AP Statistics Chapter 8   Estimating with Confidence

8.1 Confidence Intervals

INTERPRET a confidence level
INTERPRET a confidence interval in context
DESCRIBE how a confidence interval gives a range of plausible values for the parameter
DESCRIBE the inference conditions necessary to construct confidence intervals
EXPLAIN practical issues that can affect the interpretation of a confidence interval

8.2 Estimating a Population Proportion

CONSTRUCT and INTERPRET a confidence interval for a population proportion
DETERMINE the sample size required to obtain a level C confidence interval for a population proportion with a specified margin of error
DESCRIBE how the margin of error of a confidence interval changes with the sample size and the level of confidence C

8.3 Estimating Population Mean

CONSTRUCT and INTERPRET a confidence interval for a population mean
DETERMINE the sample size required to obtain a level C confidence interval for a population mean with a specified margin of error
DESCRIBE how the margin of error of a confidence interval changes with the sample size and the level of confidence C
DETERMINE sample statistics from a confidence interval

8.1 Confidence Intervals

• Goal:

• What we Know:

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Point Estimator:

Point Estimate:

An ideal point estimator will have ___________________________ and _________________________________**.**

Example: From golf balls to graphing calculators
Problem: In each of the following settings, determine the point estimator you would use and calculate the value of the point estimate. Here are the distances (in yards):

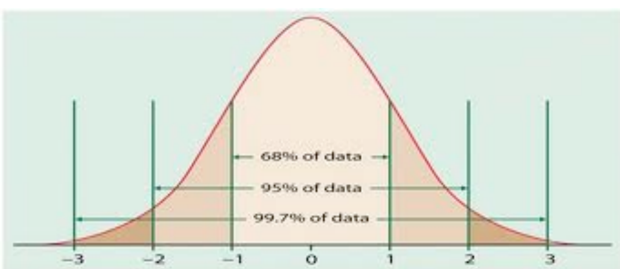
285 286 284 285 282 284 287 290 288 285

(a) The makers of a new golf ball want to estimate the median distance the new balls will travel when hit by mechanical driver. They select a random sample of 10 balls and measure the distance each ball travels after being hit by the mechanical driver.

(b) The golf ball manufacturer would also like to investigate the variability of the distance travelled by the golf balls by estimating the interquartile range.

(c) The math department wants to know what proportion of its students own a graphing calculator, so they take a random sample of 100 students and find that 28 own a graphing calculator.

To Estimate μ:



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Therefore: The interval from ____________________ to ______________________ will "capture" _________

about _______________ of the time.

The Big Idea:

Confidence Interval:

Margin of Error:

Confidence Level (C):

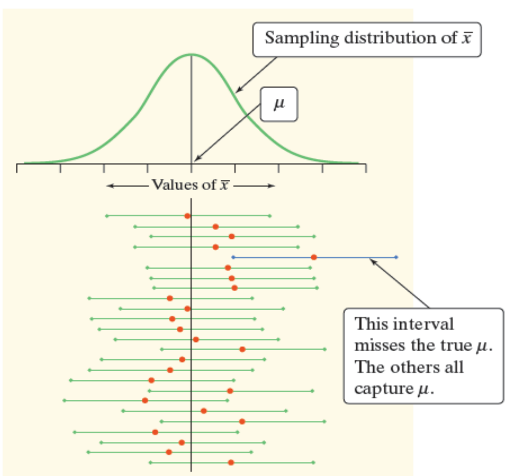
The most common confidence level is: ________________

Interpreting Confidence Levels:

To say we are "95% confident" means:

To interpret a C% confidence level is to say:

We are *C%* confident that the interval from _____ to _____ captures the actual value of the [population parameter in context]."



The confidence level does NOT:

The confidence level DOES give us a

General Form for a Confidence Interval:

Properties of Confidence Intervals:

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The margin of error gets smaller when:

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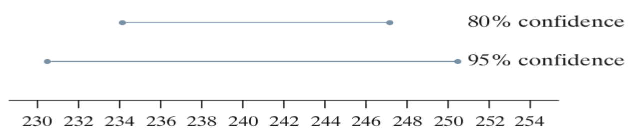

3 Conditions Must be met before Calculating Confidence Intervals:

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-
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8.2 Estimating a Population Proportion

Activity: The Beads
Your teacher has a container full of different colored beads. Your goal is to estimate the actual proportion of red beads in the container.

Sample:

Conditions for Estimating ***p*** (population proportion)

•Random:

•Normal:

•Independent:

Example: Ms. Smith's class wants to construct a confidence interval for the proportion *p* of pennies more than 10 years old in their collection of over 2000 pennies. Their sample had 57 pennies more than 10 years old and 45 pennies that were at most 10 years old.

Problem: Check that the conditions for constructing a confidence interval for *p* are met.

Random:

Normal:

Independent:

Constructing a Confidence Interval for *p*:

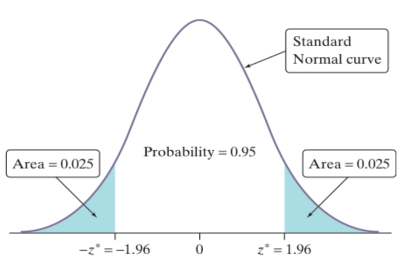
statistic ± (critical value) · (standard deviation of statistic)

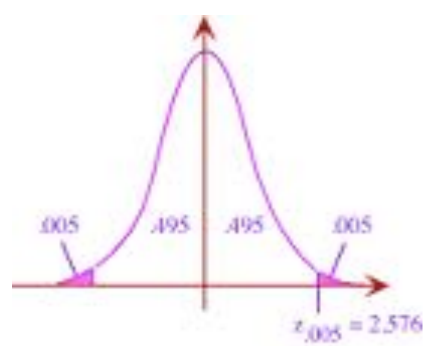
Standard Deviation becomes ______________________________ when we use p-hat in place of the true population proportion.

SE =

Finding the Critical Value: (just a z-score!)

95% confident: **99% confident:**





Example: Finding a Critical Value

Use Table A to find the critical value *z** for an 80% confidence interval. Assume that the Normal condition is met.

Example: Finding a Critical Value

Use Table A to find the critical value z^* for a 96% confidence interval. Assume that the Normal condition is met.

One-Sample *z* Interval for a Population Proportion

Once we find the critical value z^*, our confidence interval for the population proportion p is:

Choose an SRS of size n from a large population that contains an unknown proportion p of successes. An approximate level C **confidence interval for *p* is**

where z^* is the critical value for the standard Normal curve with area C between $-z^*$ and z^*.
Use this interval only when the numbers of

____________________ and __________________________ in the sample are both greater than 10 and the population is at least 10 times as large as the sample.)

One-Sample *z* Interval for a Population Proportion

Calculate and interpret a 90% confidence interval for the proportion of red beads in the container. (Example 1). Your teacher claims 50% of the beads are red. Use your interval to comment on this claim.

Sample proportion of red beads = 107/251 = 0.426
Random/Normal/Independent verified in earlier example

State: What *parameter* do you want to estimate, and at what confidence level?
Plan: *Identify* the appropriate inference *method*. Check *conditions*.
Do: If the conditions are met, perform *calculations*.
Conclude: *Interpret* your interval in the context of the problem.

Example: Kissing the Right Way?

According to an article in the *San Gabriel Valley Tribune* (2-13-03), "Most people are kissing the 'right way'." That is, according to the study, the majority of couples tilt their heads to the right when kissing. In the study, a researcher observed a random sample 124 couples kissing in various public places and found that 83/124 (66.9%) of the couples tilted to the right. Construct and interpret a 95% confidence interval for the proportion of all couples who tilt their heads to the right when kissing.

State:

Plan:

Do:

Conclude:

Choosing the Sample Size for a specific Margin of Error:

To determine sample size for a specific ME, we have to GUESS what $\hat{p}$ will be.

To do this:

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To determine the **sample size *n*** that will yield a level C confidence interval for a population proportion p with a maximum margin of error ME, solve the following inequality for n:

Example: Read the example on page 493. Determine the sample size needed to estimate p within 0.03 with 95% confidence.

Example: Tattoos
Suppose that you wanted to estimate the p = the true proportion of students at your school that have a tattoo with 95% confidence and a margin of error of no more than 0.10.

Problem: Determine how many students should be surveyed to estimate p within 0.10 with 95% confidence.

Determining Sample Size for means ***and*** *proportions.*

8.3 Estimating Population Mean:

Confidence interval for μ ***knowing the population standard deviation****.*
Excellent re-teaching of whole concept of confidence intervals.

Choose an SRS of size n from a population having unknown mean μ and known standard deviation σ. As long as the Normal and Independent conditions are met, a level C confidence interval for μ is

The critical value z^* is found from the standard Normal distribution.

ME =

To determine the sample size n that will yield a level C confidence interval for a population mean with a specified margin of error ME:

-
-
-

Example: How Many Monkeys?

Researchers would like to estimate the mean cholesterol level μ of a particular variety of monkey that is often used in laboratory experiments. They would like their estimate to be within 1 milligram per deciliter (mg/dl) of the true value of μ at a 95% confidence level. A previous study involving this variety of monkey suggests that the standard deviation of cholesterol level is about 5 mg/dl.

Critical Value:

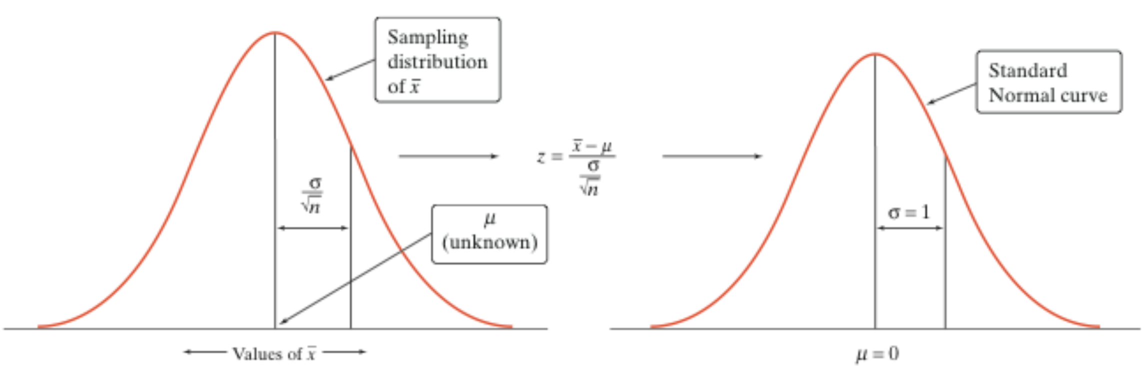
Example: How much homework?

Administrators at your school want to estimate how much time students spend on homework, on average, during a typical week. They want to estimate at the 90% confidence level with a margin of error of at most 30 minutes. A pilot study indicated that the standard deviation of time spent on homework per week is about 154 minutes.

Problem: How many students need to be surveyed to estimate the mean number of minutes spent on homework per week with 90% confidence and a margin of error of at most 30 minutes?

When σ is unknown: The *t* Distributions

When we don't know ___________ we can estimate it using the _______________________



The new statistic is **not** ________________________.

When we standardize based on ____________________________________, our statistic has a new distribution called a ___________________________.

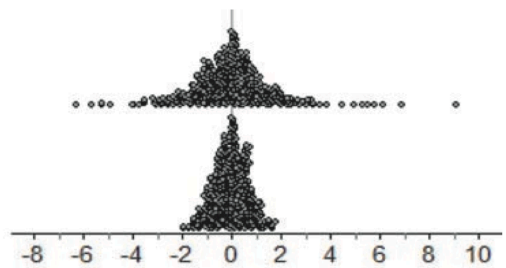
It has a *different shape* than the standard Normal curve:

√

√

$$t = \frac{\bar{x} - \mu}{s_x / \sqrt{n}}$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



Like any standardized statistic:

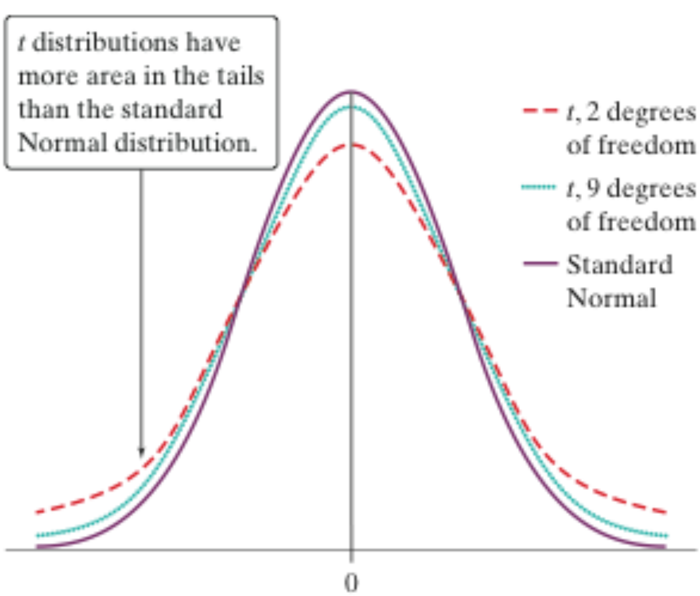
However, there is a **different *t* distribution for each sample size**, specified by its

____

The *t* Distributions; Degrees of Freedom

Draw an SRS of size *n* from a large population that has a Normal distribution with mean μ and standard deviation σ. The statistic has the ***t* distribution** with **degrees of freedom** _______________________. The statistic will have approximately a t_{n-1} distribution as long as the

____.



- The density curves of the *t* distributions are _____________________ in shape to the standard ___________________curve.
- The ______________of the *t* distributions is a bit __________________than that of the standard Normal distribution.
- The *t* distributions have ___________probability in the _____________and ___________in the _______________ than does the standard Normal.
- As the degrees of freedom _________________ the *t* density curve approaches the standard Normal curve ever more closely.

*We can use Table **B** in the back of the book to determine critical values t* for t distributions with different degrees of freedom.*

Example: Suppose you want to construct a 95% confidence interval for the mean μ of a Normal population based on an SRS of size n = 12. What critical t^* should you use?

Upper-tail probability *p*				
df	**.05**	**.025**	**.02**	**.01**
10	1.812	2.228	2.359	2.764
11	1.796	2.201	2.328	2.718
12	1.782	2.179	2.303	2.681
z*	1.645	1.960	2.054	2.326
	90%	95%	96%	98%
	Confidence level *C*			

Example: Suppose you want to construct a 90% confidence interval for the mean μ of a Normal population based on an SRS of size n = 10. What critical t^* should you use? (use Table B in your book)

Constructing a Confidence Interval for μ

Standard Error of the Sample Mean ($\bar{X}$) is ____________________ where ________ is the sample standard deviation.

It describes how far __________ will be from ______________ on average, in repeated SRS's of sample size n.

To construct a confidence interval for μ,

- Replace the standard deviation of $\bar{x}$ by its standard error in the formula for the one-sample z interval for a population mean.
- Use critical values from the t distribution with n - 1 degrees of freedom in place of the z critical values. That is:

Calculating a t-statistic

We have to verify three important conditions before we estimate a population mean:

• **Random**: The data come from a random sample of size *n* from the population of interest or a randomized experiment.
• **Normal**: The population has a Normal distribution or the sample size is large ($n \geq 30$).
• **Independent**: The method for calculating a confidence interval assumes that individual observations are independent. To keep the calculations reasonably accurate when we sample without replacement from a finite population, we should check the *10% condition*: verify that the sample size is no more than 1/10 of the population size.

Example: Can you spare a square?

As part of their final project in AP Statistics, Christina and Rachel randomly selected 18 rolls of a generic brand of toilet paper to measure how well this brand could absorb water. To do this, they poured 1/4 cup of water onto a hard surface and counted how many squares it took to completely absorb the water. Here are the results from their 18 rolls:

29	20	25	29	21	24	27	25	24
29	24	27	28	21	25	26	22	23

State: We want to estimate μ = the mean number of squares of generic toilet paper needed to absorb 1/4 cup of water with 99% confidence.

Plan:

Random:
Normal:
Independent:

Do:

Conclude:

Example: How much homework?

The principal at a large high school claims that students spend at least 10 hours per week doing homework on average. To investigate this claim, an AP Statistics class selected a random sample of 250 students from their school and asked them how many long they spent doing homework during the last week. The sample mean was 10.2 hours and the sample standard deviation was 4.2 hours.

Problem:
(a) Construct and interpret a 95% confidence interval for the mean time spent doing homework in the last week for students at this school.

(b) Based on your interval in part (a), what can you conclude about the principal's claim?

State:

Plan:

Random:

Normal:

Independent:

Do:

Conclude:

Using *t* procedures wisely:

An inference procedure is called ________________________if the probability calculations involved in the procedure remain fairly accurate when a condition for using the procedures is violated.

the t procedures are quite robust against __________________________________ *of the population except when* _________________________ *or* _________________________ *are present.* ___________________________ *improve the accuracy of critical values from the t distributions when the population Is not Normal.*

Practical guidelines for the Normal condition when performing inference about a population mean:

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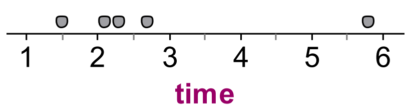
Example: Coffee and SAT scores

Problem: Determine whether we can safely use a one-sample *t* interval to estimate the population mean in each of the following settings.

(a) The dotplot below shows the amount of time it took to order and receive a regular coffee in 5 visits to a local coffee shop.



(b) The boxplot below shows the SAT math score for a random sample of 20 students at your high school.