

MAX PLANCK UCL CENTRE
for Computational Psychiatry and Ageing Research



Models of Behavior and Neuroimaging Data

Educational course on computational neuroscience and the
modeling of neurodynamics

June 14, 2015

OHBM Annual Meeting 2015,

Honolulu, Hawaii

Christoph Mathys

What the brain is about

- What do our imaging methods measure?
 - Brain activity.
- But when does the brain become active?
 - When predictions (or their precision) have to be adjusted.
- So where do the brain's predictions come from?
 - From a model.

What does this mean for neuroimaging?

If brain activity reflects model updating, we need to understand **what model** is updated **in what way** to make sense of brain activity.

The Bayesian brain and predictive coding

Model-based prediction updating is described by Bayes' theorem.

⇒ the Bayesian brain



Hermann von Helmholtz

This can be implemented by **predictive coding**.

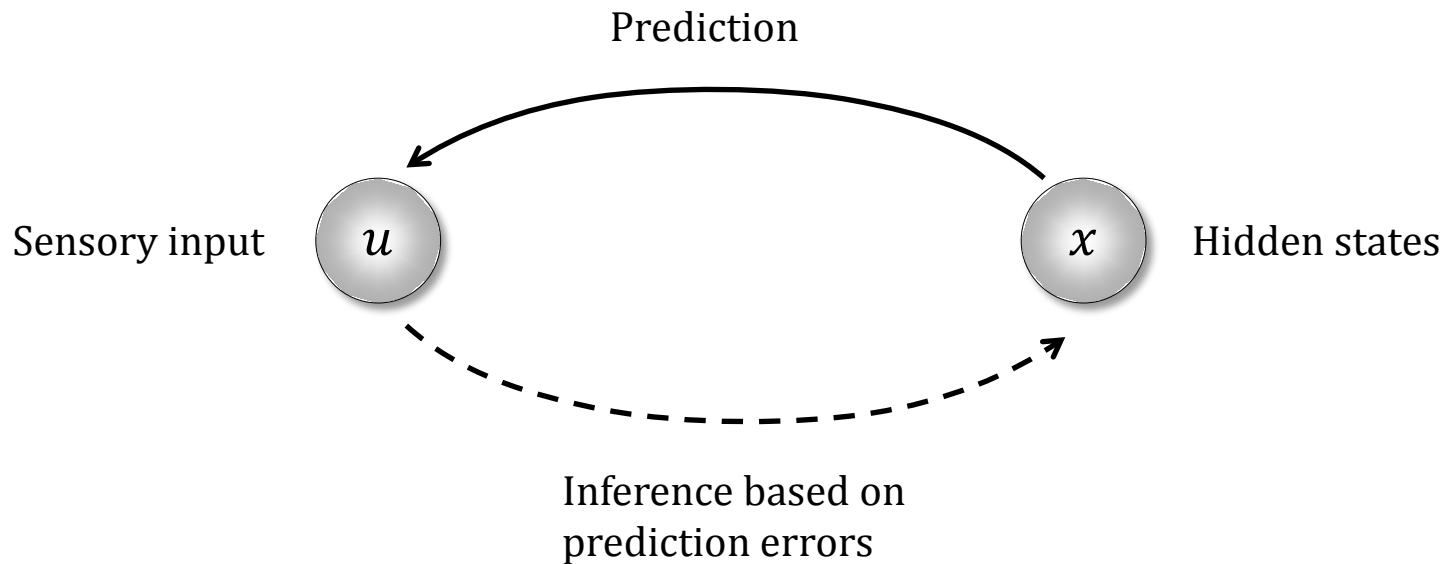
Advantages of model-based imaging

Model-based imaging permits us to

- **infer** the computational (predictive) mechanisms underlying neuronal activity.
- **localize** such mechanisms.
- **compare** different **models**.

How to build a model

Fundamental ingredients:



Example of a simple learning model

Rescorla-Wagner learning:

$$\mu^{(k)} = \mu^{(k-1)} + \alpha (u^{(k)} - \mu^{(k-1)})$$

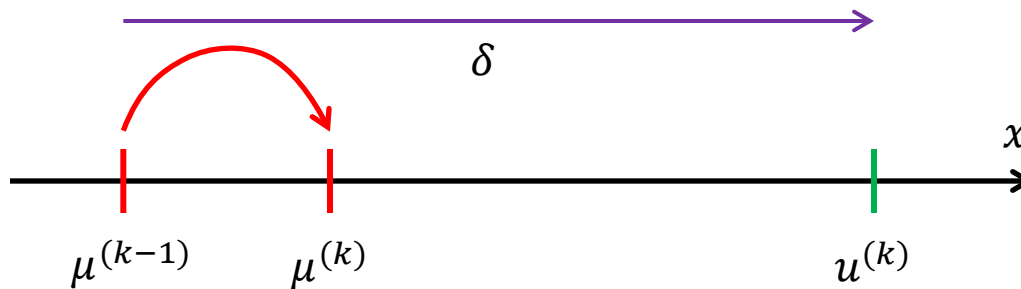
Learning rate

Previous value (prediction)

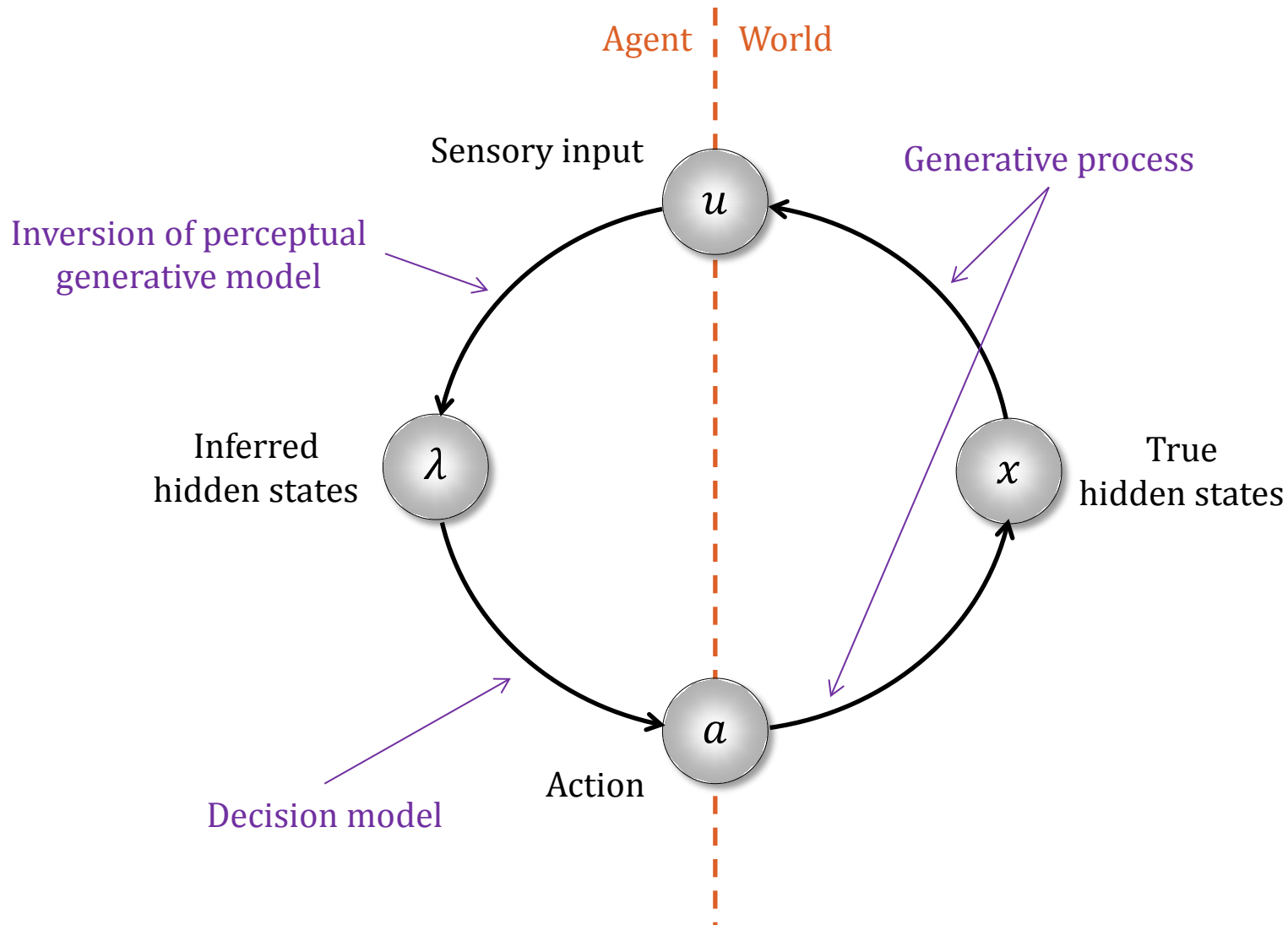
Prediction error (δ)

New input

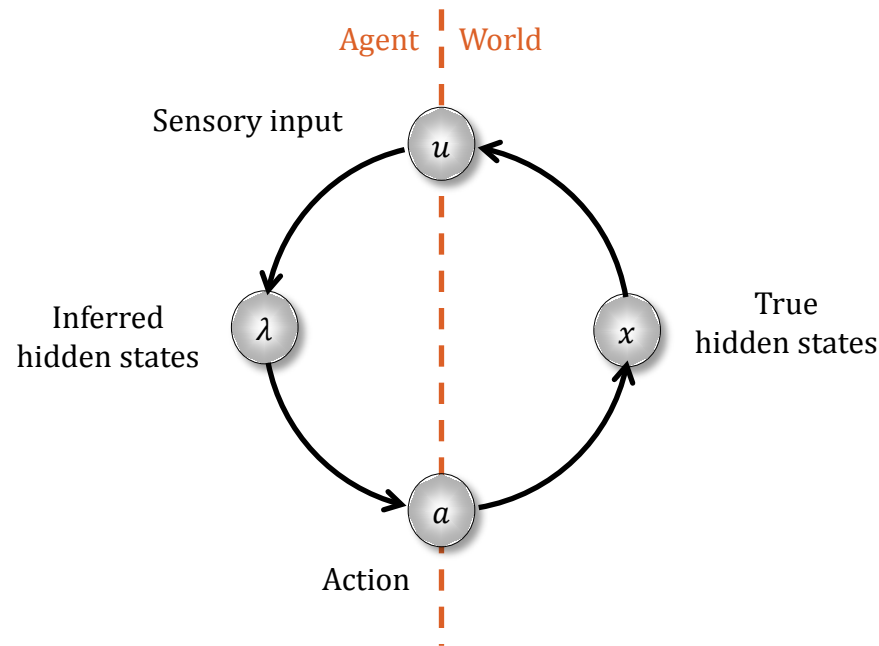
Inferred value of x



From perception to action



From perception to action



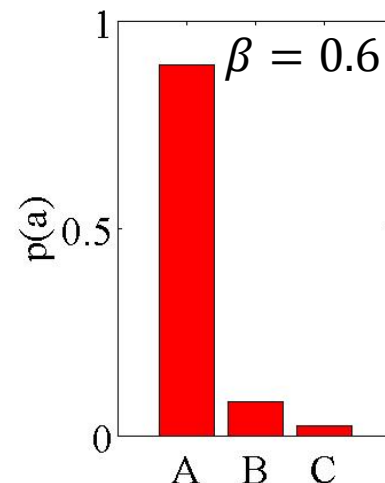
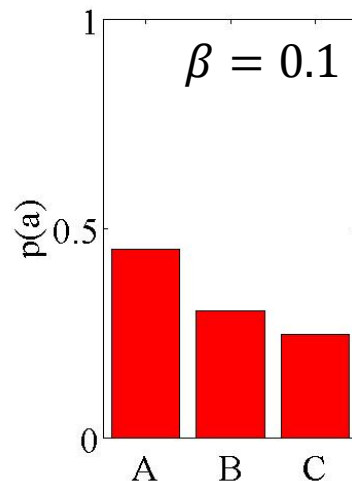
- In behavioral tasks, we observe **actions** (a).
- How do we use them to infer **beliefs** (λ)?
- We invert (i.e., estimate) a **decision model**.

Example of a simple decision model

- Say 3 options A, B, and C have values $v_A = 8$, $v_B = 4$, and $v_C = 2$.
- Then we can translate these values into action probabilities via a «softmax» function:

$$p(a = A) = \frac{e^{\beta v_A}}{e^{\beta v_A} + e^{\beta v_B} + e^{\beta v_C}}$$

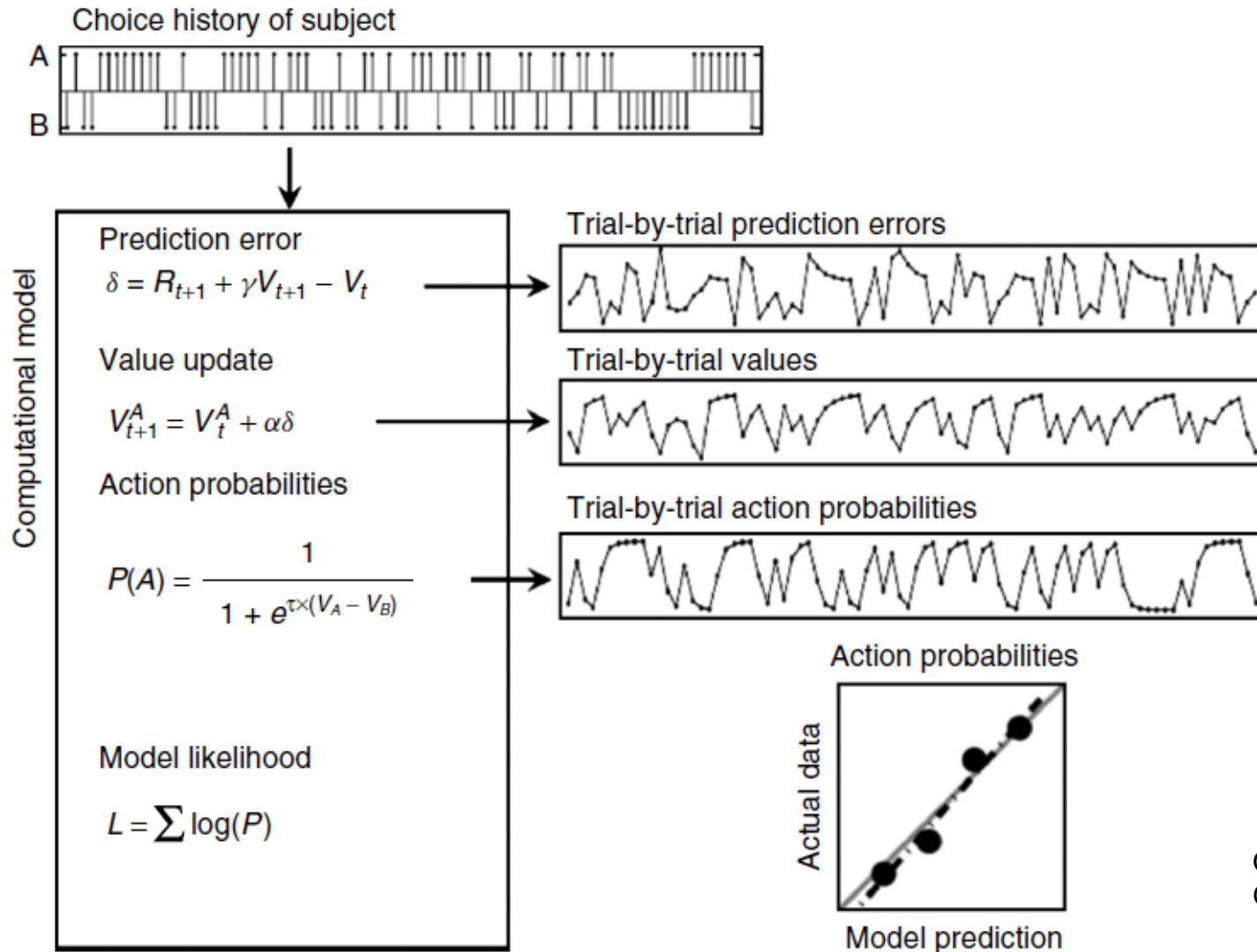
- The parameter β determines the sensitivity to value differences



All the necessary ingredients

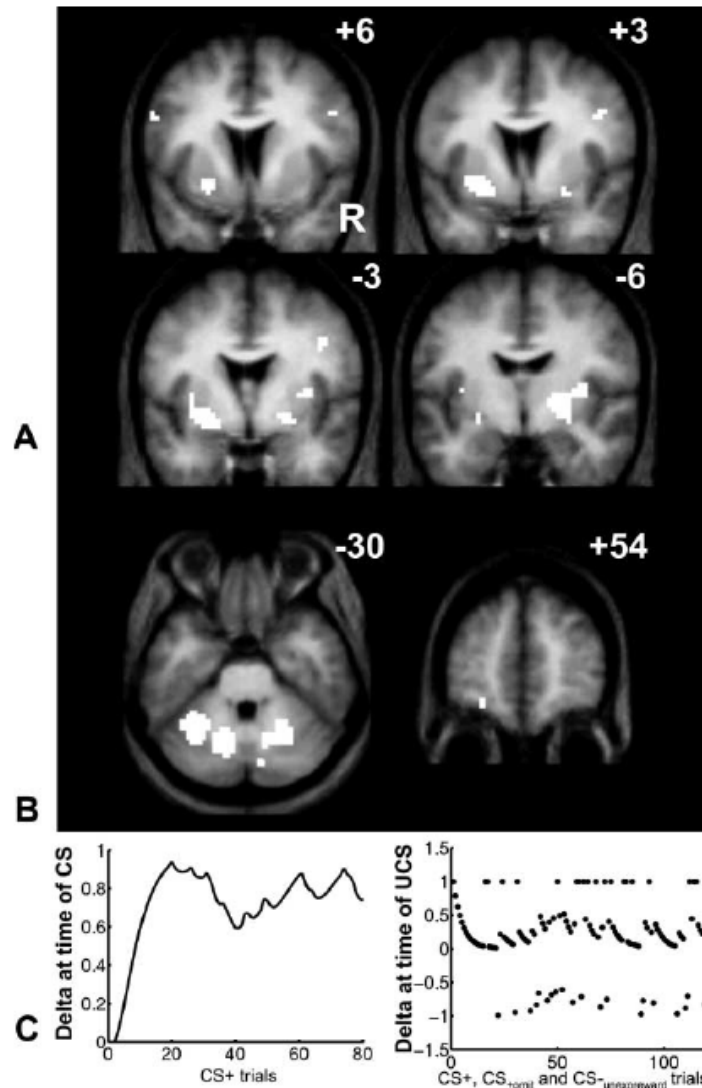
- Perceptual model (updates based on prediction errors)
- Value function (inferred state \rightarrow action value)
- Decision model (value \rightarrow action probability)

Reinforcement learning example (O'Doherty et al., 2003)



O'Doherty et al. (2003),
Gläscher et al. (2010)

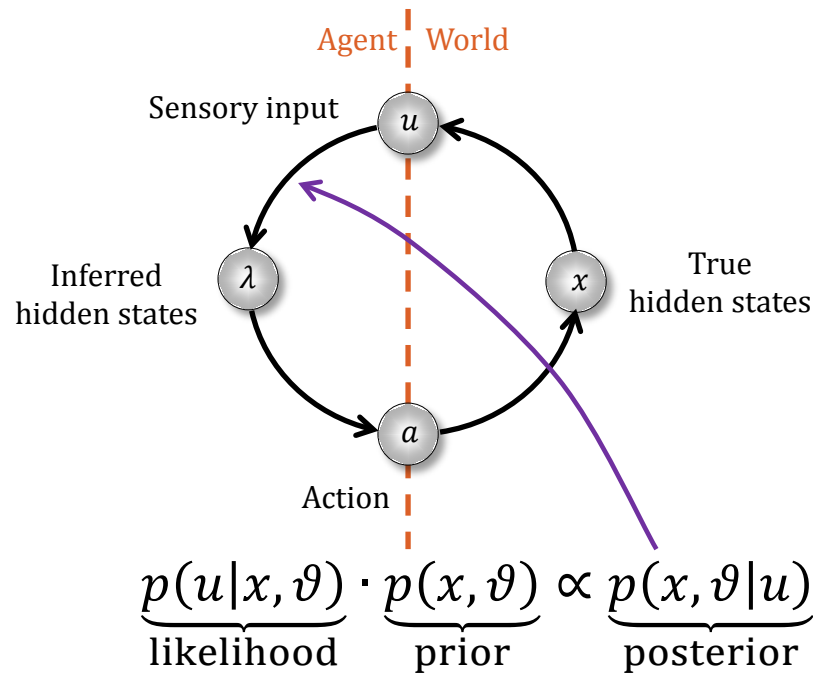
Reinforcement learning example



Significant effects of prediction error with fixed learning rate

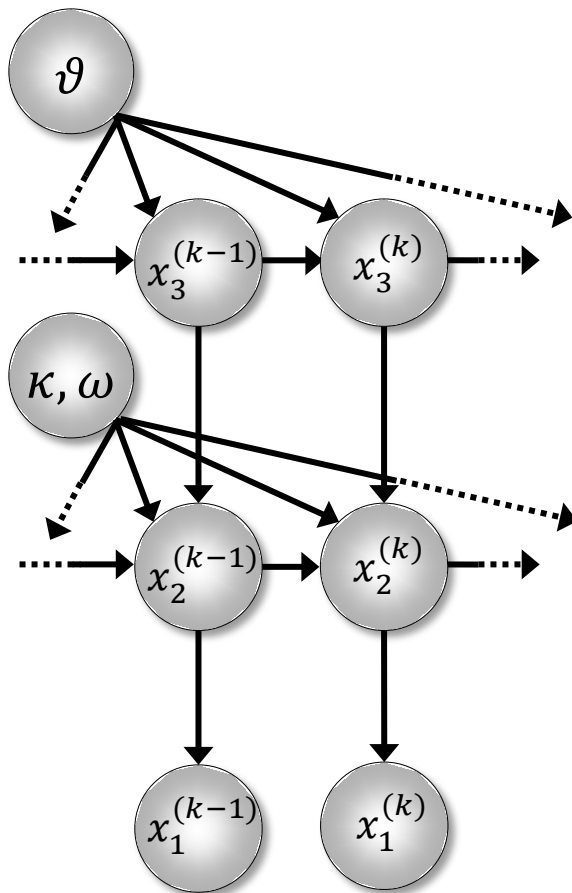
O'Doherty et al. (2003)

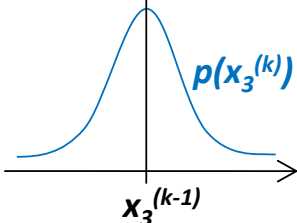
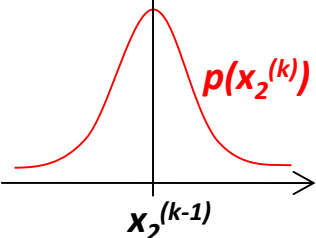
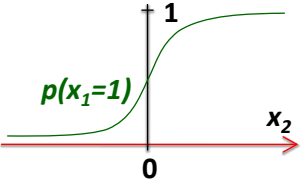
Bayesian models for the Bayesian brain



- Includes **uncertainty** about hidden states.
- I.e., beliefs have **precisions**.
- But how can we make them computationally tractable?

The hierarchical Gaussian filter (HGF): a computationally tractable model for individual learning under uncertainty



State of the world	Model
Log-volatility \mathbf{x}_3 of tendency	$p(x_3^{(k)}) \sim N(x_3^{(k-1)}, \vartheta)$ Gaussian random walk with constant step size ϑ 
Tendency \mathbf{x}_2 towards category "1"	$p(x_2^{(k)}) \sim N(x_2^{(k-1)}, \exp(\kappa x_3 + \omega))$ Gaussian random walk with step size $\exp(\kappa x_3 + \omega)$ 
Stimulus category \mathbf{x}_1 ("0" or "1")	$p(x_1=1) = s(x_2)$ $p(x_1=0) = 1-s(x_2)$ Sigmoid transformation of x_2 

HGF: variational inversion and update equations

- Inversion proceeds by introducing a mean field approximation and fitting quadratic approximations to the resulting variational energies (Mathys et al., 2011).
- This leads to **simple one-step update equations** for the sufficient statistics (mean and precision) of the approximate Gaussian posteriors of the states x_i .
- The updates of the means have the same structure as value updates in Rescorla-Wagner learning:

$$\Delta\mu_i \propto \frac{\hat{\pi}_{i-1}}{\pi_i} \delta_{i-1}$$

Predictions determine learning rate

Prediction error

- Furthermore, the updates are **precision-weighted prediction errors**.

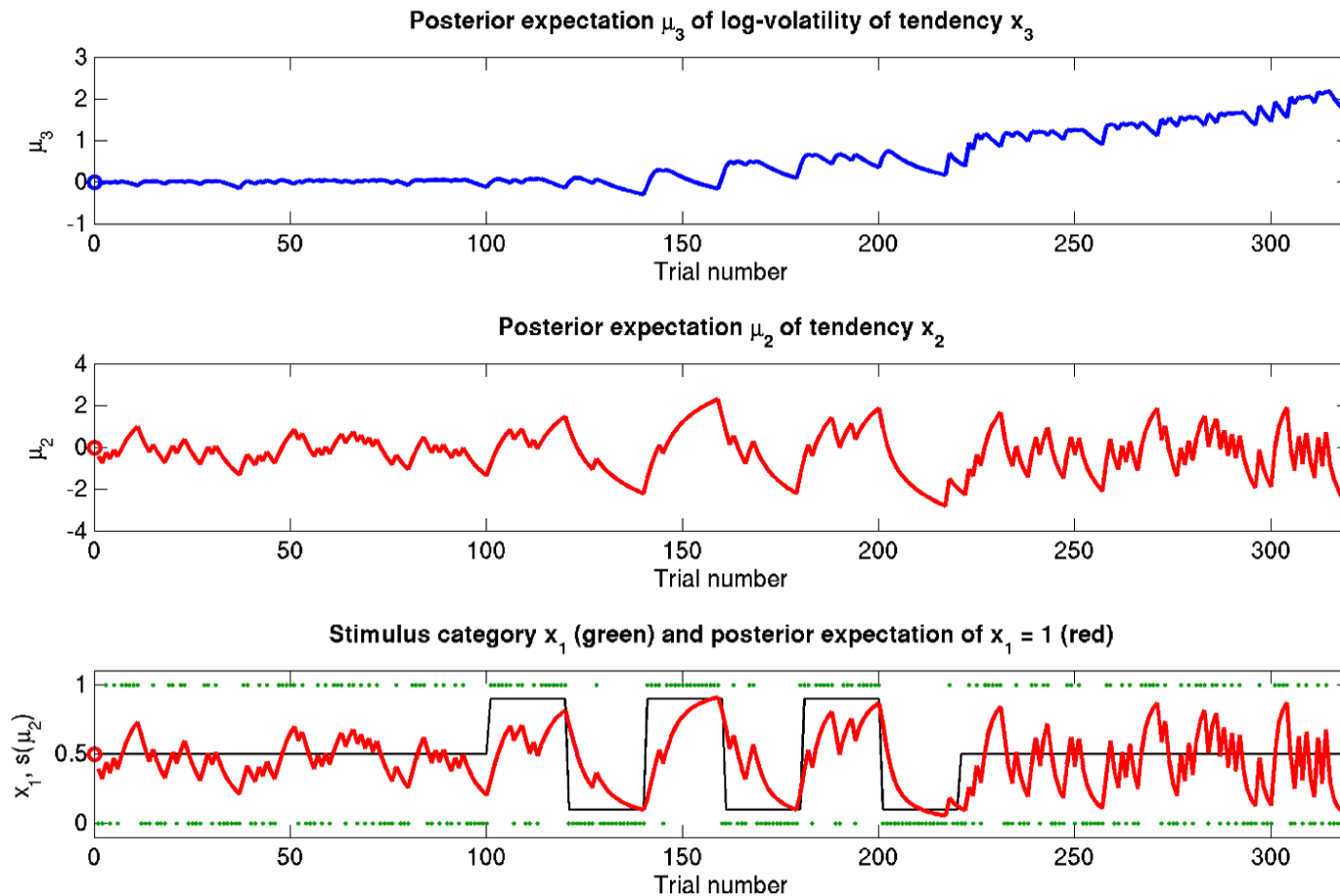
Example: Iglesias et al. (2013)

Model comparison:

BMS results	Behavioral study		fMRI study 1		fMRI study 2	
	PP	XP	PP	XP	PP	XP
HGF1	0.8435	1	0.7422	1	0.7166	1
HGF2	0.0259	0	0.0200	0	-	-
HGF3	0.0361	0	0.1404	0	0.1304	0
Sutton	0.0685	0	0.0710	0	0.0761	0
RW	0.0260	0	0.0264	0	0.0769	0

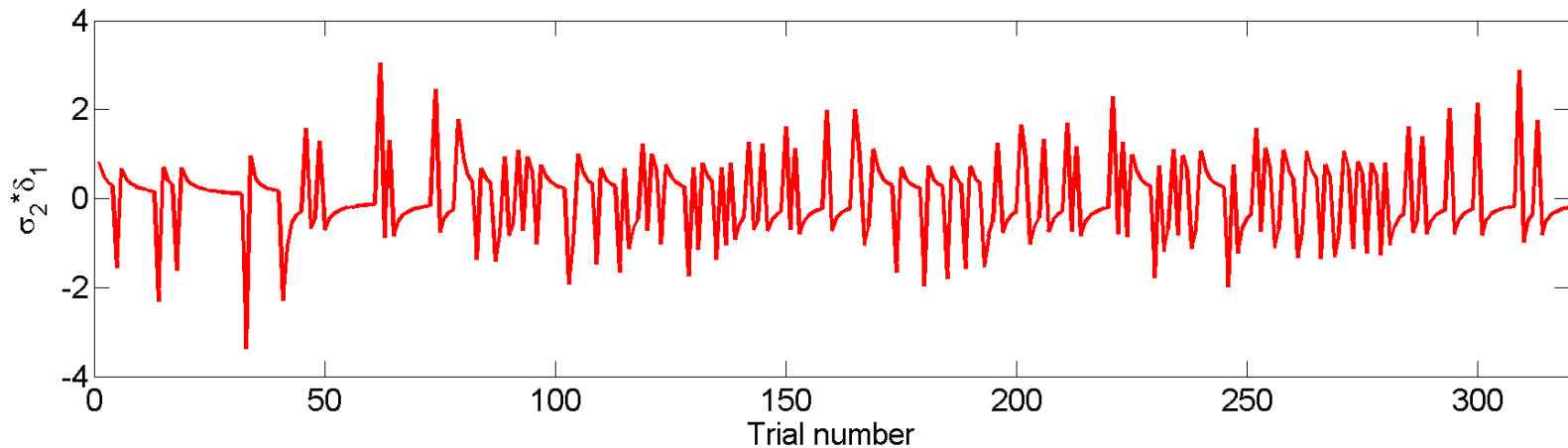
HGF: adaptive learning rate

Simulation: $\mathcal{G} = 0.5$, $\omega = -2.2$, $\kappa = 1.4$

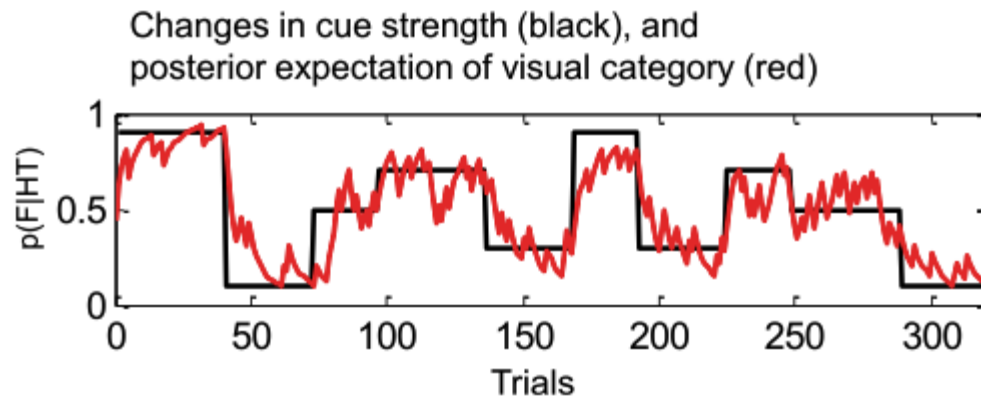
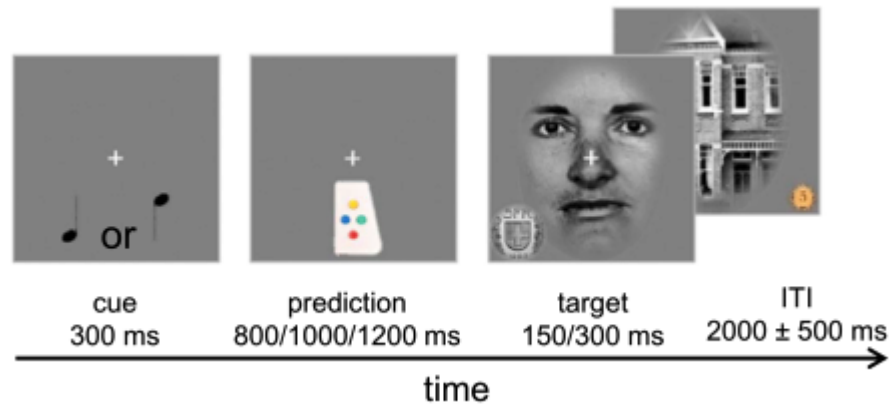


Individual model-based regressors

Uncertainty-weighted prediction error $\sigma_2 \cdot \delta_1$



Example: Iglesias et al. (2013)



Example: Iglesias et al. (2013)

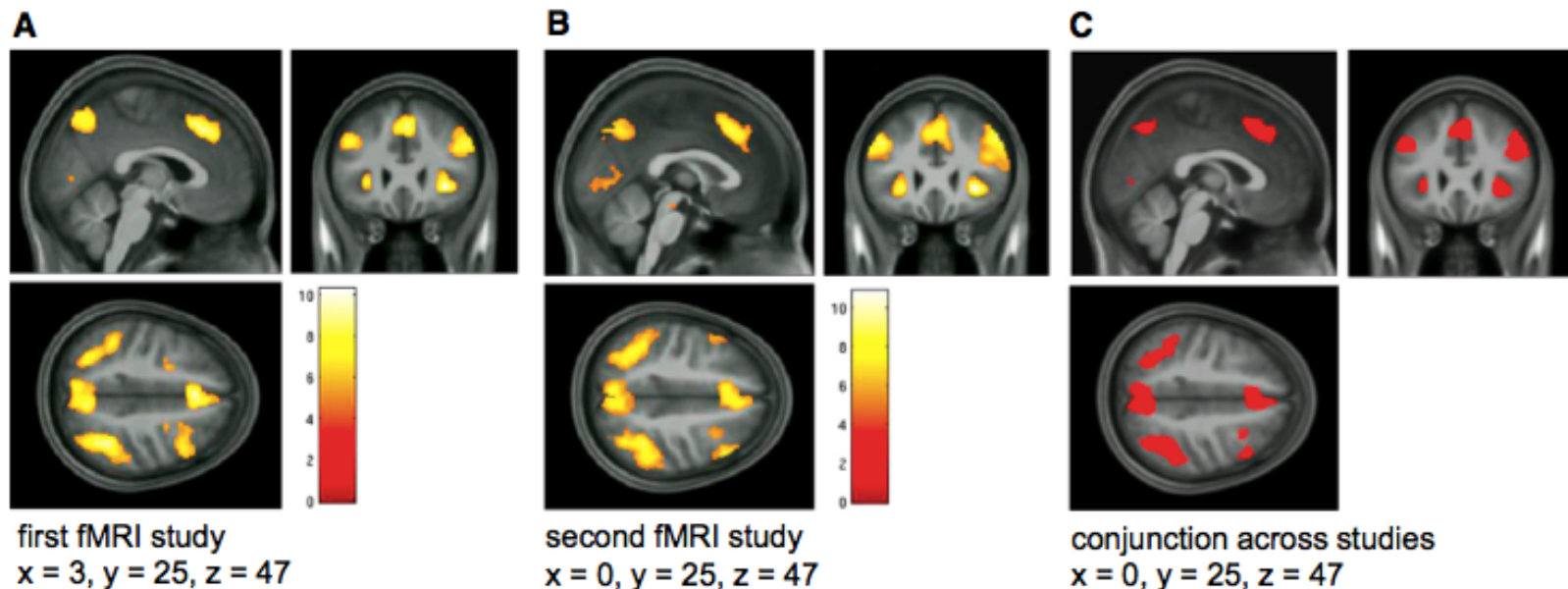


Figure 2. Whole-Brain Activations by ε_2

Activations by precision-weighted prediction error about visual stimulus outcome, ε_2 , in the first fMRI study (A) and the second fMRI study (B). Both activation maps are shown at a threshold of $p < 0.05$, FWE corrected for multiple comparisons across the whole brain. To highlight replication across studies, (C) shows the results of a “logical AND” conjunction, illustrating voxels that were significantly activated in both studies.

Example: Iglesias et al. (2013)

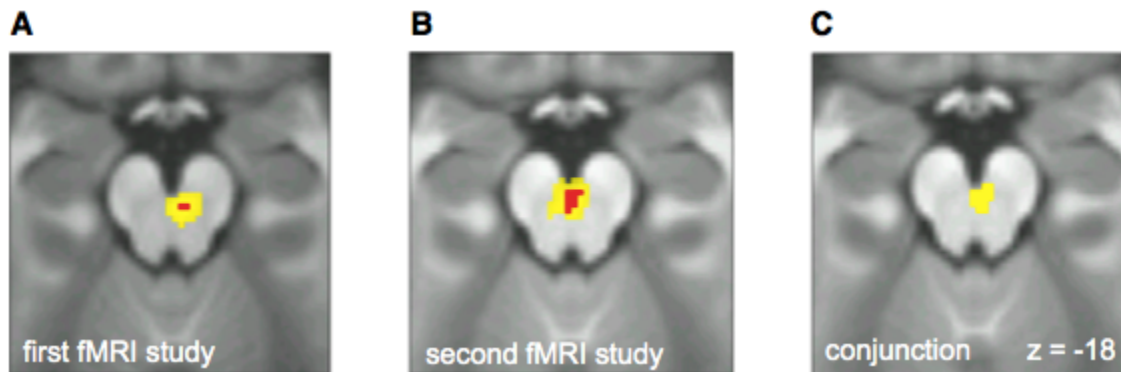


Figure 3. Midbrain Activation by ϵ_2
Activation of the dopaminergic VTA/SN associated with precision-weighted prediction error about stimulus category, ϵ_2 . This activation is shown both at $p < 0.05$ FWE whole-brain corrected (red) and $p < 0.05$ FWE corrected for the volume of our anatomical mask comprising both dopaminergic and cholinergic nuclei (yellow).
(A) Results from the first fMRI study.
(B) Second fMRI study.
(C) Conjunction (logical AND) across both studies.

Example: Iglesias et al. (2013)

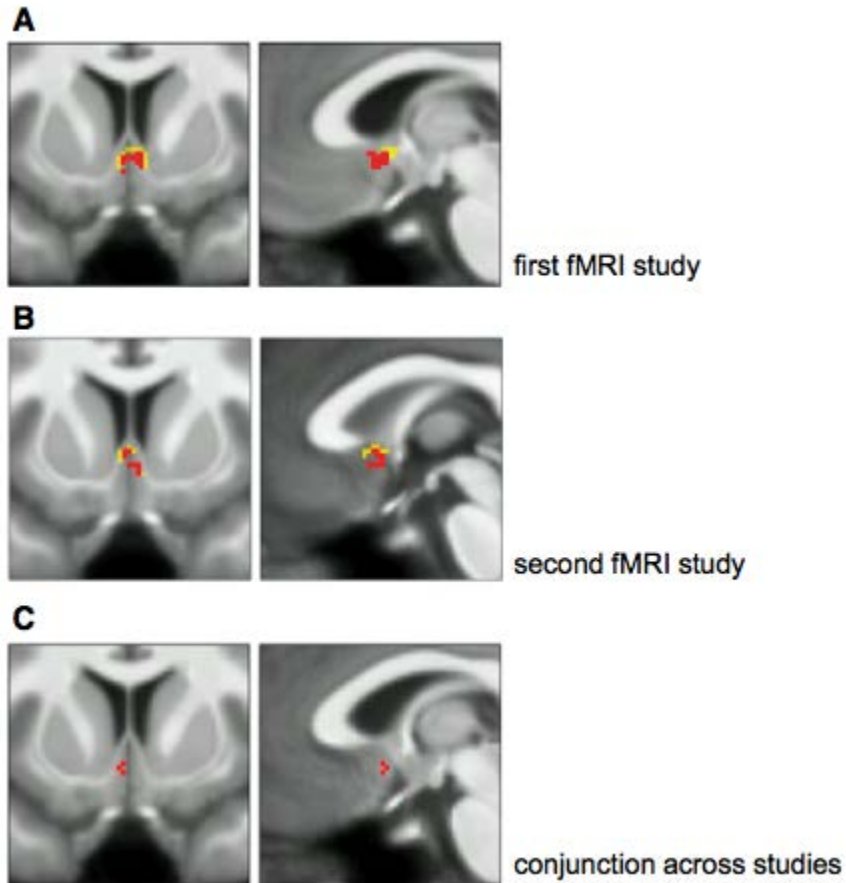


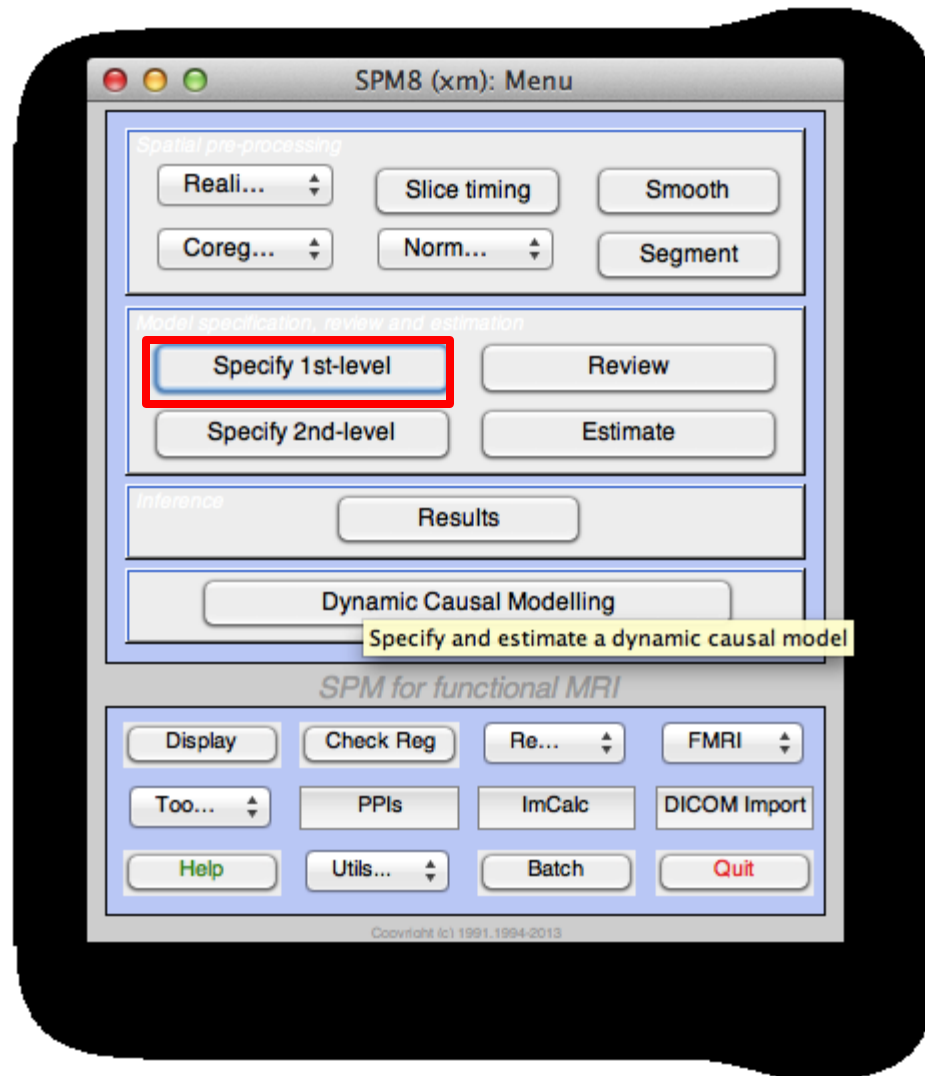
Figure 6. Basal Forebrain Activations by ε_3

Activation of the cholinergic basal forebrain associated with precision-weighted prediction error about stimulus probabilities ε_3 within the anatomically defined mask. For visualization of the activation area we overlay the results thresholded at $p < 0.05$ FWE corrected for the entire anatomical mask (red) on the results thresholded at $p < 0.001$ uncorrected (yellow) in the first (A: $x = 3, y = 9, z = -8$) and the second fMRI study (B: $x = 0, y = 10, z = -8$). (C) The conjunction analysis ("logical AND") across both studies ($x = 2, y = 11, z = -8$).

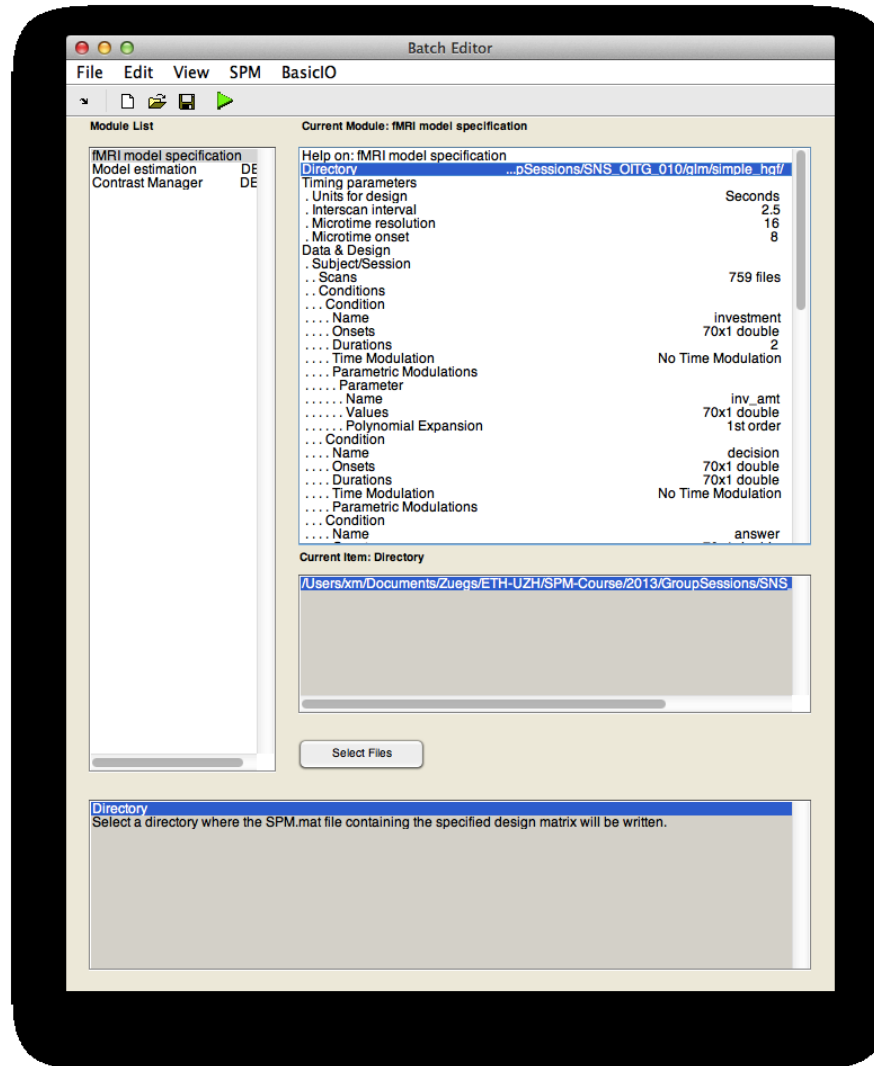
How to estimate and compare models: the HGF Toolbox

- Available at
<http://www.translationalneuromodeling.org/tapas>
- Interactive demo and manual
- Modular, extensible
- Matlab-based

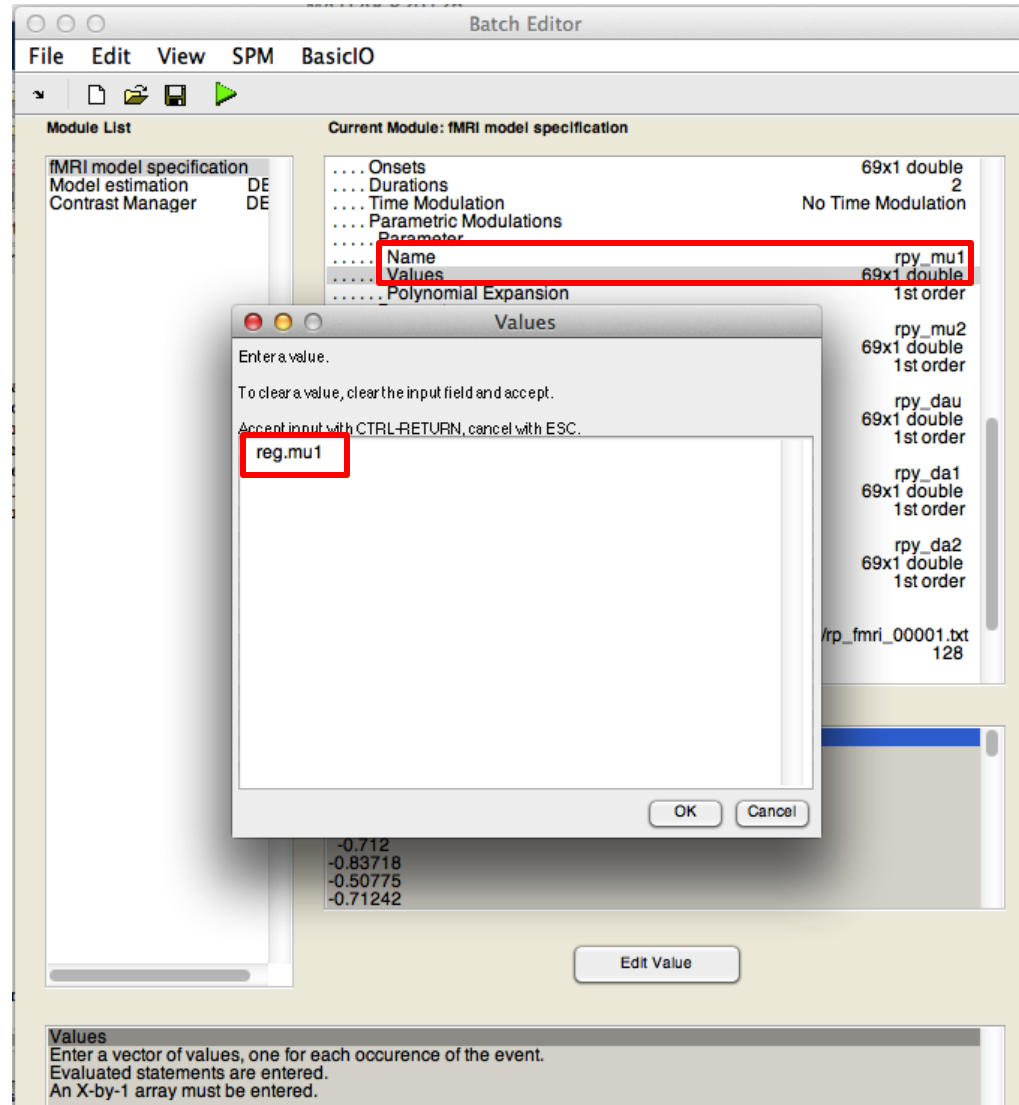
How it's done in SPM



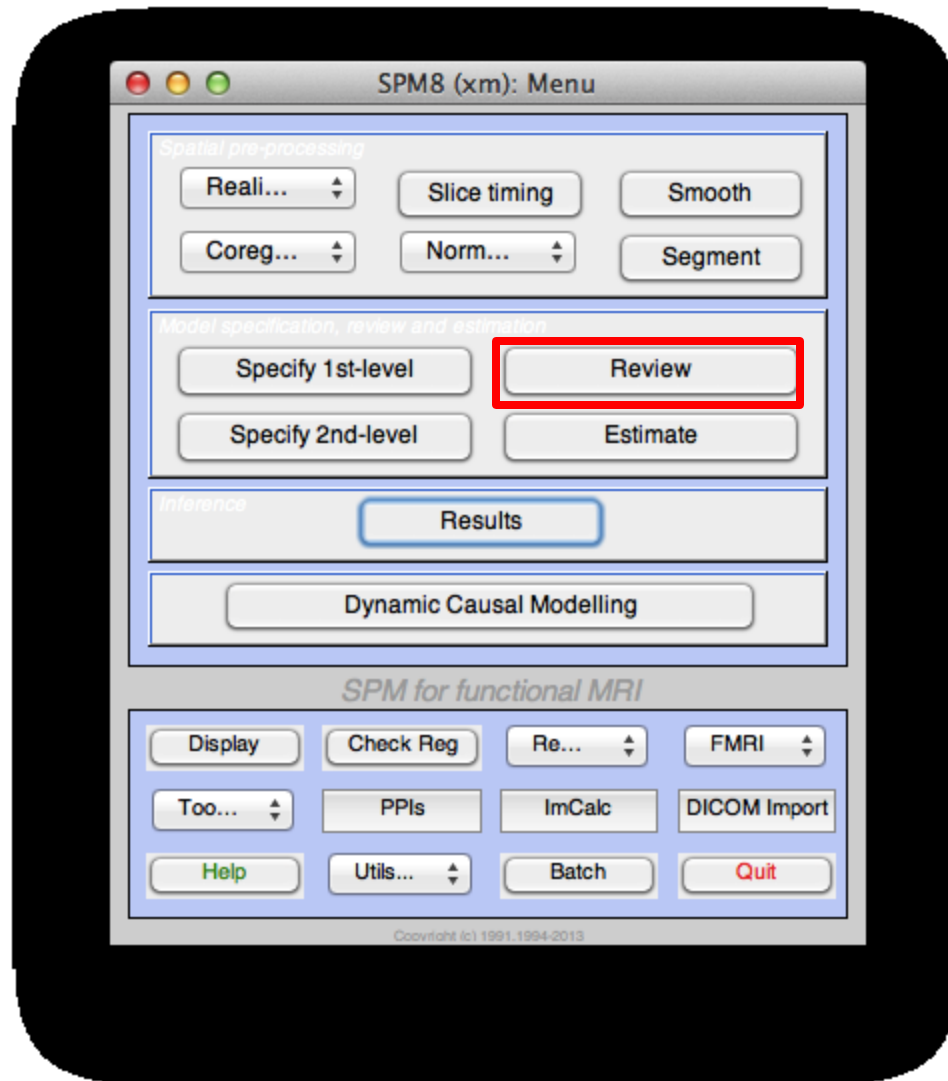
How it's done in SPM



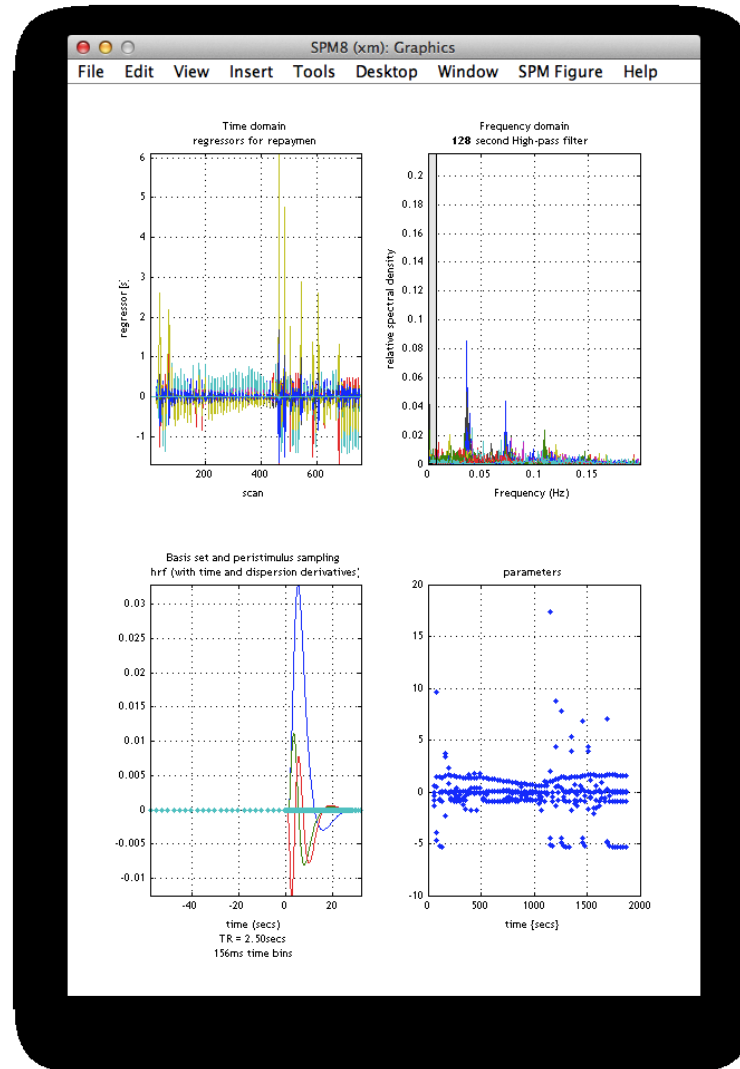
How it's done in SPM



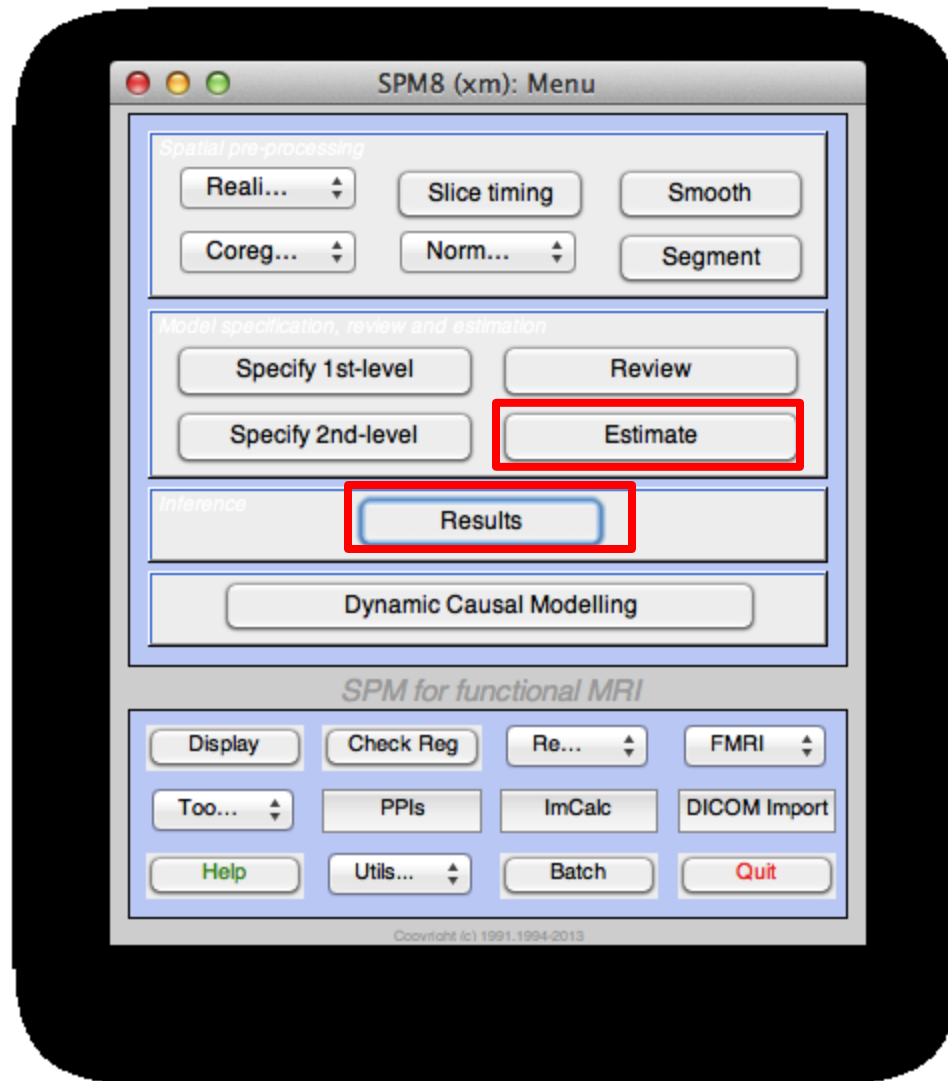
How it's done in SPM



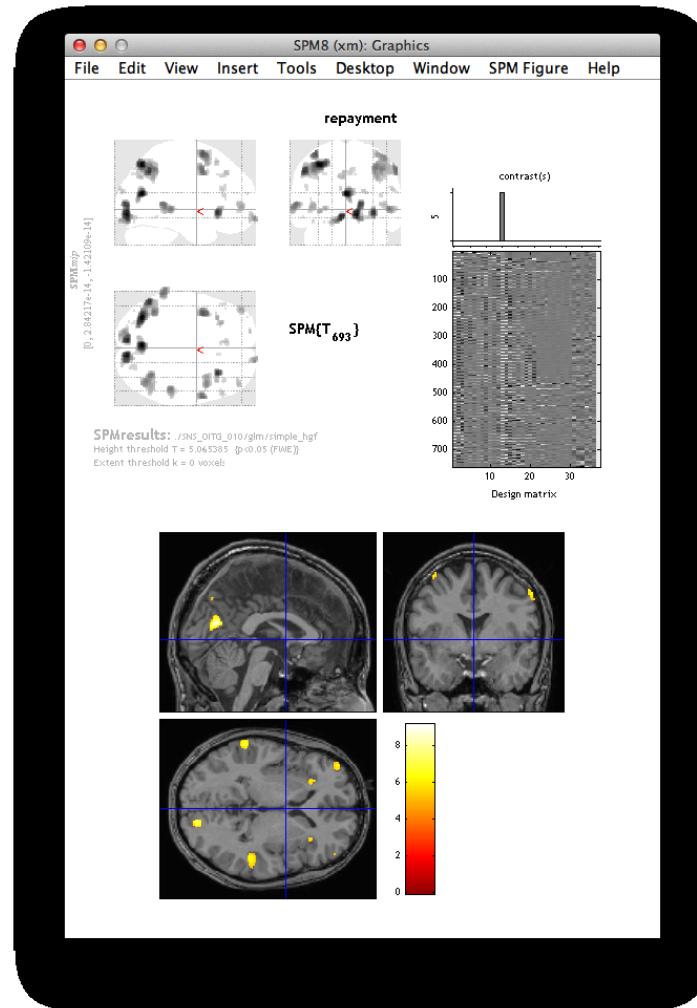
How it's done in SPM



How it's done in SPM

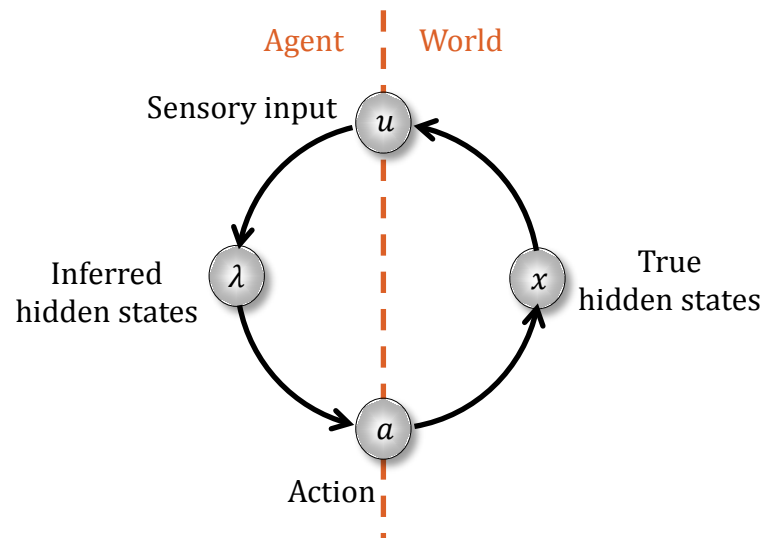


How it's done in SPM



Take home

- The brain is an organ whose job is prediction.
- To make its predictions, it needs a model.
- Model-based imaging infers the model at work in the brain.
- It enables **inference on mechanisms, localization of mechanisms, and model comparison.**



Thank you