearances of an outside note (**o**) are unavoidable in order not to disrupt the flow. The result is a smoothly flowing sequence of thirds that's easy to play and sounds good. This simplicity is obscured in the written music by the clutter of sharps, flats and naturals on different staff lines and spaces required to represent the notes relative to the written key signature.

YOU MUST BELIEVE IN SPRING

My source for this piece is *The Michel Legrand Songbook*, 1997, Warner Bros. Publications. The strongly systematic organization of this piece is difficult to see in the written music, which is a full arrangement spread over 3 pages with 2 key signatures (2 sharps, 3 flats) and 32 tritone chords of many varieties (next page) that establish two parallel modes with all notes a half tone apart (bottom of this page). This notation brings the systematic organization forward to the eye.



Modes (tonics highlighted)

	D	E	F#	G	A	B	C#	D
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	Y	L	$	.	.	.	.	@	
//O.MY	x	.	x	.	x	x	.	x	x	x	.	x	x	bebop major of D
//R.YL	x	x	.	x	.	x	x	.	x	x	x	.	x	same of Eb (all notes 1/2 tone up)
//R.YL@p7	x	x	.	x	.	x	x	.	x	x	x	.	x	relative mode resolves to C

Tritone chords (roots highlighted)

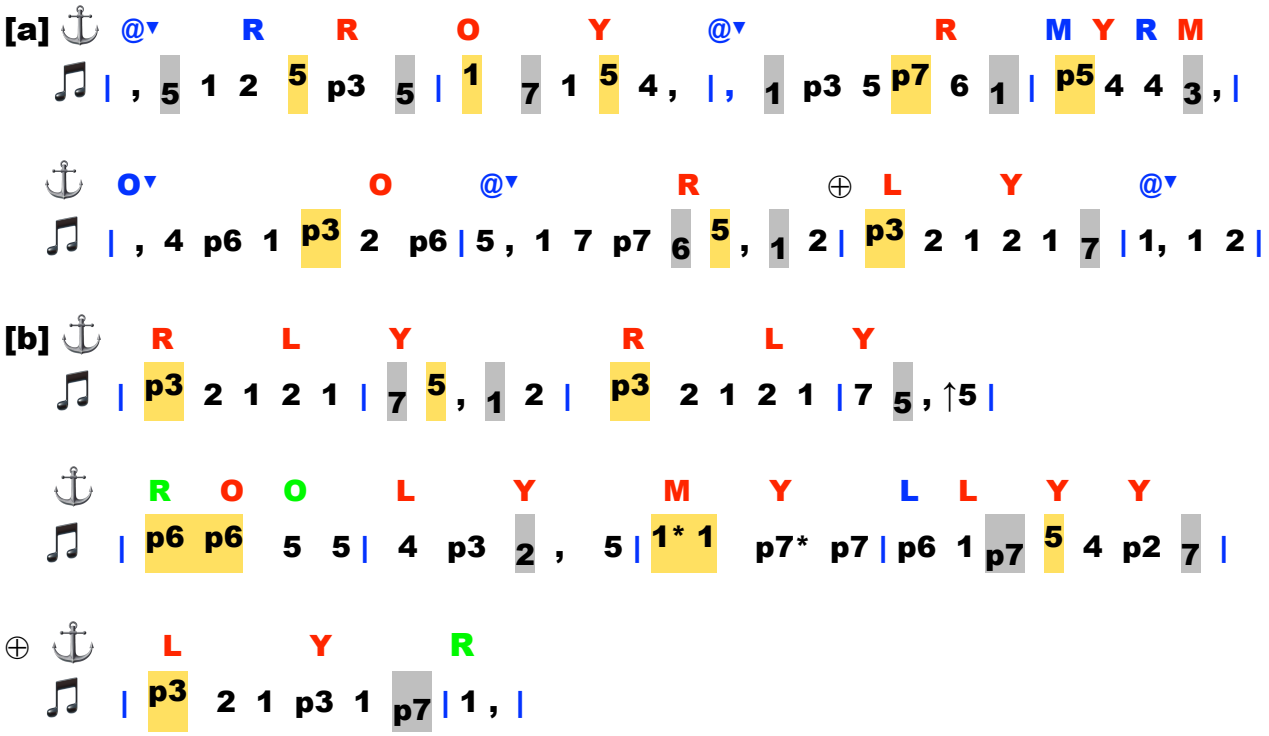
The 32 tritone chords provide core harmony consisting of 6 tritones and 2 triple-tritone mode signatures offset by a half tone. The notes of the full chords are provided by context. The fifo chords are not shown here because the core fifos are context-determined morphs of the tritones.

			D . E . F#G . A . B . C#D	
			1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
			@ P O R M Y L $ . . . . @	
[a]	1	C#m7b5	. . x . . x . . . x . x .	Y
		F#7b9	. . x . x x . . x . . x .	OY
	2	F#7b9/B	. . x . x x . . x x . x .	OY
	3	A7	. . x . . x . x . . . x .	Y
	4	A7b9/D	x . x . . x . x x . . x .	OY
[b]	1	G#m7b5	x . . . x . x . . x . . x	L
		C#7b9	. . . x . . x . . x . x x	RL
	2	F#m7b5	. . . . x . x . . x . . .	M
		B7b9	. x . . x . . x . x x . .	PM
	3	A7	. . x . . x . x . . . x .	Y
	4	C#m7b5	. . x . . x . . . x . x .	Y
		F#7	. . x . x . . . x . . x .	O
[c]=[a]				
[d]	1	G#m7b5	x . . . x . x . . x . . x	L
		C#7b9	. . . x . . x . . x . x x	RL
	2	Gm7b5	. . . x . x . . x . . x .	Y
		C7b9	. . x . x x . . x . x x .	OY
	3	Bb7	x . . x . . x . x . . . x	L
	4	Dm7b5	x . . x . . x . . x . . .	L
		G7	x . . x . x . . . x . . x	R
[e]	1	Dm7b5	x . . x . . x . . . x . x	L
		G7b9	x . . x . x x . . x . . x	RL
	2	G7b9/C	x . . x . x x . . x x . x	RL
	3	Bb7	x . . x . . x . x . . . x	L
	4	Bb7b9/Eb	x x . x . . x . x x . . x	RL
[f]	1	Am7b5	. x . . . x . x . . x . .	P
		D7b9	x x . . x . . x . . x . .	PM
	2	Gm7b5	. . . x . x . . x . . x .	Y
		C7b9	. . x . x x . . x . x x .	OY
	3	Bb7	x . . x . . x . x . . . x	L
[g]	1	Dm7b5	x . . x . . x . . . x . x	L
		Dm7b5/G	x . . x . x x . . . x . x	L
		G7	x . . x . x . . . x . . x	R

ROUND MIDNIGHT

The source for this evocative piece in Eb minor by Monk are *The Ultimate Jazz Fakebook*, Hal Leonard, 1988, p. 322 for the main body of the piece (shown first) and *Standards Real Book*, Sher Music, 2000, p. 369 for an optional introduction (next page). The following summary is easy to play without a downbeat line if you've ever heard the tune.

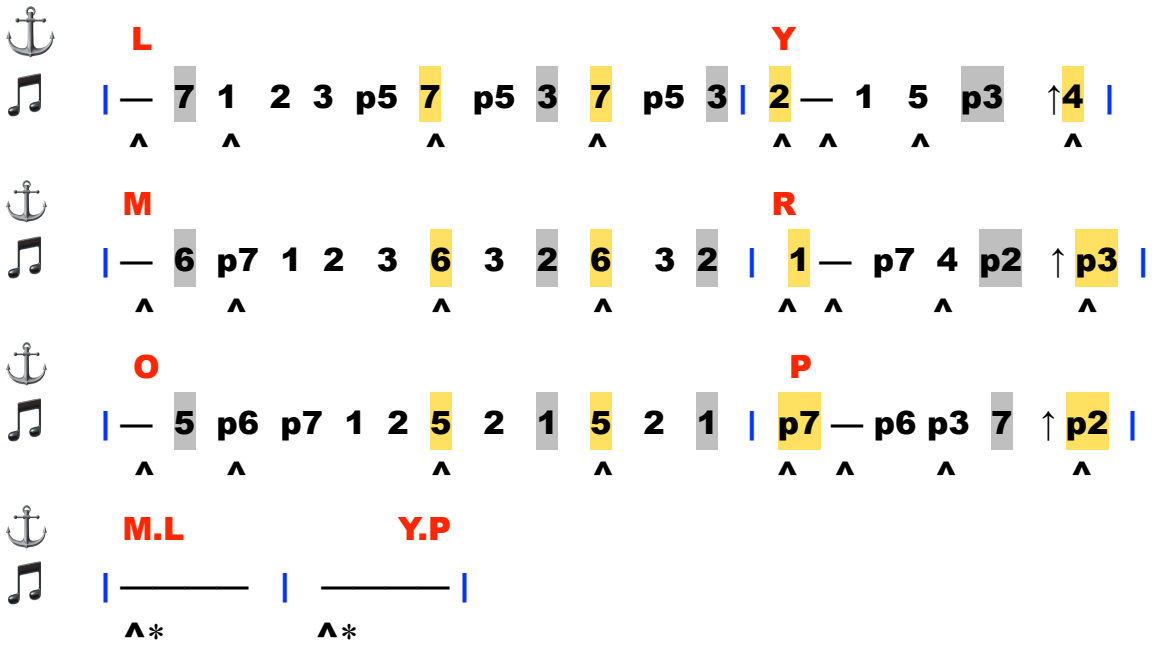
Round Midnight (original home tonic = Eb)



The melody scale is the minor family scale with passing notes, as follows. Much of the harmony is from this scale, with a few ornamental elements that fit the flow (e.g., **L-Y**, **Y-M**).

	Eb	F	Gb	Ab	Bb	B	C	Db	D	Eb
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	Y	L	$	x	x	x	x	@
//OR.Y⊕	@	.	x	x	.	x	.	$	x	x	+	x	@
ornamental		+		+			+						

Here follows an optional, 8-bar introduction (or conclusion) that's visibly based on whole-tone intervals and tritones in a highly structured way. Each 2-bar segment repeats the previous one a whole tone down. The harmony tritones are in the reverse alphabet order **LYMROP**. The final 2-bar segment is a rhythmic pattern with no specific melody notes. The up arrows say go up an octave.

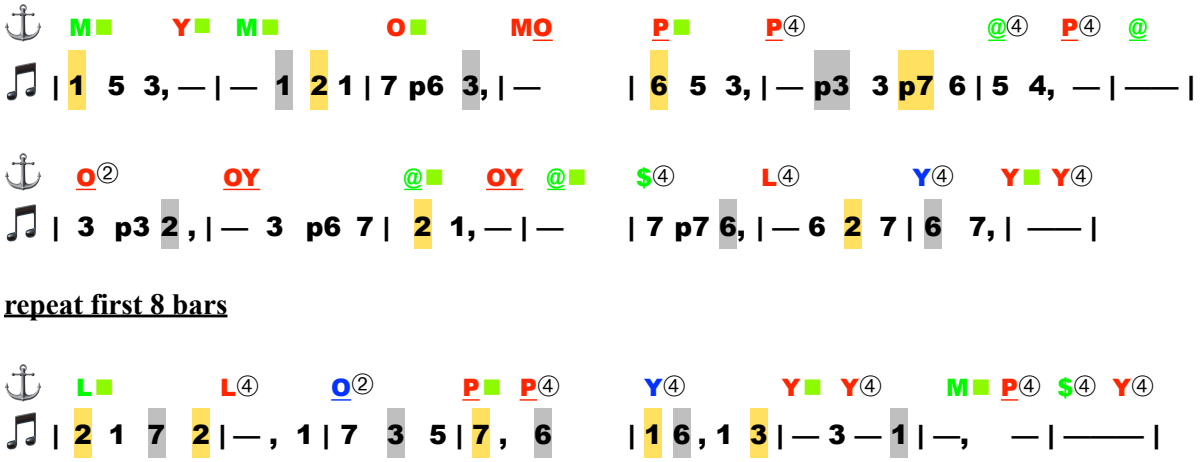


ALL OF ME

I learned this version of *All of Me* (along with *No Greater Love* coming up) in Susan Muscarella's piano comping course some years ago at the then Jazz School in Berkeley (now the Jazz Institute). The objective was learning about 4-note "open" voicings of extended and altered seventh chords often used to accompany soloists. The home tonic is C and the key signature empty, nominally identifying the Ionian mode, but the melody line uses all but two notes of the chromatic scale and the harmony uses all of them. Open voicings of the harmony chords rearrange the notes and sometimes omit the roots. The standard way of explaining such voicings is by rearranged degree numbers of chord scales, in which the same notes from adjacent chords have different degree numbers relative to different roots, which are sometimes omitted. Such representations are indirect relative to the keyboard, difficult to comprehend as a whole or remember, and too cumbersome to annotate routinely on the written music. This complexity goes away when the shapes are understood in terms of building blocks.

The following summary shows the melody line with 3-note shapes to voice all chords (except double tritones). The melody line is from the mode **//O.MI⁺**, with two appearances of minor note **p3** as ornamental passing notes. Harmony tritones **P** and **L** are ornamental substitutes for **O** and **Y**. The 3-note shapes are incomplete versions (top notes omitted size) the 4-note open voicings shown on the next two pages. Grey boxes for voicing intervals of of uncommitted size (■) are replaced by circled numbers ④, ③ and ② that commit to sizes of 4, 3 or 2 half tones. Many of the voicings sound interesting; some are easy-to-play "all fourths" shapes (counting tritones as augmented fourths). Play octave shapes based on the anchors to begin with (except for double tritones) and then shrink them into the shapes shown. The latter are easy to expand into the 4-note open voicings by adding one obvious note. Rhythm is implied swing-feel 4/4 time.

All of Me (original home tonic = C)



repeat first 8 bars

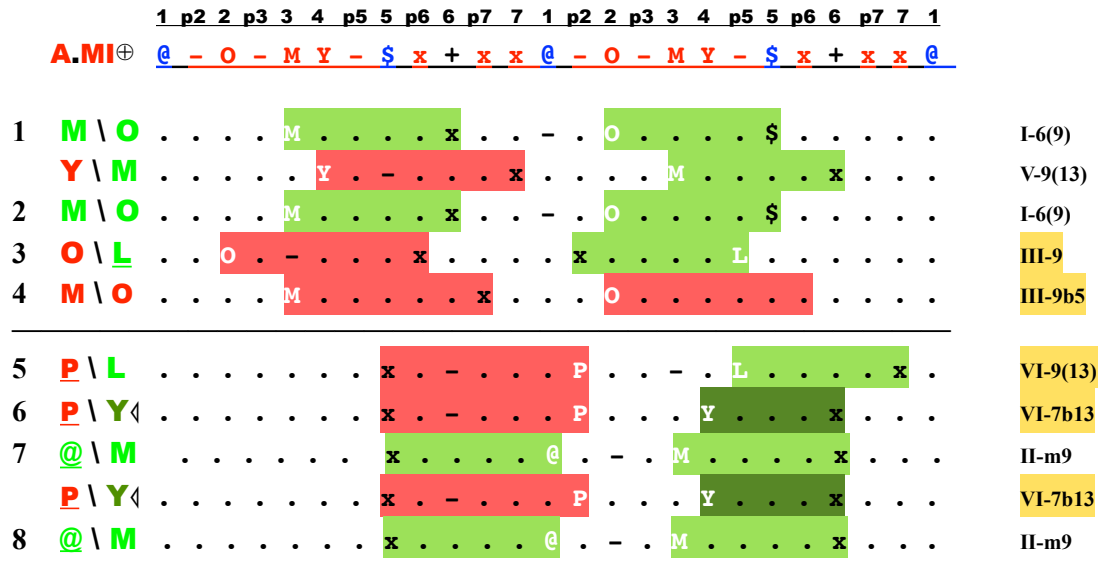
Open Voicings

Four-note "open" voicings of the written chords are shown next. The **U\V** notation identifies stacks of non-overlapping building blocks. The shapes are voicings of the chords shown on the right. The chord symbols are all satisfied serially in the flow. The four notes are difficult to play with the left hand, but easy-to-play 3-note voicings of the same chords follow from playing the core shown in the foregoing skeleton summary with a note added on top (in other words by knocking off the top note

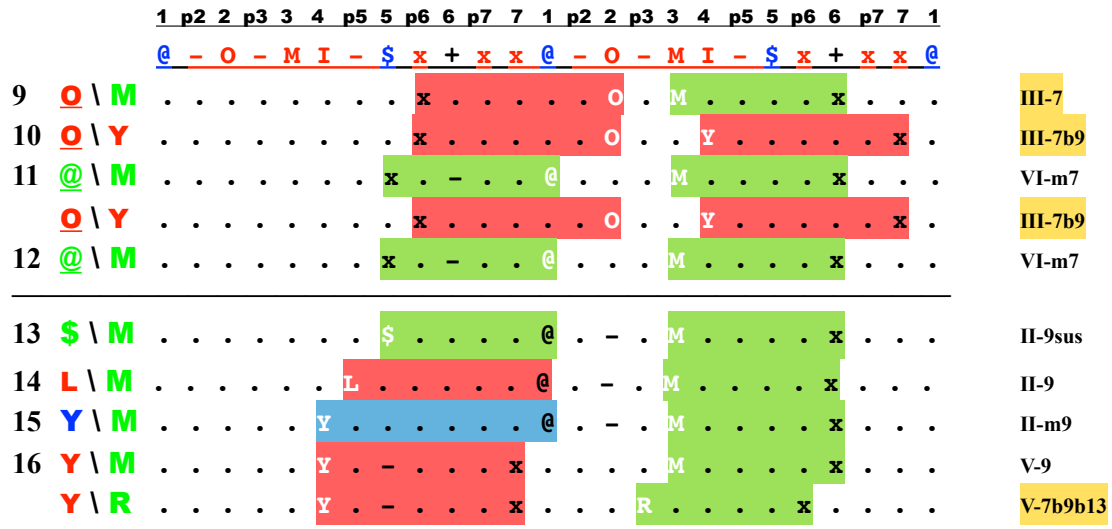
below); the top notes are easily added by the right hand under the melody line, if desired.

Chords on roots I, II and V are from the Ionian mode. The other chords (highlighted in yellow) provide visibly simple segues between the Ionian voicings, when seen in building-block terms.

Bars 1-8

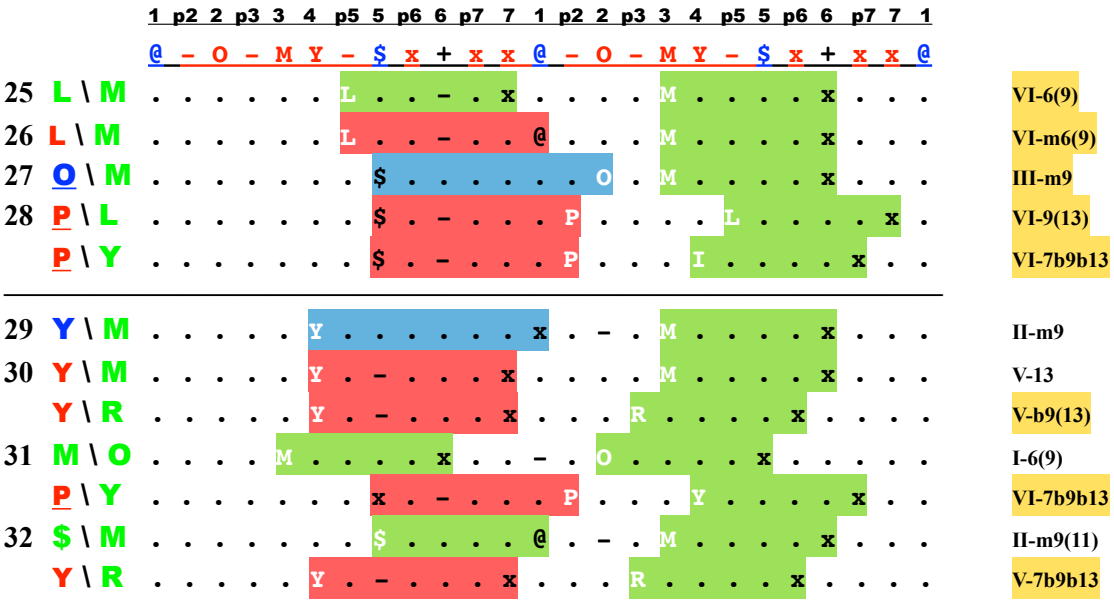


Bars 9-16



Bar 17-24 - repeat bars 1-8

Bars 25-32

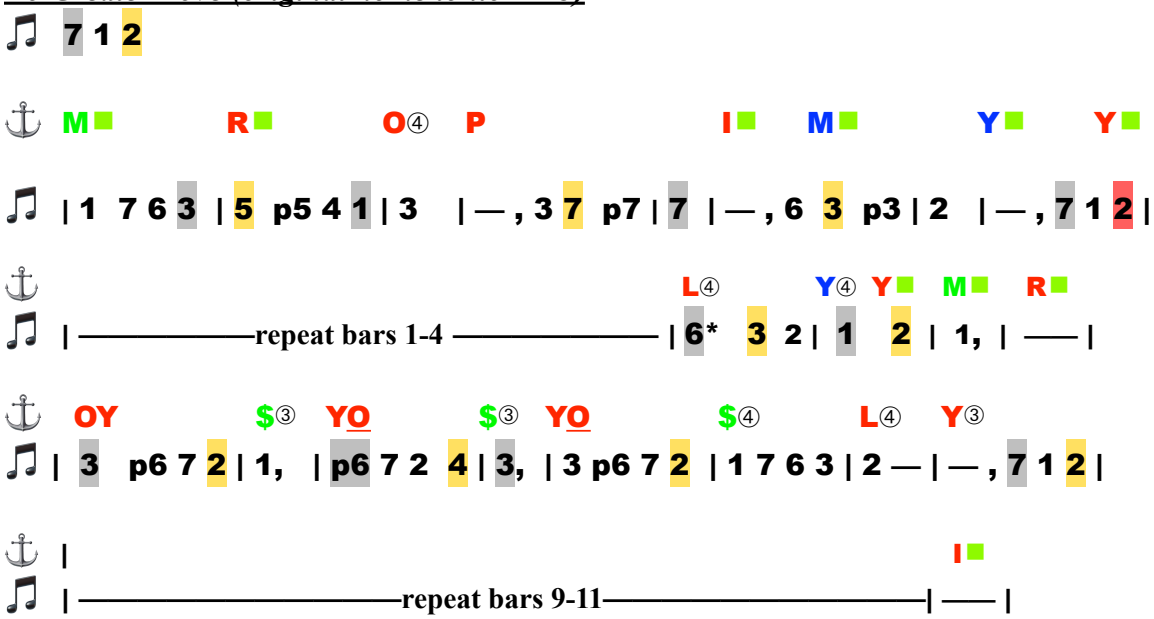


Understanding how the shapes satisfy the chord symbols requires knowing the relationship between the numeric suffixes of the chord symbols and the chromatic scale. I explain this only to make clear that the shapes satisfy the written chords, not to suggest figuring the shapes out this way. In bars 3-4, chord suffix "9" is chromatic scale note **p5** and chord suffix "b5" is chromatic scale note **p7**. The 3-note voicings in bars 3-4 don't include **p5** but it's included in bar 5, thus satisfying the bar-4 chord in the flow. The bar-4 shape jumps up a minor third in bar 5, with a wiggle that puts a fourth on top instead of a major third. The wiggle provides degree "13" of the chord. It also provides degree "9" of the bar-4 chord, thus satisfying two chord symbols at one stroke.

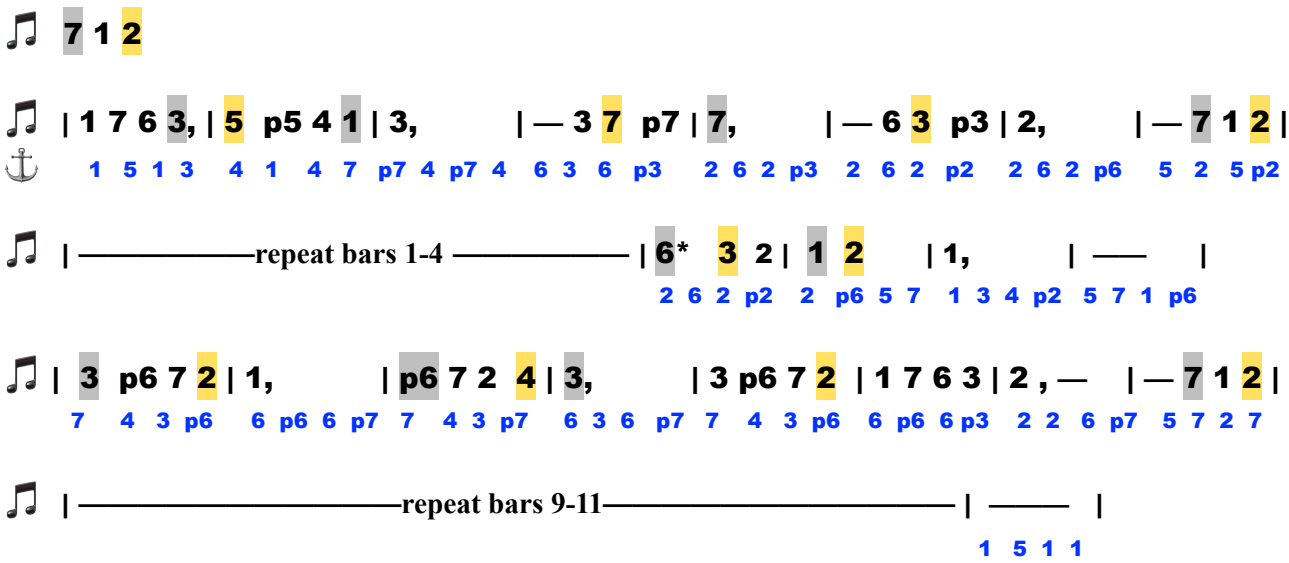
NO GREATER LOVE

I learned this piece in the same piano comping course as *All of Me*. It uses the same major family mode but is interestingly different.

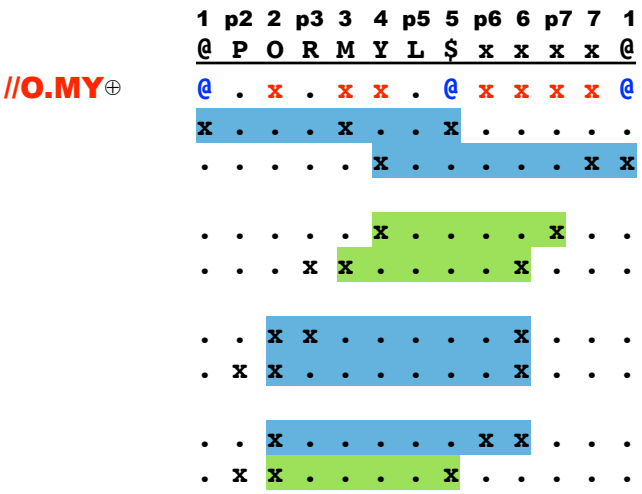
No Greater Love (original home tonic = Bb)



Here's the same thing with a walking bass line provided in class (blue text).



Here follows a development of the bass line for bars 1-4. The line outlines a building block and then transitions to the next one via an internal or external note. The only passing notes outside the major family scale are **p2** and **p3** at the ends of bars.


1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
@ P O R M Y L $ x x x x @
//O.MY⊕

BODY AND SOUL

This piece is strongly chromatic in a way that's particularly complex in music notation, namely multiple key signatures of different kinds (sharp vs. flat). My source for the written music is *The Ultimate Jazz Fakebook*, Hal-Leonard (1988), p67. I worked out the details before I heard the most astounding jazz performance of this piece I have ever heard, by Esperanza Spaulding on the CD *Ezperanza* in the cd HUCD 3140, 2008 from wwi.headsup.com. Following the changes in this performance is amazing and inspiring. Three successive sections have three different key signatures: A 5 flats (Db-Ionian); B 2 sharps (D-Ionian); C 1 flat (D-Aeolian, but actually Dorian due to a natural in the written melody line). Ionian and Dorian of tonic D are nearby parallel modes (only 2 notes different). Ionian of Db is a distant from these (5 notes different from B). The obvious home tonic is Db because A starts and ends the piece. The large distance between A and B is more than compensated by the simplicity of every note of B being a half tone up from every note of A.

Body and Soul (original home tonic = Ab)

A ┌ //Y (Ionian@1)

O P M M R LR

| — 2 3 2 3 2 | 6* — , | — 5 6 5 6 5 | 2 1 7 5 , |

1_____

O Y O M YO Y

| — 1 6 4 2 | 3 2 , | — 1 2 3 5, 5 6 p3 | 1 — | 1 — |

B ┌ altL@p2 (Ionian@p2)

2_____

L M L

| — | p2 p3 4 p6* | p2* 4 p6* — p5 | 4* p2 p3 1 p6 , | — |

C ┌ altM@p2 (Dorian@p2)

M O RL M M R O P PR

| — p2 p3 3 p6* | p3* p7 p2* p6 | 7* p6 p7* p5 | p3 2 p2 6 |

The melody scales are classical modes. The strongly chromatic harmony is not, but is consistent with the melody. When I first encountered this piece, I struggled with trying to understand how this harmony implied the melody scales and finally realized it doesn't.

		Db		Eb		F	Gb		Ab		Bb		C	Db	
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	Y	L	$	x	x	x	x	@	
A	//Y	@	.	x	.	x	x	.	$	.	x	.	x	@	Ionian of tonic 1
B	altL@p2	x	x	.	x	.	x	x	.	x	.	x	.	x	Ionian of tonic p2
C	altM@p2	.	x	.	x	x	.	x	.	x	.	x	x	.	Dorian of tonic p2

EVERY TIME WE SAY GOODBYE

This Cole Porter piece is hauntingly lovely, with major-minor changes that go well with the words about love and loss. The home tonic is Eb and the key signature is 3 flats, identifying the Ionian mode. The melody line sequence **5-4-3-2-1-7-1** at the end confirms this as the resolution mode. Otherwise, much of the of the melody line is in the major family mode **//O.MY⊕**. This and much of the harmony are strongly chromatic. The changes are often subtle and understated.

Although the main tonality is major, minor or minor-ish segments appear in both melody and harmony. Section (d) is solidly minor. The words in section (f) include the famous phrase "how strange the change from major to minor." The change may be interpreted as between family modes **//O.MI⊕** (grey highlighting) and **//OR.Y⊕**. The melody makes no explicit change of tonality here but could. The sound of a fourth going down a half tone in the harmony conveys this change to the ear, in the context.

Every Time We Say Goodbye (original home tonic = Eb)

(a) M Y Y M PM Y Y

| 3 3 3 — | — 3 3 3 | 3 — — 3 | 4 — 2 1 7 |

(b) M PM Y Y M M R O

| — 5 5 5 | — 5 5 5 | 5 — — 3 4 5 | p6 — 4 3 2 |

(c) R R O Y R M M

| — 1 1 1 | 1 7 — 5 | p7 — 3 4 5 | 6 —— |

(d) R O O O L L O Y

| — p6 p6 p6 | p6 5 — 1 2 | p3 p3 — 1 p3 2 | ——— |

(e) repeat (a)-(b)

(f) ⌐ //O.MY⊕ ⌐//OR.I⊕

$ RL O Y M M R O

| — 1 1 2 1 | 1 7 — 5 6 | p7 3 6 2 | 1 7 1 5 4 |

(g) M M L Y M RL OY

| 3 3 3 — | — 2 1 7 | 1 —— | ——— |

The major to minor bars of (f) are shown next. The harmony lead-in to these bars is **RL-OY** down a half tone. The family modes follow from filling in **OY** differently. The chords shown are from these modes.

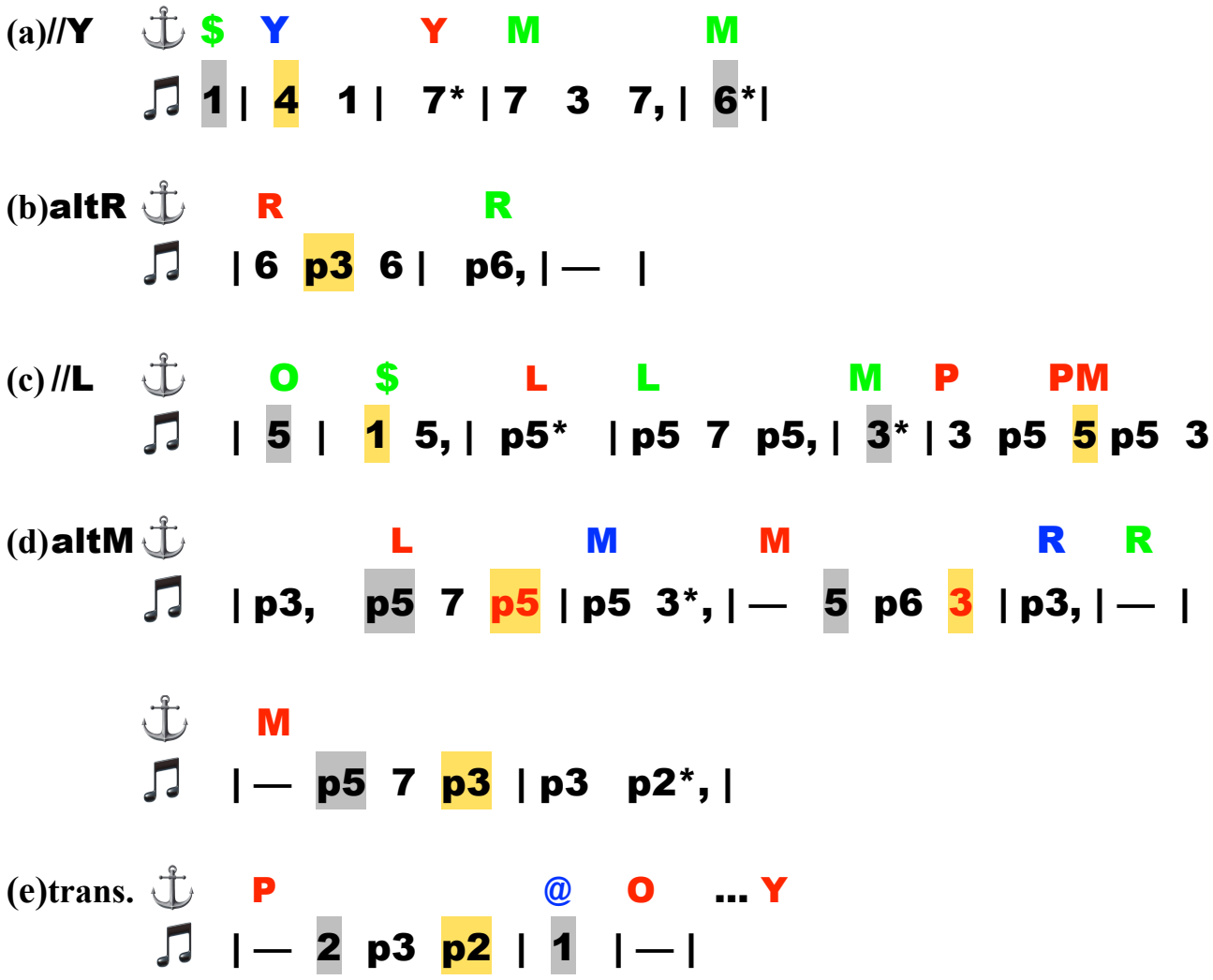
			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		
			@	.	O	R	M	Y	.	$	x	+	x	x	@		
(f)	M	*I-7*	–	.	.	.	M	.	.	.	.	.	x	.	.	|	//O.MY⊕
	M	*IV-M7*	.	.	.	.	M	–	.	.	.	x	.	.	.	|	
	R	*IV-m7*	.	.	.	R	.	–	.	.	x	.	.	.	.	|	//OR.Y⊕
	O	*pVII-7*	.	.	O	.	.	.	.	.	x	.	–	.	.	|	

ALL THE THINGS YOU ARE

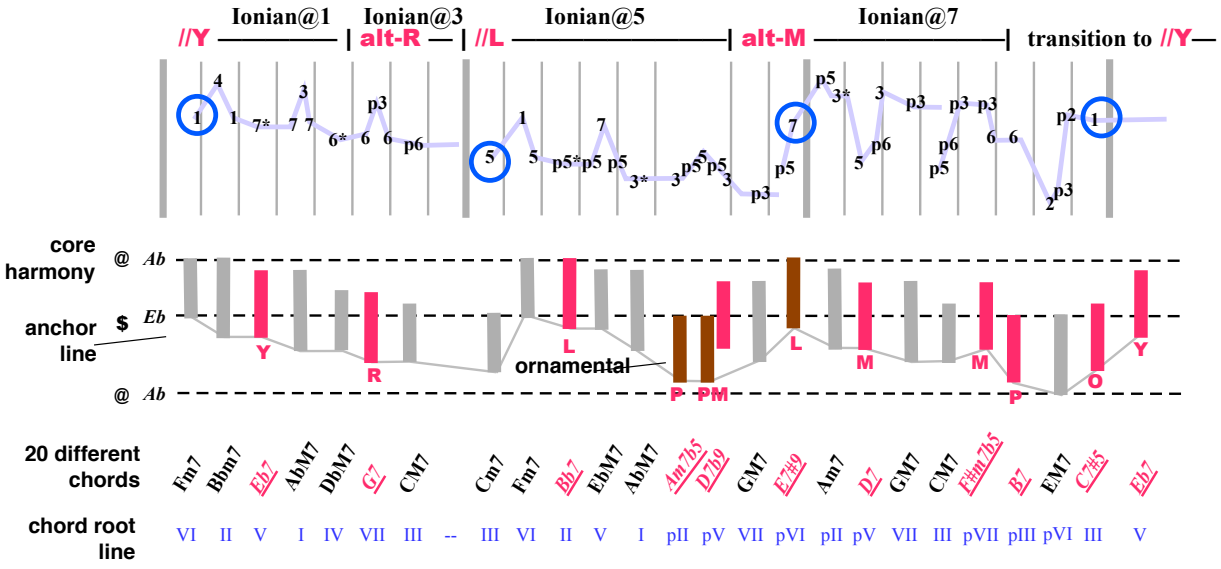
I learned this beautiful piece early in my musical adventure, and found it easy to learn because the keyboard sequences are simple, but difficult to understand in music notation. The source is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). The home tonic is Ab and the key signature is four flats, identifying the starting and ending melody modes as Ionian. In the following interpretation, the piece visits successive parallel and alt classical modes. The determining tritones of these modes in (a)-(b)-(c)-(d) are **Y-R-L-M**. The implied Ionian tonics are **1-3-5-7** but this isn't helpful because the passages don't all resolve to them, but transition to the next passage via shared notes. Better to think in terms of the home-tonic modes because they provide everything needed to play the changes.

The skeleton melody line and associated core harmony of this interpretation are shown next for bars 1-26, where all the changes occur.

All The Things You Are (original home tonic = Ab)



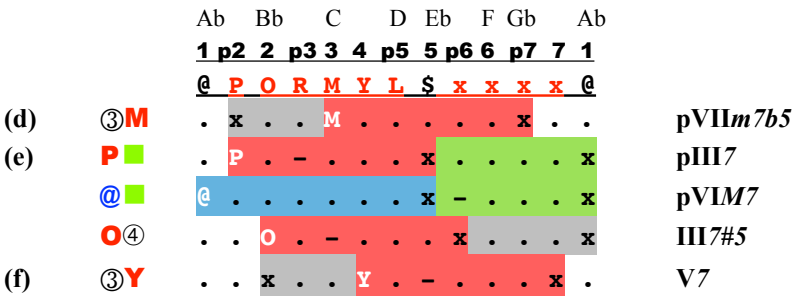
The following overview picture is helpful in getting a handle on the piece as a whole.



Here follows a summary of the modes of this interpretation ("notes" in the left column means actual melody notes). The sometimes-sparse melody lines are open to different interpretations but this interpretation provides a useful handle on the piece.

		Ab		Bb		C		D	Eb		F		G	Ab	
		1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**	
		@	P	O	R	M	Y	L	$	x	x	x	x	@	
(a)	notes	x	.	.	.	x	x	.	.	.	x	.	x	x	
	//Y	x	.	x	.	x	x	.	x	.	x	.	x	x	**Ionian@1 (Ab)**
(b)	notes	.	.	.	x	.	.	.	.	x	x	.	.	.	
	altR	.	x	.	x	x	.	x	.	x	x	.	x	.	**Ionian@3 (C)**
(c)	notes	x	.	.	.	x	.	x	x	.	.	.	x	x	
	//L	x	.	x	.	x	.	x	x	.	x	.	x	x	**Ionian@5 (Eb)**
(d)	notes	.	x	.	x	x	.	x	.	x	.	x	x	.	
	altM	.	x	.	x	x	.	x	.	x	.	x	x	.	**Ionian@7 (G)**
(e)	notes	x	x	x	x	.	.	.	.	.	.	.	.	x	**sparse, ambiguous**
(f)	//Y	x	.	x	.	x	x	.	x	.	x	.	x	x	**Ionian@1 (Ab)**

The (d)-(e)-(f) harmony transition shown next is "interesting." My piano teacher at the time suggested I just memorize it. And so I did, but I kept worrying away at understanding it in more fundamental terms. This shows that it shares notes between modes, resulting in an unusual sequence of chord symbols.

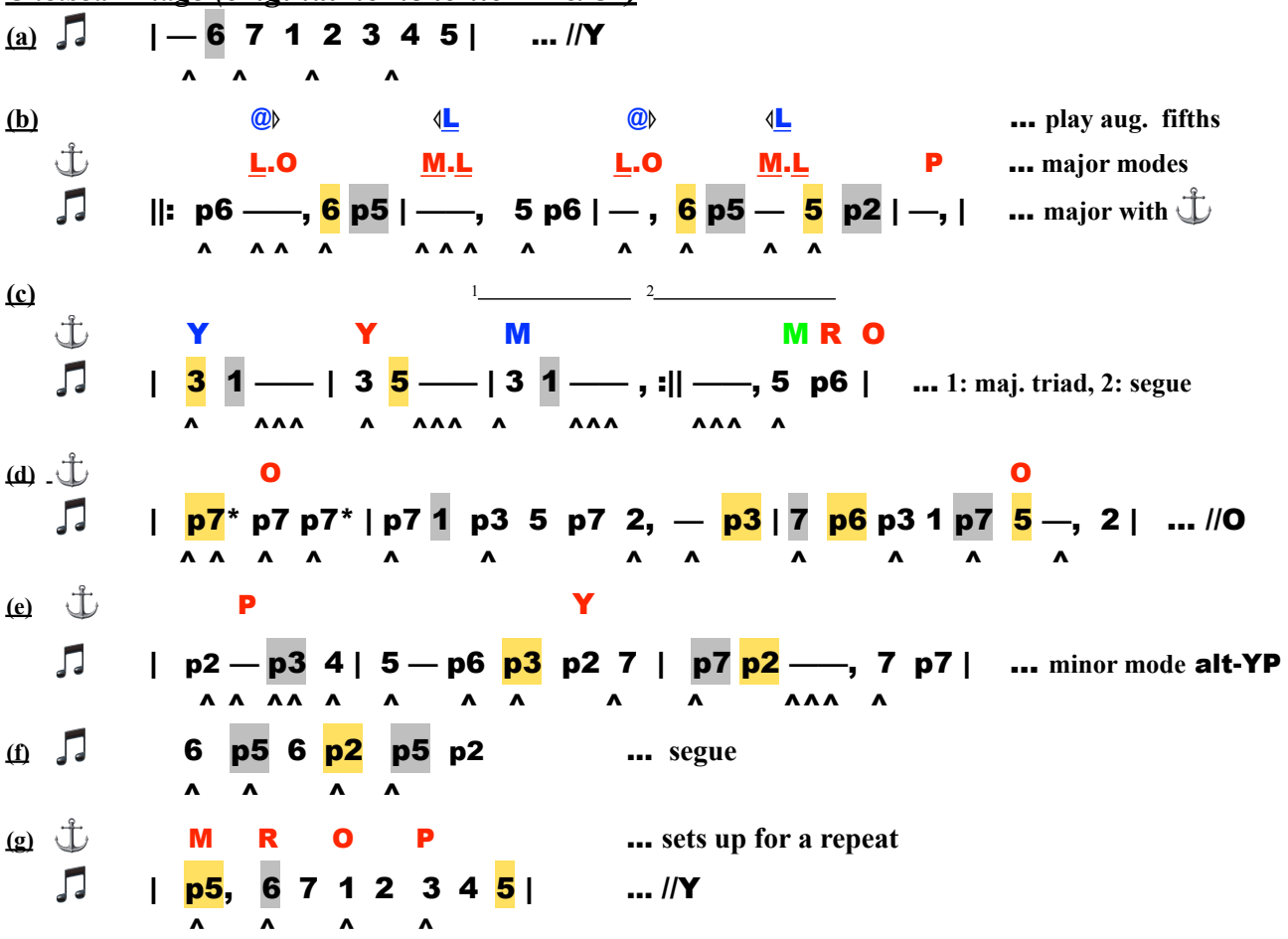

Ab Bb C D Eb F Gb Ab
1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
@ P O R M Y L $ x x x x @
(d) ③M . x . . M . . . . . x . . pVII*m7b5*
(e) P . P . – . . . x . . . . x pIII*7*
@ . . . . . . x – . . . x pVI*M7*
O④ . . O . – . . . x . . . x III*7#5*
(f) ③Y . . x . . Y . – . . . x . V*7*

CHELSEA BRIDGE

My source for this hauntingly beautiful Strayhorn piece is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). There are two written key signatures but only one home tonic on the piano, the black key immediately above middle C. The piece moves from Ionian major of this tonic (Db major, 5 flats) to Aeolian minor of the same tonic (C# minor, 4 sharps), and then back to Ionian major. This is complex to begin with and the complexity is multiplied by staying with the 4 sharps key signature to return to Ionian major. Between these parallel classical modes of the same tonic are chromatic passages that multiply the notational complexity once more. The result is a confusing muddle of many more than twelve note symbols relative to the two written key signatures, with tricky timing in some places. The piece is musically sophisticated by any measure, but the notational complexity is out of proportion to the sophistication.

I first learned this piece by rote with great difficulty from the written music, but thought that anything that sounds this good must have simple musical logic behind it. Tritones identify scales; fifos implied by them are mostly omitted. Digging this kind of thing out of written music is a struggle, but doing it sharpens the ability to think in terms of keyboard intervals instead of the notes of key signatures. It may look complex at first glance, but it's valid for any home tonic, and shines a light on commonly used patterns, which are obscured in music notation by showing the same piano key in different places by sharps or flats or naturals in spaces or lines of a musical staff.

Chelsea Bridge (original home tonic = Db/C#)



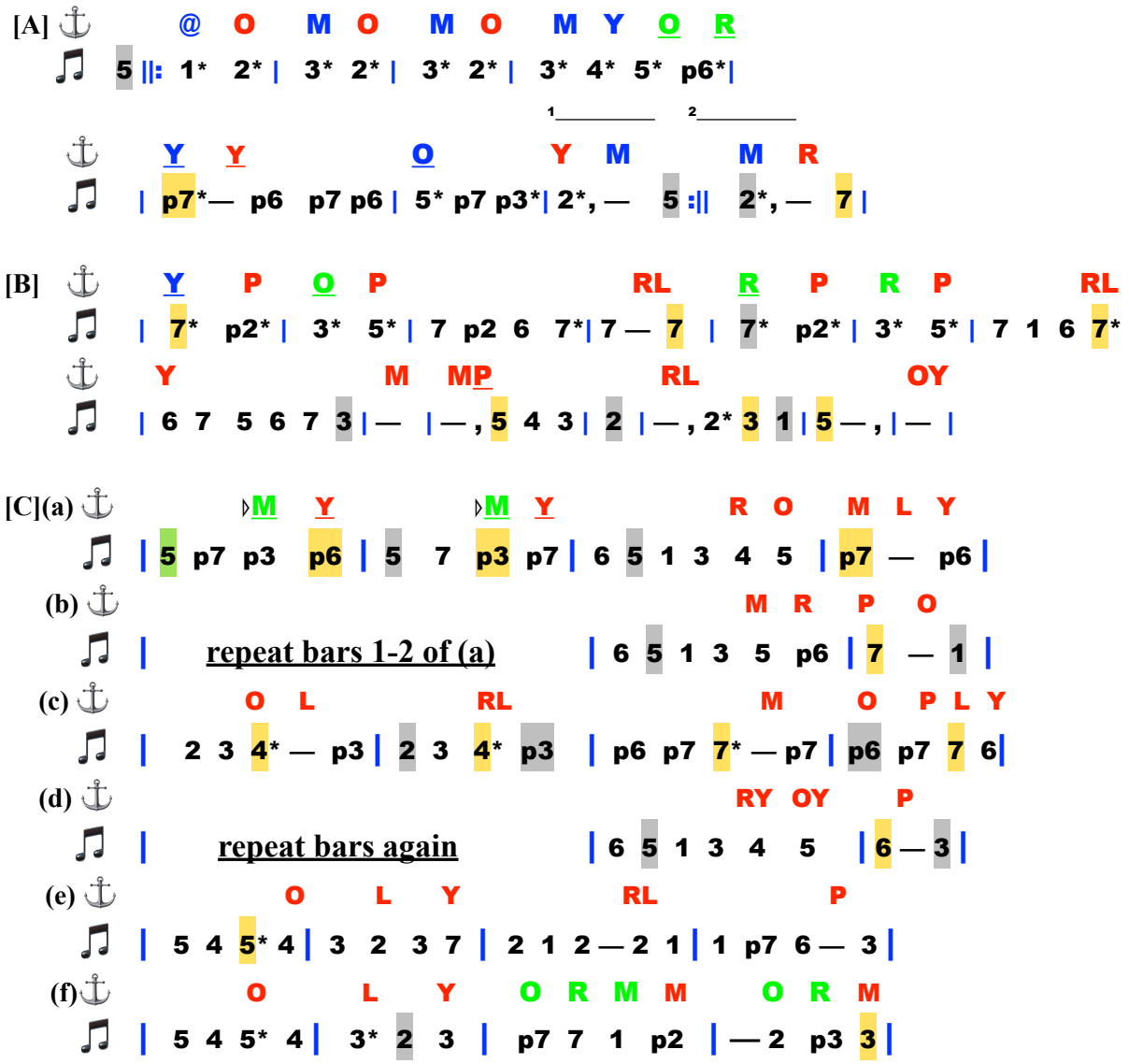
Here are the melody modes and transitions between them. The main sequence is **//I-//O-//I**. The in-between modes and segues are related to them in simple ways. The changes are musically interesting but easy to understand and play as shown here, once you "get it." I can imagine that virtuoso pianists are able to read the written music and understand these changes instinctively in whatever terms they use to conceptualize music, but this leaves everyone else stuck with learning the piece laboriously by rote, without any real understanding. Knowing the changes below enables even inexpert pianists to approach the piece with understanding.

	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1		
	@ P O R M Y L $ x x x x @		
(a)	x . x . x x . x . x . x x	//Y	Ionian
(b)	x . x . x . x . x x . x x	//LO	major mode
	@ . . . . x		imply by aug. fifth @♭
	x . x . x . x x . x x . x	//ML	major mode
	x . . . . L		imply by aug. fifth ♢L
(c)	x . . . x . . x x	//Y	major triad
	minor of home tonic		
	@ . . . . x . . . x		segue to minor via aug. fifth @♭
(d)	x . x x . x . x x . x . x	//O	Aeolian
(e)	. x . x . x . x x . x x .	alt-YP	Phrygian flat one (minor)
(f)	. x . . . . x . x		ambiguous but suggests Locrian next
	x x . x . x x . x . x . x	alt-L	Locrian goes down 1/2 tone to Ionian
	major of home tonic		
(g)	x . x . x x . x . x . x x	//Y	Ionian

LUSH LIFE

This beautiful Strayhorn piece is melodically and harmonically rich, and challenging to play from the written music because there are often two or more chords per bar, many of them chromatic relative to the written key signature of five flats (major tonic Db). The piece is so chromatic that there is more than one way of understanding it. It can be seen as daisy-chained, short segments of successive parallel classical modes, but this quickly bogs down in complexity. The view of it here is in terms of two non-classical parallel modes of the home tonic for both melody and harmony: one is the 10-note **//ORMY** minor-major mode in [A] and [C]; the other is the tonic-less **//IP** mode (Ionian#*1*) in [B].

Lush Life (original home tonic = Db)



These modes lead naturally to short, easy-to-remember segments in classical and other modes that follow from the flow. For example, **alt-YP** morphs into **//Y** halfway through [B] by altering one note.

From time to time, the harmony of the different sections substitutes **P** and **L** for **O** and **Y**, but thinking of these as ornamental relative to the **//ORMY** keeps things conceptually simple.

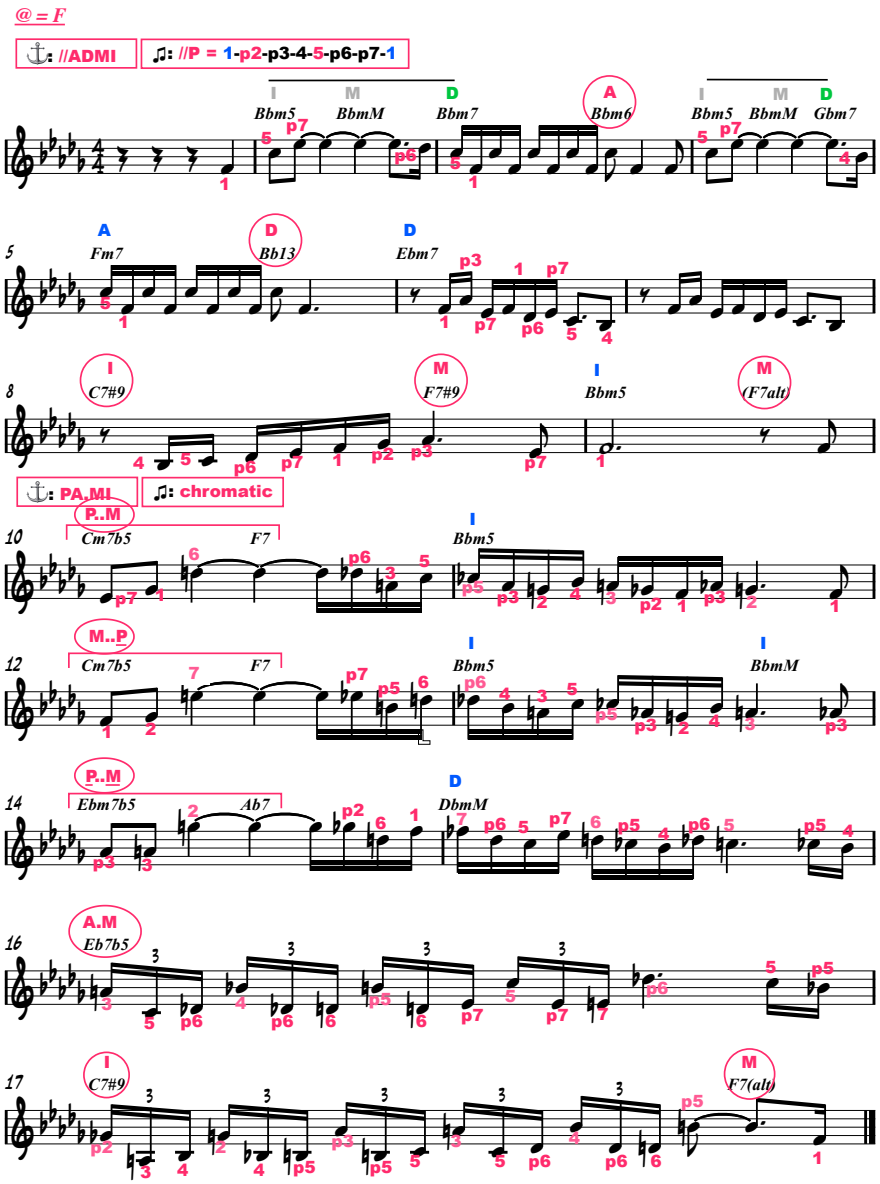
Harmony fifos are mostly omitted for simplicity, except for characteristic sequences of them in [A] and the ending bars of [C].

The double tritones **PM**, **OY** and **RL** that appear in many places in the harmony are diminished seventh chords, commonly used ornamentally in any context in which they sound good. The double tritones have circular symmetry (all intervals are minor thirds), so only three of them with different notes exist. They can be played as four notes but they can also be voiced with three notes (a tritone with a minor third on top, or the opposite) or even with two notes (outer notes a major sixth apart), when the context implies the other note(s). A sequence of two or three of them, often used as an ornamental transition between modes, can be played as as sequence of major sixths anchored by any three of six tritone anchors (e.g., **P-O-R** going up and **L-Y-M** going down, to name just two possibilities — others require inversions of individual tritones).

The chromatic melody-line run in the final two bars is a signature of the piece.

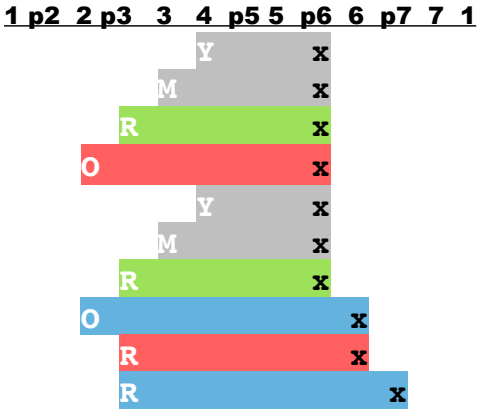
THE PEACOCKS

This hauntingly beautiful piece by Jimmy Rowles wraps up the chapter with an example in annotated music notation (in handwritten annotations, circling tritone anchors, or their chords of origin, or both is helpful). This is a Sibelius score created from a borrowed fake book to which I no longer have a reference. The annotations use the original **PADMIL** alphabet, requiring **ADI** to be understood as **ORY**. The piece sounds so "right" as written that jazz improvisations rarely stray far from it. The home tonic is determined to be F by the final notes of bars 9 and 17. In bars 1-9, the combination of the 5 flats key signature, the home tonic and the lack of accidentals in the melody line identifies the melody scale as Phrygian of the home tonic (mode signature **//P**). The melody line in bar 8 runs up through all the notes of this scale starting a fourth up (fifth down) from the tonic. The contrasting harmony is chromatic. The over-lines in bars 2 and 4 indicate held treble notes.



The ups and downs of the melody line are directly visible in the notes on a staff but the drawback is this commits to a particular tonic and key signature, and gives little direct sense of context.

The core harmony in opening bars is visibly from the **//ORMY** scale (which is minor-major relative to the home tonic).



The melody of the second section consists mainly of individually simple sequences from the full chromatic scale that are prompted by the harmony. These are better remembered in terms of simple patterns shown by **x** entries in a mode table than from note sequences of any kind.

The two groups of four sixteenth notes in bars 11-15 follow a repeated pattern of down a minor third, down a half tone, and up a minor third (net down a half tone).

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	Y	L	$	x	x	x	x	@	
bar 11	.	.	.	x	.	.	x	.	.	.	.	.	.	down
	.	.	x	.	.	x	.	.	.	.	.	.	.	up
	.	x	.	.	x	.	.	.	.	.	.	.	.	down
	x	.	.	x	.	.	.	.	.	.	.	.	.	up
bar 13	same, up a whole tone													
bar 15	same, up a minor third													

The eighth note triplets of bar 16-17 follow a repeated pattern of down a minor seventh and up a half tone, (net down a major sixth).

CHAPTER 5: OBSERVATIONS & CONCLUSIONS

Music notation is here to stay and must be lived with, but this this doesn't preclude alternate views of piano music from a different perspective. I looked for an alternate view of harmony and found one based on intervals.

The central role of tritones in PKP tends to seem inappropriate to people savvy in music notation, for several reasons. In general, it goes against received musical wisdom. More specifically, tritones cannot even be written down in music notation until a key signature is established, so imagining them as identifiers of scales is difficult. Anything involving multiple tritones is complex in music notation because it implicitly brings in other key signatures — the more tritones, the more complexity. The very concept of fifos is unconventional and even more so is the concept that they can be treated as morphed tritones, understood from context. The payoff is insight into deep structure that is difficult to see in music notation. Appreciating these things requires stepping outside the box of music notation, at least momentarily.

Thinking in notes and thinking in intervals *is* different in kind. Many experts who have learned the hard way to think in notes apparently find thinking in intervals too alien to contemplate. Comments from them have run the gamut from "obviously wrong" to "wrong headed" to "naively simplistic" to "overwhelmingly complex." One expert said, dismissively, I had "found a tritone hammer and saw everything as a nail." The book *Modalogy*, devotes 50 or so complex pages to parallel modes without ever mentioning the possibility that tritones might simplify things.

Because I started out seeing intervals and notes as dual views of the same thing, I came to find it natural to switch back and forth. I have made my best effort in this document to convey the simplicity of this. I believe, based on my personal experience as an adult beginner, that any beginner could easily learn to think this way.

I know from personal experience that PKP helps with learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. It turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

The jumpy chord root lines in many of the examples demonstrate that referring chords to constantly moving roots creates misleading complexity. This complexity is both conceptual (the underlying, smoothly flowing core harmony is obscured) and actual (playing the jumpy harmony is technically difficult because all the fingers must be lifted and moved by large intervals). It's easier to learn the core harmony and introduce the jumps later, if desired, via inversions, which are always easy on the keyboard, given an anchor line.

GENERAL BENEFITS

PKP annotations above the staff tell you many things. Combined with context provided by the melody line, they tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not right at any point by identifying tonic scales in play in a very immediate way. They tell you about transitions between major and minor tonality and between different tonics. The provide a basis for enrichment and improvisation.

Scale, modes and chords are seen as part of a continuum expressed in the same terms. This seems to me to be a very powerful benefit, compared to seeing everything beyond classical modes as special cases identified by accidentals (which includes chromatic chords with suffixes that imply

accidentals). The representations of scales and chords in terms of tritone clusters are unfamiliar to expert pianists but the interval stacks that they imply on the keyboard are completely familiar.

With PKP, playing music is guided by note-based symbology on the music page (one dual view) plus PKP annotations above the staff (the other dual view) that suggest core harmony and make changing tonic scales and tonics directly visible to the eye in terms of changing alphabet words.

A simple mode hierarchy covers much ground. The mode signatures provide a novel, high-level, conceptual view of tonic scales that's easier to hold in the mind's eye than the very much larger number of spelled out tonic scales in music notation. The modes cover 12x39=468 scales of 12 different tonics in music notation (more than is covered by scale dictionaries such as *The Source*). Words of 1-4 letters from the 6-letter alphabet identify, by tritone content, modes with 7 or more notes, 1-4 tritones, and no intervals larger than a minor third. The count of 39 modes includes 11 single modes covered by single words and 4x7=28 parallel modes covered by transpositions of 1-2 letter master words. A selected home tonic anchors all of this to the keyboard.

The words are analogous to the words of biological DNA in the sense that they identify deep structure without spelling out the details. Words of biological DNA are "expressed" as proteins, the building blocks of life. **PORMYL** words are "expressed" as building blocks of scales and harmony. Biological DNA is "extracted" from biological samples and interpreted by sophisticated machines called "sequencers." **PORMYL** scales and building blocks are "extracted" and "sequenced" from musical lines by sophisticated machines called human eyes. Biological DNA is "inherited" from parents. **PORMYL** words are "inherited" from general knowledge about them encoded in the hierarchy.

Symmetry breaking in the scale hierarchy and symmetry breaking in core harmony are two sides of the same coin. In the hierarchy, it yields scales with fewer tritones. In harmony, it releases the tension of dissonance to produce consonance. Seeing the simplicity of this requires forming mental images of it relative to the schematic tonic octave that are independent of specific assignments of the octave to the keyboard.

Understanding harmony is possible independently of practicing. Keyboard symmetries centering around tritones enable high-level representations of harmony that are simple in their own terms and deep in harmonic meaning in any terms.

Music is freed from its key-signature straightjacket. Harmony scales are understood at a glance from alphabet words aggregated by eye, instead of having to be figured out or remembered from key signatures, accidentals and chromatic chord symbols. Tonic changes are no more difficult than any other changes.

Anchor lines that define harmonic cores replace chord root lines as the fundamental musical lines of harmony. Chord roots are visibly secondary elements in the flow of the music because inversions move them into the body of the chord, where they become no more than another chord note that may be omitted if implied by context.

Sight reading anchor lines provides deep context. Anchor lines tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not by identifying tonic scales in play. They tell you about transitions between major and minor tonality and between different tonics.

Multiple explicit or implicit key signatures become a non-issue. This is true whether the key signatures imply tonic changes or only scale changes for the same tonic. Tonic changes amount to no more than moving (conceptually) a tonic pointer within the home octave and reinterpreting the alphabet

letters relative to it, without changing any notation.

Chord symbols become a non-issue. Chord complexities that result from banging square pegs (chord symbols) into round holes (places in scales where they don't quite fit) are misleading. Chords described by chord symbols are, on the keyboard, almost universally composed of combinations of PKP's two kinds of building blocks, of which only tritones are always core. A melody line plus an anchor line above the staff imply a voicing of an unidentified chord progression. Specific chords are identified by assigning roots.

Relating the notation to the keyboard is simple. Different home octaves look very different on the keyboard because of different mixes of black and white piano keys, but the difference is manageable for two reasons: mapping the notation to the keyboard boils down to assigning a 6-letter alphabet to 6 adjacent piano keys; and the C octave provides a mental reminder of the meaning.

Learning blues prepares you for anything music notation can throw at you. Blues is an example of "folk music" that evolved independently of music notation and therefore tends to be regarded as naive when measured against its depth and complexity. This condescending view is unwarranted. Blues is harmonically sophisticated in a highly organized way that makes it, to my mind, a better starting point than classical modes of key signatures for understanding many forms of music.

Sharps and flats are not needed. Building blocks are the same number of half tones no matter how they are notated. This enables PKP to dispense with sharps and flats without any loss of musical accuracy. Expert pianists tend to think they are still needed to identify context because they have learned to use them for that, but note positions relative to tonics are sufficient.

The simple flow of harmony in harmonic cores, in which building blocks slide or morph into other building blocks, is universal. Nothing constrains harmony to be played this way but the flow is so natural and easy to play that much is to be gained by learning pieces this way and then transitioning to other playing styles if desired.

Tritone-intensive harmonic cores of much music provide so much harmonic variety that voicing them requires only adding depth. Depth is easily added by doubling the treble line of the harmonic core an octave down, or the bass line an octave up, and then making adjustments to move points on the line closer to the core, if desired.

No more piano wizardry is required to have fun playing music than the ability to sight-read melody lines in music notation. Anyone with eyes to see intervals on the keyboard can play harmonic cores from anchor lines annotated above the staff, and remember them.

The simple way harmony is represented provides great freedom in choosing how to play it. Playing styles may range between the extremes of smoothly flowing harmony within the octave of the home tonic of piece to jumpy harmony in which all harmony notes go up from a chord root line. Improvisation follows naturally and is fun, even if not expertly done.

The simplification provided by the alphabet is different in kind from simplifying chord symbols. Any sophistication in the harmony is in the tonic scales of origin of the chords.

Mode signatures provide an above-the-staff markup notation for the concept of "modal jazz" which is otherwise a concept without a notation.

PKP annotations above the staff are not always necessary. With experience, straightforward chord progressions may be sight-read in PKP terms.

SOME REFERENCES

1. Barta, *The Source: The Dictionary of Contemporary and Traditional Scales*, Hal Leonard (1995), for helping me to be sure I was not missing important scales.
2. Mehegan, *Jazz Improvisation 1: Tonal and Rhythmic Principles*, Watson-Guptil (1984), for Roman-numeral chord notation, and for teaching me (unintentionally) that it is not a solution for complex chromatic chord progressions, but a problem if pushed beyond its basic function of specifying chord root lines.
3. Eskelin, *Lies My Music Teacher Told Me*, Stage Three Publishing (1994) for insight into the nature of scales and musical "perfection," and for encouraging me to think outside the box.
4. Dmitri Tymoczko, *A Geometry of Music* (2011) for stimulating discussions of how to think about music from different angles.
5. Mark Levine, *The Jazz Theory Book*, Sher Music Co. (1995) for providing examples of well known jazz scales and harmonic forms in conventional notation, against which to verify PKP coverage.
6. George Russell, *The Lydian Chromatic Concept of Tonal Organization*, http://www.georgerussell.com/lc.html, for making me aware that PKP covers the concept, because nothing is changed by replacing the Ionian mode by the Lydian mode as the default reference major mode for any piece of music.
7. Edward Frenkel, *Love and Math: The Heart of Hidden Reality*, Perseus (2013) (on Kindle), for many insights into the usefulness of symmetry.
8. Jeff Brent with Schell Barkley, *Modalogy — Scales, Modes & Chords: the Primordial Building Blocks of Music,* Hal Leonard (2011), for the most comprehensive treatment I have found of this subject. The result is painstakingly accurate but overwhelmingly complex because it sticks to representations based on sharps and flats that follow from the misalignment of pitches of notes of overlapping octaves.
9. Ross W. Duffin, *How Equal Temperament Tuning Ruined Harmony (and Why You Should Care)*, W.W. Norton (2007), for an understanding of the piano's deviation from musical perfection.
10. Stephon Alexandar, *The Jazz of Physics: The Secret Link Between Music and The Structure of the Universe*, Basic Books (2016). This amazing book resonates strongly with my own ideas. The author is a jazz musician and physicist who speaks of doing physics as informing the playing of jazz and playing jazz as informing the doing of physics. Both require rising above formalisms to form conceptual representations, with symmetries highlighted as particularly important.

ACKNOWLEDGMENTS

I did not take this musical journey alone. I received comments and help from many people while the ideas were germinating and consolidating.

Music theorist Paul Steinbeck encouraged me to continue writing at a time when I was becoming discouraged. Jazz pianist, teacher and composer Taylor Eigsti has been an inspiration to me. Although I have never been a piano student of his, I have learned much from him in sporadic discussions in person and by email. I am deeply grateful for his willingness to take time away from a busy schedule to engage in these discussions. A short series of piano lessons from SF jazz pianist Michael Parsons helped me to see more clearly the relationship between my ideas and standard jazz-piano practice.

Thanks to SMT (Society for Music Theory) members Charise Hastings, Peter Shultz and Neil Newton for insightful email comments and encouragement following announcements of my website on an SMT mailing list. Thanks to music professor Robert Rawlins for email encouragement, and for helpful examples and comments. Aaron Blumenfeld and Susan Muscarella gave helpful courses at the Jazz School in Berkeley (now the Jazz Institute) that provided many examples to chew on (including, from Susan Muscarella's course, some interesting variations on *Summertime* in D minor that helped me understand ornamental scales). Amateur pianist and Jazz School Board Chair Susan Brand, and concert pianist and music entrepeneur Robert Taub, provided early encouragement.

My piano teacher in San Francisco for several years, Ken Fishler, provided inspiration and how-to information on chord voicings, while patiently tolerating and responding to my question-everything approach. My first piano teacher in Ottawa, Canada, Sally Robinson, started me off right as an adult beginner by helping me to learn favorite harmonically sophisticated pieces by following her fingers on the keyboard without understanding anything about what I was doing except that it sounded right — being able to play these pieces, even if clumsily and by rote, enabled understanding to seep in gradually. I think that without this particular way of starting out — fingers on the keyboard first, written music later — I would not have started thinking about keyboard harmony in the way I did. It forced to my attention the large gap between simplicity on the keyboard and complexity of the full notation that represents it.

I wore out the patience of many music professionals by bombarding them with unsolicited email requests for comments on my ideas. I would like to thank four in particular, who politely responded to numerous emails in spite of being uncomfortable with my ideas: Harry Likas, Dmitri Tymoczko, Jeff Brent and Daniel Glover. Their criticisms helped me change my explanations without changing my mind.

Thanks to friends Marva Black, Mike Budde, Peter Marchant and Selinda Spugies for various comments on this project along the way.

My wife Sheila's accurate musical ear helps me know what does and does not sound good on the piano, in addition to her making my life generally interesting. My grandsons Joshua and Ethan Feiber provided encouragement and comments; Joshua set up the website www.pianotheoryman.com as birthday gift; Ethan, who learned very young to play the piano impressively well by ear, has experimented with the notation and found it helpful for approaching written music.

SOME COMMENTS FROM READERS

The following comments provide a kind of history of the development of PKP: the dates on the left identify when the commenters read different drafts of this material; the material has evolved considerably since then. The unconventionality of putting tritones on center stage has tended to get in the way of expert musicians accepting the ideas at all, let alone seeing them as simple, which has not been helped by the fact that it took me a long time to find the simple way of explaining the ideas that appears in this document. I continue to think the ideas are important for learning the piano in a less restrictive manner than is conventional but have not yet been able to find an audience. At the very least, the more recent comments show that the PKP method is more than just a fantasy of an over-enthusiastic amateur.

(2016) Taylor Eigsti more recently than below. "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

(2011) Paul Steinbeck. Assistant Professor of Music Theory; Washington University, St. Louis

"The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music."

(2009) Robert Rawlins. University Music Department Chair (Rowan University); jazz musician; teacher; author of several books on jazz

"I became aware of Raymond Buhr's novel method for analyzing and voicing chromatic chord progressions in 2008 through a draft of a paper he wrote on the subject. I have kept up to date on developments of the method and we have had many email exchanges discussing issues of interpretation and application. I am a member of his intended target audience—a jazz musician who is not a pianist who needs to work out harmonic patterns on the piano from time to time. I am also a music teacher who has actually tried out aspects of his method on students. I can vouch from personal experience for his method's helpfulness in dealing with complex chromatic chord progressions."

(2009) Susan Brand. Board Chair, The Jazz School in Berkeley; amateur pianist

"When Raymond Buhr consulted me about his theory of chromatic chord progressions, I was immediately struck by his ability to analyze and attempt to simplify this complex subject. Mr. Buhr brings a unique perspective and a great deal of enthusiasm, depth of understanding and originality of viewpoint to the subject. Over the years I have watched the continuous work that he has put into editing and rethinking his work. He has had ongoing consultations with many knowledgeable musicians/ teachers/editors and all have contributed to the development of the method described in this book. His ideas offer a way of understanding musical theory that will add greatly to the field and will be extremely helpful to musicians and music educators."

(2008) Taylor Eigsti. Jazz pianist, composer, teacher, former Artist in Residence at The Jazz School in Berkeley

"Through the brilliant lens of an engineer, Raymond Buhr has laid out an analysis of harmony that is a unique and complex look at the right-brain from the left-brain's perspective."

APPENDIX A : UNCONVENTIONAL ELEMENTS

TERMINOLOGY & NOTATION

- **anchor:** identifies a building block by the position of its bottom end relative to the home tonic
- **anchor set:** set of tritone and fifo anchors that define a scale by construction
- **anchor line:** anchor sequence written above the staff
- **alphabet: PORMYL** identifies anchors by the first letters of the names of classical modes (internal letters in the case of **O** and **R**)
- **building block:** tritones or fifos (fifths or fourths) anchored at alphabet positions (size distinctions determined by color coding: red for tritones, blue for fifths, green for fourths).
    - words determine combinations, with secondary letters parenthesized (the others are **core**)
    - superscripts ▼(minor), ▲(major), ● (dim) identify building-block splits that provide triads
    - attachments ■, ■, and ■ identify octave completion building blocks that form octave **stacks**
    - attachment ■ identifies an interval smaller than a building block to be determined from context
    - attachment ⏵ or ⏴ identifies an expanded or shrunk building block
- **chromatic scale of the home octave: 1-p2-2-p3-3-4-p5-5-p6-6-p7-7-1**
    - **chord roots use RN symbols**
- **context:** provided by mode signatures for melody and harmony plus flow of both
- **core:** set of octave shapes morphed asymmetrically by a half tone from a tritone shape
- **family:** a set of sub-scales of a family scale defined by a single master mode
- **fifo:** fifth or fourth that are opposite inversions (add up to an octave)
- **flow:** formed by morphs and slides of building blocks
- **frame:** defined by the tonic **@** and pitch center **$** of a tonic octave
- **morph:** change in the size of a building block by a half tone at one or both ends
- **mode signature:** word prefixed by **//** or **alt**
    - plus superscript identifies a family mode with an implied extra note
    - a transposed mode signature transposes a mode, e.g, **tr[//I]@6**
- **outside:** not in a given tonic scale, as distinct from "chromatic" meaning not in a key-signature scale
- **pattern:** organized arrangement of intervals on the keyboard or over time
- **pitch center:** note identified by a fifth/fourth octave split, symbolized by **$**
- **shape:** an object on the keyboard determined by split or combined **building blocks**
- **slide:** size-preserving movement of a building block
    - **wobbly slide:** combined morph and slide
- **phlat:** prefix **p** identifying chromatic-scale notes in the whole tone gaps of the major scale of a tonic
- **tonic pointer:** symbol of form **@t** indicating a secondary tonic
- **word:** set of alphabet letters
    - optional dots indicate skipped letters
    - underlining indicates inversions of building blocks
    - parentheses indicate non-core building blocks
    - backslashes indicate non-overlapped building blocks

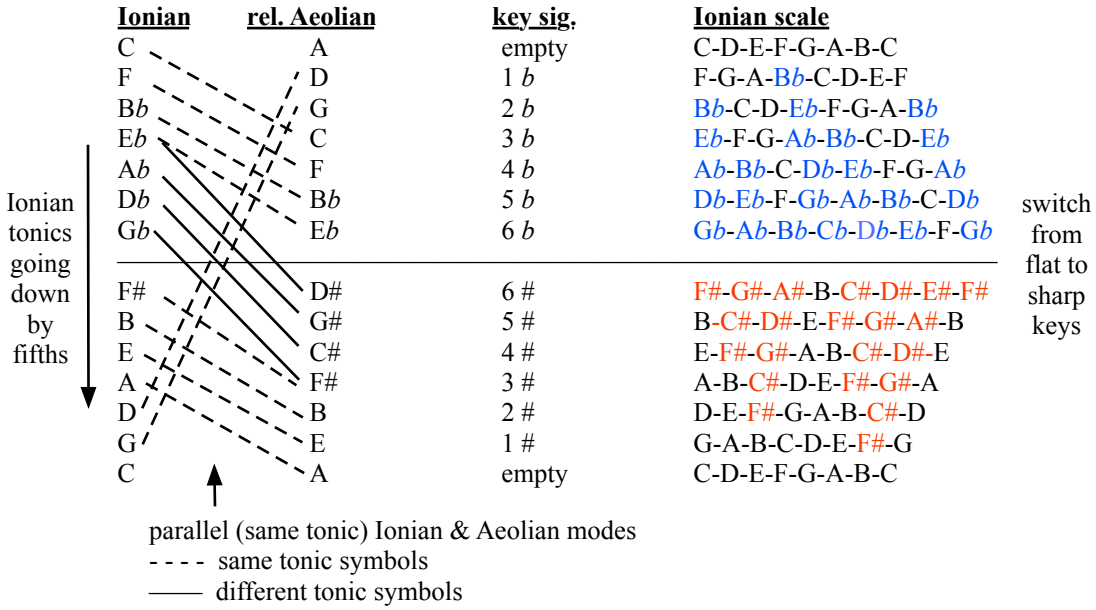
APPENDIX B: ABOUT SCALES

KEY-SIGNATURE SCALES

The following summary of the standard interpretation of the major and natural minor tonic scales of key signatures is a helpful reference. It shows the sharped notes of sharp scales in red and the flatted notes of flat scales in blue. Although there are only five black piano keys, scales with six flats or sharps exist because of the scale-spelling rule that the same letter note cannot appear on both sides of a half-tone interval. For example, note B is C*b* in a 6-flats scale.

Major Ionian and relative minor Aeolian) scales are shown on the same line because they have the same key signature. Parallel minor scales (diagonal lines) have the same tonic, which does not necessarily mean the same note symbol because the same piano key may be represented by different note symbols (e.g., the parallel D*b* major and C# sharp minor scales, which have the same black-key tonic on the piano, have different note symbols for it).

Major and minor scales have a fundamental note in the lower fifth of the scale that's a major or minor third above the tonic (PKP makes this explicit by symbols **3** and **p3** for these note positions).



Avoiding multiple written key signatures by using accidentals to identify implicit key changes replaces one kind of complexity by another because accidentals have many more uses than this (e.g., ornamental passing notes, or identifying scales that originate in no key signature). Sorting out the possibilities requires considerable head-scratching at a very low conceptual level.

Key-signature-based music notation has stood the test of time and is here to stay because of the huge legacy of music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation.

CROSS REFERENCES

The following table cross-referencing music notation and PKP notation. This is the simplest case. More generally, naturals may enter the note-symbol picture to cancel sharps or flats of key signatures,

and white piano keys may be represented by sharps or flats (B=Cb, C=B#, F=E#, E=Fb) to satisfy the scale-spelling rule that the same letter note must not appear twice. Experts develop rules of thumb about sharps and flats as indicators of what's going on, and see PKP as discarding these clues — not so, the clues are present in PKP in a different way.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7
@	P	O	R	M	Y	L	$	x	x	x	x
C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B
C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C
D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db
D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D
E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb
F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E
F#/Gb	G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F
G	G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb
G#/Ab	A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G
A	A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab
A#/Bb	B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A
B	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb

APPENDIX C: ABOUT CHORDS

Chords identified by chord symbols with four or more notes boil down to combinations of the two kinds of building blocks. Here follow some examples. Two fifos (fifths or fourths) form major-7, minor-7 or major-6 chords. One fifo and one tritone form dominant-7, half-diminished-7 (a.k.a. minor-7-♭5) or minor-6 chords. Three fifos or two fifos and a tritone form 9th, 1lth and 13th extensions of these chords. When tonic scales depart from key signatures, the same kinds of building blocks are available from the scales but now more than one tritone is available. For example, two tritones form diminished-7 or dominant-7-♭5 chords. Two tritones and one fifo (that may share a note with a tritone) form variations of other chord types such as dominant-7♭9. Omitting inner notes of combinations may yield different intervals, or thinned voicings of chords. For example, an augmented fifth (8 half tones) is combination of two tritones a whole tone apart with their inner notes omitted; and a 3-note voicing of a 4-note seventh chord is composed of the outer notes plus the most important inner note.

ANCHOR LETTERS FROM TRITONE CHORDS

The following table of tritone chords summarizes the simplest examples of chords with tritones in different positions.

offset of tritone bass note above root R	suffixes on root symbol R										
	7	7♯9(13)	9♭13 (or ♯5)	7♭9	7♭5 (or ♯4 or ♯11)	dim7 (or o7)	m7♭5	m6	M7(11)	♭9	M7♯11 (or ♭5)
fourth									x		
major third	x	x	x	x	x						
minor third		x				x		x			
whole tone			x								
half tone				x						x	
0					x	x	x				x

Only chord variations that alter tritone content are included. For example, R7, R9, R7#9, R13 and R9(13) are all variations of R7 with the same tritone content and so are all represented in the table by R7 (variations are left to context). Diminished seventh chords (Rdim7) have no counterpart in classical modes. Sus chords are not shown because they have no tritones. They are typically V-7♯3 chords. A Vsus-V-I progression is a substitute for a II-V-I progression in which one note is altered between the first two steps.

Inverting the tritone of a chord into the home octave (if it isn't already there) determines its PKP anchor. For example, The tritone of a V-7 chord is partially outside the home octave but its inversion is within it. The bass note of its inversion is a fourth above the home tonic which makes its anchor ▌.

Double tritones are the normal upper limit for tritone content of chords encountered above the staff in fake books, lead sheets and sheet music.=

FIGURED BASS NOTATION; EXTENDED CHORDS

Figured bass notation provides a simple representation of chords from scales with 7 notes and no adjacent half tones. It represents chords by stacks of numbers going up from a bass note, in which each number is a count of the scale steps to the next note up. The stacks are annotated on a staff as literal stacks of numbers (vertical lists). I write them here as horizontal lists separated by plus signs. For classical modes, a major or minor third is always two scale steps and whole tone is always one scale step (e.g., a seventh chord is **2+2+2** and a sixth chord is **2+2+1**).

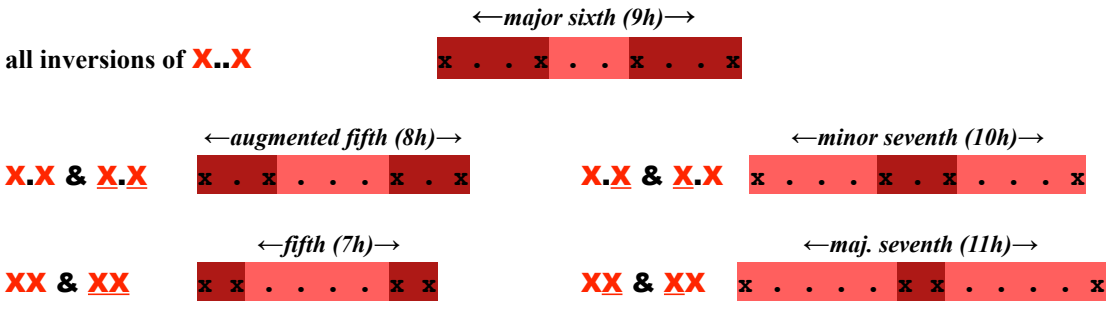
Figured bass notation can be helpful for understanding extended chords of classical modes that follow from splitting chord steps of size **2** into **1+1** sequences and then moving the added note up an octave. The **+4** on top of two of the extended chords on the right is a consequence of avoiding **1+1+1** sequences in the in-place forms. The corresponding chord symbols have degree-number suffixes 6, 9, 11 or 13 added to basic symbols to identify the extensions.

chord type	(2) = split	in place	extended
seventh	2+2+2	2+2+2	–
ninth	(2)+2+2	1+1+2+2	2+2+2+2
eleventh (seventh+11th)	2+(2)+2	2+1+1+2	2+2+2+2+4
thirteenth (ninth+13th)	(2)+2+(2)	1+1+2+1+1	2+2+2+2+2+4

DOUBLE TRITONES

The number and variety of chords formed of or voiced by double tritones is startling. The chord symbols are all over the map but the double tritones have only three basic keyboard shapes exemplified by **RM**, **M.L** and **R..L** from the blues family mode, namely two tritones offset by a half tone, a whole tone or a minor third. Let's symbolize these shapes by **XX**, **X.X** and **X..X** , where the **X**s represent alphabet letters and the dots represent skipped letters. The same shapes appear for different mixes of tritones on both sides of the mode hierarchy. Double tritones with whole tone and half tone separations are progressively more dissonant than single tritones when their notes are sounded together. Double tritones with minor third separations have a "sweet" sound that's somewhere between dissonance and consonance.

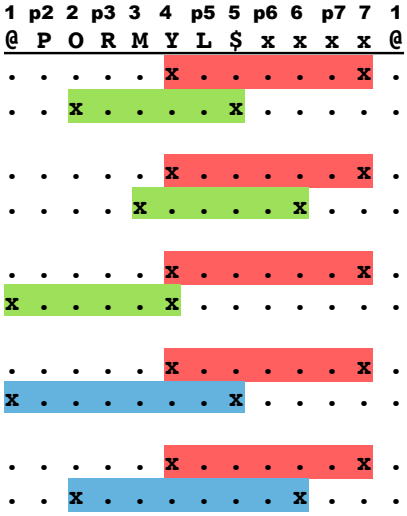
The basic shapes are simple but finding any particular inversion of one on the keyboard can be difficult if you try to do it by finding the individual inversions and combining them." Two things make it difficult: one is different mixes of black and white keys in different home octaves; the other is different outer and inner intervals for different inversions. Better to work inwards from the outer intervals, as shown next.



Find the bass note, establish the treble note, then move inwards from both ends by the offset interval. Either all inversions have same shape or all inversions have only two shapes.

MISLEADINGLY COMPLEX CHORD SYMBOLS FROM CLASSICAL MODES

This illustrates, for the Ionian mode, that a startling variety of chords are voiced by simple shapes formed of a tritone combined with different fifos (some are rootless, all are in the mode).



V*7*, IIm***7(13)*, I*M9(11)***

V*9(13)*, IV*M7b5*

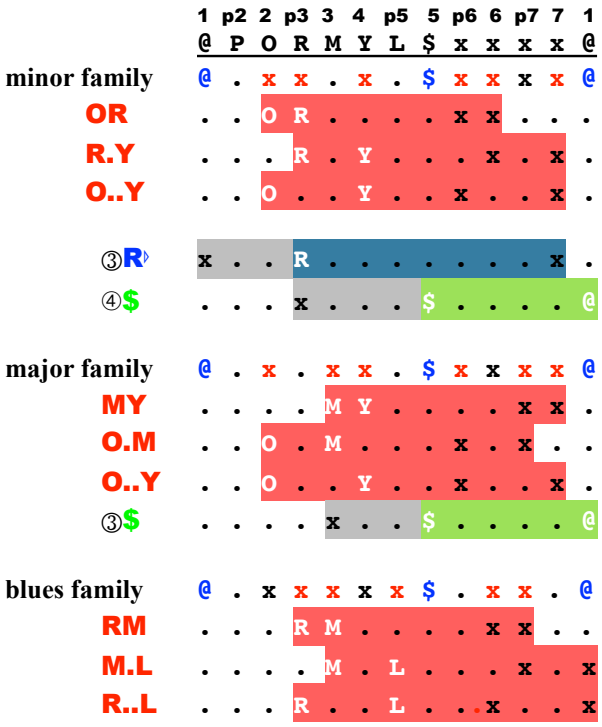
V*7(11)*, II*m7(13)*

V*13(11)*, I*M7b5*

VII*m7b5*, IV*M6b5*, II*m6*

CHORDS FROM NON-CLASSICAL MODES.

This illustrates that non-classical modes provide many new chords.



IV*7#9(13)*, VII*7#9(13)*

IV*7b5*, VII*7b5*

II*dim7*, IV*dim7*, pVI*dim7*, VII*dim7*
rootless *7b9* rel. to roots a 1/2 tone down

I*mM7*, pIII*M*⁺

I*m* (minor triad @▾, spread out)

pII*7#9(13)*, V*7#9(13)*

III*7b5*, pVII*7b5*

as above

I (major triad)

I*7#9(13)*, pV*#9(13)*

I*7b5*, pV*7b5*

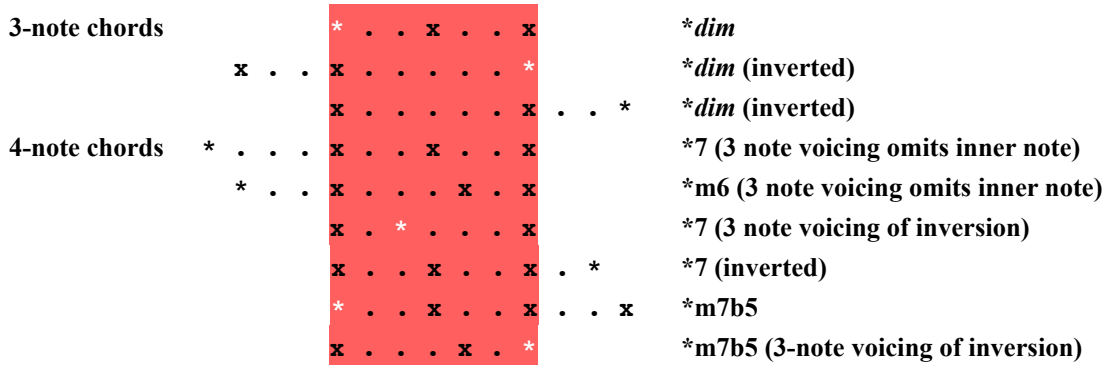
analogous to O..I

The strongly dissonant chords formed of two tritones offset by half tone would tend to be voiced in

context as a stacked tritone and fourth with no internal half tones. This shape has a rich sound with a dissonant edge. The weakly dissonant chords formed of two tritones offset by a whole tone can be played with all notes in any inversion. The chords formed of two tritones offset by a minor third have a circular symmetry (same shape in all inversions) that produces a unique sound I have come think of as "sweet."

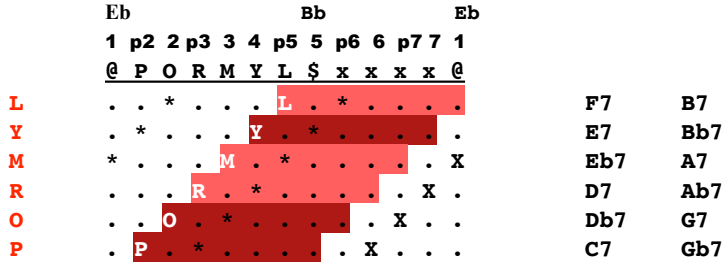
CHORDS DIRECTLY FROM TRITONES ON THE KEYBOARD

Tritone chords may be identified directly on the keyboard as indicated by the following selection of examples (* identifies the root). Context would determine which one fits.



TRITONE SUBSTITUTE CHORDS

The tritones of all possible dominant seventh chords and their tritone substitutes for any home tonic are easily determined from a table of the following form, the top line of which is the actual scale frame of the home tonic on the keyboard (**Eb-Bb-Eb** provides an example). The chord roots are the * entries in the table. All possible chords are listed on the right for the home tonic.



APPENDIX D: ABOUT PARALLEL MODES

MODALITY VERSION

The enriched classical modes identified as "melodic" and "harmonic" presented in the book *Modalogy*, for which I have coined the name **bridge mode**s, are summarized here, except with notes in numeric-chromatic-scale notation, instead of in the RN (Roman Numeral) notation with sharps, flats and naturals used in the book.

The purpose is twofold. One is to verify the correctness of the PKP view of these modes. The other is to highlight the complexity of the conventional way of knowing them.

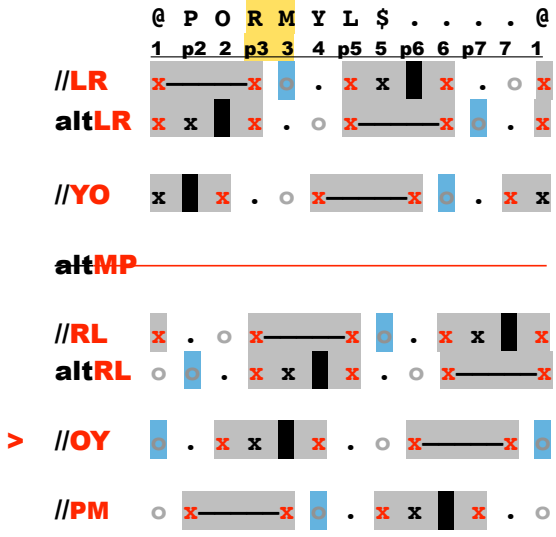
One source of complexity is the clothes of music notation (sharps, flats, naturals). For example, the tritone anchored by **R** that is a component of many of the minor modes is understood as **p3-6** in the symbolic chromatic scale, and that's it. Inversions in different places in harmony are left to context. In *Modalogy*, tritone anchor **p3** is ♭III or ♯II and tritone anchor **6** is VI or ♭VII, and that's only the anchors.

A second source of complexity is the alphabet soup of names and the intricate details of the scales makes them very difficult to distinguish functionally, or to remember as enumerated scales. Seeing the possibility that tritones might be fundamental scale-defining objects for these modes is effectively impossible in the terms used in *Modalogy*. This impossibility is demonstrated by a discussion of defining and non-defining notes of the many and various modes that never mentions tritones.

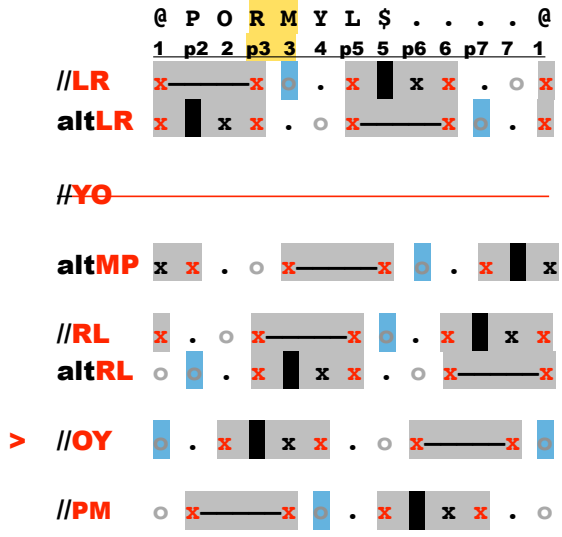
A third source of complexity is grouping the modes according to the tonality of the master modes, which has no simple relationship to the contexts in which they may appear.

PKP's way of knowing the modes by 2-letter mode signatures from a 6-letter alphabet is simple and unambiguous. The 7-note harmonic modes follow from removing the blacked-out notes from the 8-note parallel modes of the harmonic-minor-major as follows. In the contexts in which these modes appear, the removals are determined by context.

minor master



major master



The above modes are cross-referenced below. The spelled-out modes are correct but a few of the names taken from Modalogy are unclear to me, so I am not sure of them. In any case, the point of listing the names here is to show the complexity of this view, not to advance the particular names.

Parallel Modes of the Melodic Minor

//LO	1-2-3-p5-p6-6-7-1	Lydian Augmented (Lydian-#5)
altLO	1-2-p3-4-p5-p6-p7-1	Aeolian Diminished (Aeolian-b5)
//ML	1-2-3-p5-5-6-p7-1	Lydian Dominant (Lydian-b7)
altML	1-p2-p3-3-p5-p6-p7-1	Jazz Altered (Locrian-b4)
//RY	1-2-p3-4-5-6-7-1	Melodic, or Jazz, Minor (Ionian-b3)
//OM	1-2-3-4-5-p6-p7-1	Jazz Mixolydian (Mixolydia-b6)
//PR	1-p2-p3-4-5-6-p7-1	Jazz Phrygian (Phrygian-#6)

Parallel Modes of the Harmonic Minor

//LR	1-p3-3-p5-5-6-7-1	Lydian Blues Major, Lydian ♯2
altLR	1-p2-p3-4-p5-6-p7-1	Jazz Phrygian Diminished
//YO	1-2-3-4-p6-6-7-1	Ionian Augmented
//RL	1-2-p3-p5-5-6-p7-1	Romanian, Dorian-#4, Mishebarakh
altRL	1-p2-p3-4-p5-p6-6-1	Leading Tone Major Diminished, Locrian ♭7, Locrian Diminished-7
//OY	1-2-p3-4-5-p6-7-1	Harmonic Minor, Aeolian ♮7, Jazz Minor ♭6, Mohammedan
//PM	1-p2-3-4-5-p6-p7-1	Phrygian Dominant

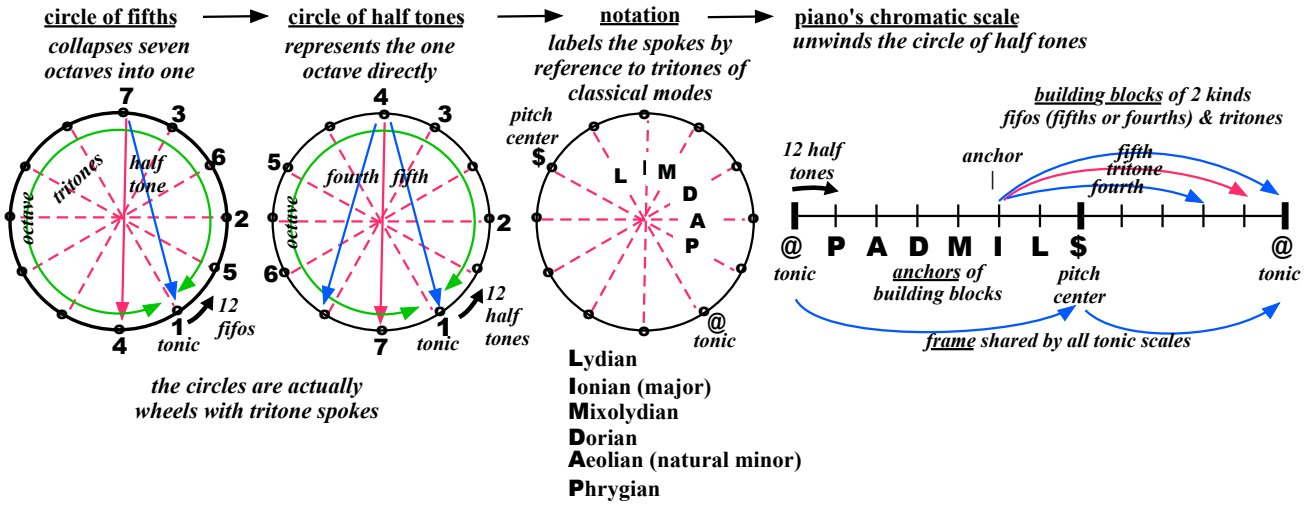
Parallel Modes of the Harmonic Major

//LR	1-p3-3-p5-p6-6-7-1	Lydian Blues Augmented, Lydian Augmented ♯2
altLR	1-2-p3-4-p5-6-p7-1	Jazz Minor ♯4, Lydian Diminished
//RL	1-2-p3-p5-5-6-7-1	Lydian Melodic Minor, Lydian b3
altMP	1-p2-p3-3-5-p6-p7-1	Altered Phrygian Dominant, Phrygian ♭4, Superlocrian ♮5, Superphrygian
altRL	1-p2-p3-3-p5-p6-6-1	Leading Tone Minor Diminished, Super Locrian ♭7
//OY	1-2-3-4-5-p6-7-1	Harmonic Major
//PM	1-p2-3-4-5-6-p7-1	Jazz Phrygian Dominant, Mixolydian ♭2

APPENDIX E: ABOUT SYMMETRY & SYMMETRY-BREAKING

The elements of PKP are summarized here in a way that provides a link between the concept of symmetry breaking in these pages and in the book *The Jazz of Physics*. This kind of thing is fascinating to anyone with a mathematical bent but possibly not to anyone else. (This uses the original version of the alphabet, namely **PADMIL**, before I replaced **ADI** with **ORY** to avoid confusion with other uses of the letters.)

The circle of fifths shown below left presents the basic elements of tonic scales and chord progressions from them in one simple picture. The points going counterclockwise around the circle are 12 notes a fifth apart (7 half tones) across 7 keyboard octaves (12 fifths x 7 half tones = 7 octaves x 12 half tones). The circle is manifestly a wheel with tritone spokes. The notes of the major scale are the numbered points around the half circle delimited by the highlighted tritone spoke (6 half tones).



The circle visibly establishes two kinds of Lego™-like **building blocks** as fundamental elements of music, namely fifths or fourths referred to jointly as **fifos** for simplicity, and tritones. The circle as labeled identifies fifths going up the keyboard through 7 octaves or fourths going down the keyboard through 5 octaves. For the major scale, it identifies a basic chord root line going down by five fifths (3½ octaves) or five fourths (2½ octaves) to the tonic (equivalent to progressively zig-zagging down a fifth and up a fourth within one octave).

The pictorial geometry of the circle of fifths and the musical geometry of its interpretation are different in a way that is counterintuitive (different numbers of octaves going around the circle for fifths and fourths, very different musical sizes of lines of almost the same pictorial length that cut across the circle). This becomes confusing for more general chord progressions of this and other scales.

The circle of half tones rearranges the spokes of the circle to put the labeled points around it in scale order. Around the circle remains an octave and across any spoke remains a tritone. The half tones across the circle now go around it and the fifos around the circle now go across it.

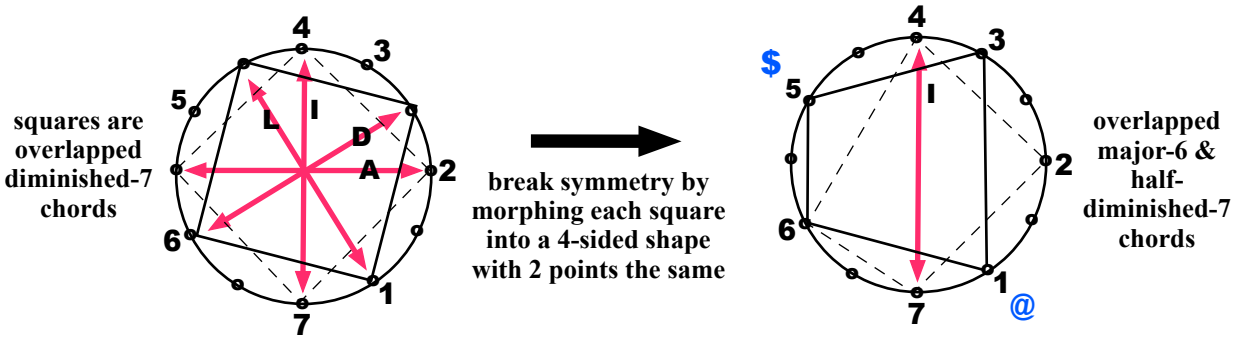
The end points of all the spokes of either circle visibly identify all the points around it, suggesting a notation that labels spokes (building blocks) instead of points (notes). This simple observation is the twist that leads to PKP's unique combination of simplicity and depth. The spoke labels, called **anchors**, directly identify tritones but also identify fifos. In the chromatic scale that unwinds from the

circle, each anchor letter identifies a tritone and two fifos morphed from it. The letters identify relative positions of the anchors in the chromatic scale. Tritone anchors are unambiguous independently of context and fifo anchors are ambiguous but the ambiguity is generally resolved by context. No anchors are needed in the top pitch half of the octave because inversions cover them.

The letters of the **PADMIL** alphabet mark the relative positions of the anchors of the single tritones of classical modes, but this is only to provide a correspondence between PKP notation and music notation. It does not bind the letters to the classical modes, or to tritones.

SYMMETRY BREAKING IN THE CIRCLE OF FIFTHS

According to *The Jazz of Physics*, symmetry-breaking is a deep feature of how both music and the universe work. Here's a view of breaking the symmetry of a diminished seventh scale to yield a major scale that corresponds to an example on page 66 of the book (in the book, the scale is diminished, not diminished seventh, so the dotted square and its transformation are missing). No mention is made of tritones).



The symmetry-breaking takes place in progressive steps that morph tritones into fifos.