A very long uniform line of charge has charge per unit length 4.80 μC/m and lies along the x-axis. A second long uniform line of charge has charge per unit length -2.4 μC/m and is parallel to the x-axis at y1 = 0.40 m .

What is the magnitude of the net electric field at point y2 = 0.20 m on the y-axis?

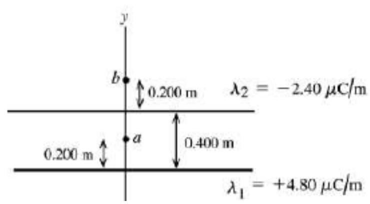
What is the direction of the net electric field at point y2 = 0.2 m on the y-axis?

What is the magnitude of the net electric field at point y3 = 0.60 m on the y-axis?

What is the direction of the net electric field at point y3 = 0.60 m on the y-axis?

IDENTIFY: Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of charge and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.19.



$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

EXECUTE: (a) At point a, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$$E_1 = (1/2\pi\epsilon_0)[(4.80\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 4.314\times10^5\ \text{N/C}$$

$$E_2 = (1/2\pi\epsilon_0)[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}$$

$$E = E_1 + E_2 = 6.47\times10^5\ \text{N/C, in the } y\text{-direction.}$$

(b) At point b, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$$E_1 = (1/2\pi\epsilon_0)[(4.80\times10^{-6}\ \text{C/m})/(0.600\ \text{m})] = 1.438\times10^5\ \text{N/C}$$

$$E_2 = (1/2\pi\epsilon_0)[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}$$

$$E = E_2 - E_1 = 7.2\times10^4\ \text{N/C, in the } -y\text{-direction.}$$

EVALUATION: At point a the two fields are in the same direction and the magnitudes add. At point b the two fields are in opposite directions and the magnitudes subtract.

An infinitely long cylindrical conductor has radius R and uniform surface charge density σ.

In terms of σ and R, what is the charge per unit length λ for the cylinder?

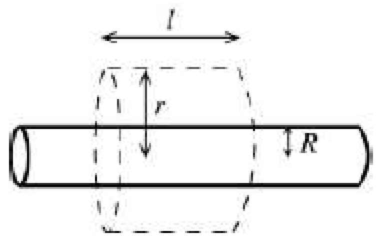
In terms of σ, what is the magnitude of the electric field produced by the charged cylinder at a distance r>R from its axis?

Express the result of part B in terms of λ

IDENTIFY: Apply Gauss's law to a Gaussian surface and calculate E.

(a) SET UP: Consider the charge on a length l of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) Apply Gauss's law to a Gaussian surface that is a cylinder of length l, radius r, and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.31.



EXECUTE:

$Q_{\text{encl}} = \sigma(2\pi Rl)$

$\Phi_E = 2\pi rlE$

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \text{ gives } 2\pi rlE = \frac{\sigma(2\pi Rl)}{\epsilon_0}$$

$$E = \frac{\sigma R}{\epsilon_0 r}$$

(c) EVALUATE: Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda/2\pi\epsilon_0 r$.

$\sigma = \dfrac{\lambda}{2\pi R}$, so $E = \dfrac{\sigma R}{\epsilon_0 r} = \dfrac{R}{\epsilon_0 r}\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{\lambda}{2\pi\epsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

A very long conducting tube (hollow cylinder) has inner radius a and outer radius b. It carries charge per unit length +α, where α is a positive constant with units of C/m. A line of charge lies along the axis of the tube. The line of charge has charge per unit length +α.

Calculate the electric field in terms of α and the distance r from the axis of the tube for r<a.

Calculate the electric field in terms of α and the distance r from the axis of the tube for a<r<b.

Calculate the electric field in terms of α and the distance r from the axis of the tube for r>b.

What is the charge per unit length on the inner surface of the tube?

What is the charge per unit length on the outer surface of the tube?

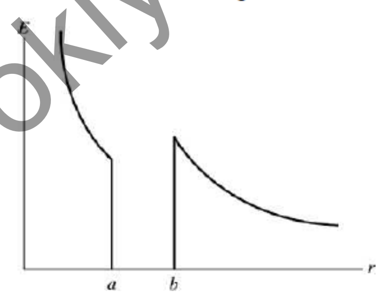
IDENTIFY: Apply Gauss's law.
SET UP: Use a Gaussian surface that is a cylinder of radius r, length l and that has the line of charge along its axis. The charge on a length l of the line of charge or of the tube is $q = \alpha l$.

EXECUTE: **(a)** (i) For $r < a$, Gauss's law gives $E(2\pi r l) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\alpha l}{\epsilon_0}$ and $E = \frac{\alpha}{2\pi\epsilon_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi r l) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{2\alpha l}{\epsilon_0}$ and $E = \frac{\alpha}{\pi\epsilon_0 r}$.

The graph of E versus r is sketched in Figure 22.40.
(b) (i) The Gaussian cylinder with radius r, for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.
EVALUATE: For $r > b$ the electric field is due to the charge on the outer surface of the tube.



A very long, solid cylinder with radius R has positive charge uniformly distributed throughout it, with charge per unit volume ρ.

erive the expression for the electric field inside the volume at a distance r from the axis of the cylinder in terms of the charge density ρ.

What is the electric field at a point outside the volume in terms of the charge per unit length λ in the cylinder?

IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi r l$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: (a) For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi r l) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\rho\pi r^2 l}{\epsilon_0}$ and $E = \dfrac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi r l) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\rho\pi R^2 l}{\epsilon_0}$ and

$E = \dfrac{\rho R^2}{2\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$, radially outward.

A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d (see the Figure (Figure 1)). The inner shell has total charge +2q, and the outer shell has charge +4q.

calculate the magnitude of the electric field in terms of q and the distance r from the common center of the two shells for r<a.

Calculate the magnitude of the electric field in terms of q and the distance r from the common center of the two shells for a<r<b.

Calculate the magnitude of the electric field in terms of q and the distance r from the common center of the two shells for b<r<c.

Calculate the magnitude of the electric field in terms of q and the distance r from the common center of the two shells for c<r<d.

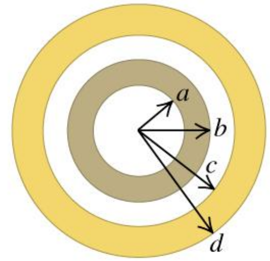
Calculate the magnitude of the electric field in terms of q and the distance r from the common center of the two shells for r>d.

What is the total charge on the inner surface of the small shell?

What is the total charge on the outer surface of the small shell?

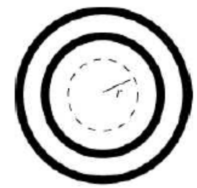
What is the total charge on the inner surface of the large shell?

What is the total charge on the outer surface of the large shell?



IDENTIFY: Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.47a.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

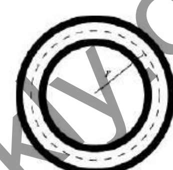
$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ says

$E(4\pi r^2) = 0$ and $E = 0$.

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.47b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.



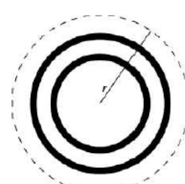
EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.47c.



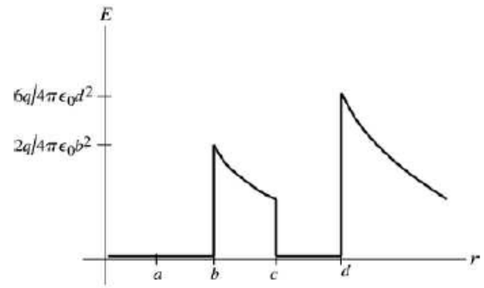
EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

$\Phi_E = \frac{Q_{encl}}{\epsilon_0}$ gives $E(4\pi r^2) = \frac{6q}{\epsilon_0}$

$E = \frac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.47d.



(b) IDENTIFY and SET UP: Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{encl} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{encl} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

An insulating hollow sphere has inner radius a and outer radius b.
Within the insulating material the volume charge density is given by
$\rho(r)=\alpha/r$,where α is a positive constant.

What is the magnitude of the electric field at a distance r from the center of the shell, where $a<r<b$?
Express your answer in terms of the variables α, a, r, and electric constant ϵ_0.

A point charge q is placed at the center of the hollow space, at r=0. What value must q have (sign and magnitude) in order for the electric field to be constant in the region $a<r<b$?
Express your answer in terms of the variables α, a, and appropriate constants.

What then is the value of the constant field in this region?
Express your answer in terms of the variable α and electric constant ϵ_0.

IDENTIFY: We apply Gauss's law in (a) and take a spherical Gaussian surface because of the spherical symmetry of the charge distribution. In (b), the net field is the vector sum of the field due to q and the field due to the sphere.

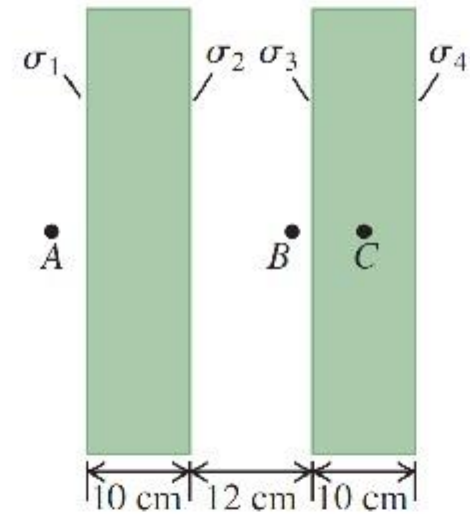
(a) SET UP: $\rho(r)=\dfrac{\alpha}{r}$, $dV=4\pi r^2 dr$, and $Q=\int_a^r \rho(r')dV$.

EXECUTE: For a Gaussian sphere of radius r, $Q_{\text{encl}}=\int_a^r \rho(r')dV=4\pi\alpha\int_a^r r'dr'=4\pi\alpha\frac{1}{2}(r^2-a^2)$. Gauss's law says that $E(4\pi r^2)=\dfrac{2\pi\alpha(r^2-a^2)}{\epsilon_0}$, which gives $E=\dfrac{\alpha}{2\epsilon_0}\left(1-\dfrac{a^2}{r^2}\right)$.

(b) The electric field of the point charge is $E_q=\dfrac{q}{4\pi\epsilon_0 r^2}$. The total electric field is $E_{\text{total}}=\dfrac{\alpha}{2\epsilon_0}-\dfrac{\alpha}{2\epsilon_0}\dfrac{a^2}{r^2}+\dfrac{q}{4\pi\epsilon_0 r^2}$. For E_{total} to be constant, $-\dfrac{\alpha a^2}{2\epsilon_0}+\dfrac{q}{4\pi\epsilon_0}=0$ and $q=2\pi\alpha a^2$. The constant electric field is $\dfrac{\alpha}{2\epsilon_0}$.

EVALUATE: The net field is constant, but not zero.

Two very large, nonconducting plastic sheets, each 10.0 cm thick, carry uniform charge densities σ1,σ2,σ3 and σ4 on their surfaces, as shown in the following figure(Figure 1) . These surface charge densities have the values σ1 = -6.00 μC/m2 , σ2=5.00μC/m2, σ3 = 2.0 μC/m2 , and σ4 = 4.00μC/m2. Use Gauss's law to find the magnitude and direction of the electric field at the following points, far from the edges of these sheets.



What is the magnitude of the electric field at point A, 5.00 cm from the left face of the left-hand sheet?

What is the direction of the electric field atpoint B, 1.25 cm from the inner surface of the right-hand sheet?

What is the magnitude of the electric field at point C, in the middle of the right-hand sheet?

IDENTIFY: The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: (a) At A: $E_A = \dfrac{|\sigma_2|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_1|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|)$.

$$E_A = \frac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82\times10^5\ \text{N/C to the left.}$$

(b) $E_B = \dfrac{|\sigma_1|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|)$.

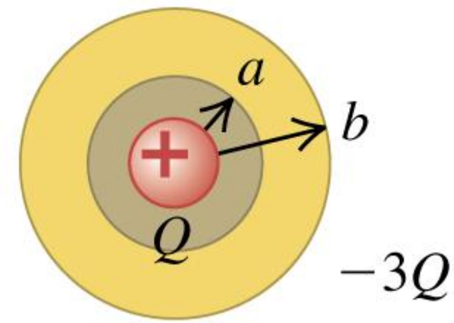
$$E_B = \frac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95\times10^5\ \text{N/C to the left.}$$

(c) $E_C = \dfrac{|\sigma_4|}{2\epsilon_0} + \dfrac{|\sigma_1|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} - \dfrac{|\sigma_3|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_4| + |\sigma_1| - |\sigma_2| - |\sigma_3|)$.

$$E_C = \frac{1}{2\epsilon_0}(4\ \mu\text{C/m}^2 + 6\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2 - 2\ \mu\text{C/m}^2) = 1.69\times10^5\ \text{N/C to the left.}$$

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

A conducting spherical shell with inner radius a and outer radius b has a positive point charge Q located at its center. The total charge on the shell is −3Q, and it is insulated from its surroundings.



Derive the expression for the electric field magnitude in terms of the distance r from the center for the region r<a. *a<r<b*. *r>b*.

What is the surface charge density on the inner surface of the conducting shell?

What is the surface charge density on the outer surface of the conducting shell?

IDENTIFY: Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

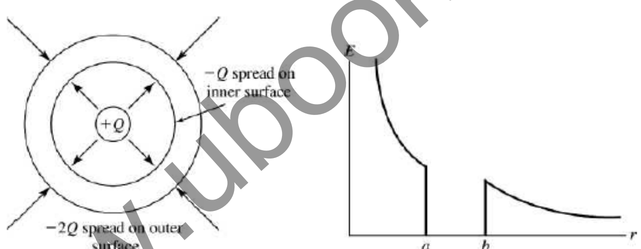
EXECUTE: (a) For $r < a$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge, the total charge on the inner surface is $-Q$ and the surface charge density on the inner surface is $\sigma = -\dfrac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\dfrac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.46a.

(e) The graph of E versus r is sketched in Figure 22.46b.



(a) (b)