Section 14: Trigonometry – Part 1

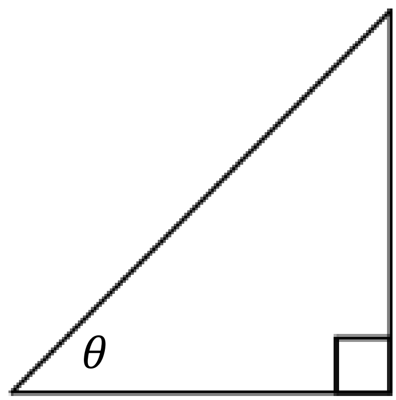
The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.F-TF.1.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle; convert between degrees and radians.
MAFS.912.F-TF.1.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MAFS.912.F-TF.1.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

Videos in this Section

Video 1: The Unit Circle – Part 1
Video 2: The Unit Circle – Part 2
Video 3: Radian Measure – Part 1
Video 4: Radian Measure – Part 2
Video 5: More Conversions with Radians
Video 6: Arc Measure

Section 14 – Video 1
The Unit Circle – Part 1

Let's review trigonometric ratios.

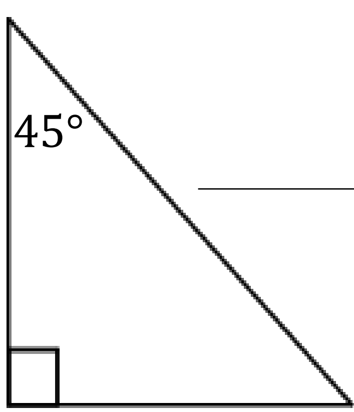


Label the hypotenuse, opposite and adjacent legs of θ.

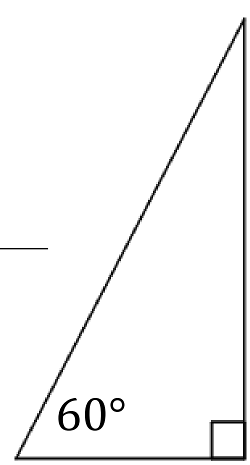
$\sin\theta =$ ________ $\cos\theta =$ ________ $\tan\theta =$ ________

Let's review special right triangles.

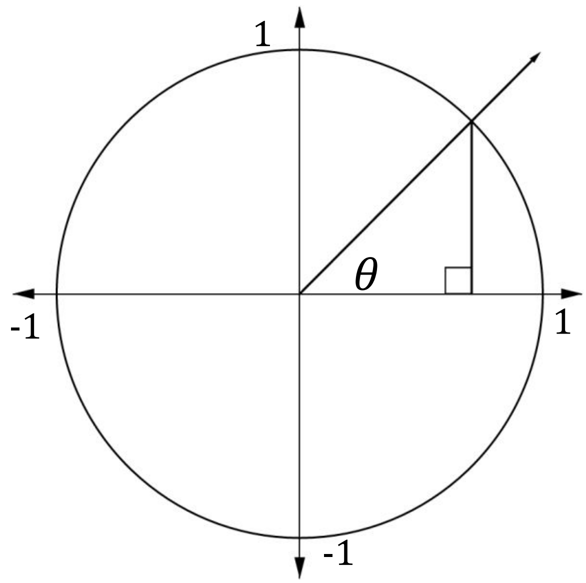
$45° - 45° - 90°$



$30° - 60° - 90°$



Consider the following diagram of a unit circle.



- The unit circle has a radius of _________ that can be used to determine values of _____________________ functions.

- The unit circle is centered at the ________________.

- __________ is the angle formed by the positive ____________________ and a ray that intersects the unit circle (also called the ____________ side).

- The angle measure is ______________if the rotation of the terminal side is counterclockwise and ___________________ if the rotation of the terminal side is clockwise.

- If the base of the right triangle is x, and the height is y, the ordered pair at which the angle intersects the unit circle is _________________.

What is the length of the hypotenuse?

Write the trigonometric functions for θ in the unit circle.

$\sin\theta =$ ____________ $\cos\theta =$ ____________ $\tan\theta =$ ____________

We can use these trigonometric functions to find ____________
__________ on the unit circle.

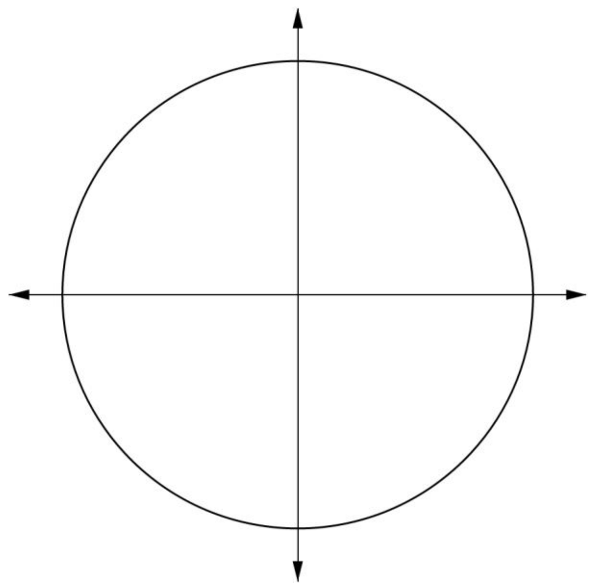
Use the unit circle to complete the following chart.

	Sign of the Function's Value in Quadrant			
Trigonometric Function	I	II	III	IV
$\sin\theta$				
$\cos\theta$				
$\tan\theta$				

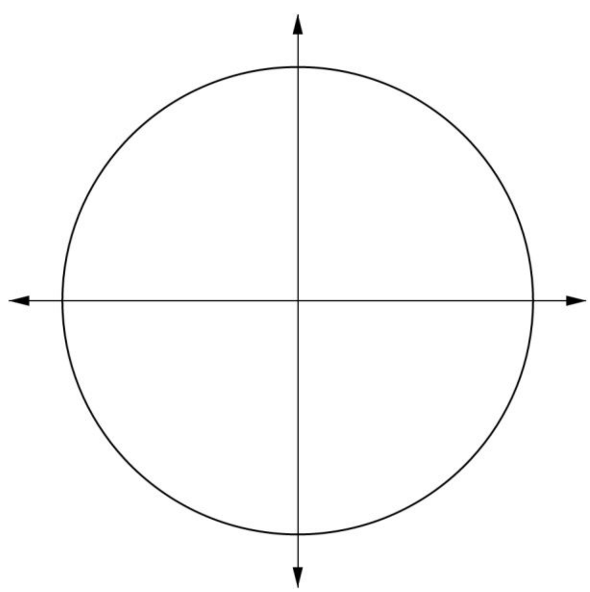
Section 14 – Video 2
The Unit Circle – Part 2

Let's Practice!

Find the coordinates of the point of intersection of the unit circle and a $30°$ angle.

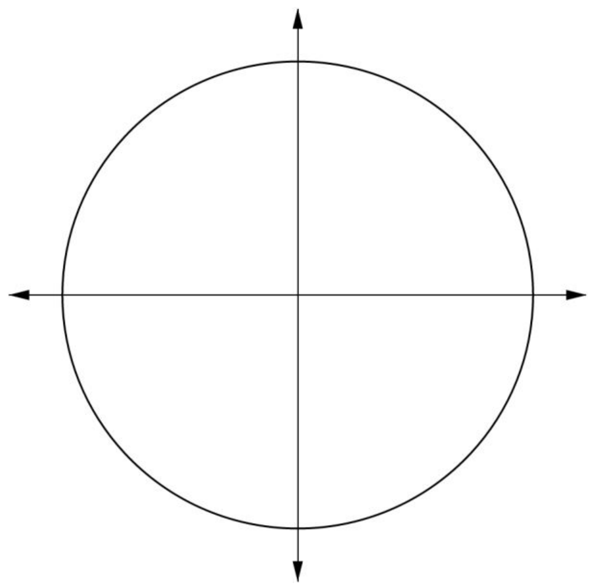


Find the coordinates of the point of intersection of the unit circle and a $-60°$ angle.

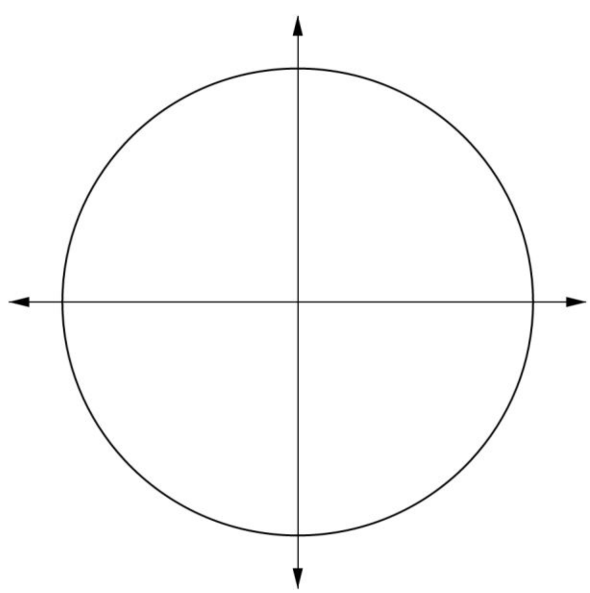


Try It!

Find the coordinates of the point of intersection of the unit circle and a $45°$ angle.

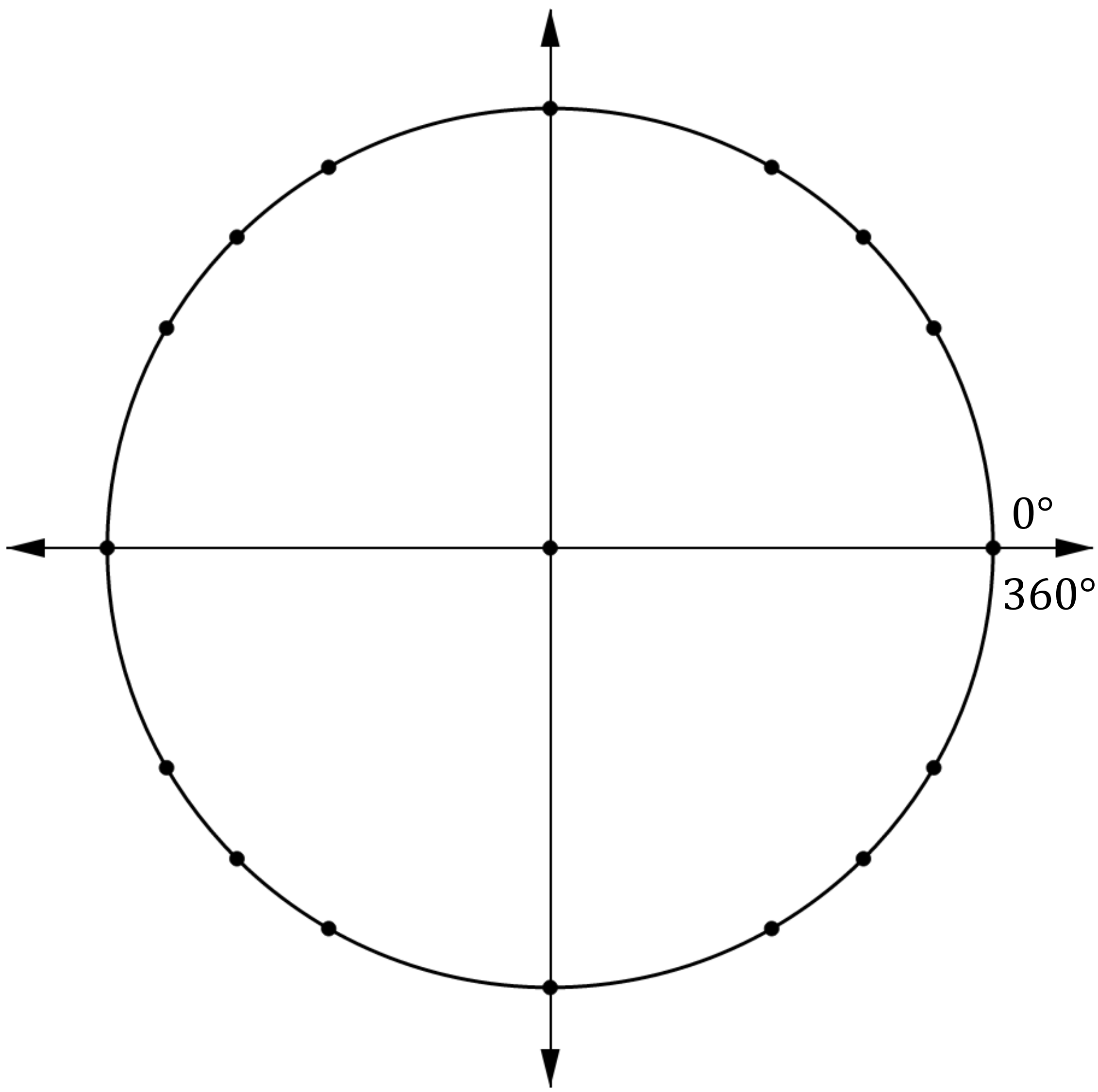


Find the coordinates of the point of intersection of the unit circle and a $-30°$ angle.



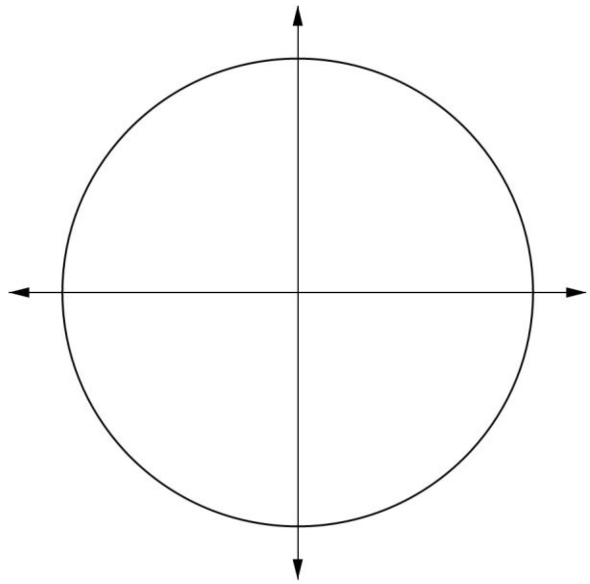
BEAT THE TEST!

1. Draw triangles in the unit circle below to illustrate an angle that has a value of $\cos\theta = -\frac{\sqrt{3}}{2}$.



Section 14 – Video 3
Radian Measure – Part 1

When measuring angles in radians, one rotation around the circle ($360°$) is equivalent to ________ radians.



What is the radian measure at 180°? Label it on the circle.

What is the radian measure at 90°? Label it on the circle.

What is the radian measure at 270°? Label it on the circle.

How can you convert degrees to radians?

How can you convert radians to degrees?

Let's Practice!

Convert 150° into radians.

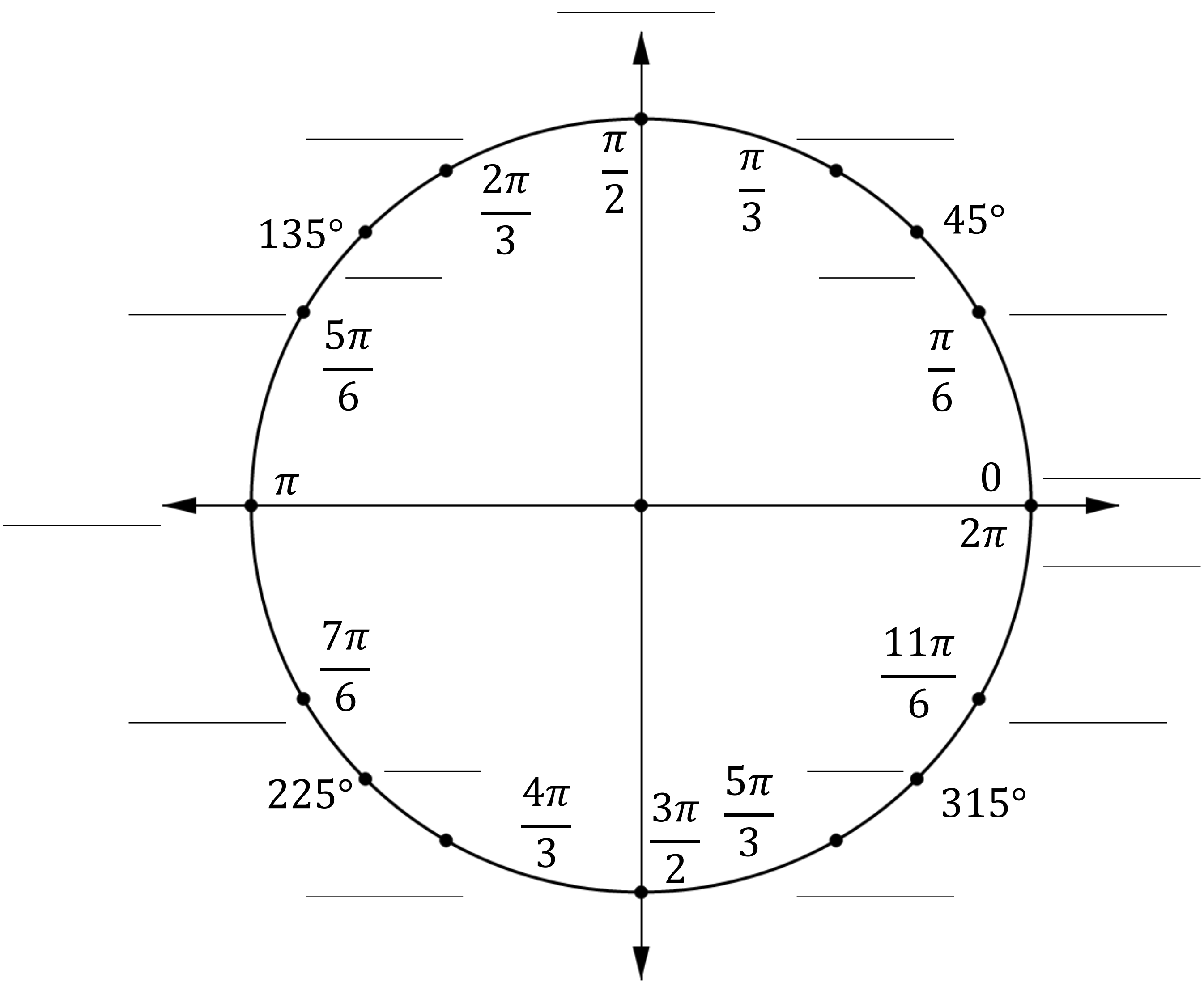
Convert $-\frac{3\pi}{4}$ into degrees.

Try It!

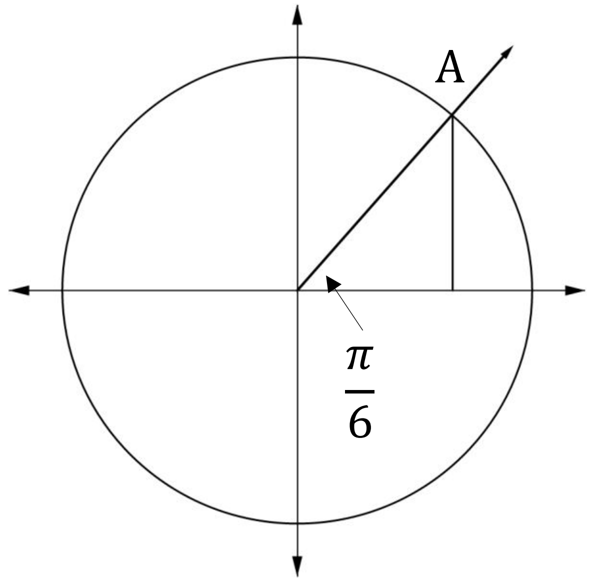
Convert -225° into radians.

Convert $\frac{7\pi}{6}$ into degrees.

Complete the unit circle by providing the missing angle measures (both degrees and radians).



Consider the unit circle diagram below.



Evaluate $\sin\frac{\pi}{6}$.

Evaluate $\cos\frac{\pi}{6}$.

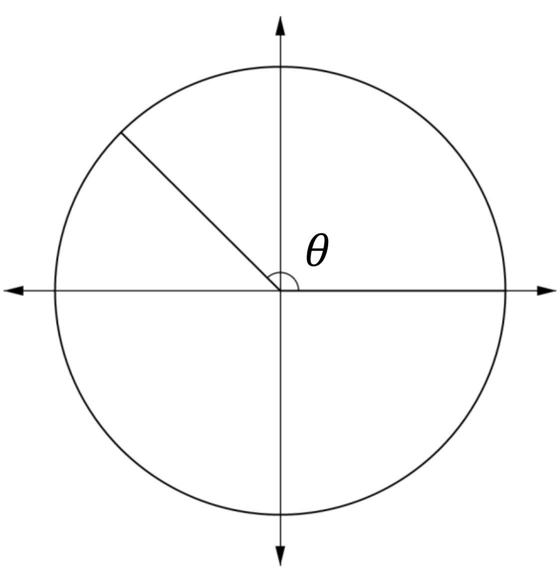
Find the coordinates of A.

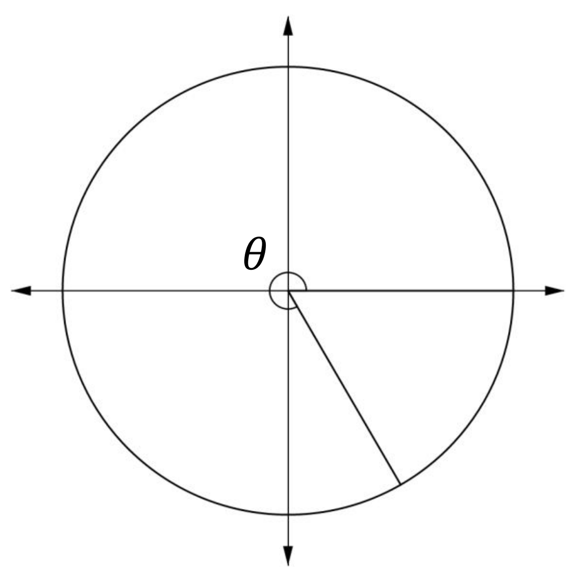
Section 14 – Video 4
Radian Measure – Part 2

- A reference angle is an ____________________ angle formed by the terminal side of a given angle and the ______________________.

- Reference triangles can be used to evaluate the trigonometric values of an angle whose terminal side is not in Quadrant _____.

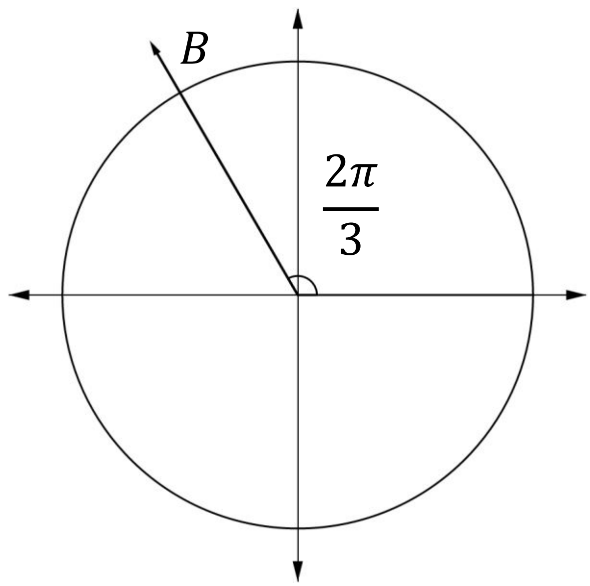
Consider the diagrams below. Draw the reference triangles that we could use to find the trigonometric functions for $\angle\theta$.





Let's Practice!

Consider the unit circle diagram below.



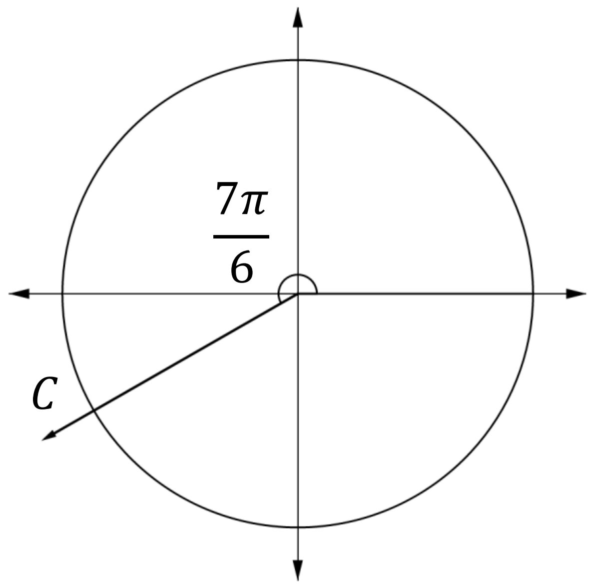
Evaluate $\sin\frac{2\pi}{3}$.

Evaluate $\cos\frac{2\pi}{3}$.

Find the coordinates for point B.

Try It!

Consider the unit circle diagram below.



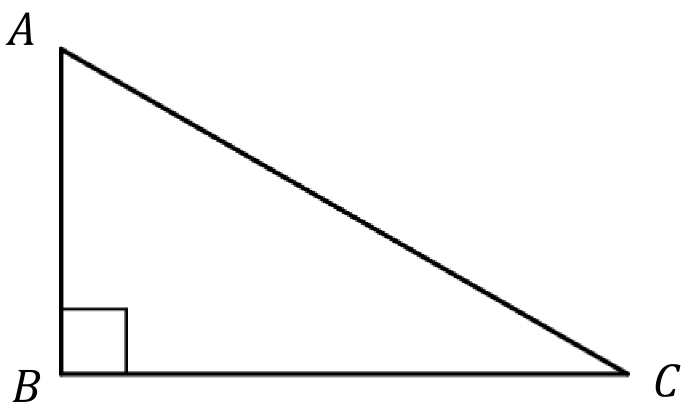
Evaluate $\sin\frac{7\pi}{6}$.

Evaluate $\cos\frac{7\pi}{6}$.

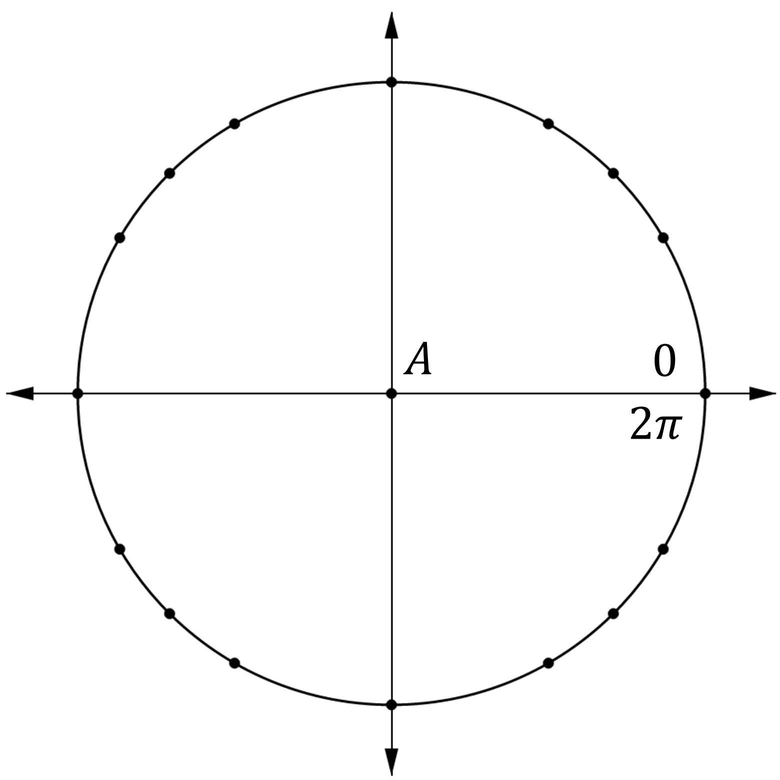
Find the coordinates for Point C.

BEAT THE TEST!

1. In ΔABC, $m\angle BAC = 60°$ and $AC = 1$ unit.



Draw triangles in Circle A to show how ΔABC can be placed in the circle to illustrate $\sin(\theta)$, where $\theta = \pm\frac{\pi}{3} \pm n\pi$ for $n = 0$ and $n = 1$.



Section 14 – Video 5
More Conversions with Radians

What is another way to write $\frac{2}{3}\pi$ radians?

How can we write radians in decimal form?

If we are given radians in decimal form, how can we write them in terms of Pi?

If we are given radians in decimal form, how can we convert to degrees?

Let's Practice!

Convert $\frac{1}{4}$ radians to degrees.

What is the radian equivalent (in decimal form) of $1°$?

Try It!

Convert $32°$ to radians (in decimal form).

What is the degree equivalent of 1 radian?

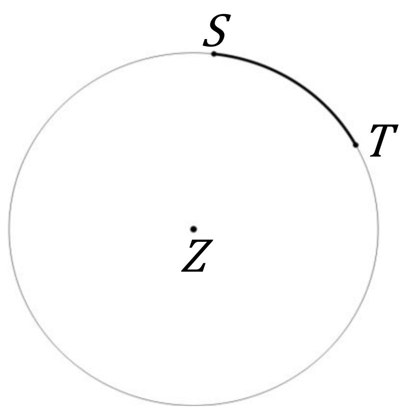
BEAT THE TEST!

1. Which of the following are equivalent or approximately equivalent to $-\frac{1}{2}$? Select all that apply.

   ☐ $\cos -60°$
   ☐ $\sin 30°$
   ☐ $\cos \frac{2\pi}{3}$
   ☐ $\sin \frac{5\pi}{6}$
   ☐ $\cos -1.047$
   ☐ $\sin 3.67$

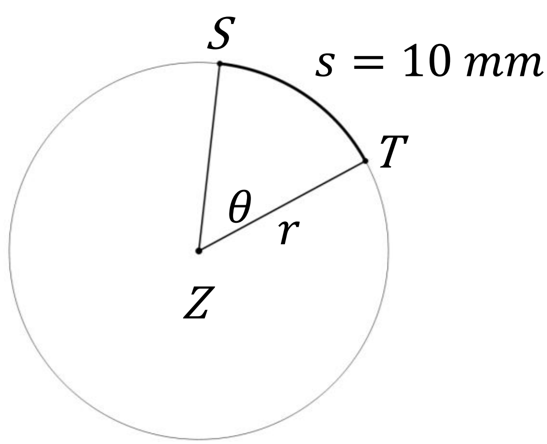
Section 14 – Video 6
Arc Measure

- An __________ of a circle is a portion of the circumference of the circle.



- The length of an arc is the length of its _____________ of the circumference. We usually use s to symbolize arc length.

Consider the diagram below, where $\theta = 45°$ and the circumference of the circle is $80\ mm$.



The word _____________ means to be the opposite of. In the diagram above, we say, "_________ subtends _______________."

- To find the length of an intercepted arc, multiply the circumference of the circle by the _____________ of the central angle to $360°$.

Complete the table below.

Rad., r	Circum., $C = 2\pi r$	Central Angle	Ratio of Central Angle to $360°$	Length of Subtended Arc, s	Ratio of arc length to radius, $\frac{s}{r}$
1		$60° = \frac{\pi}{3}$			
2		$90° = \frac{\pi}{2}$			
3		$120° = \frac{2\pi}{3}$			
4		$240° = \frac{4\pi}{3}$			

What pattern do you notice in the table?

Write a formula that uses the radius and subtended arc length to find a central angle.

Let's Practice!

Find the length of the arc subtended by a central angle of $315°$ in a circle whose radius is 20 in.

Try It!

Find the measure of the central angle of a circle with radius 3 cm that subtends an arc whose length is 2π cm.

BEAT THE TEST!

1. Lisette and Ailani are running a relay race. The circular track on which they are running has a diameter of 60 meters. Ailani is positioned $89°$ from her teammate. How far will Lisette have to run before passing the baton to Ailani?