# THE EMPEROR HAS NO CLOTHES: <br> <br> MUSIC AS IT ACTUALLY IS ON THE PIANO KEYBOARD 

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## INTRODUCTION

I approached the piano as an adult beginner interested in learning to play jazz, which eventually led me to wonder how jazz pianists can improvise in terms of a music notation that seemed to me to be misleadingly complex for the piano. ${ }^{1}$ I thought improvisation must tap into deeper musical structures that are obscured by this notation. Music notation has stood the test of time and is here to stay for piano music, even if for no more reason than the huge legacy of piano music written in it. But the piano has also stood the test of time without needing variable-pitch piano keys to play notes exactly as specified by the sharps and flats of music notation. In effect, "the emperor has no clothes" - piano music is much simpler than the "clothes" of this notation make it seem.

The piano is not just for experts, but the conventional approach to teaching and learning it tends to make it so. Understanding is expected to emerge by osmosis from the same extensive practicing that develops "chops." Thousands of hours at the keyboard are required to master innumerable details across a range of exercises and pieces before understanding deeper structures becomes possible. I wondered if this could be reversed by learning deeper structures first, in a manner independent of music notation.

I was encouraged to think this might be possible by the simplicity of the way people with musical ears understand melody lines as sequences of pitch intervals going up and down from a starting pitch, independently of the actual starting pitch, of durations of notes and rests, and of rhythm. Let's call such a line a skeleton melody line. Skeleton melody lines are directly playable on the piano because pitch intervals on the piano are measured in half tones, the pitch intervals played by adjacent piano keys.

Bear with me while I summarize some simple facts that follow from this, to set the stage. A skeleton melody line of tonal music establishes a particular piano key as a home tonic, which in turn establishes a 12-half-tone home octave as the source of the tonic scales that supply the melody notes. Melody notes that go outside an established home octave go to corresponding scale positions in a stack of home octaves that look the same and are harmonically equivalent. The piano provides twelve stacks of overlapping home octaves, offset by half tones, in which all the piano keys in the overlapped parts are shared, which means all the half tones are shared. Everything about a piece of tonal music may be understood in terms of a single conceptual home octave determined by a symbolic home tonic that may be assigned to any piano key. Visits to secondary tonics are conceptually simple in interval terms because of the shared half tones.

The tonic scales of the conceptual home octave provide the notes of both melody and harmony. Harmony in these terms is simpler than melody, in the sense that different arrangements on the keyboard of the same notes are harmonically equivalent. Shapes that voice chords are formed by

[^0] A7\#11. There are 24 chords, an average of 2 per bar for 12 bars, only 5 of which (highlighted in blue) don't contain tritones. The tritone content is the basis of deep structure of startling simplicity that is the subject of this document.
combining or splitting Lego-like building blocks from the conceptual home octave: intervals of different sizes measured in half tones. Three different sizes of building blocks that split octaves in half in two different ways are sufficient for most purposes. Fifths (7 half tones) and fourths ( 5 half tones) split octaves into equal pitch halves. Tritones ( 6 half tones, 3 whole tones) split octaves into equal geometric halves. These simple building blocks provide harmonic richness that follows from different combinations of consonance (fifths, fourths) and dissonance (tritones). Smaller and larger intervals are inner or outer notes of shapes. For example, the harmonically rich chord progression footnoted on the previous page is no more than a sequence of combinations of these building blocks in pairs (see Chapter 5). Shapes formed of combinations of building blocks are easily separable into core and enrichment parts, where the core part provides the essential sound and the enrichment part fills in details to fit context. An important element of tonal music is the provision of context that implies enrichment details. Harmony understood within the home octave may be played in many different ways by inverting selected building blocks up or down one or more times (an inversion puts the notes in the opposite order on the keyboard). Notating all the possibilities is complex but the complexity is avoidable because inversions are harmonically equivalent. Choosing an inversion is a detail that can be left to context in tonal music.

The new thing here is that tonic scales are also shapes formed of combinations of the same building blocks, leading to a view of tonal music in which scales of melody and the chords of harmony are understood in the same terms. The terms are very simple, as described next. A building block within the home octave can be understood in terms of only two pieces of information, namely its anchor position (its closest note above the home tonic) and its size (7, 6 or 5 half tones). A home octave has a fundamental scale frame shared among most scales, composed of the octave notes and the note at the pitch center of the octave that splits it into a fifth and a fourth (scales without pitch centers exist but are understood by reference to scales with them). The scale frame plus six anchors within its bottom fifth are sufficient to identify all possible building blocks within the home octave. The identification of the six anchors requires an alphabet of six anchor symbols. Tonic scales and chords are both determined by words from the alphabet. The different sizes of building blocks are distinguished by color coding the anchor symbols. The alphabet is analogous to that of biological DNA in the sense that words from it identify deep structure.

Thus piano music is understood in terms of parallel modes of the home tonic of a piece of music, independently of key signatures, instead of in terms of relative modes determined by key signatures. Modes are scales defined by interval sequences. Relative modes have the same notes and parallel modes have the same starting note. Key signatures provide a complex view of parallel modes.

I call the resulting system for representing and understanding piano music in these terms PKP, standing for "Picturing Keyboard Patterns." PKP's representation of the shapes of scales and chords in the same terms enables simple annotations on the written music to provide contextual cues that help in learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. This turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

Is this combination of simplicity and depth only a serendipitous side effect of the organization of the piano keyboard? Or is it a fundamental property of music that's obscured by music notation?

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Either way, it works in practice. To discover it, I had to enter uncharted territory. Nothing I read said anything about it and no expert I approached knew anything about it.

## GENESIS OF PKP

Any thoughtful person approaching the piano for first time can see that the "emperor has no clothes" but the perception is soon swamped by the intense focus needed to learn the piano in terms of music notation. I have been asked, if the ideas are so good, why have they not been discovered before by some talented young piano student? I suggest the answer is a lack of the powerful combination of factors that got me started and kept me going against the conventional wisdom. The first factor was experience as a university professor developing notations for software design. Software, like piano music, is described by abstract notations (programming languages) that imply things to be performed on hardware. Like piano music, the combination in execution is complex. I thought that my experience developing notations to deal with this complexity would help me in trying to do the same thing for piano music. The second factor was training in math and physics that made me confident of the existence of a "dual" representation of piano music based on intervals instead of notes. Dual representations are well known in math and physics to provide insight into complex phenomena (e.g., frequency-response/time-response and wave/particle). The third factor was an early curiosity about the complexity of music notation, developed while playing trumpet in school bands as a youth, which resurfaced when I took up the piano. The fourth factor was time to pursue my curiosity provided by retiring from my job as a university professor exactly when I was becoming motivated to develop a different view. Developing this view and writing about it became an absorbing retirement hobby over many years.

My admitted lack of musical credentials is cause for skepticism that I have anything useful to say, so the opinions of PKP of a couple of music professionals are worth quoting. Musical theorist Paul Steinbeck: "The hook ..., at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music." Jazz pianist/ composer/teacher Taylor Eigsti: "... a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

## GUIDE TO READERS

The potential audience includes novices like I was when I started out, pop and jazz musicians who are not pianists but want to explore harmony on the piano, music teachers interested in a different approach to teaching piano music, "wannabe" experts interested in understanding music at a higher conceptual level than notes, and curious experts.

Chapter 2 develops the basic concepts and notation. Chapter 3 provides examples of a few basic pieces represented in these terms. Chapter 4 explores the building-block world in depth. Chapter 5 provides some more advanced pieces that initially puzzled me and I suspect might puzzle anyone (the first example is the piece with the footnoted chord progression on the previous page). Chapter 6 provides observations and conclusions. References, acknowledgements and comments from some readers follow. Appendix A summarizes unfamiliar terminology, Appendices B-E are about scales (B), chords (C), non-classical parallel modes (D) and the importance of hidden symmetries (E).

## CHAPTER 2: CONCEPTS \& NOTATION

## A SIMPLE CONCEPTUAL MODEL

A simple but accurate conceptual model of the home octave on the emperor-has-no-clothes piano is shown below.

and so on

A half tone is the musical interval played by adjacent piano keys. A single home octave is represented by a line marking off into twelve equal parts representing half tones. The bottom @ is the home tonic and the top @ is at once the top note of the same octave and the tonic of the next octave up, which looks the same on the keyboard and is harmonically equivalent. Thus one conceptual home octave represents a stack of actual home octaves. The piano keyboard provides overlapping home octaves offset by half tones. The vertically aligned dots represent the same piano keys in these overlapped home octaves. A piece of music has a single home tonic but may visit secondary home octaves with different tonics. Such visits are very simple in this conceptual picture because the dots represent the same piano keys.

This simple picture is complicated by the fact that the pitch sizes of the 12 half tones must increase within the octave to make the pitch of the top note double that of the bottom note (the increasing pitch sizes are represented by equal divisions of the line because they're musically equal). If overlapped home octaves are all to provide the same pitch increases, the dots cannot be exactly vertically aligned. The sharps and flats of music notation represent this misalignment. The piano would have to provide variable pitch piano keys to capture it. It compensates for not doing so by using equal temperament tuning to provide a uniform sound across the board. The worldwide popularity of the piano as a general purpose musical instrument is evidence that this is good enough (see the book How Equal Temperament Tuning Ruined Music for a contrary view).

Two different theories of music follow from two different judgements of what's most important here, namely the perfect pitch sizes of half tones in individual octaves (music notation), or the alignment of half tones between overlapping octaves (the piano). PKP provides a theory of the second kind. The first chapter identified two kinds of building blocks as fundamental to PKP, namely dissonant tritones and consonant fifths or fourths. In the PKP view, tritones are primary in a structural sense and fifths or fourths are secondary because they're morphed from tritones. This primacy of dissonant tritones is analogous to the primacy of dissonant half tones in music notation. This does not mean that dissonance is primary to the sound of music. Wholly consonant music without either either half tones or tritones exists, provided by pentatonic scales. It only means that tritones and half tones provide fixed structural references for music that includes both consonance and dissonance. Half tones provide it for music notation because they're the fundamental divisions of octaves. Tritones provide it for PKP because they're the only building blocks that have a fixed size (they're the same size in either inversion). The use of tritones as structural references is not even imaginable in music notation because
every tritone originates in a different key signature, in which it's identified by a pair of notes tied to a written key signature. It's simple for the piano keyboard.

The anchor alphabet mentioned in the opening chapter is primarily a way of identifying all possible tritones, from which all possible fifths and fourths are understood to be morphed. This warrants a new term, fifo (standing for "fifth or fourth") to identify the result of the morphing as a building block of known type but unspecified size. The traditional terms are used when the size is known but the new term enables leaving the size to context. The concept of fifos morphed from tritones is independent of the presence or absence of the tritone in particular scales because the alphabet that determines tritone anchors is a property of the home octave as a whole.

Before proceeding with the development of these ideas, it will be helpful to know that viewing music through the lens of the notation to come provides strong evidence of the fundamental importance of tritones in music that includes them. Classical modes that define the scales of key signatures contain single tritones that are functionally equivalent to key signatures as identifiers of the modes. This generalizes to multiple tritones of non-classical modes being functional generalizations of key signatures that identify these more general modes. Chords containing tritones are signposts of important musical events such as resolving to the tonic of a mode, changing to a parallel mode (e.g., major to minor) or changing to a different tonic for the same mode (e.g., the same major mode). "Tritone substitute" chords (same tritones in opposite inversions, all non-tritone notes different) are a staple of chord substitution in jazz. The many different kinds of chords containing tritones are summarized in a table in Appendix C. The sounds of tritones, fifths and fourths in harmony sliding to different keyboard positions and morphing into each other within the home octave as the music moves forward are fundamental to much music. The notes may be spread out in the music but the essence of their musical contribution is captured by their harmonically equivalent presence within the home octave.

The following extension of the picture of the conceptual home octave on the keyboard brings forward the fundamental structural importance of tritones.


The symmetrically split home octave is a stack of two tritones that are opposite inversions. The asymmetrically split octave is a stack of a fifth with a fourth on top. This gives the first hint of the importance of symmetry and asymmetry in PKP representations of piano music. The lower tritone morphs into the lower fifth and upper tritone into the upper fourth, illustrating the nature of morphing by one possibility among several. The pitch center is in most tonic scales, and so joins with the lower and upper tonic to define a consonant scale frame that's an important starting point for understanding tonic scales in general (tonic scales without a pitch center exist but they're understood by reference to scales with pitch centers). The letter $\mathbf{L}$ is one of six tritone anchors within the lower fifth of the scale frame. The scale-frame symbols @ and $\mathbf{\$}$ are two of eight possible fifo anchors (the other six are shared with tritones). The notes of the scale frame may be bass notes of tritones going up from them but are

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never tritone anchors (e.g., the tritone with bass note @ is an inversion of the tritone anchored by $\mathbf{L}$ ). This avoids ambiguity in the identification of tritones - they can only be identified by six anchors.

This picture gives concrete meaning to the concept that tritones and fifos are the fundamental building blocks of music. Octaves are fundamental and these building blocks fall out of them in a way that demonstrates the fundamental morphing relationship between them. These building blocks are not, of course, the only intervals of music, but other intervals are largely determined as inner or outer notes of shapes formed of building blocks that determine scales and voicing of chords. This does not exclude other intervals from having occasional fifo-like roles (e.g., major thirds and augmented fifths), but these are easily identified special cases). Tritones alone are uniquely fundamental.

## THE ALPHABET

As illustrated next, the alphabet PADMIL provides six letters identifying the anchors of the six possible different tritones going down by half tones from the geometric center of the home octave identified by $\mathbf{L}$. The alphabet letters are the first letters of the names of the classical modes containing tritones anchored in these relative positions. Two modes are associated with tritone $\boldsymbol{L}$ because opposite inversions are both within the home octave, and so are associated with two parallel modes (as will explained, there's no ambiguity in this because knowing one mode means knowing the other, and the choice is determined by context).


This naming convention is useful because it connects these unfamiliar concepts with music notation via the modes that determine the key-signature scales of music notation, and gives the anchor symbols intuitive meaning. It does not restrict the tritones to these modes - their existence is identified independently of any mode, classical or non-classical, in which they find themselves. Any set of six letters could perform the same identification. The anchor letters are normally in a special boldfaced font (Arial Black) that makes them stand out as different in kind from some of the same letters used in music notation. The entries in this and future tables of this kind are in a different, fixed-width font (Courier) that enables tables with aligned columns to be constructed in text, but there's no change of meaning - the Courier font is reserved for tables.

Anyone familiar with classical modes will recognize that the PADMIL ordering of classical modes is unconventional. The conventional ordering is IDPLMAL, identifying successive relative modes of the Ionian mode. In this order, $\mathbf{L}$ appears twice because the Lydian and Locrian relative modes have different starting notes. The PADMIL order is different because it's based on parallel modes. This ordering is practically useful because it puts parallel classical modes that differ by one note adjacent to each other.

There are no anchors in the top fourth of the home octave because inversions of tritones anchored in the bottom fifth provide tritones with bass notes (not anchors) in the top fourth. Bass notes of inverted
tritones are not anchors. For example, the bass notes of inversions of tritones $\mathbf{P}$ and $\mathbf{L}$ are the pitch center and the home tonic, which are never identified as tritone anchors.

The six anchors are also of fifos morphed from the tritones. The sizes of building blocks are identified by colored letters: $\mathbf{X}$ for tritones, $\mathbf{X}$ for fifths and $\mathbf{X}$ for fourths, where $\mathbf{X}$ is any alphabet letter. This is unnecessary in any context in which the nature of an anchor is understood, but is otherwise necessary. Underlining an anchor letter means a building block of the indicated size goes down from the anchor (the default is up). For tritones only, $\mathbf{X}$ and $\mathbf{X}$ identify opposite inversions. Fifos are different because opposite inversion are of different sizes: $\mathbf{X}$ and $\mathbf{X}$ are opposite inversions, as are $\mathbf{X}$ and $\underline{\mathbf{X}}$. It can sometimes be useful to use a neutrally colored placeholder letter (e.g., $\mathbf{X}$ ) for the anchor of a fifo to be determined entirely from context (both the size and the inversion).

The color coding identifies the building blocks at a glance, but this leaves open the question of how to pronounce the symbols to oneself when reading them. The simplest answer is to pronounce the letter with an attached numeric prefix or suffix that identifies the size in half tones going up or down. For example, $\mathbf{M}$ is pronounced "em-6," $\underline{\mathbf{M}}$ is pronounced " $6-\mathrm{em}$," $\mathbf{M}$ is pronounced "em-7" and $\mathbf{M}$ is pronounced " 5 -em."

There's a confusing mismatch between the conventional meanings of "fifth" or "fourth" as 5 or 4 scale steps in a classical mode, and their meanings as sizes in half tones. A fifth has 7 half tones, not 5; a fourth has 5 half tones, not 4 ; and a major third has 4 half tones. Pronouncing the sizes identified by the color-coded symbols keeps the meaning in mind.

The 6-letter alphabet is a circular loop in the sense that traversing it in one direction goes off one end to proceed in the same direction from the other end. Each step in such a traversal identifies a tritone offset from the previous one by a half tone. This is true even between opposite ends of the alphabet because $\mathbf{P}$ and $\underline{\underline{L}}$ (or $\underline{\mathbf{P}}$ and $\mathbf{L}$ ) are offset by a half tone.

The conceptual home octave and its mapping to the keyboard are illustrated next for two of the twelve possible home octaves. The same conceptual home octave looks very different in different places on the keyboard, but the difference is manageable because it's independent of music notation and depends only on six easily recognizable tritones, the relative positions of which are identified by a simple alphabet. When playing a particular piece - whether learning it or revisiting it - keeping the piano key of its home tonic fixed in the mind is essential, to avoid inadvertently interpreting things relative to the different home tonic of some recently played piece that's still in the mind. Putting a temporary label (e.g., a small stick-on circle) on the home-tonic piano key can be helpful.


## Music in Motion

The goal is to learn and internalize interval-based conceptual models of piano music in motion of the general nature shown next, in which fundamental musical patterns heard by the ear are visible to the eye without reference to note symbols or music notation. To be clear, such pictures represent mental models, not actual pictures to be drawn. Such pictures may be drawn for explanation, as here and later, but the basic intent is that they're determined by very simple annotations next to melody notes on the staff and chord symbols above the staff. A chromatic scale notation for identifying piano keys of melody lines will be introduced later in this chapter, but the concepts are independent of it or of any particular notation for notes.

Music in motion is understood in terms of visual patterns on a grid that represents the piano keyboard from left to right on the page and successive time steps going down the page. Melody notes are identified by " $\mathbf{x}$ " marks on the grid. Associated core harmony is provided by building blocks in line with the melody notes (also possibly in line with spaces between successive melody notes). Core harmony is provided by Lego-like building blocks, namely tritones (red), fifths (blue) and fourths (green). Tritones are core by definition, fifos may be core. The core determines modes, which determines enrichment fifos to combine with core building blocks (always fifos because all the tritones are in the core). The harmonic core here is determined by the anchor sequence M-D-D-@. Durations of notes and inter-note rests are left to context, only the points at which notes first appear are identified.


There are nuances but this is the essence. The depth of helpful understanding provided by conceptual models of this simple form, for music of all kinds, is startling.

## PARALLEL MODES \& MODE SIGNATURES

The alphabet that provides the anchors of the building blocks is the source of mode signatures for all the parallel modes that can be played on the piano, as well as for all relative modes of them determined by secondary tonics in their scales. A mode signature consists of a single- or multi-letter word from the alphabet prefixed by the symbol " $l /$ " standing for "parallel." Classical modes are identified by single-letter words. Except for a small number of ambiguous cases, the words identify modes by tritone content, which is sufficient to identify most modes. This provides more coverage than scale dictionaries such as The Source.

The fixed size of tritones in either inversion leads to the following powerful property of mode signatures: parallel mode changes determined by tritone changes and same-mode tonic changes are "two sides of the same coin." Here's why. Inverting a tritone is equivalent to transposing it by an interval of a tritone, so transposing a mode signature by a tritone may be interpreted as a same-mode tonic change by a tritone. The two modes of the same kind with tonics a tritone apart are tritone substitute modes because they have all tritones inverted and all non-tritone notes different. The concept of tritone substitute modes is a simple generalization of the well known concept of tritone substitute chords. For modes identified by tritone content, the concept means that results of tritone
transpositions up or down by an interval less than a tritone, and of tritone transpositions in the opposite direction by a tritone minus the same interval, may be understood as pairs of tritone substitute modes. One mode is a primary mode identified by the prefix // and the other mode is a tritone substitute mode identified by the prefix alt. For example, the classical mode $/ / \mathbf{L}$ (Lydian) has a tritone substitute mode alt-L (Locrian), which is also a parallel mode because the $\mathbf{L}$ tritone contains the home tonic. Primary modes without $\mathbf{L}$ have alt modes that are not parallel modes because they don't contain the home tonic. This is the basis for a remarkably simple way of understanding changes that can be very complex in music notation. Details are developed by in Chapters 3 and 4 but hold the concept in mind.

That such deep coverage and powerful insights are provided by a simple tritone-based notation suggests that the conventional wisdom that tritones cannot be fundamental to music is only true for the particular notation provided by key signatures.

## Parallel Classical Modes

The following Lego-like view of the building blocks of the two parallel classical modes that determine the default major and minor modes of key signatures provides deep insight into the nature of parallel modes in general. The complete set of parallel classical modes is presented in Chapter 4, but its essence is captured here. The Lego-like view implies the scales at the bottom. Turning this around, knowing these scales from their mode signatures means knowing the building blocks.

## //I: Ionian mode

|  | @ P ADM\|LS.... @ |
| :---: | :---: |
| \$ | . . . . . . . \$ . . . . x |
| I | . . . . . I . . . . . . x |
| I | . . . . . I . . . . x . |
| M | . . M . . . . . . x . |
| M | . . . M . . . . x . . . |
| A | . . A . . . . . x . . . |
| A | . . A . . . . x . . . . . |
| @ | @ . . . . . . x . . . . . |
| scale | @ . $\mathbf{x}$. x x . \$ . x . x @ |

//A: Aeolian mode
@ PADMILS.... @


This view provides menus of building blocks for constructing chords; it brings forward core harmonic sequences to the eye; it provides insight into constructing shapes to voice chords; it illustrates, in a graphic way, a big reason for introducing the fifo concept - mutual substitutability of fifos. The anchor symbols at the top are repeated in the building blocks for readability, but the building block positions relative to the header line are visibly sufficient without this.

The terms "mode" and "scale" mean somewhat different things here: a mode is defined by a mode signature; a scale is defined by a note sequence. In music notation, a note sequence is represented by note symbols. The note sequences here are implied by intervals. A 12 -symbol chromatic scale will be introduced later that can be used to write out scales identified by intervals, but our focus first is on understanding modes and chords in interval terms, independently of the notes of which they're formed.

These modes are visibly determined by the symmetric dispositions of their only two half tones (highlighted in yellow) relative to the single tritone (red text); all the other scale intervals are whole tones. The building blocks that result are morphs of the tritone into fifos and of the fifos into each other
(the fifos may also be understood as morphed from tritones not in the scale but with anchors in the scale). The fifos of the scale frame are shared. The tritone and the fifos on either side of it are core building blocks because they determine the scale. The other three fifos (lighter shading) are secondary.

The core fifos are mutually dissonant and therefore functionally different. The fifo that establishes major or minor tonality ( $\mathbf{M}$ or $\mathbf{D}$ ) is a resolution fifo, automatically making the other fifo nonresolution. Core harmonic resolution sequences are visibly I-I-M (eye-7, eye-6, em-7) for Ionian, or A-A-D (eh-5, eh-6, dee-5) for Aeolian. The core sequences I-I or A-A anticipates resolution in the mode of the tritone and the core sequences I-M or A-D provides resolution. These core sequences are from only two classical modes but their nature is general for both classical and non-classical modes.

The lightweight fifo notation exploits the mutual substitutability of fifos in harmony. Core fifos are not substitutable for each other ( $\mathbf{I}$ and $\mathbf{M}$ of Ionian, $\mathbf{A}$ and $\mathbf{D}$ of Aeolian), but non-core fifos are substitutable for each other and for core fifos, because of mutual consonance. This enables commitments to many fifos in harmony to be deferred to context. Fifos are a source of both harmonic richness in music and notational complexity in music notation. The lightweight notation keeps the former and avoids the latter.

## Chords from Building Blocks

Shapes that form or voice chords follow from splitting or combining building blocks, as illustrated next for selected core building blocks from these modes (lighter shading indicates secondary fifos).

This only dips a toe into deep chord waters - see Chapters 3-5 and Appendix C for more - but is sufficient to give a sense of how chords of all kinds can be understood independently of chord symbols in terms only of building blocks identified by the PADMIL alphabet (the chord roots on the right are RN, Roman Numeral, symbols that identify root positions in the modes relative to the home tonic).


Example (a) forms major and minor triad chords by splitting fifths from the different modes, asymmetrically, into a minor third with a major third on top or the opposite. A very simple notation suffices for this: $\mathbf{A}^{\boldsymbol{V}}$ and $\boldsymbol{I}^{\boldsymbol{\Delta}}$ indicates split fifths with, respectively, the smaller or larger interval on the bottom. These are triad chords with the anchor symbols as roots, and so can be pronounced as "eh-minor-triad" or "eye-major-triad," keeping in mind that this is a special case and anchors are not chord
roots in general.
The double anchor notation covers all chords formed of the same notes in any inversion (four possible bass notes), two of which have different chord symbols. The shape is a combination of unsplit fifths which automatically brings in center notes of the triads. The double anchor notation is accurate but makes a postponable commitment to the secondary fifth (all that's really needed here is the core fifth and one outside note).

Example (b) forms a diminished chord from the Aeolian mode by splitting its tritone symmetrically into a stack of two minor thirds symbolized by $\mathbf{A}^{\bullet}$ (this may be pronounced "eh-dim," keeping mind that this is a special case and anchors are not chord roots in general). The circle indicates the split is symmetric. Asymmetric splits of tritones may occur as a result of combining a tritone and a fifo, but not normally for independent triad chords.

A combination of the un-split A tritone and I fifth forms a half diminished seventh chord that has different names in different, harmonically equivalent inversions. The combination automatically brings in the center notes of the triads. The double anchor notation makes a postponable commitment to a secondary fifth.

Example (c) combines a fourth and a tritone to form an inversion of dominant seventh chord rooted on the pitch center of the home octave. The combination splits the tritone asymmetrically. The double anchor notation makes a postponable commitment to a secondary fourth.

Example (d) goes outside the domain of classical modes by combining the two tritones into a circularly symmetric double tritone Al that forms a diminished seventh chord on each of its notes as roots. The double anchor notation is not a postponable overcommitment because all tritones are core.

## Stepping Stones to Chords

Octave shapes provide harmonic depth without adding new notes. They provide stepping stones from core building blocks to chords formed of overlapped building blocks. A sequence of octave shapes for the core Ionian sequence I-I-M is shown next. The color-coded box prefixes on the left identify octave completion building blocks going down from the anchors. The same anchor line with the same box symbols as suffixes instead of prefixes (I■-I■-M■, pronounced eye-7-5, eye-6-6, em-7-5) identifies an inversion of these octave shapes with the anchor line on the bottom, which is equivalent to having the anchor line on top because the bottom and top are an octave apart. Either way, octave shapes are easily morphed into 3-note scale shapes with all notes different. The result may be a stack of a core building block and a secondary building block, in which case the same notation applies.

## notation <br> ■I <br> $\square!$ <br> - M



## pronunciation <br> 5-eye-7 <br> 6-еуе-6 <br> 5-em-7

 the final two shapes are "all fourths" (counting the tritone as an augmented fourth). This sequence is very simple on the keyboard - one note drops a scale step and then all notes drop a scale step - but not so simple in chord notation. "All-fourths" shapes of this kind are common in non-classical modes. For example, I■ (eye-6-5) transposes down a half tone to $\mathbf{M}$ ■ (em-6-5), a common shape in blues that
voices a I7\#9 chord from a blues family scale (coming up).
The pronunciation of these shapes suggests a simple general notation for a harmonic core with an added treble or bass line offset up or down by an interval smaller than a building block. The sequence
 core with major thirds on top as one possible variation. Numbers 4, 3, 2 and 1 in prefixes or suffixes to anchor symbols would indicate different intervals on the bottom or top. The intervals might imply fifths or fourths overlapping the core, but this notation postpones this commitment. In principle, the simple colored-box notation for added building blocks could be generalized to different colors for major thirds, minor thirds, whole tones, and half tones, but too many different colors become confusing. Blue, red and green are easy to keep straight, and keep the eye focused on fundamentals.

This is enough to give a sense of where we're headed. Details are best left to examples in upcoming chapters.

## CHROMATICISM

Successive parallel classical modes bring chromaticism (deviation from a written key signature) into the domain of classical modes in a very simple way: one alphabet step (up or down) in a mode signature brings in a new tritone that alters one scale note; two steps alters two scale notes; three steps alters three scale notes; and so on. In each step, one altered note is provided by the tritone (the other tritone note is in both modes). The new modes are chromatic relative to the key signature of the original mode.

The chromaticism of such changes is identified in music notation by accidentals in melody lines and chromatic chords in harmony, without any explicit indication of its origin, which may be classical modes, non-classical modes, or ornamentation that has no scale implications. Explicit key-signature changes indicate the origin is in classical modes, provided there are no accidentals in the melody and no chords that go outside the key signatures, but much strongly chromatic music is written with a single key signature. Even the simplest classical mode changes can be complex in music notation.
"How strange the change from major to minor" - these words from the song Every Time We Say Goodby (covered in Chapter 5) express, incidentally, the extremes of complexity that are possible for a simple parallel mode change such as $/ / I-/ / \mathbf{A}$ in music notation. Suppose the home tonic is the first black key above C , which is either $\mathrm{C} \#$ or Db in music notation, depending on context. Then this change is from 5 flats for Db major to 4 sharps for $\mathrm{C} \#$ minor. The change naturalizes 5 notes and then sharps 4 notes - 9 symbol changes to alter 3 notes! A particularly confusing feature of this change is the different symbols for the minor tonic $(\mathrm{CH})$ and the major tonic $(\mathrm{Db})$ that seem to imply slightly different pitches for the same tonic. They don't - what they actually imply is slightly different pitches for the notes C and D , when used as references for sharps or flats. The pitch differences are real independently of the piano but not for the piano, and so overcomplicate written piano music.

Going beyond classical modes requires thinking of mode signatures as a kind of functional generalization of key signatures. The contribution of PKP is understanding all such changes in a simple way in terms of tritone changes.

These are deep and complex waters in music notation. PKP concepts and notation enable probing these depths in a way but that provides insight without becoming overwhelmed by details. That said, the probing remains challenging because music is challenging, as illustrated by the advanced examples in Chapter 5.

## A HIERARCHY OF PARALLEL MODES

Classical modes provide the key-signature scales of music notation, and the symbols of the anchor notation, but simpler pentatonic modes are the foundation of a hierarchy of parallel modes that includes both classical and non-classical modes. Pentatonic modes are simple because they have no half tones or tritones. Anyone with a musical ear can sing melodies in them. Harmonizing the melodies is simple because of the absence of dissonance.

## Pentatonic Modes

The Lego-like view of pentatonic modes shown next follows from the earlier view of the Ionian (major) and Aeolian (minor) classical modes by deleting half tones and tritones. It also follows from first principles by splitting the bottom fifth of the scale frame by a note that identifies major or minor tonality, and then splitting the remaining intervals by notes that add neither half tones nor tritones. This leaves minor-third gaps in the scales, identified by horizontal lines. Minor thirds are very distinctive scale intervals in the grand scheme of things, and so are worth identifying this way in any scale containing them. The signature of each mode is a letter identifying a core fifo (indicated by $>$ ) that determines the scale in the context of the scale frame, subject to the constraint of no half tones.


A lot of fun can be had, and insight gained, by improvising pentatonic melody and harmony on the black piano keys. Pentatonic major and minor modes formed of all five black piano keys are easy to see on the keyboard. The tonic of the all-black-key pentatonic-major mode is the black key at the geometric center of the C octave. The tonic of the all-black-key, relative pentatonic-minor mode is the next black key down. This experimentation can lead straightforwardly to blues because basic blues modes add one note to these modes.

## Blues Modes

The path from pentatonic modes to blues modes is simple, in contrast to the path via classical modes of key signatures. Blues modes emerge from a mashup of the parallel pentatonic modes. Viewing blues this way appears to be novel, based on the reactions of experts I consulted. The novelty is worth exploring because it leads to seeing blues as a logical, coherent musical genre, in contrast to the impression music notation gives of an ad-hoc combination of disparate elements.

A mashup, as the term is used in the music business, means combining different musical things to make a new musical thing that's different in kind. A mashup of parallel pentatonic modes combines all the notes to form new parallel modes that are different in kind, as shown next. The 8 -note mashup of the two pentatonic modes may be understood to follow from singers of simple pentatonic tunes "bending" selected notes by a half tone to give a sad twist to the major mode or a happy twist to the minor mode (equating "sad" with minor and "happy" with major). The effect is to switch to the
opposite pentatonic mode. The extended mashup adds one more bent note, often called the "flatted 5 th" because it's a half tone down from the 5th note of a classical mode. The three bent notes of the blues are the 3 rd, 5 th and 7 th notes of the default major scale of a key signature (the Ionian mode). Blues modes are different in kind from classical modes - mixed tonality, different numbers of notes, multiple tritones, multiple half tones (including adjacent half tones forbidden in classical modes). I learned the 9 -note mode in conventional terms some years ago in a blues piano workshop at the then Jazz School in Berkeley (now the Jazz Institute). The novelty here is knowing it by a simple, tritoneonly mode signature that's independent of both specific home tonics and music notation.

|  | @ PADMILS....@ |  |
| :---: | :---: | :---: |
| I/M | @ . x . M- ${ }^{\text {S }}$. $\mathrm{x}-\mathrm{C}$ | pentatonic major (5 notes) |
| //D | D . x . \$- x . @ | pentatonic minor (5 notes) |
| /IDM | x D M x . $\mathrm{S}^{\text {. }} \mathrm{x} \times$ | mashup (8 notes) |
| //DM.L | @ . x D M x L \$ . x x . @ | extended mashup (9 notes) |

As shown next, basic 6-note blues modes taught to beginners as "the" blues scales are, at once, one-note extensions of the pentatonic modes and sub-modes of the 9 -note family mode. "Major blues" is called that because it originates in the pentatonic major mode, but the added note makes it minormajor. In each case, the extension is a tritone anchor because it and a scale note form a tritone (red text). The fifo anchor letters in the mode signatures distinguish the modes from classical modes with the same tritone. The two 6 -note modes are more than just sub-modes because their mashup forms the family mode. The extra $\mathbf{M}$ tritone of the family mode is formed of one note from each of these modes.


The following Lego-like picture provides an overview of the family mode.


The three core tritones and the scale frame provide 7 of the 9 scale notes, leaving 2 notes to be
implied by the origin of the mode in parallel pentatonic modes (the added 2 notes are optional for the 9note scale but become important for sub-scales). The picture suggests core harmony sequences just as an earlier picture suggested them for classical modes. The difference here is tritones dominate the core harmony, which is therefore very simple because tritones are very simple on the keyboard.

A few examples of chords shown next will give a sense of how this goes. The combinations of building blocks are simple but the implied chord symbols can be complex, not only individually, but because the same shape may be a voicing of different chords on different roots. Possible chord roots are identified in the table by dashes and on the right by RN (Roman Numeral) symbols that give their position in the Ionian mode of the tonic as a reference mode (more on this notation later). The double tritone DM is too strongly dissonant for most purposes, but its anchors a half tone apart express a fundamental feature of blues, namely mixed minor-major tonality. This feature is captured by a 3-note shape formed of anchors $\mathbf{M}$ and $\mathbf{D}$ in different octaves with a note in between that provides a tritonefourth stack (sometimes called "all fourths" because a tritone is an augmented fourth). Sequences of such shapes are often used to provide ornamental transitions between modes. See Appendix C and Chapters 3-5 for more.


## Non-Classical Major and Minor Modes

Non-classical major and minor family modes follow from a mashup of the $/ / \mathbf{A}$ and I/I classical modes. Mashups of these modes cover so much ground in a useful way that there seems little need to consider other possible mashups of classical modes.

The immediate result of the mashup is a 4-tritone, 10 -note parent mode identified by IIADMI. The mixed minor-major tonality of this mode provides a solid basis for sub-modes of one or the other tonality. These sub-modes follow from morphing either the $\mathbf{D}$ or $\mathbf{M}$ tritone into a scale fifo with the same top note. The result is pair of 3-tritone, 9 -note modes of minor or major tonality identified by a compound tritone/fifo signature, in which a single fifo anchor identifies the fifo morphed from the missing tritone (the fifth $\mathbf{D}$ is morphed from missing tritone $\mathbf{M}$, and the fourth $\mathbb{M}$ is morphed from missing tritone $\mathbf{D}$ ).

```
|/ADMI
|/ADD.| or I/AD.|+
/IA.MMI or //A.MI+
```

I/ADMI
||ADD.| or I/AD.|+
/IA.MMI or /IA.MI+
@ PADMILS....@
@ PADMILS....@

$$
\begin{aligned}
& \text { @ . A D M I . } \$ \mathrm{x} x \mathrm{x} \mathrm{x} \text { @ } / / \mathrm{I}+/ / \mathbf{A} \text { mashup } \\
& \text { @ . } \mathrm{x} \text { X. } \mathrm{x} \text {. } \$ \mathrm{x} \times+\mathrm{x} \text { @ minor family } \\
& \text { @ . } \mathrm{x} \cdot \mathrm{x} \mathrm{x} \cdot \mathrm{\$} \mathrm{x}+\mathrm{x} \mathrm{x} \text { @ major family }
\end{aligned}
$$

The only difference between the 9-note family modes is the highlighted tonality in the bottom fifth.

The double appearances in the signatures of the same anchor as both a tritone and a fifo anchor are accurate but I prefer the equally accurate but simpler all-tritone signatures, with a plus superscript that symbolizes filling in the top fourth. The forms of the family modes say something important about tonality in general, namely that it's determined by the bottom fifth of the home octave, leaving the top fourth to context. Tritones $\mathbf{P}$ and $\mathbf{L}$ and the corresponding classical modes are excluded from this picture but often appear ornamentally in music from these scales as functional substitutes for tritones $\mathbf{A}$ and I (e.g., in sequences such as P-A or L-I).

A sampling of important 8 -note and 7 -note sub-modes is provided next (see Chapter 4 for more). The melodic and harmonic modes are like classical modes in having seven notes and no adjacent half tones. They appear in pieces of music as sub-modes of not only major and minor families but also of the blues family seen earlier. The modes are very simple, each in its own way. The melodic modes are simple because they're almost whole tone scales (yellow highlighting brings forward sequences of four whole tones). The harmonic modes are simple because their double tritones have circular symmetry, manifested as a stack of three minor thirds. An empty outer minor third (identified by a horizontal line) establishes the opposite minor third as the determiner of the master tonality (identified by a major or minor fifo anchor). The rest of the mode follows from the scale frame and the rule of no adjacent half tones.


The melodic and harmonic modes are the only non-classical modes that have been formally developed in music notation into a set of 7 parallel modes that are analogous to the 7 parallel classical modes. The book Modalogy develops these modes in conventional terms, with results that are painstakingly correct but overwhelmingly complex for all but experts (Appendix D). A simple view of these modes is developed in Chapter 4. The harmonic major and minor modes are represented by an 8note mashup of them that I call "harmonic minor-major." This mode determines a total of 8 parallel modes (instead of $2 \times 7=14$ ), and context generally determines the corresponding 7 -note mode, if needed.

A Lego-like picture of the building blocks is not shown here because it follows the general form of the blues picture, with obvious differences of detail.

## CHROMATIC SCALE

The use of note symbols has been avoided so far to develop a view of music based on intervals as musical objects that may be understood independently of the pairs of notes of which they're formed. Developing the same concepts in terms of note symbols bogs down in complexity. However note symbols are needed for melody lines, among other things.

It may seem ironic that an interval-based view of music requires a note-based representation of melody lines, but there's a good reason for it. Purely interval-based representations of melody lines are
ruled out by the difficulty of mentally tracking successive note positions without making errors. The chromatic scale coming up adds no significant complexity to the enterprise because its 12 symbols for 12 piano keys are in direct correspondence with the symbols of the conceptual home octave. A glance at one gives the meaning in the other.

Music notation does not provide a 12 -symbol chromatic scale but the piano's C-octave provides a simple basis for one: number the seven white keys 1-2-3-4-5-6-7 and identify the five black keys by prefixed numbers $\mathbf{p 2 - p 3 - p 5} \mathbf{- p 6 - p} 7$, where $\mathbf{p}$ is a position indicator standing for "phlat" and meaning next piano key down from the numbered key. The prefix is not a flat symbol of music notation because it applies only to these five piano keys ( $\mathbf{p} \mathbf{1}$ and $\mathbf{p} \mathbf{4}$ are not alternate symbols for $\mathbf{7}$ and $\mathbf{3}$ ), and there are no symbols corresponding to sharps. The numbers are not degree numbers that count scale notes, but position indicators in the chromatic scale of any home octave. The notation enables understanding the functions of the piano keys of any octave by mental reference to the C octave. It's simple enough to annotate on the staff next to note symbols of a written melody line.

## chromatic scale of the home octave $1 \begin{array}{llllllllll}1 & \text { p2 } & 2 & p 3 & 3 & 4 & \text { p5 } & 5 & \text { p6 } & 6\end{array}$ p7 71 conceptual home octave <br> @ P A D M L \$ x x x $\mathbf{x}^{(1)}$

Why not avoid the additional alphabet notation by identifying anchors by scale symbols? The answer is the powerful concept of an anchor is different in kind and needs its own notation. Scale symbols used for anchors would have to be identified as such. The alphabet symbols are PKP's way of doing this.

## Mode Scales

A first step in understanding a piece in PKP terms is annotating chromatic scale symbols next to melody notes on a staff. The annotations are an aid to interpretation. Melody modes are understood by marking the chromatic scale positions so identified in a 12-column table of the kind seen many times so far. Such a table provides a basis for identifying parallel modes in a melody line. As already explained, parallel mode changes may be understood as tonic changes if the melody indicates such changes. The notes are the same, only the order is different.

## Melody Plus Harmony

A skeleton melody line is a separate shorthand summary of what is, or could be, annotated on the staff. Such lines provide a useful basis for understanding melody plus harmony. Here follows an example skeleton melody line for a familiar piece, Happy Birthday to You.


Asterisks indicate repeated notes, arrows indicate ups and downs by differently colored arrows, commas indicate phrasing, and highlighting (optional) indicates melody peaks within phrases, to help the eye see them in a line of text (the mental model is of the movement of position markers on a grid in which the keyboard is horizontal and time goes down the page).

The addition of bar lines from a specific written piece connects a skeleton melody line to it, while still leaving details of timing within bars open. For example, the following bar lines leave open the playing of this melody line in $3 / 4$ time or $4 / 4$ time.


The essence of an entire piece of music may be represented by a skeleton melody line with an associated harmony line, which could be an alphabet-based anchor line or a walking bass line represented in the same notation as the skeleton melody line. These are the ways examples are presented in Chapters 3 and 5.

## Chords

Full chords represented by chord symbols are results in PKP, not starting points. Identifying implied chords requires a notation for chord roots. A suitable notation is provided by replacing the numbers of the chromatic scale notation with RN symbols as follows (tonic root "I" is not the same as Ionian anchor "I" in a mode table): I-pII-II-pIII-III-IV-pV-V-pVI-VI-pVII-VII. This is an adaption of a notation for chord roots used by Mehegan in his piano jazz instruction book.

## OBSERVATIONS

Very little notation is needed to use these ideas for actual pieces of music. Annotations are required on the written music to show anchor symbols above the staff next to chord symbols (the starting point is a table of tritone chords in Appendix C), and chromatic scale symbols on the staff next to melody notes. The only separate essential notation is a 12 -column mode table used to understand melody and harmony modes. Shorthand representations of melody plus harmony are optional extras that provide overviews of pieces that don't become bogged down in the details of music notation for different home tonics (Chapters 3, 5), or help to figure out tricky passages.

## CHAPTER 3: SOME SIMPLE EXAMPLES

The examples in this chapter are basically simple but exploring them plumbs surprising depths of musical sophistication. Anyone who has followed this so far should be able to understand and play the examples. The objective is not to show what a piece "actually is," or how the composer or arranger might have viewed it, but to arrive at a simple way of understanding it in purely keyboard terms, and annotating the understanding on the written music to guide playing it. The result is what the piece "is" to the player.

It's useful to think of music as divided into domains identified as basic classical (inherently not chromatic), chromatic classical (inherently chromatic) and non-classical (inherently chromatic). The term "classical" refers to the modes of key signatures, not divisions of music into categories such as classical, jazz and pop. The domains determine how chromaticism enters the picture. In the chromatic classical domain, chromaticism enters via successions of parallel classical modes (and possibly corresponding tonic changes that are the opposite side of the same coin). In the non-classical domain, chromaticism enters via scales that are more general than classical modes. Some chromatic pieces may be interpreted as being in either the chromatic classical or the non-classical domain, with the choice depending on the helpfulness of the interpretation.

For strongly chromatic pieces, the core harmony is often so tritone-intensive that core fifos may be omitted; their inclusion in some examples is sufficient to give a sense of how to include them.

I think of PKP as freeing the understanding of music from the straightjacket of music notation. This is a hard sell to musicians already expert in playing from music notation, because of the many thousands of hours at the keyboard required to get there, and because of the shared language provided by music notation for all musicians. The long hours are needed to become an expert pianist at some level, but not everyone has this ambition. It's possible to enjoy playing the piano for pleasure, without being expert at it. For people so inclined, PKP provides access to pieces that would otherwise be inaccessible. For the musically ambitious, PKP can provide helpful insight into the deep structure of piano music, independently of practicing. Nothing here requires abandoning music notation, only looking at it from a different, complementary angle.

The examples are, in order;
Happy Birthday
Backwater Blues
Straight No Chaser (walking-bass-line version)
Blue Monk (walking-bass-line version)
Summertime
I Got Rhythm
Over the Rainbow
Traumerai

## BASIC CLASSICAL DOMAIN : "HAPPY BIRTHDAY"

## Home Tonic - F (1 flat key signature: Ionian of F)

Here follows a 2-line summary of this piece. The symbols on the left identify the harmony anchor line and the melody line (seen at the end of Chapter 2).


The aim of a skeleton melody line is to provide a framework for adding harmony, not a detailed specification of melody. As illustrated next for the first few bars, this melody line is playable in $3 / 4$ time or $4 / 4$ time by different assignments of downbeats $(\downarrow)$ to notes. The above notation deliberately leaves the difference to the pianist.


The harmony anchor line follows from knowing the mode of the melody line, which requires marking the positions of all the melody notes in a 12 -column home-octave table, as shown next (the table is independent of the specific home tonic but the notes for tonic F are shown for concreteness). The selected building blocks of this mode are the ones that appear in the above anchor line.


The mode tritone is positioned relative to the melody line to anticipate resolution in the mode. This is important because the melody line by itself is ambiguous until major tonality is established by the first appearance of note $\mathbf{3}$ in the third phrase. The postponement of resolution to the tonic is signified in the first and third phrases by the tritone harmonizing the final melody note of the phrase. Immediate resolution at the end of the second and fourth phrases is signified by the tritone harmonizing the preresolution melody note. The multiple appearances of the mode tritone "seed" the core, which is completed by fairing in fifos from the building-block menu. Core fifo anchors of suitable functionality are faired into the line of tritone anchors ( $\mathbf{I}$ before $\mathbf{I}, \mathbf{M}$ or $\mathbb{M}$ after I). The choice of $\mathbb{M}$ at points of
melodic resolution avoids putting a dissonant half tone below the tonic, assumed to be played in the next octave up. The lightweight notation enable fifo substitutions to be made easily, if desired.

Most transitions in this core harmony are slides or morphs. A slide moves a building block to a different keyboard position while holding its size. A morph changes its size while holding one end fixed. The exception is the wobbly slide I-M which changes both position and size in one step. The unusual term represents the hand movements exactly - slide the hand while moving the fingers ("wobbling" them) for the size change. The wobbly slide I-M may be understood as a contraction of $\mathbf{I}-\mathbf{M}-\mathrm{M}$ (a tritone slides down a half tone and then morphs into a fourth). Thinking this way can be useful because it suggests a 2 -step anchor line that may identify mode changes (e.g., I-M stays in the Ionian mode, and I-M-M goes to the parallel Mixolydian mode).

Different added root lines for the same core yield voicings of different chord progressions. The number of different possible chord progressions with this same core tends to be startling to the uninitiated. Chord roots are only references for specifying notes, and not actually fundamental to the sound of a chord progression (more on this later). In contrast, the core building blocks are fundamental to the sound.

## From Octave Shapes to Chords

As illustrated next, octave shapes formed from anchored building blocks are stepping stones to shapes representing chords. This particular example illustrates "voice leading" harmony, in which the harmony follows the melody as closely as possible below it ( $\boldsymbol{\mathcal { K }}$ entries identify melody notes). The shape $\mathbf{M} \square$ in bar 5 is $\square \mathbf{M}$ from bar 4 inverted upwards to follow the melody line.


The home tonic and the mode tritone determine the scale that supplies the fifos. Chord roots identified by dashes within the shapes, are visibly provided by context. Different choices of roots yield different chords but these choices are simple and obvious. This is voice-leading harmony overall, not just in bar 4. The top harmony notes in bars $1,2,4$ and 5 fall on top of melody notes, depriving the harmony of an extra note, which is easily fixed by shrinking the building blocks down to the vertical lines. Vertical lines in other bars suggest other shrinkages that replace other doubled notes. The shrinkages are nuances that don't change the essence of the sound. Different roots yield different chords for the same shapes (e.g., the first two shapes could voice chords IVM7 and IVM7b5, among many other possibilities - see Appendix C).

## Walking Bass Lines

There are two ways of implying a scale early in a piece of music. One is by harmony that implies the scale (e.g., the mode tritone in bar 2). The other is by a walking bass line that complements the melody to bring in all the scale notes early. Walking bass lines provide a popular alternative to chordal harmony for many kinds of music. Switching between chordal harmony and a walking bass line is good way of providing variety.

A simple, 3-downbeats-per-bar walking bass line is shown next for this piece in $3 / 4$ time. The line is identified by an anchor symbol on the left because it replaces the anchor line (or perhaps supplements it), and is shown below the melody line, where it's actually played, because showing it above like core harmony would be too confusing on a note-by-note basis.



```
    \(|1>3>5|>4>3>2 \quad|>3>4>7|>6>3>1 \mid\)
    \(|>5>1|>7>6,74|>3>1>2|>1, \quad \mid\)
    \(|>1>3>5|>4>3>2 \quad|\nearrow 1>4>7|>5>3>1 \mid\)
```

Walking bass lines are easy to create using a mode table, as illustrated next. The basic idea is to outline or transition between building blocks.


## NON-CLASSICAL DOMAIN: BACKWATER BLUES

## Home Tonic - F (2-flats key signature: Mixolydian of F)

This is a simple, 3-chord, 12-bar blues in F, which I learned some years ago in a blues piano workshop at the then Jazz School in Berkeley, as representative of "probably half the blues pieces played by pop and jazz musicians." It's a simple piece but exploring it plumbs musical depths. This shows only core harmony, which will be filled in as we go along. Except for bar lines, commas delimiting phrases, and an assumption of 4 beats to a bar, timing and rhythm are left open. Placement of notes on downbeats or upbeats is left open. Commas at the ends of phrases only identify the last note, leaving open whether it's sustained or not. The concept is to convey the essence with minimum visual clutter. Nuances of timing are implicitly understood to follow from relationships between downbeats and upbeats (e.g., for swing feel, delay melody notes on upbeats to give the effect that notes on downbeats are bounced off them, also known as "long-short").


The main scales plus selected harmony building blocks are summarized below. As for classical modes, the building blocks arranged in order going down the keyboard suggest possible core harmony sequences. There are more building blocks but these are the only ones used by this core harmony.


Substituting I for $\mathbf{L}$ follows from conventional thinking about chord roots. Dominant-7 chords on roots I, IV and $\mathbf{V}$ are conventionally regarded as standard blues chords: the tritones $\mathbf{M}, \mathbf{D}$ and $\mathbf{I}$ are from these chords. The problem with this is tritones are more fundamental than chord roots to the blues. For example, a variety of blues chords with different roots emerge from tritone substitutions for dominant-7 chords (same tritone, root a tritone away, all non-tritone notes different). Chords I7, IV7 and V7 have tritone substitutes pV7, VII7 and pII7. Using these substitutions muddies the chord waters, but core harmony remains clear. Tritone substitution is not limited to dominant-7 chords (or
even to chords - recall alt modes). The $\mathbf{V}$ chord containing $\mathbf{L}$ is $\mathbf{V M 7 ( 1 1 )}$, of which chord pIIM7(11) is a tritone substitute. These chords and others like them may appear in blues pieces - for example, the footnoted chord progression on the first page of Chapter 1 - defeating all but experts by their complexity in music notation.

Given the general case, it's best to start off thinking of I/DM.L as the fundamental blues scale, and of the substitution of $\mathbf{I}$ for $\mathbf{L}$ as a means of simplifying chord progressions of simple blues pieces. In either case, the function of the single appearance of the tritone in this harmony is signaling a "turnaround" - the beginning of the last 4 bars of a 12-bar blues - as an aid to improvisors.

Harmony based on un-inverted and inverted tritone cores is shown next. It's easy to see how these simple voicing lines follow from altering tritone-based octave shapes. The simple voicing lines are sufficient because the tritones provide sufficient harmonic variety. Omitted chord roots are identified by dashes.


All-tritone core harmony is a relatively common feature of blues pieces. An all-tritone core is not only simpler than a mixed tritone/fifo core, it's also musically more definite. Assuming the family blues mode with no substitution of $\mathbf{L}$, the fundamental core of this piece over 12 bars is remarkably simple, as shown next.

## M-(D-M-M), D-(D-M-M), L-(D-M-M)

Chord substitution is a favorite trick of jazz musicians, often having the contrary effects of simplifying the keyboard flow of the harmony while complicating its symbolic representation in chord notation. Chord substitution for this piece boils down to holding the above core and changing a bass or treble line. For example, raising the bass line a whole tone for the last four bars on the left above,
voices the chord sequence V7-IV7b13-I9-I9. Tritone substitute chords discussed earlier are another example. Make enough substitutions and symbolic chord progressions can quickly become difficult to comprehend for all but experts (again, see the footnoted chord progression on page 1 of Chapter 1).

## Walking Bass Lines

Walking bass lines are popular for blues. Almost the first pieces I learned when I started out were walking-bass-line versions of Monk's Straight No Chaser and Blue Monk. I learned them from my first piano teacher, without reference to music notation, by copying her finger movements on the keyboard. After a bit more experience, I started to notate such lines for myself using the skeleton-melody-line notation. Walking bass lines are easy to represent in this notation, are easily portable in this form between different pieces, are easy to create knowing mode scales, and are easy to play. Nothing beats experimenting with walking bass lines as a way of learning to think in intervals.

All of these properties are illustrated by a walking bass line I learned for Straight No Chaser, shown next as harmony for Backwater Blues (yellow highlighting identifies ornamental passing notes). The initial four bars establish the 8-note, minor-major blues scale that's a mashup of the parallel pentatonic modes.


Understanding the bass line as $4 / 4$ downbeats provides the timing reference for the melody. Melody notes on upbeats between downbeats are for swing feel and also to avoid direct dissonances in some bars (the avoidance is a style choice because transient dissonances go with the territory).

Walking bass lines are easy to create using a mode table, as illustrated next for bars 1-4 .


## NON-CLASSICAL DOMAIN: TWO BLUES BY MONK

Although these pieces are relatively difficult in music notation because of their strong chromaticism, they're simple on the keyboard and easy to learn in this notation. They were among the first pieces I learned when I started out, and were so easy to learn that I think they belong in a set of introductory pieces. There's nothing in them requiring advanced musical knowledge or expert chops, and playing them is great fun for anyone who likes blues. A source for these and other Monk Pieces is Thelonius Monk Fakebook, Hal Leonard, 2002. The melody lines here are from this source, but I can cite no source for the walking bass lines.

## Straight No Chaser (Home Tonic F)

This is the piece that supplied the walking bass line for Backwater Blues. The walking bass line provides the timing that places melody notes on downbeats or upbeats. The interleaving of melody and harmony makes the whole sound like more than the sum of the parts.


## Blue Monk (Home Tonic Bb)

The walking bass line for this piece is similar in character but different in detail, to fit the different melody line. The bass line, played an octave below the melody line, provides four downbeats per bar. The piece features a double melody line. The second melody line is not shown in note symbols below because it follows a simple pattern that's better understood without them. The second melody line is offset down from the main melody line by minor thirds, except for major thirds below the notes highlighted in grey (the double melody line is illustrated on the next page for bars 1-4).


The double melody line of the first four bars, shown next, demonstrates the simple pattern. The entire piece is remarkably easy to play once you "get" this pattern. Seeing it clearly in the written music is remarkably difficult because of the clutter of sharps, flats and naturals required to represent the chromatic intervals relative to the 2 -flats key signature. The main melody line moves by half tones except for a few jumps by whole tones. The major thirds down from it (solid horizontal lines) appear when the main line jumps a whole tone. After these jumps, the second line either also jumps to keep the minor thirds going, or doesn't jump to keep the major thirds going.


## MIXED DOMAINS : "SUMMERTIME"

## Home Tonic - D (1 flat key signature: Aeolian of D)

I learned this well known Gershwin piece in the form presented here some years ago in a piano comping course given by Susan Muscarella at the then Jazz School in Berkeley. In the summary below, the anchor line is from a chord progression that will be presented later as a result. The only reference I have for this version of the piece is my course notes. Like Backwater Blues, it's a simple piece, the exploring of which plumbs musical depths.

The 6-note melody line is pentatonic minor except for a single half tone at end of bar 8 that could imply many minor scales, but the classical assumption would be Aeolian or Dorian. The harmony is strongly chromatic but is more than ornamental because, with the exception of bar 5, the tritone core identifies a succession of sub-modes of the minor family mode identified by $/ / \mathbf{A D I}{ }^{+}$(recall that the plus supersript means fill in the top fourth). The sub-modes are shown above the anchor line, providing an example of how modal harmony may be notated. This is a minor context in which the I/AI mode is the harmonic minor.


The modes are summarized next. The arrows at the right indicate some possibilities for inter-mode segues that are exploited in the above anchor line.


The skeleton melody line and the anchor line shown above are sufficient to play the melody with 3note octave shapes for the harmony. The result has depth because of the three notes, and variety because of the tritones.

Adjusting the harmony to provide more shape variety is easy, as illustrated next for bars 1-8 plus the resolution bar 16. The numeric suffixes or prefixes on anchor symbols indicate offsets of voicing notes above or below the core, measured in half tones. These would be added to the above anchor
symbols to indicate desired voicings. These shapes voice the chords on the right, if not always in place then in the flow (omitted chord roots are identified by dashes). All the voicings except for the II chord are rootless, demonstrating once again that chord roots are not inherently fundamental to the sound of a chord progression.


The numeric prefixes or suffixes for the offsets down or up of voicing notes have the useful property of telling the size of a shape directly: it's the size if the building block plus the size of voicing interval. For example, a tritone with a fourth on top (often called "all fourths" because a tritone is an augmented fourth) has a size of $6+5=11$ half tones, a half tone less than an octave. This makes finding the shape on the keyboard easy - find the treble note a half tone below the bass note an octave up, and complete the shape by adding an internal note a tritone above the bass note. All-fourths shapes generally imply complex chord symbols because this is not a basic chord shape from a classical mode.

As summarized next, there's a strong contrast between the simplicity of these shapes and the complexity of the chord symbols. The adjustment of basic chord symbols to fit scales, or places in scales, they don't quite fit is like "banging square pegs into round holes" - the results tend to be messy.

| chord scale notes (bars 1-8) | chromatic scale notes |
| :---: | :---: |
| root of $\boldsymbol{I}-\boldsymbol{m 6 6 ( 9 )}$, "5" of IV-13, " 7 " of III-m7 | 1 |
| root of II-m7b5, "9" of I-m6(9), "13" of IV-13 | 2 |
| "7" of $\boldsymbol{I V}-7$, "\#5" of $\boldsymbol{V}$, "\#9" of $\boldsymbol{I}-7 \# 9$ | p3 |
| "b5" of $\boldsymbol{I I}$, "b9" of $\boldsymbol{V}$-7b9 | p6 |
| "b5" of $\boldsymbol{I V}$-7b5 5 (not in the above but could be) | 7 |

## CHROMATIC CLASSICAL DOMAIN : "I GOT RHYTHM"

## Home Tonic - Bb (2-flats key signature: Ionian of Bb)

This Gershwin piece is the origin of widely copied chord changes called "Rhythm Changes" by jazz musicians. The source is The Standards Real Book, Sher Music (2000), p191. The melody line is Ionian of the home tonic $(\mathrm{Bb})$ except for one highlighted "outside" note that isn't a passing note. This note is the trigger for parallel mode changes in the second eight bars (between the double bar lines).


The parallel mode changes are summarized next. The other side of the same coin is Ionian tonic changes going down by fifths to the home tonic (highlighted sequence 6-2-5-1). Because this is the other side of the same coin, there's no need to think in terms of a full Ionian scale being transposed down by successive fifths. The parallel mode changes highlighted in yellow provide the same notes. The tonic changes and corresponding note changes are easy to know going backwards from the end.


There's nothing new in bars 1-8, but it's worth taking a quick look at the simplicity of bars 4-7


A particularly simple harmonic sequence is shown next for bars $9-16$ where the mode changes occur. Each 2-bar segment is nominally in the Ionian mode of a different secondary tonic, but the modes are daisy-chained together from one to the next via shared notes instead of resolving to the secondary tonics. In terms of the 3-note shapes shown, the tritones are as ornamental as they are in the previous bars because all the non-tritone notes are from the home Ionian mode. There's one departure in the full chords - the VIIm7 chord in bar 9 includes note p5.


This sequence doesn't explore the scale changes to the degree it could, but the scales are available to be explored in improvisations.

## ORNAMENTED BASIC CLASSICAL: "OVER THE RAINBOW"

## Home Tonic - Eb ( $\mathbf{3}$ flats key signature: Ionian of Eb)

This is an exercise in creating strongly chromatic harmony from scratch, illustrating the extent to which chromatic harmony can be purely ornamental. The first eight bars of the piece are sufficient for the purpose, which are so well known to everyone that there's no need for a reference. The melody line is straight Ionian, trending downwards in zig-zags over an octave range.

As shown next, the downward trending melody line is given a downward trending tritone anchor line consisting of the alphabet in reverse order starting on $\mathbf{L}$ and wrapping around: L-I-M-D-A-P-L-I. The tritones are spread out over the melody line and positioned for consonance with it.


The simple tritone "seed" on the left below provides a framework for adding core fifos and some other variations on the right. The double tritones provide some elegant symmetric relationships between successive shapes (e.g., PM-A shrinks the outer notes of the double tritone inwards a half tone; and A-AI-M moves a fifth aligned with the bottom note of AI to a fifth aligned with the top note).


Here's a summary of the melody and final core harmony.


## MIXED : "TRAUMERAI" (SCHUMANN)

## Home Tonic - F (1 flat key signature: Ionian of F)

The reference for this beautiful classical piece is the Classical Fake Book, 2nd Edition, Hal Leonard (2013). It provides chord symbols that yield the anchor line shown (the chord symbols are not shown because there's nothing new in them). It opens and closes with four bars in the Ionian mode of the home tonic. In between, it can be understood as a succession of classical modes, or as a single, extended non-classical mode - the 9-note major family scale $\| \mathbf{A M I} \mathbf{I}^{+}$. The latter is simpler for this piece because it provides all the melody and most of the harmony, and minimizes the number of mode changes to think about. Ornamental elements of the harmony (highlighted in yellow) are the common $\mathbf{P}$ and $\mathbf{L}$ functional substitutions for tritones $\mathbf{A}$ and $\mathbf{I}$. Notes highlighted in red bring forward the top notes of their phrases to the eye.

repeat bars 1-8


The melody scales and building blocks from them are summarized next. The scales provide more building blocks than this but the others aren't used. Given this picture, reading the above anchor line is straightforward. As always, the single building blocks of the core may be played as octave shapes that add depth without adding new notes.


## CHAPTER 4: THE BUILDING-BLOCK WORLD

This chapter goes into the details of using building blocks to identify tonic scales and to form shapes that voice chords. There are no new concepts here, only development of ones already presented, so the chapter may be skimmed to see what's in it, as a preparation for knowing where to look for details when needed, and then to go on to the more advanced examples in the next chapter.

## TONIC SCALES FROM BUILDING-BLOCK CONTENT

The following short dictionary of mode signatures for tonic scales summarizes in half a page all the scales and more in scale dictionaries such as The Source. The mode signatures on the left identify scales by tritone content.

| signature | 1 p 22 p 334 p 55 p 66 p 771 | scale type |
| :---: | :---: | :---: |
| PADMIL | $\mathrm{x} P \mathrm{~A}$ D M I L $\mathrm{x} \times \mathrm{x} \mathrm{x} \times \mathrm{x}$ | chromatic (12 notes) |
| //P.DM.L | X P . D M . L x - x x . x | diminished (8-notes, min-maj) |
| //AD.IL |  | diminished (8 notes, min) |
| PA.MI | P A . M I . $\mathrm{x} \times$. x x . | diminished (8 notes, maj) |
| ADMI | . A D M I- - $\times$ x $\times$ x. | no name (8 notes, min-maj) |
| //A.M.L | X . A . M . L - X . X . X | whole tone (6 notes, maj) |
| P.D.I | P • D - I • X - X - X | whole tone ( 6 notes, min) |
| //DM.L | @ . X D M X L \$ . x X . @ | min-maj blues family (9 notes) |
| //DMI | @ . X D M I . \$ . x x x @ | variation |
| //ADM |  | variation |
| //P.DM | @ P X D M X . \$ . X ¢ X . @ | variation |
| \|/ADD.I | @ . A D . I . \$ $\mathrm{x} x+\mathrm{x}$ ¢ | minor family (9 notes): aka //AD. ${ }^{+}$ |
| /\|A.MMI | @ . A M M . \$ $\mathrm{x}+\mathrm{x} \mathrm{x}$ @ | major family (9 notes): aka //A.MI+ |
| //DM | @ . A D M I . \$ . x x . @ | pentatonic union (8 notes), basic blues scale |
| //D.I | @ . A D . I . \$ . x . x @ | melodic minor (7 notes): master of 7 modes |
| /IAD.I | @ . A + I . \$ x - x @ | harmonic minor (7 notes) |
| //A.MI | @ . A . + I . \$ x-m @ | harmonic major (7 notes) |
| \|/A.M| | @ . A P + I . \$ $\mathrm{x}+\mathrm{l}$. C | "bebop" major (8 notes) |
| //ADMI | @ . $\mathrm{A}+\mathrm{C}^{\text {I }}$. \$ $\mathrm{x}-\mathrm{x}$ @ | harmonic min-maj (8 notes): master of 8 modes |
| //I | @ . A . M I . \$ . x . x @ | Ionian (7 notes): master of 7 modes |
| //M | @ . A . M-_ \$ . X | pentatonic major |
| IID | @--D . I . \$--x . @ | pentatonic minor |

The scales above the double line are atonal, with mirror symmetry (same interval sequence going up and down). The ones without the $/ /$ prefix are not true parallel modes because they don't contain the home tonic. The scales below it are asymmetric and tonal. These scales are completed by populating
the anchor sets in the lower fifth of the scale frame with fifo-only anchors. The letters ADMI are fundamental to these anchor sets, with different omissions of letters or different identifications of them as fifo-only anchors determining different scales. Minor-third intervals of scales are shown as solid lines to make them stand out to the eye. These are by no means the only possible scales but the dictionary is easily extended if anyone sees a need.

The focus of PKP is tonal music but atonal scales are included because shared letters of their defining words make them visible structural parents of scales lower down in the dictionary; and also because they may be used ornamentally in tonal music. Atonal scales have no minor, major or minormajor tonality by themselves but representing them by words that place them in the context of the home tonic gives them the relative tonality shown on the right (the presence of $\mathbf{D}, \mathbf{M}$ or $\mathbf{D M}$ identifies minor, major or minor-major tonality).

There are no 5-letter words because the implied scales would be too close to the chromatic scale to be usefully distinguished from it. As scale identifiers, single-letter words are reserved for scales with seven or more notes, which includes classical modes but not 6 -note blues scales containing single tritones that are understood as sub-scales of higher level blues scales.

The zone within the dictionary ranging from 4-letter words down to 2-letter words that identify parallel modes is complex in music notation. Scales must be understood as enumerated notes relative to one of more than twelve possible different key signatures. The role of classical modes as the defining scales of key signatures makes music from scales that differ markedly from them misleadingly complex. The dictionary makes this zone simple by pushing note symbols and interval inversions down to a lower conceptual level. The three scales in the middle formed around three asymmetric triple tritones provide important families of tonal scales.

The table covers all possible mode changes. As explained in Chapter 2, tonic changes are understood to be implied by mode changes, when a melody line requires them.

## SCALE FAMILIES

Scale families provide a simple, unified view of a very large number of scales of music notation of different kinds in different key signatures that are, in aggregate, overwhelmingly complex for all but experts.

## Blues Family

The blues hierarchy is shown next.


## D, M

At the top is the diminished scale P.DM.L that is a kind of "parent." This scale morphs into the DM. $\mathbf{L}$ scale (the $\mathbf{P}$ tritone morphs into a fifo while holding its upper note, the pitch center, fixed). The
morphing breaks the symmetry of the diminished scale, yielding an asymmetric tonal scale that contains the other scales as sub-scales that follow from pruning anchors.

The words M.L and D..L determine parallel "melodic" and "harmonic" modes that share with classical modes the properties of seven notes and no adjacent half tones.

Conspicuously missing are tritones $\mathbf{A}$ and I from the classical modes conventionally regarded as the default minor and major modes of key signatures. Fifo anchors $\mathbb{A}$ and $\|$ are in the scales but the associated tritones would be ornamental extras.

Blues has high tritone content but it also has high fifo content (five possible fifo anchors from the DM.L scale). A strong blues sound follows from core harmony that has correspondingly high tritone content (for example, the footnoted chord progression of Goodbye Porkpie Hat in the opening chapter). Weakening the core tritone content in favor of in-scale fifos weakens the blues sound.

I learned the 9 -note scale in conventional terms some years ago in a blues piano workshop at the then Jazz School in Berkeley, but this intuitively simple way of understanding and remembering it is my own invention and appears to be novel. Pianists and music teachers I have asked don't seem to know about it. Books I have consulted don't say anything about it.

## Minor and Major Families

The union of the parallel Aeolian and Ionian modes forms a 10 -note scale of mixed minor-major tonality determined by the word ADMI that is a master scale of family scales of major and minor tonality. As shown next, the minor and major hierarchies are slightly more complex than the I/DM.L blues hierarchy because ADMI is not the only possible parent atonal scale, and the classical modes at the bottom are morphs of the modes above them, not sub-scales.


The ADMI scale reduces to 9 -note minor or major family scales by morphing the $\mathbf{M}$ or $\mathbf{D}$ tritone into a fifo with the same top note. The morphed scales are identified by the augmented words AD.I+ or A.MI+ where the plus superscript indicates the top note of the missing tritone is retained. The result is both interesting and simple: an all-half-tone top end for both scales, with bottom ends of different tonalities.

The double tritones D.I and A.II on the left, and A.M and A..I on the right, determine parallel "melodic" and "harmonic" modes that share with classical modes the properties of seven notes and no adjacent half tones.

## PARALLELMODES

The systematic development of the concept of parallel modes in music notation is restricted to single-tritone classical modes and to double-tritone, non-classical modes identified by the terms "melodic" and "harmonic" that share with classical modes the properties of seven notes and no adjacent half tones.

## Classical Modes

The complete set of parallel classical modes is summarized next. There's a lot of information here, provided for reference, but the meaning is very simple: a tritone determines a primary mode (signatures of the form $I / \mathbf{X}$ ); mode changes are determined by tritone changes; changes between adjacent primary modes alter one note, provided by the new mode tritone; knowing a primary means knowing its alt mode (all non-tritone notes are different); only alt-L is a true parallel mode because only its tritone contains the home tonic; counting relative modes, all possible key-signature scales with tonics provided by the twelve piano keys of a home octave are covered.


The highlighted symmetric shapes determine the modes. The shapes follow from the symmetric arrangement of half tones relative to the mode tritone (both inside it or both outside it). Symmetric shapes formed by and around tritones are a fundamental feature of the PKP way of knowing scales. Symmetry is well known in math and physics to be a cornerstone for understanding complexity, so it's logical that it might do so for music, which is complex by any measure. The only hurdle to overcome is the unfamiliar notation. The notation is symbolic but there's no math here and everyone has an intuitive understanding of symmetry from everyday life.

The alt modes are "tritone substitute modes" (same tritones, tonics a tritone apart). The term is inspired by the term "tritone substitutes" for chords with the same tritone and roots a tritone apart. Learning that such chords are often used by jazz pianists for chord substitutions in improvisations provided my first glimpse of the possibility that tritones might be fundamental to an interval-based
view of piano music. That they are fundamental is demonstrated by the fact that they keep popping up as such in every interval-based representation of modes and chords.

Why include alt modes that are not true parallel modes in a table of parallel modes? The answer is they and the primary modes provide the means of seeing parallel mode changes and tonic changes as "two sides of the same coin."

## Two Sides of the Same Coin

Parallel mode changes for classical modes and same-mode tonic changes were said earlier to be two sides of the same coin. Both are interpretations of the same tritone change.

For example, the tritone change I-D down a whole tone may be interpreted as a parallel IonianDorian mode change or as tonic change down a whole tone to a relative Ionian mode. The parallel change alters 2 notes and the relative change alters zero notes.


A change to a new Ionian tonic that's not in the primary parallel mode of the new tritone requires an alt mode with all non-tritone notes different. In the following continuation of the same example, the tonic is a tritone up from a whole tone down, putting it a major third above the home tonic. The mode change alters 4 notes and the tonic change alters zero notes. The new Ionian tritone is the opposite inversion of the original Ionian tritone.

$$
\begin{array}{lllllllllllllll} 
& @ & P & A & D & M & \mathbf{L} & \mathbf{S} & . & . & . & . & @ \\
\text { I/I } & @ & \cdot & x & \cdot & x & I & \cdot & \$ & \cdot & x & \cdot & x & @ & \text { Ionian } \\
\text { alt-D } & \cdot & x & \cdot & D & x & \cdot & x & \cdot & x & x & \cdot & x & & \text { the new Ionian tritone is } \underline{D} \\
& & & & & \text { Ttonic of relative Ionian mode up a major third }
\end{array}
$$

The same simple principles cover all possible mode and tonic changes, as summarized in a notation-free way in the following simple table.

| anchor change | tonic <br> change | altered notes | tonic <br> change | altered notes |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| +3 | -3 | 3 | +3 | 3 |  |
| +2 | -4 | 4 | +2 | 2 |  |
| +1 | -5 | 1 | +1 | 5 |  |
| 0 | 0 | 0 | $\pm 6$ | 5 |  |
| -1 | +5 | 1 | -1 | 5 |  |
| -2 | +4 | 4 | -2 | 2 | <- example above |
| -3 | +3 | 3 | -3 | 3 |  |

The changes in the left column are from an established mode (parallel or alt) identified by a tritone anchor position in the alphabet, to a parallel mode identified by a different position a given number of steps up $(+)$ or down (-) in the alphabet as a circular loop (steps off one end wrap around to the other
end). Steps in opposite directions add up to a tritone. The yellow highlights identify changes that alter the fewest notes. This is sufficient to identify the mode signature of a result because changes between primary modes and alt modes alter the most notes.

The table generalizes simply and directly to non-classical modes, but is also less important for them because they're already so chromatic that extra chromaticism provided by tonic changes tends to be avoided.

## Overview of "Melodic" and "Harmonic" Modes

The melodic and harmonic modes are different in kind because each of the words of the mode signature is unique for the former, and half of the words are repetitions of the other half in the opposite order for the latter. This is because the double tritones of the latter are circularly symmetric (all internote intervals are minor thirds in any inversion). The harmonic modes can be dauntingly complex in conventional terms (Appendix D) because the irregularity of the master mode makes rotated transpositions doubly irregular. The irregularity can be finessed by making the master mode mixed minor-major, leaving the choice of its tonality in a particular parallel mode to context.

| melodic minor modes |  |  |
| :--- | ---: | :---: |
| PD | alt PD |  |
| AM | alt AM |  |
| DI | alt D! |  |
| ID | alt ID |  |
| ML | alt ML |  |
| LA | alt LA |  |


crossed-out modes don't contain the home tonic but may be used ornamentally

The three words A.M, D.I and M.L that fall out of the earlier scale hierarchies determine four parallel modes of the melodic minor mode identified by D.I. The three words determine four modes because any word containing $\mathbf{L}$ determines a primary parallel mode and an alt mode with all nontritone notes different.

The two words A..I and D..L that also fall out of the foregoing scale hierarchies determine eight possible parallel modes of a harmonic minor-major master mode.

Details follow for information, but most of the modes fall directly out of family scales in the context of particular pieces of music.

## Parallel "Melodic" Modes

These modes are summarized below in the same format as for parallel classical modes to enable easy comparison. As for classical modes, there are seven parallel modes (checked, with the master mode double checked). The unchecked modes are pseudo modes that don't contain the home tonic. Neither IP mode contains the home tonic but the major mode is designated //IP because of its place in the mode table. The twelve modes are so close to whole-tone scales that they are easy to know on the keyboard. A whole-tone scale is formed of five stacked whole tones, and these scales contain four stacked whole tones highlighted in yellow for the six primary modes. The notes highlighted in blue that provide references for the whole tone stacks are the tonics going down by fifths (6-2-5-1-4-p7) of the relative master mode.

As for classical modes, the alt modes are tritone substitutes with all non-tritones notes different,
and the ones containing $\mathbf{L}$ are parallel modes because the $\mathbf{L}$ tritone contains the home tonic (the difference here is there are two of these modes). Some useful relationships to classical modes are shown on the right, but keep in mind that the best way of understanding how the modes emerge in music is as sub-modes of higher level family modes in the scale hierarchy. It's also useful to remember that these scales are morphed from the whole tone scales identified by A.M.L or P.D.I by splitting one scale note into two notes a half tone above and below it (the lower note is the one highlighted in blue).


## Parallel "Harmonic" Modes

Making the minor-major mode the master reduces fourteen parallel modes that are often intricate and difficult to comprehend or remember (Appendix D), into the eight simple parallel modes check marked below (double checked for the master mode). The four unchecked modes are tritone partners that don't include the home tonic. The secondary tonics of the primary modes are highlighted in blue; the ones highlighted in grey are offset a tritone from these. The pattern is very simple: alt modes switch the empty and full minor thirds; the empty minor third is always bookended by half tones. Twoletter words with the same letters that identify different primary modes have the letters in opposite order.

|  |  | $\begin{array}{llllllllllll} @ & P & A & D & M & I & L & \$ & x & x & x & x \end{array} \text { @ }$ |
| :---: | :---: | :---: |
| $\checkmark$ | //DL | x . x D-L L . x x x x |
| $\checkmark$ | alt-DL | x x . D x x L . x x- x |
| $\checkmark$ | //LLD | $\mathrm{x}-\mathrm{D} \mathrm{x}$. L $\mathrm{x} x \mathrm{x}$. $\mathrm{x} x$ |
| $\checkmark$ | alt-LD | x x x D . x L- x x . x |



An example of the I/DL determined by blues family context is shown next. The highlighted segment shows one note selected by context from the two notes above.

```
1p2 2 p3 3 4 p5 5 p6 6 p77 1
//DM.L x . x D M x L $ . x x . x
    //DL x . x D-L $ . x x . x e.g., "Romanian"(Appendix D)
```


## CHORDS FROM BUILDING BLOCKS

When piano music is fully written out on the grand staff, the flow of the music is directly visible. The problem is the amount of complex detail is overwhelming for all but experts. When music is separated into melody on a staff and harmony represented by chord symbols above the staff, only the flow of melody is visible because chords may be voiced in different ways to give different harmonic flows that complement the melody line in different ways.

A piano teacher once told me to learn chords by picturing the chord symbols on the keyboard. I bought a chord symbol wall chart that did exactly this. As I looked at it while trying to learn pieces with these chords, I began to see it as hiding simple keyboard shapes that move in simple ways behind a facade of misleading complexity. This led me to see voicings of chord progressions as built up from harmonic cores without reference to chord symbols beyond identifying tritones. Knowing the scales of origin of the harmony from mode signatures provided by the tritones makes transforming cores into shapes to voice chords simple.

Mode signatures provide the building blocks that combine to make chords, as illustrated below for seventh chords from the Ionian mode that provide the basic symbols of chord notation.


The core building blocks, shown in color, are analogous to "guide intervals" used in standard chord voicing practice. Guide intervals are usually the 3rd-7th intervals of chord scales. These are the core building blocks of chords I, II and V here, but not of the others. The core building block of a chord in PKP is the most unique building block of the chord in the tonic scale. This is always a tritone for chords containing a tritone. Otherwise it's one of the two fifos of the scale anchored a half tone apart (e.g., fifths anchored by $\boldsymbol{I}$ and $\mathbf{M}$ for the Ionian mode). Think of the bottom or top segments shown in grey as the visible parts of enrichment fifos, of sizes to be determined, "snapped onto" the core building blocks underneath them at scale points (the o-o fifos are all fifths for basic seventh chords going up from these roots, but these are not the only possibilities).

The shapes in the second column represent interval stacks created by the combinations of building blocks. The numbers are counts of half tones: $\mathbf{3}$ represents a minor third, 4 represents a major third. This is a variation of a notation called "figured bass notation" (Appendix C), in which different numbers represent counts of scale steps, not half tones, between chord notes. In this variation, the building blocks are partial stacks: $\mathbf{3 3}$ is a tritone, $\mathbf{4 3}$ and $\mathbf{3 4}$ are fifths. The shapes are exact for any position on the keyboard. Pinpointing the position is conventionally accomplished by specifying chord roots (e.g., the IVM7 chord could be written IV434). In PKP, the pinpointing is accomplished by core anchors. A core anchor with a a single outside voicing note above or below it provides a 3-note voicing of a chord that's often sufficient in context. The voicing note for these chords and this mode is always a major or minor third above or below the building block, with the choice determined by the scale.

## Chords From Core Building Blocks

Putting the chord roots in down-by-fifths order brings forward to the eye the simple way in which chord progressions for the Ionian mode may be formed around two I-I-M core anchor lines joined by morphing the final fifth $\mathbf{M}$ of the first line into the fourth $\mathbf{M}$. The transtions are all morphs except for one wobbly slide. Three-note voicings are completed by adding major or minor thirds determined by the scale above or below the core, and notated by adding a numeric prefix or suffix to the anchor symbol, specifying the size of the thirds.

The chords are core building blocks (anchored) with overlapped enrichment fifos (0-0). The 3-note voicings omit the inner note of the enrichment fifos, which in this case is either the pitch center of the octave or a note a whole tone above it, both of which tend to be implied by context in passages of music from the scale. The intervals above and below the core are not not themselves building blocks, but the projecting ends of overlapped fifos.


Transforming this so the chords go up from the roots yields the difficult-to-play result shown next. This is difficult to play because the hand must be lifted and moved by large jumps, while also adjusting the fingers for "wobbles" between minor and major thirds. Such shape progressions may also be understood as sliding a fixed "scale shape," with 2 scale steps between each note, to different positions in the scale. This is conceptually simple but doesn't represent the wobbles, which are specific to particular modes and must be learned from practicing. Understanding such progressions in terms of actual keyboard shapes enables knowing the shapes for any mode and root sequence independently of practicing them.


There's obvious benefit in learning the easier-to-play morphed version first and then switching to this jumpy version, if desired, by inverting selected building blocks (which is always easy when building blocks are known by their anchors).

## Triad Chords

Seventh chords and their inversions, sixth chords, are the workhorse chords of 4-part harmony. Three-note voicings of them just shown are still seventh or sixth chords. Triad chords are different in kind because they're fundamentally three notes. Triad chords are formed by splitting fifths (but not fourths) asymmetrically to form stacks of opposite-sized thirds (major or or minor), or splitting tritones symmetrically to form stacks of minor thirds.

A very simple notation for triad chords is provided by adding a superscript to the anchor of fifth or tritone that identifies a symmetric or asymmetric split: $\boldsymbol{\Delta}$ or $\boldsymbol{V}$ for asymmetric splits ( $\boldsymbol{\Delta}$ indicates the major third is on the bottom and $\boldsymbol{\nabla}$ indicates it's on top); and ${ }^{\bullet}$ for symmetric splits. For example, A $\boldsymbol{A}$ is a major triad (chord symbol II, notes 2-p5-6), $\mathbf{A}^{\boldsymbol{V}}$ is a minor triad (chord symbol $\mathbf{I I} \boldsymbol{m}$, notes 2-4-6), and $\mathbf{A}^{\bullet}$ is a diminished chord (chord symbol IIdim, notes 2-4-p6). Triad chords may be voiced in a more spread-out fashion by holding the inner note and inverting the building block: e.g., the major triad @ (notes 1-p3-5) may be voiced as the fourth $\$$ with a major third underneath: $\mathbf{4} \$$ (notes $\mathbf{p 3 - 5} \mathbf{- 1}$ ).

Augmented fifths (8 half tones) may, like tritones, be split symmetrically. The result is a stack of two major thirds (4 half tones each) that forms an augmented major triad. If an augmented fifth on any anchor $\mathbf{X}$ is represented by $\mathbf{X}^{+}$, then $\mathbf{X}^{+\bullet}$ represents a stack of two major thirds. The chord so formed is an augmented major triad.

The fact that stacks of minor and major thirds form 3-note chords suggests that stacks of the building blocks may provide 3-note shapes that voice chords with more notes.

## Stacked Building Blocks

Stacked building blocks that voice chords with four or more notes are remarkably simple objects on the keyboard. They may be represented in anchor notation in a simple way that matches their actual simplicity. One of the building blocks is represented by a color-coded anchor symbol ( $\mathbf{X}, \mathbf{X}$ or $\mathbf{X}$ ). The other is represented by a symbolic prefix or suffix ( $\square$, $\square$ or $\square$ ) that represents an attached building block of the color-coded size. For example, $X ■$ is a stack of two fourths that spans a minor seventh, $\mathbf{X} \square$ is a stack of a tritone and a fourth that spans a major seventh (often called an "all fourths" shape, considering a tritone as an augmented fourth), and $X \square, X \square$ or $X \square$ are octave shapes. $X \square$ is larger than an octave. The symbols in all cases may include inversions of the core building blocks. and may be in either order.

This notation is a more visual substitute for stacks of building blocks than the notation presented earlier that uses numeric suffixes or prefixes for offsets of voicing notes (for example, $X \square$ and $\mathbf{X} \square$ mean the same as $\mathbf{X 5}$ and $\mathbf{X 5}$ ). The numbers remain necessary for non-building-block voicing intervals..

## Open Voicings

Returning to the original 4-note chords, the 4-note open voicings of them in (a), next, follow from inverting the enrichment fifos upward into the next octave and identifying the inversions by anchors The shapes are represented by symbols of the form $\mathbf{X I Y}$ in which the backslash emphasizes that the building blocks are one above the other, not overlapped. The shapes could be identified by the notation shown on the left in parentheses, but parsing the interval stacks is cumbersome and error prone.

Examples (b) and (c) are simple variations that provide voicings of a variety of complex-looking chord symbols. This vividly illustrates the banging-square-pegs-into-round-holes nature of chord symbols. Simple variations in the building blocks yield complex variations in the chord symbols because of having to adjust them to "holes" in scales they don't quite fit.

Example (c) is particularly simple: move one note down a scale step and then move all notes down a scale step. The shapes in the last two steps are "all fourths" (counting the tritone as an augmented fourth). These are far from the only possible variations but they give a good sense of the often deceptively complex nature of chord progressions.


The double-anchor notation covers the 3-note shapes described earlier: simply omit the top note of the upper building block. That said, it's simpler to work with core building blocks and offsets from them if 3-note shapes are sufficient.

## OBSERVATIONS

Chord symbols in tonal music often way over-specify notes because many of their notes are provided by context (mode scale, melody line, neighboring chords). A chord root is no more than a reference for an accurate specification of a set of notes. Inversions are often notated as radically different chords on different roots. Many different roots may provide chord symbols that do the job, perhaps with the addition of suffixes indicating extended or altered notes (often this is like "banging square pegs into round holes" - the result is messy). The mutual substitutability of many scale fifos offers many mutually consonant or harmonically equivalent chords.

Tritones are the only fixed elements provided by chord symbols (same size in either inversion, not substitutable in the ordinary sense of mutual consonance or harmonic equivalence), and so can be relied upon to provide a fixed element of any chord containing them. They identify parallel modes from which building blocks may be chosen to form shapes that provide suitable voicings of, or substitutes for, written chords. Knowing the scales makes it easy to fill in core fifos that morph to or from tritones, and to add enrichment fifos determined by bass or treble lines below or above the core.

Early in my musical adventure, I learned about simple "open" voicings of complex chord progressions in a "piano comping" course given by jazz pianist Susan Muscarella at the Jazz School in Berkeley (now the Jazz Institute). The voicings were based on chord progressions that were complex to begin with, making figuring out the often-rootless voicings in terms of reorganized degree numbers a complex process with difficult-to-remember results.

At the time, jazz pianist Taylor Eigsti was an artist in residence at the Jazz school who, I heard, was teaching a chord-symbol-free way of understanding and playing chord progressions. This resonated with my developing ideas about building blocks, so I contacted him to learn about his method. He told me that he recommends, to beginning jazz piano students, a practicing regime of moving fixed scale shapes (determined by fixed sequences of counts of scale steps between successive notes) to different positions in classical modes, without reference to chord symbols. The objective is developing the instinctive moves required of jazz pianists. The method has no notation and is learned from exercises. The lack of a notation makes it difficult to generalize the method to more irregular modes that violate the constraints of seven notes and no adjacent half tones. Fixed keyboard shapes enter the picture (e.g., the "all fourths" shapes mentioned several times up to now).

I decided to find for myself a simple, common notation for such different ways of understanding and playing chord progressions. PKP is what I found.

## CHAPTER 5: ADVANCED EXAMPLES

This chapter explores a smorgasbord of example pieces that I found difficult to understand when I first encountered them in music notation because of strong departures from classical modes, often combined with difficult or multiple written key signatures. The idea of a smorgasbord is to offer something for everyone.

A takeaway from this chapter is confirmation that tritones are structurally fundamental to music understood in interval terms, and that a DNA-like alphabet is sufficient to cover any tonic scale or chord progression that may be played on the piano, independently of the key signature(s) in which a piece may be written. This is different in kind from "the score is sacred" view of many classical musicians because it enables variations of harmony based on common cores and implied modes.

In a famous session of PBS's Piano Jazz, Bill Evans, in conversation with Marian Macpartland, said words to the effect that he advocated taking a piece apart to understand its architecture before putting it back together in an improvisation. I thought this was a great concept but wondered how "architecture" could be conceptualized. I suggest that the concept of musical domains introduced in Chapter 3 provides a way. The domains identified there, namely basic classical (not chromatic), chromatic classical (chromaticism introduced via parallel classical modes) and non-classical (uses non-classical modes that are inherently chromatic), are explored in some depth in this chapter.

I have heard it said, and believe it to be true, that composition is frozen improvisation. Therefore it, like improvisation, requires a sense of architecture. I suggest that exactly such a sense of architecture is revealed by the examples in this chapter. The fact such a sense can be extracted from such a wide variety of pieces without any information other than that provided by the piece itself suggests that it was put there by a composer or arranger, based on an intuitive understanding that's no doubt different in kind from PKP, but that can be represented simply by PKP.

The examples are intended to be understood by playing them, not just looking at pages, and so should be approached in front of a piano on which they can be tried. Start with the melody line and the tritone core to establish basic flow. Play the tritone core as octave shapes first. Then play the actual shapes shown, including interpolated fifo shapes. Then try variations.

The examples are as follows, in order of presentation:

Goodbye Pork Pie Hat<br>Prelude to an Afternoon of a Faun<br>Giant Steps<br>Round Midnight<br>Body and Soul<br>Laura<br>When Sunny Gets Blue<br>Every Time We Say Goodbye<br>All of Me<br>No Greater Love<br>All the Things You Are<br>Chelsea Bridge<br>Lush Life<br>The Peacocks (the only one in annotated music notation)

## NON-CLASSICAL DOMAIN : "GOODBYE PORKPIE HAT"

## Home Tonic - Eb ( $\mathbf{3}$ flats key signature: Ionian of Eb)

This blues in Eb is a poster child for chromatic music that's difficult in music notation for all but experts. My source for this piece is Mingus Fakebook, Hal Leonard (1991). Trying to learn this piece from this source was one of the stimuli that sent me down the path to PKP.

The key signature of 3 flats and the home tonic of Eb jointly identify the Ionian mode $/ / \mathbf{I}$ as the reference mode for the accidentals that determine blues notes. This is an example of the often misleading nature of key signatures. A 5-flats or 6-flats key signature indicating Dorian or Aeolian modes of Eb would be closer to the blues.

The simple melody line shown in the following summary of the piece is mostly in the 6 -note minor blues scale that's an extension of the pentatonic minor mode. The melody notes highlighted in red bring forward the melody peaks to the eye, which otherwise tend to disappear into the background in the linear textual representation of this simple melody line. Over-lined bars 6-7 are a variation that brings in ornamental chords and ornamental melody notes (the latter highlighted in yellow). In bar 7, the 1.p2 notation means the first note is "crushed" into the second note on the same beat.


Tritone sequences provide the "seed" around which the rest of the harmony is organized. This is contrary to conventional wisdom, which does not recognize tritones as fundamental to the structure of music. The tonic and tritones from the chords identify the mode, and the fifo shapes from the mode are morphed into or from the tritones to fit context. The harmony line captures the result

The harmony line determines the harmonic flow shown next, which is almost all 3-note rootless voicings of written chords (the omitted roots are identified by dashes). The idea is to hold a picture like this is in the mind's eye while playing, not necessarily to write it down. The free use of tritone substitute chords (e.g., I7/pV7, II7/pVI7, pIIM7\#11/VM7\#11) makes for a complex written chord progression. These chords have all non-tritone notes different (including roots a tritone apart) but the same harmonic function; in other words, they combine different fifos with the same tritone.

The all-green shapes are stacks of two fourths ( $\square \|$ ) that voice an unnecessarily complex variety of transition chords. For a start, the pII roots of the major seventh chords are byproducts of tritone substitutions, not indicators of visits to a mode containing this note. Beyond that, so much harmonic variety is provided by the tritone chords that fifo chords need only provide smooth transitions between them. The stack of two fourths does the job. A more accurate voicing of the sus chords would be a stack of fourths up a whole tone from this ( $\square$ ) but the stack shown is sufficient to get started.

The bass line provided by the chord roots is missing but could easily be added back underneath if
desired. That said, this harmony evokes the haunting, rich sound of the piece in a very satisfying way - small note changes in a smoothly flowing keyboard pattern make big sound changes.


The variation in bars 6-7 is an example of a sophisticated harmonic sequence formed from fixed keyboard shapes that's easy to play and sounds good in context. The three tritones not in the blues scale emphasize the melodic departure from it, while their "all fourths" voicings (augmented fourth with a fourth on top) provide a bluesy sound in a blues context. The segue from bar 5 slides its final shape (a tritone with a major third on top) up a whole tone. The shape then slides down a minor third while morphing into the first all-fourths shape. The final all-fourths shape segues smoothly back to the original blues by morphing into the same shape as at the end of bar 5 . Thinking this way guides playing without reference to complex chord symbols.

## NON-CLASSICAL DOMAIN : "PRELUDE TO AN AFTERNOON OF A FAUN"

## Home Tonic - C\# (4 sharps key signature: Aeolian of C\#)

This piece and Goodby Porkpie Hat have something unexpected in common, namely the I/DM.L blues scale. I was motivated to investigate this piece by things said about its violation of music convention in the article Beauty in the Void, Alex Ross, The New Yorker, Oct. 29, 2018. I was curious about the non-classical modes it might use. My source for the written music is the Classical Fake Book, 2nd Edition, Hal Leonard (2013), page 222. The key signature is 4 sharps and the home tonic is C\#, identified by the final note of the melody line.

The melody lines of [A] and [B] differ only in the final bar. The only unusual notational feature below is the double arrow at the end of [A], indicating a jump into the second octave up. Yellow highlighting identifies ornamental passing notes.


Transposing all the melody notes outside the home octave into corresponding positions in it (below) reveals the scales in play. The melody of [A]-[C] uses the blues scale plus passing notes highlighted in yellow above. The harmony of [A]-[C] is from this scale and also borrows from the nearby diminished scale. Appearances of the $\mathbf{A}$ and I tritones in bars 7-8 are ornamental. These excursions outside the blues scale are analogous to similar excursions in Goodbye Porkpie Hat.


The blues family scale provides a structural handle on the piece that's missing otherwise. That music notation provides no structural handle is verified by the amazement expressed in the referenced article at Debussy's "departures from music convention."

The annotated harmony shown next in Lego form satisfies the written chords mostly in place, and always in the flow. The harmony building blocks from the family scale that appear in Goodbye Porkpie Hat also appear here.

These passages use many triad chords, which are 3-note shapes consisting of symmetrically split tritones (e.g., dim chord $\mathbf{A}^{\bullet}$ ) or asymmetrically split fifths (e.g., major triad $\mathbf{P}^{\mathbf{\wedge}}$ ). Fitting the triads into the flow sometimes inverts them or converts them into seventh chords (e.g., $\boldsymbol{I}^{\bullet}$ in bar 8 is converted into a diminished seventh chord, the outer notes of which drop a whole tone to bar 9).


## CHROMATIC CLASSICAL DOMAIN : "GIANT STEPS"

## Home Tonic - F\# (empty key signature plus accidentals)

Coltrane's jazz classic (The Real Book, 6th Edition, Hal-Leonard) is famously difficult. The melody notes and chord sequences are from distant Ionian scales with tonics G, B and Eb that differ by 4 notes from each other (their implicit key signatures are 1 sharp, 4 sharps and 3 flats). Tonic changes are rapid, often one every bar.

The melody line and incomplete core harmony are shown next for home tonic $\mathrm{F} \#=\mathbf{1}$ (highlighting is of segments from different modes explained following this summary). Resolutions to home tonic $\mathbf{1}$ and to secondary tonics $\mathrm{A} \#=\mathbf{3}$ and $\mathrm{Eb}=\mathbf{6}$ establish the primary melody tonality relative to the home tonic as major to the ear. The core harmony includes all of the tritones but only selected fifos. The remaining fifos follow from obvious morphings. Learning the piece from this summary and then adding the morphed fifos by eye and ear is easier than trying to learn everything at once because the alignments between melody and full core harmony are sometimes counter-intuitive.


As shown in the following table, the melody scale considered by itself is an altered harmonic-minor-major mode of tonic 6: the unaltered mode is determined by double tritone A..I; the alteration substitutes tritone $\mathbf{D}$ for tritone $\mathbf{A}$. This only substitutes the anchor because the other note is already in the scale. This is the clearest way of identifying the alteration (identifying it by an altered degree number would be confusing because the 8 -note scale has an extra degree number in it its bottom fifth). The melody scale is actually an assembly of fragments of the Ionian modes but it's useful to know it from a mode signature. Playing the melody line knowing this mode is easy by itself. Playing the full core harmony is easy by itself. The only difficulty is the sometimes confusing alignment between the two.


The unhighlighted segments in the melody line are from $/ / \mathbf{M}$ and the highlighted ones from the alt modes. The different tritones of the melody line cross over between these modes. The highlighted melody segments in bars 1-2 and 5-6 are $\mathbf{4 h}>\mathbf{3 h}>\mathbf{4 h}$ sequences from alt-L and alt-A that are also from the overall melody mode.

The core harmony is best learned first without the light-shaded fifos, which can be added later by simple morphing. Melodic resolutions to the home and secondary tonics are marked " $>$ " on the left. The morphs are all downward, with one exception (A-A in bars 14-15 because the context leading up to it is different from before). This core harmony fits the written chords as shown on the right. Dashes indicate omitted roots. Octave shapes that go well with the flow of the melody are indicated on the left.


Improvisations could be based on the single melody mode. Or they could be based on the two main harmony modes $/ / \mathbf{M}$ and alt-L. The alt-A mode may be considered ornamental because it has only one note not in the other two modes (7). Among other possibilities, this suggests blues improvisations.

## NON-CLASSICAL DOMAIN : "ROUND MIDNIGHT" (MONK)

## Home Tonic - Eb ( 6 flats key signature: Aeolian of Eb)

The sources are The Ultimate Jazz Fakebook, Hal Leonard, 1988, p. 322 for the main part of the piece (shown first) and Standards Real Book, Sher Music, 2000, p. 369 for an optional introduction.

The main melody scale is as follows, relative to which the highlighted notes p5, $\mathbf{3}$ and $\mathbf{p 2}$ are ornamental. Much of the harmony is from this scale, with a few highlighted ornamental elements that fit the flow (e.g., L-I, I-M).

$$
\begin{aligned}
& \mathrm{Eb} \quad \mathrm{~F} \quad \mathrm{~Gb} \quad \mathrm{Ab} \quad \mathrm{Bb} \mathrm{Cb} \quad \mathrm{Db} \quad \mathrm{~Eb} \\
& 1 \text { p2 } 2 \text { p3 } 3 \text { 4 p5 } 5 \text { p6 } 6 \text { p7 } 7 \text { 1 } \\
& \text { @ P A D M I L } \$ \mathrm{~S} X \mathrm{X} X \mathrm{X} \text { @ }
\end{aligned}
$$

Here follows an optional, 8-bar introduction (or conclusion) that's visibly based on whole-tone intervals and tritones in highly structured ways. The three successive 2-bar segments repeat the previous interval sequence a whole tone down. Each starts on the non-anchor note of harmony tritone of the second bar and ends on the anchor note in the next octave up. The final 2-bar segment is a rhythmic pattern with no specific melody notes.

| L | \｜L | ｜I |
| :---: | :---: | :---: |
| J |  | ｜ 2 2， $1 \times 5 \times p 3>4, ~ \mid$ |
| 这 | ｜M | \｜D |
| $\int$ | $1-7 \mathrm{p} 7,72 \times 3 \times 2$ | $\|>1, \backslash p 7 \backslash 4 \backslash p 2>p 3$, |
| 过 | \｜A | ｜P |
| $\int$ |  |  |
| 込 | ｜ML I IP｜ |  |
| J | I－－－I－－－｜ |  |

## CHROMATIC CLASSICAL DOMAIN : "BODY AND SOUL"

## Home Tonic Db ( 5 flats key signature: Ionian of Db)

This piece is strongly chromatic in a way that's particularly complex in music notation. My source for the written music is The Ultimate Jazz Fakebook, Hal-Leonard (1988), p67. Three successive sections have three different key signatures: A 5 flats (Db-Ionian); B 2 sharps (D-Ionian); C 1 flat (DAeolian, but actually Dorian due to a natural in the written melody line). Ionian and Dorian of tonic D are nearby parallel modes (only 2 notes different). Ionian of Db is a distant from these, with many notes different ( 5 notes different from B ). The obvious home tonic is Db because A starts and ends the piece. The large distance between $A$ and $B$ is more than compensated by the simplicity of every note of $B$ being a half tone up from every note of $A$.


The melody scales are classical modes. The strongly chromatic harmony is not, but is consistent with the melody. When I first encountered this piece, I struggled with trying to understand how this harmony implied the melody scales and finally realized I had it backwards. Here follow the melody scales.



## NON-CLASSICAL DOMAIN: "LAURA"

## Home Tonic - C (empty key signature)

My source for this piece is The Jazz Book, John Brimell, CPP/Belwin, 1989, p24. The source book is sub-titled Today's Easy Adult Piano but this piece is "easy" only in the sense that the density of notes on the page is low. Playing it without thinking about the changes is easy, but provides no handle on the changes. Getting a handle on the changes is made difficult by the sparseness of some of the melody lines, and the repetition of harmony that's determinative in one section and ornamental in others. The highlighted elements of harmony and melody are determinative for the interpretation presented here. This is easy to play without thinking about what they mean. The interpretation follows.

go to (a) to repeat or to (e) to finish


The interpretation based on the highlighted elements is as follows (see table on next page). All sections, except the turnaround section (d) in pentatonic major, are in the harmonic-minor-major mode of the tonic sequence 5-4-p3-1 highlighted in blue in the melody line. Except for the final tonic, the melody lines don't resolve to these tonics because the modes are daisy chained via shared notes. The circularly symmetric double tritones highlighted in grey are determiners of 8-note, harmonic-minormajor modes of the tonics of the respective sections. This mode is very simple: the double tritone provides a stack of 3 minor thirds going up from a whole tone above the highlighted tonic; the bottom minor third (closest to the tonic) is filled with half tones and the top minor third (farthest away) is empty; the scale frame adds two notes. The parallel modes of the home tonic are relative modes of these modes. Not every section visits all the notes, but section (e) does, thus establishing a logical basis
for the rest. A useful feature of this interpretation is it includes the adjacent half tones of the filled-in minor thirds as scale notes, thus providing a clean representation of every section, free of ornamental passing notes, with one exception in (f).

The following table shows the actual melody notes and the notes of the corresponding harmonic-minor-major mode determined by the indicated double tritone. Harmony tritones in a section that are not in the mode double tritone are ornamental. This interpretation provides a handle on the piece that enables, for example, filling in core fifos in the harmony and enriching the harmony without reference to music notation, and improvising.

(b) notes
mode of PM

(c) notes
alt mode of AI

(d) notes pentatonic major
 x pentatonic major .
(e) notes

note from context
(f) notes mode of AI

## NON-CLASSICAL DOMAIN: WHEN SUNNY GETS BLUE

## Home Tonic - G (1 flat key signature: Dorian of G)

My source for this piece is sheet music by Jack Segal and Marvin Fisher from Hal-Leonard (1956), publication number HL00351105. The melody line from this source is a I/DM.L blues in G, with a 4 bar section in the bridge that visits $/ / \mathbf{L}$. The harmony shown below is a modification of the written harmony, which is a mix of fifo and tritone chords that sound fine but are confusingly irregular. Ornamental touches are highlighted in yellow - the ones in the melody line are from the written music and the ones in the harmony are part of the reharmonization.



Play the harmony in the first instance as octave shapes by doubling the treble line of the core an octave down. The 5 -tritone runs down by half tones in bars 4-5 and 6-7 are very easy to play and sound good with the melody line. They include ornamental tritones $\mathbf{I}, \mathbf{A}$ and $\mathbf{P}$ that are easily morphed into inscale fifos, if desired.

The Lego-like harmony is shown next. Keep in mind that this is only for illustration because it's implied by the anchor line. The harmony of the first four bars of the bridge is in the $/ / \mathbf{L}$ mode (Lydian). The final four bars of the bridge return to the original blues scale. The transition between them in bars $13-14$ is marked by the distinctive sound of a fourth-fourth slide down a half tone.


## NON-CLASSICAL DOMAIN: EVERY TIME WE SAY GOODBYE

## Home Tonic - Eb (3-flats key signature: Ionian of Eb)

This piece is hauntingly lovely, with ambiguous changes that go well with the words about love and loss. The melody line comes to rest at the end on the home tonic $\mathbf{1}$ in the unmistakeable Ionian-mode sequence 5-4-3-2-1-7-1 and much of the melody line is in the major family scale $/ / \mathbf{A} . \mathbf{M I}+$ of which Ionian is a sub-scale. Minor (or minor-ish) segments in melody and harmony are highlighted in yellow. Except for one, all-minor, 4-bar section, minor touches are brief and sometimes in opposition (minor in melody and major in harmony, or vice versa). There's sufficient ambiguity in much of this piece that there's little point in overthinking scale implications, which is why there are no annotated mode signatures or identified secondary tonics. The ambiguity is a deliberate feature of the piece by a creative composer.



(c)

(d)

(e) repeat (a)-(b)


The relative timing of the melody line and the rich, mixed harmony is organized in an irregular way that I have always found impossible to remember in chord terms. The chords sometimes harmonize melody notes and sometimes provide transitions between them, in patterns that are easy to get wrong. It's much easier to get the core harmony right because the flow cues what comes next to the eye and ear.

In phrase (f) shown next, the two melody bars with the famous words "how strange the change
from major to minor" stay resolutely in major for the melody, while the highlighted harmony, in these bars only, goes from major to minor. The harmony of the first two of these bars is intricate in chord terms but simple on the keyboard. The final voicing for the second bar implies the double tritone AI that's a half tone down from DL in the first bar. In effect, the core sequence is DL-AI down a half tone. Octave voicings for the remaining bars are sufficient.


The sound of the piece is relatively insensitive to the exact placement of the harmony relative to the melody. When the melody is on a downbeat, playing the harmony on the following upbeat - and vice versa - often sounds fine. For this reason, the harmony placements shown don't necessarily align exactly with the chord placements in the written music. Many of the harmony shapes are rootless voicings of the written chords.

## NON-CLASSICAL DOMAIN : "ALL OF ME"

## Home Tonic - C (empty key signature: Ionian of C)

I learned this version of All of Me in C (along with No Greater Love in Bb coming up) in Susan Muscarella's piano comping course some years ago at the then Jazz School in Berkeley (now the Jazz Institute), as an example of using 4-note "open" voicings of extended and altered seventh chords to accompany soloists. The chord progression is strongly chromatic and therefore complex in chord symbol terms. The open voicings rearrange the chord notes and sometimes omit the roots. The standard way of explaining such voicings is by rearranged degree numbers of chord scales, in which the same notes from adjacent chords have different degree numbers relative to different roots that are sometimes omitted. Such representations are indirect relative to the keyboard, difficult to comprehend as a whole, or remember, and difficult to annotate on the written music because they require too many symbols. This complexity goes away when the shapes are understood in terms of core building blocks.

The skeleton melody line and harmony anchor line are shown next. Trying to understand this as a succession of classical modes bogs down in complexity. The simplest way of remembering it is this: melody and harmony are both from the 9 -note, major family mode A.MI+ (/II plus notes p6 and p7), with the passing ornamentation shown highlighted in yellow. The two appearances of minor note p3 in the melody line are passing notes because omitting them has no substantive effect on the sound. The appearances of $\mathbf{P}$ and $\mathbf{L}$ in the harmony are ornamental substitutes for $\mathbf{A}$ and $\mathbf{I}$ because substituting the latter for them sounds fine.

repeat first 8 bars


Scales

melody \& harmony (first 16 bars)
melody \& harmony (last 8 bars) ornamental subs for $A$, I (all harmony)

The next picture shows what this looks like in terms of successive building blocks on the keyboard. This intended to represent a picture in the mind's eye implied by the above, not something to be written
down. Play this harmony in the first instance as octave shapes formed by doubling the treble line of the core an octave down. The only exception is bars 3-4 in which the tritones establish outer notes a minor seventh apart that may be held for both, only moving the middle note.

1, 17

2, 18
3, 19
4, 20
5, 21
6, 22


1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
A.MI' ${ }^{\text {@ }}$ - A $-\mathrm{MI}-\$ \mathrm{~S}+\mathrm{x} \times \mathrm{X}$ @

25
26
27
28

29
30


## Open Voicings

Four-note "open" voicings developed from the same core are shown next. This is intended to represent a picture in the mind's eye implied by the notation in the below left column (which would be annotated above the staff). The XIY notation identifies non-overlapping building blocks $\mathbf{X}$ on the bottom and $\mathbf{Y}$ on top (the backslash distinguishes this from from overlapped building blocks represented by $\mathbf{X Y}$ ). The shapes are voicings of the chords shown on the right. Chords on roots I, II and V are from the Ionian mode. The other chords (highlighted in yellow) provide visibly simple segues between the Ionian voicings. The chord symbols are all satisfied serially in the flow. This is remarkably simple, conceptually, compared to the standard way of understanding the same thing in terms of degree numbers of chord scales. The four notes are difficult to play with the left hand, but easy-to-play 3-note voicings of the same chords follow from omitting the top notes; the top notes can be added by the left
hand under the melody line.
Bars 1-8
$\begin{array}{lllllllllllllllllllllllll}1 & p 2 & 2 & p & 3 & 4 & p & 5 & p 6 & 6 & p 7 & 7 & 1 & p 2 & 2 & p 3 & 3 & 4 & p & 5 & p 6 & 6 & p 7 & 7 & 1\end{array}$



Bars 9-16

| 1 | $p 2$ | 2 | $p$ | 3 | 4 | $p 5$ | 5 | $p 6$ | 6 | $p 7$ | 7 | 1 | $p 2$ | 2 | $p 3$ | 3 | 4 | $p 5$ | 5 | $p 6$ | 6 | $p 7$ | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



| 13 \$ 1 . . . . . . . . \$ . . . . @ . - . . . . . x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14 L\M | - | - | - | - | - | - | IT | - |  | - | - | - | - | @ |  | - | - | - |  |  | - | - |
| 15 \|\M | - | - | - | - | - | I | - | - |  | - | - | - | - | @ |  |  | - | - |  |  | - | - |
| 16 IMM | - | - | - | - | - | I | - | - |  | - | - | - | X | . |  |  | - | - |  |  | - | - |
| I\D | - | - | - | - | - | I | - | - |  | - | - | - |  | - |  |  | - | D |  |  | . | - |

Bar 17-24 - repeat bars 1-8

Bars 25-32


Understanding how the shapes satisfy the chord symbols requires knowing the relationship between the numeric suffixes of the chord symbols and the chromatic scale. I explain this only to make clear that the shapes satisfy the written chords, not to suggest figuring the shapes out this way. In bars 3-4, chord suffix " 9 " is chromatic scale note $\mathbf{p 5}$ and chord suffix "b5" is chromatic scale note $\mathbf{p} 7$. The 3note voicings in bars 3-4 don't include p5 but it's included in bar 5, thus satisfying the bar-4 chord in the flow. The bar-4 shape jumps up a minor third in bar 5, with a wobble that puts a fourth on top instead of a major third. The wobble provides degree " 13 " of the chord. It also provides degree " 9 " of the bar- 4 chord, thus satisfying two chord symbols at one stroke.

## NON-CLASSICAL DOMAIN : "NO GREATER LOVE"

## Home Tonic - Bb ( 2 flats key signature: Ionian of Bb)

I learned this piece in Bb major in Susan Muscarella's piano comping course at the then Jazz School in Berkeley. The melody mode is major-family //A.MI+ throughout, with passing notes highlighted in yellow. The strongly chromatic harmony is in this mode, with the addition of ornamental tritones $\mathbf{P}, \mathbf{D}$ and $\mathbf{L}$ that don't change the melody mode (this includes the altered harmony pattern in the bridge). The numeric suffixes in the anchor line identify voicing notes by the number of half tones they are above the core (i.e., above the anchored building block). There's nothing new in the harmony so no Lego-like view of it is presented.


## Scales

//A.MI+ (major family)

ornamental

## CHROMATIC CLASSICAL DOMAIN : "ALL THE THINGS YOU ARE"

## Home Tonic - Ab (4 flats key signature: Ionian of Ab)

I learned this strikingly beautiful piece early in my musical adventure, and found it easy to learn because the keyboard sequences are simple, but difficult to understand in music notation. The source is The Ultimate Jazz Fakebook, Wong, Hal Leonard (1988). The skeleton melody line and associated core harmony for bars 1-26, where all the changes occur, are shown below. In this interpretation, the determining tritones of the melody modes are I-D-L-M highlighted in grey in the harmony. The modes - summarized on the next page - are interpreted for melody phrases (a)-(b)-(c)-(d) as Ionian modes of tonics 1-3-5-7. Tonics $\mathbf{1}$ and $\mathbf{5}$ are obvious from the melody lines but this is less so for tonics $\mathbf{3}$ and 7. Tonic $\mathbf{3}$ is not even in the melody line in (b) and tonic $\mathbf{7}$ is only a passing note in (d). These tonics are provided by the determining tritones. The low visibility of the secondary tonics is a result of resolutions to them being replaced by transitions to the next mode via shared notes. Passage (e) provides an "interesting" transition back to Ionian of the home tonic. There is no definite melody mode.


Here follows a summary of the modes of this interpretation ("notes" in the left column means actual melody notes). The sometimes-sparse melody lines are open to different interpretations but this interpretation is solid because it provide all the melody notes. The half-tone tonic change in (e) is difficult to pin down to a single melody mode. The melody is sparse. The harmony suggests modes I/P and $/ / \mathbf{A}$ but the melody moves between them. My piano teacher at the time said he had always thought something was odd about this transition, and suggested I just memorize it. And so I did, but I kept
worrying away at understanding it in more fundamental terms.


Here's the (d)-(e)-(f) harmony transition (missing chord roots are identified by dashes).
$\mathrm{Ab} \quad \mathrm{Bb} \quad \mathrm{C} \quad \mathrm{D} \quad \mathrm{Eb} \quad \mathrm{F}$ Gb Ab
1 ph 2 ph 34 pf 5 pf 6 ph 71 @ P_ADMELSXXXX@
(d)
(e)

VIm 7 bs
pIII7
pVIM7
III7\#5
V7
(f) I

The following overview picture is helpful in getting a handle on the piece as a whole.



20 different chords
chord root
line


## NON-CLASSICAL DOMAIN : "CHELSEA BRIDGE"

## Home Tonic - C\#/Db (4 sharps for Aeolian of C\#, 5 flats for Ionian of Db)

The source for this hauntingly beautiful Strayhorn piece is The Ultimate Jazz Fakebook, Wong, Hal Leonard (1988). There are two key signatures but only one home tonic, the black key identified by both Db and CH . The notational complexity that follows from this has been discussed elsewhere in these pages. This piece multiplies the complexity by returning to the 5 -flats Ionian mode at the end via naturals relative to the 4 -sharps key signature. Between the parallel classical modes are chromatic passages that multiply the notational complexity once more. The result is a confusing muddle of many more than twelve note symbols relative to the two written key signatures. The piece is musically sophisticated by any measure, but this notational complexity is way out of proportion to the sophistication.

I first learned this piece by rote with great difficulty from the written music, but thought that anything that sounds this good must have simple musical logic behind it. It does, as shown below: the basic structure is major-minor-major where the major and minor sections open with I/I and I/A and then branch out into non-classical modes of the same tonality. The concluding major section is I/I. Yellow highlighting identifies ornamental elements relative to identified scales. Parallel modes of the melodic minor determine melody and harmony scales in bars 2-5. All the scales are shown on the next page.


In bars 2-5, the harmony double tritones may be implied by playing their outer notes as augmented fifths moving down and up by whole tones, as shown next. Playing these augmented fifths with the
sparse melody line is very easy, and sounds good.


Here are the melody scales. In bars 2-9, the various parallel modes, all with major tonality, are determined jointly by the melody and harmony. In bars 2-5, the melody line sequences p6-6-p5-5 and $\mathbf{p 5} 5-\mathbf{- p 6} \mathbf{- 6}$ move back and forth between the L्A and ML modes. The slide of an augmented fifth up and down by a whole tone captures the signature note difference. In bars 6-9, the sparse melody line exercises the major triad of the I/I mode identified by the harmony. In bars 13-15, the IP minor mode follows the //A minor mode like the several major modes follow the opening //I major mode.


The melody and harmony have an elegant simplicity in these terms that's not difficult to understand, remember or play.

## NON-CLASSICAL DOMAIN : "LUSH LIFE"

## Home Tonic - Db ( 5 flats key signature: Ionian of Db)

This beautiful Strayhorn piece is melodically and harmonically rich, and challenging to play from the written music because there are often two or more chords per bar, many of them chromatic relative to the written key signature of five flats (major tonic Db ). The piece is so chromatic that there is more than one way of understanding it. It can be seen as daisy-chained, short segments of successive parallel classical modes, but this quickly bogs down in complexity.

The view of it here is in terms of two non-classical parallel modes of the home tonic for both melody and harmony: one is the 10 -note /IADMI minor-major mode in the verse [A] and chorus [C]; the other is the alt-IP mode (Ionian sharp one) in the bridge [B].


These modes lead naturally to short, easy-to-remember segments in classical and other modes that follow from the flow. For example, alt-IP morphs into I/I halfway through [B] by altering one note. From time to time, the harmony of the different sections substitutes $\mathbf{P}$ and $\mathbf{L}$ for $\mathbf{A}$ and $\mathbf{I}$, but thinking of these as ornamental relative to the IIADMI keeps things conceptually simple. The grey shading in [C] highlights differences.

There are no voicing extensions because the intent is to play octave shapes for single tritones and for fifos, which adds depth without adding new notes (the piece is so melodically and harmonically rich that the voicings don't have to add variety). Harmony fifos are mostly omitted for simplicity, except for characteristic sequences of them in [A] and the ending bars of [C].

The double tritones PM, AI and DL that appear in many places in the harmony are diminished seventh chords, commonly used ornamentally in any context in which they sound good. The double tritones have circular symmetry (all intervals are minor thirds), so only three of them with different notes exist. They can be played as four notes but they can also be voiced with three notes (a tritone with a minor third on top, or the opposite) or even with two notes (outer notes a major sixth apart), when the context implies the other note(s). A sequence of two or three of them, often used as an ornamental transition between modes, can be played as as sequence of major sixths anchored by any three of six tritone anchors (e.g., P-A-D going up and L-I-M going down, to name just two possibilities - others require inversions of individual tritones).

The chromatic melody-line run in the final two bars is a signature of the piece.

## MIXED DOMAIN : "THE PEACOCKS"

## Home Tonic - F ( 5 flats key signature: Phrygian of F)

This wraps up the chapter with an example in annotated music notation (in handwritten annotations, circling tritone anchors, or their chords of origin, or both is helpful). This strongly chromatic piece is one of the most haunting jazz pieces I have heard or learned to play. It sounds so "right" as written that jazz improvisations rarely stray far from it.

The home tonic is determined to be F by the final notes of bars 9 and 17. In bars 1-9, the combination of the 5 flats key signature, the home tonic and the lack of accidentals in the melody line identifies the melody scale as Phrygian of the home tonic (mode signature $/ / \mathbf{P}$ ). The melody line in bar 8 runs up through all the notes of this scale starting a fourth up (fifth down) from the tonic. The contrasting harmony is chromatic. The over-lines in bars 2 and 4 indicate held treble notes.


Harmony in bars 2-9 contrasts strongly with the melody. The core harmony in these bars, the first part of which is shown next, is visibly from the atonal ADMI scale (which is minor-major relative to the home tonic).


The melody of the second section consists mainly of individually simple sequences from the full chromatic scale that are prompted by the harmony.

| bar 11: | p5 p3 2 4 | $\geqslant 3 h>h>3 h$ | net $>\mathrm{h}$ |
| :---: | :---: | :---: | :---: |
|  | $3 \backslash p 2$ 1 p 3 |  |  |
| bar 13: | p6 4 $3>5$ |  |  |
|  | p5 ${ }^{\text {P }} 3$ 2 2 |  |  |
| bar 15: | $7 \backslash p 6 \times 5 \sim p 7$ |  |  |
|  | $6 \backslash p 5 \backslash 4 \sim p 6$ |  |  |
| bar 16: | 3 5 ${ }^{\text {d }} 6$ | 9h/h | net $\backslash 8 \mathrm{~h}$ (aug. fifth) |
|  | 4 p6 6 |  |  |
|  | p5 6才p7 |  |  |
|  | 5 $\mathrm{p}^{\text {7 }} 77$ |  |  |
| bar 17: | p2 3 4 |  |  |
|  | 2 4 4 5 |  |  |
|  | p3 p5 5 |  |  |
|  | 3 5 ${ }^{\text {d }} 6$ |  |  |
|  | 4 p6 6 |  |  |

## CHAPTER 6: OBSERVATIONS \& CONCLUSIONS

Music notation is here to stay and must be lived with, but this this doesn't preclude alternate views of piano music from a different perspective. I looked for an alternate view of harmony and found one based on intervals.

The central role of tritones in PKP tends to seem inappropriate to people savvy in music notation, for several reasons. In general, it goes against received musical wisdom. More specifically, tritones cannot even be written down in music notation until a key signature is established, so imagining them as identifiers of scales is difficult. Anything involving multiple tritones is complex in music notation because it implicitly brings in other key signatures - the more tritones, the more complexity. The very concept of fifos is unconventional and even more so is the concept that they can be treated as morphed tritones, understood from context. The payoff is insight into deep structure that is difficult to see in music notation. Appreciating these things requires stepping outside the box of music notation, at least momentarily.

Thinking in notes and thinking in intervals is different in kind. Many experts who have learned the hard way to think in notes apparently find thinking in intervals too alien to contemplate. Comments from them have run the gamut from "obviously wrong" to "wrong headed" to "naively simplistic" to "overwhelmingly complex." One expert said I had "found a tritone hammer and saw everything as a nail" as if the very existence of tritones as useful size tools was unthinkable. The book Modalogy, devotes 50 or so complex pages to parallel modes without ever mentioning the possibility that tritones might simplify things by providing defining notes.

Because I started out seeing intervals and notes as dual views of the same thing, I came to find it natural to switch back and forth. I have made my best effort in this document to convey the simplicity of this. I believe, based on my personal experience as an adult beginner, that any beginner could easily learn to think this way.

I know from personal experience that PKP helps with learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. It turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

The jumpy chord root lines in many of the examples demonstrate that referring chords to constantly moving roots creates misleading complexity. This complexity is both conceptual (the underlying, smoothly flowing core harmony is obscured) and actual (playing the jumpy harmony is technically difficult because all the fingers must be lifted and moved by large intervals). It's easier to learn the core harmony and introduce the jumps later, if desired, via inversions, which are always easy on the keyboard, given an anchor line.

## GENERAL BENEFITS

PKP annotations above the staff tell you many things. Combined with context provided by the melody line, they tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not right at any point by identifying tonic scales in play in a very immediate way. They tell you about transitions between major and minor tonality and between different tonics. The provide a basis for enrichment and improvisation.

Scale, modes and chords are seen as part of a continuum expressed in the same terms. This seems to me to be a very powerful benefit, compared to seeing everything beyond classical modes as
special cases identified by accidentals (which includes chromatic chords with suffixes that imply accidentals). The representations of scales and chords in terms of tritone clusters are unfamiliar to expert pianists but the interval stacks that they imply on the keyboard are completely familiar.

With PKP, playing music is guided by note-based symbology on the music page (one dual view) plus PKP annotations above the staff (the other dual view) that suggest core harmony and make changing tonic scales and tonics directly visible to the eye in terms of changing dictionary words.

A scale dictionary of less than a page covers much ground. The PKP dictionary provides a novel, high-level, conceptual view of tonic scales that's easier to hold in the mind's eye than the very much larger number of spelled out tonic scales in music notation. The dictionary covers $12 \times 39=468$ scales of 12 different tonics in music notation (more than is covered by scale dictionaries such as The Source). Words of 1-4 letters from the 6-letter alphabet identify, by tritone content, 39 tonic scales/ modes with 7 or more notes, 1-4 tritones, and no intervals larger than a minor third. The count of 39 scales/modes includes 11 single scales covered by single words and $4 \times 7=28$ parallel modes covered by transpositions of 1-2 letter master words (only the master mode is identified in the dictionary). A selected home tonic anchors all of this to the keyboard.

The words are analogous to the words of biological DNA in the sense that they identify deep structure without spelling out the details. Words of biological DNA are "expressed" as proteins, the building blocks of life. PADMIL words are "expressed" as building blocks of scales and harmony. Biological DNA is "extracted" from biological samples and interpreted by sophisticated machines called "sequencers." PADMIL scales and building blocks are "extracted" and "sequenced" from musical lines by sophisticated machines called human eyes. Biological DNA is "inherited" from parents. PADMIL words are "inherited" from general knowledge about them encoded in the dictionary.

Symmetry breaking in the scale dictionary and symmetry breaking in core harmony are two sides of the same coin. In the dictionary, it yields scales with fewer tritones. In harmony, it releases the tension of dissonance to produce consonance. Seeing the simplicity of this requires forming mental images of it relative to the schematic tonic octave that are independent of specific assignments of the octave to the keyboard.

Understanding harmony is possible independently of practicing. Keyboard symmetries centering around tritones enable high-level representations of harmony that are simple in their own terms and deep in harmonic meaning in any terms.

Music is freed from its key-signature straightjacket. Harmony scales are understood at a glance from dictionary words aggregated by eye, instead of having to be figured out or remembered from key signatures, accidentals and chromatic chord symbols. Tonic changes are no more difficult than any other changes.

Anchor lines that define harmonic cores replace chord root lines as the fundamental musical lines of harmony. Chord roots are visibly secondary elements in the flow of the music because inversions move them into the body of the chord, where they become no more than another chord note that may be omitted if implied by context.

Sight reading anchor lines provides deep context. Anchor lines tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not by identifying tonic scales in play. They tell you about transitions between major and minor tonality and between different tonics.

Multiple explicit or implicit key signatures become a non-issue. This is true whether the key
signatures imply tonic changes or only scale changes for the same tonic. Tonic changes amount to no more than moving (conceptually) a tonic pointer within the home octave and reinterpreting the alphabet letters relative to it, without changing any notation.

Chord symbols become a non-issue. Chord complexities that result from banging square pegs (chord symbols) into round holes (places in scales where they don't quite fit) are misleading. Chords described by chord symbols are, on the keyboard, almost universally composed of combinations of PKP's two kinds of building blocks, of which only tritones are always core. A melody line plus an anchor line above the staff imply a voicing of an unidentified chord progression. Specific chords are identified by assigning roots.

Learning blues prepares you for anything music notation can throw at you. Blues is an example of "folk music" that evolved independently of music notation and therefore tends to be regarded as naive when measured against its depth and complexity. This condescending view is unwarranted. Blues is harmonically sophisticated in a highly organized way that makes it, to my mind, a better starting point than classical modes of key signatures for understanding many forms of music.

Sharps and flats are not needed. Building blocks are the same number of half tones no matter how they are notated. This enables PKP to dispense with sharps and flats without any loss of musical accuracy. Expert pianists tend to think they are still needed to identify context because they have learned to use them for that, but note positions relative to tonics are sufficient.

The simple flow of harmony in harmonic cores, in which building blocks slide or morph into other building blocks, is universal. Nothing constrains harmony to be played this way but the flow is so natural and easy to play that much is to be gained by learning pieces this way and then transitioning to other playing styles if desired.

Tritone-intensive harmonic cores of much music provide so much harmonic variety that voicing them requires only adding depth. Depth is easily added by doubling the treble line of the harmonic core an octave down, or the bass line an octave up, and then making adjustments to move points on the line closer to the core, if desired.

No more piano wizardry is required to have fun playing these kinds of music than the ability to sight-read melody lines in music notation. Anyone with eyes to see intervals on the keyboard can play harmonic cores from anchor lines annotated above the staff.

The simple way harmony is represented provides great freedom in choosing how to play it. Playing styles may range between the extremes of smoothly flowing harmony within the octave of the home tonic of piece to jumpy harmony in which all harmony notes go up from a chord root line. Improvisation follows naturally and is fun, even if not expertly done.

The simplification provided by the alphabet is different in kind from simplifying chord symbols. Any sophistication in the harmony is in the tonic scales of origin of the chords.

Mode signatures provide an above-the-staff markup notation for the concept of "modal jazz" which is otherwise a concept without a notation.

PKP annotations above the staff are not always necessary. With experience, straightforward chord progressions may be sight-read in PKP terms.

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3. Eskelin, Lies My Music Teacher Told Me, Stage Three Publishing (1994) for insight into the nature of scales and musical "perfection," and for encouraging me to think outside the box.
4. Dmitri Tymoczko, A Geometry of Music (2011) for stimulating discussions of how to think about music from different angles.
5. Mark Levine, The Jazz Theory Book, Sher Music Co. (1995) for providing examples of well known jazz scales and harmonic forms in conventional notation, against which to verify PKP coverage.
6. George Russell, The Lydian Chromatic Concept of Tonal Organization, http:// www.georgerussell.com/lc.html, for making me aware that PKP covers the concept, because nothing is changed by replacing the Ionian mode by the Lydian mode as the default reference major mode for any piece of music.
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10. Stephon Alexandar, The Jazz of Physics: The Secret Link Between Music and The Structure of the Universe, Basic Books (2016). This amazing book resonates strongly with my own ideas. The author is a jazz musician and physicist who speaks of doing physics as informing the playing of jazz and playing jazz as informing the doing of physics. Both require rising above formalisms to form conceptual representations, with symmetries highlighted as particularly important.

## ACKNOWLEDGMENTS

I did not take this musical journey alone. I received comments and help from many people over the decade or so the ideas were germinating and consolidating.

Music theorist Paul Steinbeck encouraged me to continue writing at a time when I was becoming discouraged about finding a way of bringing my ideas before the music community. Jazz pianist, teacher and composer Taylor Eigsti has been an inspiration to me. Although I have never been a piano student of his, I have learned much from him in sporadic discussions in person and by email. I am deeply grateful for his willingness to take time away from a busy schedule to engage in these discussions. A short series of piano lessons from SF jazz pianist Michael Parsons helped me to see more clearly the relationship between my ideas and standard jazz-piano practice.

Thanks to SMT (Society for Music Theory) members Charise Hastings, Peter Shultz and Neil Newton for insightful email comments and encouragement following announcements of my website on an SMT mailing list. Thanks to music professor Robert Rawlins for email encouragement, and for helpful examples and comments. Aaron Blumenfeld and Susan Muscarella gave helpful courses at the Jazz School in Berkeley (now the Jazz Institute) that provided many examples to chew on (including, from Susan Muscarella's course, some interesting variations on Summertime in D minor that helped me understand ornamental scales). Amateur pianist and Jazz School Board Chair Susan Brand, and concert pianist and music entrepeneur Robert Taub, provided early encouragement.

My piano teacher in San Francisco for several years, Ken Fishler, provided inspiration and how-to information on chord voicings, while patiently tolerating and responding to my question-everything approach. My first piano teacher in Ottawa, Canada, Sally Robinson, started me off right as an adult beginner by helping me to learn favorite harmonically sophisticated pieces by following her fingers on the keyboard without understanding anything about what I was doing except that it sounded right being able to play these pieces, even if clumsily and by rote, enabled understanding to seep in gradually. I think that without this particular way of starting out - fingers on the keyboard first, written music later - I would not have started thinking about keyboard harmony in the way I did. It forced to my attention the large gap between simplicity on the keyboard and complexity of the full notation that represents it.

I wore out the patience of many music professionals by bombarding them with unsolicited email requests for comments on my ideas. I would like to thank four in particular, who politely responded to numerous emails in spite of being uncomfortable with my ideas: Harry Likas, Dmitri Tymoczko, Jeff Brent and Daniel Glover. Their criticisms helped me change my explanations without changing my mind.

Thanks to friends Marva Black, Mike Budde, Peter Marchant and Selinda Spugies for helpful insights on aspects of this material that helped me to understand how to get the ideas across better. Thanks to singer extraordinaire and friend Lorna Kollmeyer for helping me figure out how to organize piano accompaniment in guitar-strumming style.

My wife Sheila's accurate musical ear helps me know what does and does not sound good on the piano, in addition to making my life generally interesting. My grandsons Joshua and Ethan Feiber provided encouragement and comments; Joshua set up the website www.pianotheoryman.com as birthday gift; Ethan, who learned very young to play the piano impressively well by ear, has experimented with the notation and found it helpful for approaching written music.

## SOME COMMENTS FROM READERS

The following comments provide a kind of history of the development of PKP (the dates on the left identify when the commenters read different drafts of this material). The unconventionality of putting tritones on center stage has tended to get in the way of expert musicians accepting the ideas at all, let alone seeing them as simple, which has not been helped by the fact that it took me a long time to find the simple way of explaining the ideas that appears in this document. I continue to think the ideas are important for learning the piano in a less restrictive manner than is conventional but have not yet been able to find an audience. At the very least, the more recent comments show that the PKP method is more than just a fantasy of an over-enthusiastic amateur.
(2016) Taylor Eigsti more recently than below. "... a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."
(2011) Paul Steinbeck. Assistant Professor of Music Theory; Washington University, St. Louis
"The hook ..., at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music."
(2009) Robert Rawlins. University Music Department Chair (Rowan University); jazz musician; teacher; author of several books on jazz
"I became aware of Raymond Buhr's novel method for analyzing and voicing chromatic chord progressions in 2008 through a draft of a paper he wrote on the subject. I have kept up to date on developments of the method and we have had many email exchanges discussing issues of interpretation and application. I am a member of his intended target audience-a jazz musician who is not a pianist who needs to work out harmonic patterns on the piano from time to time. I am also a music teacher who has actually tried out aspects of his method on students. I can vouch from personal experience for his method's helpfulness in dealing with complex chromatic chord progressions."

## (2009) Susan Brand. Board Chair, The Jazz School in Berkeley; amateur pianist

"When Raymond Buhr consulted me about his theory of chromatic chord progressions, I was immediately struck by his ability to analyze and attempt to simplify this complex subject. Mr. Buhr brings a unique perspective and a great deal of enthusiasm, depth of understanding and originality of viewpoint to the subject. Over the years I have watched the continuous work that he has put into editing and rethinking his work. He has had ongoing consultations with many knowledgeable musicians/ teachers/editors and all have contributed to the development of the method described in this book. His ideas offer a way of understanding musical theory that will add greatly to the field and will be extremely helpful to musicians and music educators."
(2008) Taylor Eigsti. Jazz pianist, composer, teacher, former Artist in Residence at The Jazz School in Berkeley
"Through the brilliant lens of an engineer, Raymond Buhr has laid out an analysis of harmony that is a unique and complex look at the right-brain from the left-brain's perspective."

## APPENDIX A: UNCONVENTIONAL ELEMENTS

## TERMINOLOGY \& NOTATION

- anchor: identifies a building block by the position of its bottom end relative to the home tonic
- anchor set: set of tritone and fifo anchors that define a scale by construction
- anchor line: anchor sequence written above the staff
- alphabet: PADMIL identifies anchors by the first letters of the names of classical modes
- building block: tritones or fifos (fifths or fourths) anchored at alphabet positions (size distinctions determined by color coding).
- chromatic scale: 1-p2-2-p3-3-4-p5-5-p6-6-p7-7-1
- context: provided by mode signatures for melody and harmony plus flow of both
- core: sequence of building blocks of harmony identified by an anchor line
- family: a set of sub-scales of a family scale defined by a single mode signature
- fifo: fifth or fourth that are opposite inversions (add up to an octave)
- flow: formed by morphs and slides of building blocks
- frame: defined by the tonic @ and pitch center \$ of a tonic octave (plus sometimes by tonality)
- mode signature: word identifying a tritone cluster and implying an anchor set via the scale dictionary
- pattern: organized arrangement of intervals on the keyboard or over time
- pitch center: note identified by a fifth/fourth octave split, symbolized by $\mathbf{\$}$
- morph: small change in the size of a building block while holding one end fixed
- outside: not in a given tonic scale (as distinct from "chromatic" meaning not in a key-signature scale)
- shape: combination of building blocks
- slide: size-preserving movement of a building block
- phlat: prefix $\mathbf{p}$ identifying chromatic-scale notes in the whole tone gaps of the major scale of a tonic
- tonic pointer: suffix of form @t attached to an anchor symbol to indicate a secondary tonic
- wobbly slide: combined morph and slide
- word: set of alphabet letters with optional dots indicating skipped letters


#### Abstract

ABOUT THE ALPHABET An effort must be made to avoid confusing letters of the PADMIL alphabet with other uses of the same letters in music theory and notation (A, D and I are particularly troublesome). In principle, any six letters would do for the alphabet, but the connection made to classical modes by this alphabet is too useful to discard. I thought of substituting the Greek letters epsilon $(\boldsymbol{\epsilon})$ and delta ( $\boldsymbol{\Delta}$ ) for $\mathbf{A}$ and $\mathbf{D}$ and the English letter $\mathbf{Y}$ for $\mathbf{I}$, to yield $\mathbf{P E \Delta M Y L}$, pronounced almost the same and therefore easy to remember as having the same meaning (epsilon represents "Ae" of Aeolian). However, the mix of different alphabets introduces a different kind of confusion. I also thought of PQRMYL as an arbitrary way of getting rid of $\mathbf{A}, \mathbf{D}$ and $\mathbf{I}$. Or even $\mathbf{U V W X Y Z}$, which has the undesirable side effect of also getting rid of useful $\mathbf{P}, \mathbf{M}$ and $\mathbf{L}$. I have not been able to think of anything better than PADMIL.


## APPENDIX B: ABOUT SCALES

## KEY-SIGNATURE SCALES

The following summary of the standard interpretation of the major and natural minor tonic scales of key signatures (Ionian and Aeolian modes) is a helpful reference. It shows the sharped notes of sharp scales in red and the flatted notes of flat scales in blue. Although there are only five black piano keys, scales with six flats or sharps exist because of the scale-spelling rule that the same letter note cannot appear on both sides of a half-tone interval. For example, note B is $\mathrm{C} b$ in a 6 -flats scale.

Major (Ionian) and relative minor (Aeolian) scales are shown on the same line because they have the same key signature. Parallel minor scales (diagonal lines) have the same tonic, which does not necessarily mean the same note symbol because the same piano key may be represented by different note symbols (e.g., the parallel $\mathrm{D} b$ major and C\# sharp minor scales, which have the same black-key tonic on the piano, have different note symbols for it).


Avoiding multiple written key signatures by using accidentals to identify implicit key changes replaces one kind of complexity by another because accidentals have many more uses than this (e.g., ornamental passing notes, or identifying scales that originate in no key signature). Sorting out the possibilities requires considerable head-scratching at a very low conceptual level.

Key-signature-based music notation has stood the test of time and is here to stay because of the huge legacy of music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation.

## CROSS REFERENCES

Cross-referencing music notation and PKP notation requires the following table. This is the simplest case. More generally, naturals may enter the note-symbol picture to cancel sharps or flats of key signatures, and white piano keys may be represented by sharps or flats ( $\mathrm{B}=\mathrm{Cb}, \mathrm{C}=\mathrm{B} \#, \mathrm{~F}=\mathrm{E} \#, \mathrm{E}=\mathrm{Fb}$ ) to satisfy the scale-spelling rule that the same letter note must not appear twice. Experts develop rules of
thumb about sharps and flats as indicators of what's going on, and see PKP as discarding these clues not so, the clues are present in PKP in a different way.

| 1 | p2 | 2 | p3 | 3 | 4 | p5 | 5 | p6 | 6 | p7 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| @ | P | A | D | M | I | L | \$ | x | x | x | x |
| C | C\#/Db | D | D\#/Eb | E | F | F\#/Gb | G | G\#/Ab | A | A\#/Bb | B |
| C\#/Db | D | D\#/Eb | E | F | F\#/Gb | G | G\#/Ab | A | A\#/Bb | B | C |
| D | D\#/Eb | E | F | F\#/Gb | G | G\#/Ab | A | A\#/Bb | B | C | C\#/Db |
| D\#/Eb | E | F | F\#/Gb | G | G\#/Ab | A | A\#/Bb | B | C | C\#/Db | D |
| E | F | F\#/Gb | G | G\#/Ab | A | A\#/Bb | B | C | C\#/Db | D | D\#/Eb |
| F | F\#/Gb | G | G\#/Ab | A | A\#/Bb | B | C | C\#/Db | D | D\#/Eb | E |
| F\#/Gb | G | G\#/Ab | A | A\#/Bb | B | C | C\#/Db | D | D\#/Eb | E | F |
| G | G\#/Ab | A | A\#/Bb | B | C | C\#/Db | D | D\#/Eb | E | F | F\#/Gb |
| G\#/Ab | A | A\#/Bb | B | C | C\#/Db | D | D\#/Eb | E | F | F\#/Gb | G |
| A | A\#/Bb | B | C | C\#/Db | D | D\#/Eb | E | F | F\#/Gb | G | G\#/Ab |
| A\#/Bb | B | C | C\#/Db | D | D\#/Eb | E | F | F\#/Gb | G | G\#/Ab | A |
| B | C | C\#/Db | D | D\#/Eb | E | F | F\#/Gb | G | G\#/Ab | A | A\#/Bb |

## SCALES AS INTERVAL STACKS

The representation of scales as interval stacks is an adaption of a notation called "Figured Bass Notation" (Wikipedia) for identifying harmony by annotating bass notes on a staff with number stacks that represent counts of scale steps going up from the bass notes. The adaption replaces counts of scale steps with counts of half tones, and lists the stacks horizontally instead of vertically. For representing scales, the numbers are restricted to $\mathbf{1}$ (half tone $\mathbf{h}$ ), $\mathbf{2}$ (whole tone $\mathbf{2 h}$ ) and $\mathbf{3}$ (minor third $\mathbf{3 h}$ ). Any scale may be spelled out as a stack of these numbers that adds up to twelve. For example, the Ionian mode is identified by $\mid \mathbf{2 2 1 2 | 2 2 1 |}$ and the Aeolian mode by $\mid \mathbf{2 1 2 2 | 1 2 2 |}$, where the vertical lines indicate the scale frame. The Ionian stack also identifies inter-note interval sequence of the singer's solfege scale do-re-mi-fa-so-la-ti-do. The interval stacks are the same for parallel or relative modes, which aids clear thinking about the nature of modes and the relationships between them. For example, the interval stack of the Aeolian mode is visibly a rotation of that of the Ionian mode.

## APPENDIX C: ABOUT CHORDS

Chords identified by chord symbols with four or more notes boil down to combinations of the two kinds of building blocks. Here follow some examples. Two fifos (fifths or fourths) form major-7, minor-7 or major-6 chords. One fifo and one tritone form dominant-7, half-diminished-7 (a.k.a. minor-7-b5) or minor-6 chords. Three fifos or two fifos and a tritone form 9th, 11th and 13th extensions of these chords. When tonic scales depart from key signatures, the same kinds of building blocks are available from the scales but now more than one tritone is available. For example, two tritones form diminished-7 or dominant-7-b 5 chords. Two tritones and one fifo (that may share a note with a tritone) form variations of other chord types such as dominant-7 b 9. Omitting inner notes of combinations may yield different intervals, or thinned voicings of chords. For example, an augmented fifth ( 8 half tones) is combination of two tritones a whole tone apart with their inner notes omitted; and a 3-note voicing of a 4-note seventh chord is composed of the outer notes plus the most important inner note.

## ANCHOR LETTERS FROM TRITONE CHORDS

The following table of tritone chords summarizes the simplest examples of chords with tritones in different positions.

| offset of tritone bass note above root R | suffixes on root symbol R |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 7 | 7\#9(13) | $\begin{aligned} & \hline 9 \quad 13 \\ & \left(\begin{array}{l} \text { or } \# 5 \end{array}\right) \end{aligned}$ | 7 b 9 | $\begin{gathered} 7 b 5 \\ \text { (or \#4 or \#11) } \end{gathered}$ | $\begin{gathered} \hline \operatorname{dim} 7 \\ (\text { or } 07) \end{gathered}$ | m7 b 5 | m6 | M7(11) | b9 | $\begin{aligned} & \mathbf{M} 7 \# 11 \\ & \text { (or b } 5 \end{aligned}$ |
| fourth |  |  |  |  |  |  |  |  | x |  |  |
| major third | $\mathbf{x}$ | X | x | $\mathbf{x}$ | X |  |  |  |  |  |  |
| minor third |  | X |  |  |  | x |  | X |  |  |  |
| whole tone |  |  | X |  |  |  |  |  |  |  |  |
| half tone |  |  |  | x |  |  |  |  |  | X |  |
| 0 |  |  |  |  | x | x | x |  |  |  | x |

Only chord variations that alter tritone content are included. For example, R7, R9, R7\#9, R13 and R 9 (13) are all variations of R 7 with the same tritone content and so are all represented in the table by R7 (variations are left to context). Diminished seventh chords (Rdim7) have no counterpart in classical modes. Sus chords are not shown because they have no tritones. They are typically V-7\#3 chords. A Vsus-V-I progression is a substitute for a II-V-I progression in which one note is altered between the first two steps.

Inverting the tritone of a chord into the home octave (if it isn't already there) determines its PKP anchor. For example, The tritone of a V-7 chord is partially outside the home octave but its inversion is within it. The bass note of its inversion is a fourth above the home tonic which makes its anchor $\mathbf{I}$.

Double tritones are the normal upper limit for tritone content of chords encountered above the staff in fake books, lead sheets and sheet music.

## MISLEADINGLY COMPLEX CHORD SYMBOLS

A startling variety of chords are voiced by simple shapes formed of a tritone combined with different fifos, as illustrated next for the Ionian mode (some are rootless, all are in the mode).


Non-classical modes provide many new chords. Here follow some examples.


The strongly dissonant chords formed of two tritones offset by half tone would tend to be voiced in context as a stacked tritone and fourth with no internal half tones. This shape has a rich sound with a dissonant edge. The weakly dissonant chords formed of two tritones offset by a whole tone can be
played with all notes in any inversion. The chords formed of two tritones offset by a minor third have a circular symmetry (same shape in all inversions) that produces a unique sound I have come think of as "sweet" because too much of it sets my teach on edge, like ingesting too much of a sugary sweet.

## FIGURED BASS NOTATION; EXTENDED CHORDS

Figured bass notation provides a simple representation of chords from scales with 7 notes and no adjacent half tones. It represents chords by stacks of numbers going up from a bass note, in which each number is a count of the scale steps to the next note up. The stacks are annotated on a staff as literal stacks of numbers (vertical lists). I write them here as horizontal lists separated by plus signs. For classical modes, a major or minor third is always two scale steps and whole tone is always one scale step (e.g., a seventh chord is $\mathbf{2 + 2 + 2}$ and a sixth chord is $\mathbf{2 + 2 + 1}$ ).

Figured bass notation can be helpful for understanding extended chords of classical modes that follow from splitting chord steps of size 2 into $\mathbf{1 + 1}$ sequences and then moving the added note up an octave. The $\mathbf{+ 4}$ on top of two of the extended chords on the right is a consequence of avoiding $\mathbf{1 + 1 + 1}$ sequences in the in-place forms. The corresponding chord symbols have degree-number suffixes 6,9 , 11 or 13 added to basic symbols to identify the extensions.

| chord type | $(2)=$ split |  | in place |
| :--- | :--- | :--- | :--- |
| seventh | $2+2+2$ |  | extended |
| ninth | $(2)+2+2$ | - |  |
| eleventh (seventh+11th) | $2+(2)+2$ | $1+1+2+2$ | $2+2+2+2$ |
| thirteenth (ninth+13th) | $(2)+2+(2)$ | $1+1+2+1+1$ | $2+2+2+2+2+4$ |

## APPENDIX D: ABOUT PARALLEL MODES

## MODES FROM MODALOGY

The non-classical parallel modes as presented in the book Modalogy are summarized next, except with notes in numeric-chromatic-scale notation, instead of in the RN (Roman Numeral) notation with sharps and flats used in Modalogy. Corresponding PKP mode signatures are shown on the left.

The purpose is twofold. One is to verify the PKP view of modes relative to Modalogy. The other is to illustrate the complexity of the conventional representations. For example, the tritone anchored by D that is a component of many of the minor modes is understood as p3-6 in the symbolic chromatic scale, and that's it. Inversions in different places in harmony are left to context. In Modalogy, tritone anchor p3 is b III or \# II and tritone anchor $\mathbf{6}$ is VI or bV VII, and that's only the anchors.

The alphabet soup of names and the intricate details of the scales makes them very difficult to distinguish functionally, or to remember as enumerated scales. Seeing the possibility that tritones might be fundamental scale-defining objects for these modes is effectively impossible in these terms. This impossibility is demonstrated by a discussion in Modalogy of defining and non-defining notes of the many and various modes that never mentions tritones.

The simple, unique mode signatures of PKP are novel.


## APPENDIX E: ABOUT SYMMETRY \& SYMMETRY-BREAKING

The elements of PKP are summarized here in a way that provides a link between the concept of symmetry breaking in these pages and in the book The Jazz of Physics. This kind of thing is fascinating to anyone with a mathematical bent but possibly not to anyone else.

The circle of fifths shown below left presents the basic elements of tonic scales and chord progressions from them in one simple picture. The points going counterclockwise around the circle are 12 notes a fifth apart ( 7 half tones) across 7 keyboard octaves ( 12 fifths $\times 7$ half tones $=7$ octaves $\times 12$ half tones). The circle is manifestly a wheel with tritone spokes. The notes of the major scale are the numbered points around the half circle delimited by the highlighted tritone spoke ( 6 half tones).


The circle visibly establishes two kinds of Lego ${ }^{\text {TM }}$-like building blocks as fundamental elements of music, namely fifths or fourths referred to jointly as fifos for simplicity, and tritones. The circle as labeled identifies fifths going up the keyboard through 7 octaves or fourths going down the keyboard through 5 octaves. For the major scale, it identifies a basic chord root line going down by five fifths ( $31 / 2$ octaves) or five fourths ( $21 / 2$ octaves) to the tonic (equivalent to progressively zig-zagging down a fifth and up a fourth within one octave).

The pictorial geometry of the circle of fifths and the musical geometry of its interpretation are different in a way that is counterintuitive (different numbers of octaves going around the circle for fifths and fourths, very different musical sizes of lines of almost the same pictorial length that cut across the circle). This becomes confusing for more general chord progressions of this and other scales.

The circle of half tones rearranges the spokes of the circle to put the labeled points around it in scale order. Around the circle remains an octave and across any spoke remains a tritone. The half tones across the circle now go around it and the fifos around the circle now go across it.

The end points of all the spokes of either circle visibly identify all the points around it, suggesting a notation that labels spokes (building blocks) instead of points (notes). This simple observation is the twist that leads to PKP's unique combination of simplicity and depth. The spoke labels, called anchors, directly identify tritones but also identify fifos. In the chromatic scale that unwinds from the circle, each anchor letter identifies a tritone and two fifos morphed from it. The letters identify relative positions of the anchors in the chromatic scale. Tritone anchors are unambiguous independently of
context and fifo anchors are ambiguous but the ambiguity is generally resolved by context. No anchors are needed in the top pitch half of the octave because inversions cover them.

The letters of the PADMIL alphabet mark the relative positions of the anchors of the single tritones of classical modes, but this is only to provide a correspondence between PKP notation and music notation. It does not bind the letters to the classical modes, or to tritones. The letters are in a special boldfaced font to distinguish them from other uses of some of them in music notation/theory (there is no actual possibility of confusion because the different notations are never mixed together).

## SYMMETRY BREAKING IN THE CIRCLE OF FIFTHS

According to The Jazz of Physics, symmetry-breaking is a deep feature of how both music and the universe work. Here's a view of breaking the symmetry of a diminished scale to yield a major scale that corresponds to an example on page 66 of the book (in the book, the dotted square and its transformation are not shown and no mention is made of tritones).

break symmetry by morphing each square into a 4-sided shape with 2 points the same


The symmetry-breaking is shown next in progressive steps that morph tritones into fifos.



[^0]:    ${ }^{1}$ This chord progression for the haunting E b blues Goodbye Porkpie Hat written by Mingus as a tribute to Lester Young, is a "poster child" for the complexity with which music notation represents things that are simple on the keyboard. It's presented here only as an example of the need for a simpler view. The progression is E b 7\#9—B9(13)—EM9—A7\#11—D b 9sus—B9(13)—D b 7sus-E b 7A bm11—B7(13)—Fm7 b 5—B b 7\#5\#9—C13\#11—F7(13)—B7—EM7—A7(13)—A b 7—B b 7—D b 7—E b 7\#9—B7—EM7\#11—

