THE EMPEROR HAS NO CLOTHES: MUSIC AS IT ACTUALLY IS ON THE PIANO KEYBOARD

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INTRODUCTION

I approached the piano as an adult beginner interested in jazz, which eventually led me to wonder how jazz pianists can improvise in terms of a music notation that seemed to me to be misleadingly complex for the piano.[1] I thought, jazz improvisation must tap into deeper musical structures that are obscured by this notation. Music notation has stood the test of time and is here to stay for piano music, even if for no more reason than the huge legacy of piano music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation. In effect, "the emperor has no clothes" — piano music is much simpler than the "clothes" of this notation make it seem.

Trying to understand the deep structure of piano music without the obscuring clothes of music notation became an absorbing hobby that eventually led me to a simple system I call **PKP** (standing for "Picturing Keyboard Patterns"). The scope of PKP is tonal music in which a melody line is harmonized at selected points by chords formed of clusters of piano keys. Before you stop reading because of my admitted lack of musical credentials, consider the opinions of PKP of a couple of music professionals. Musical theorist Paul Steinbeck: "The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music." Jazz pianist/composer/teacher Taylor Eigsti: "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

Modes are scales defined by interval sequences that may have different realizations on the keyboard: parallel modes of different kinds have the same tonic; relative modes have the same notes and different tonics. PKP differs from music notation in putting parallel modes on notational center stage, determined by words from a 6-letter alphabet, instead of relative modes, determined by the key signatures of music notation. This enables exploiting two simple facts to yield a simple picture: any octave on the keyboard has the same shape (12 half tones), independently of the mix of black and white keys in it; and the half tones of overlapping octaves on the keyboard are aligned. The alphabet is analogous to the alphabet of biological DNA in the sense that the words identify deep structure. The identification is of Lego-like **building block** intervals determined by the letters. **Mode signatures** of 1-4 letters identify parallel modes. Sequences of letters or short words annotated above the staff represent the flow of building blocks in melody and harmony. Mode signatures are identified in this flow by the presence of their letters or words.

The alphabet represents building blocks relative to the tonic of a single conceptual home octave

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[1] What follows can only be dealt with in its own terms by experts. It's a chord progression written by Mingus for the hauntingly beautiful E♭ blues *Goodbye Porkpie Hat*. This piece is a "poster child" for complex music notation. The progression is E♭7♯9—B9(13)—EM9—A7♯11—D♭9sus—B9(13)—D♭7sus—E♭7—A♭m11—B7(13)—Fm7♭5—B♭7♯5♯9—C13♯11—F7(13)—B7—EM7—A7(13)—A♭7—B♭7—D♭7—E♭7♯9—B7—EM7♯11—A7♯11. There are 24 chords, an average of 2 per bar for 12 bars, only 5 of which (highlighted in blue) don't contain tritones. The tritone content is the basis of deep structure of startling simplicity that is the subject of this document.

that's harmonically equivalent to a stack of home octaves on the keyboard. Many pieces of music have an easily recognizable home tonic: it's the piano key on which the melody line comes to rest at the end, or starts, or both. Secondary tonics may be visited along the way but their parallel modes are easily understood by reference to the single home tonic. The home tonic of a piece as a whole is sometimes ambiguous but it's always possible with tonal music to identify an important passage with a definite tonic that can serve as a reference for the piece as a whole.

PKP represents shapes of scales and chords in the same terms, enabling simple annotations on the written music to provide contextual cues that help in learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. This turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

Is this combination of simplicity and depth only a serendipitous side effect of the organization of the piano keyboard? Or is it a fundamental property of music that's obscured by music notation? Either way, it works in practice. To discover it, I had to enter uncharted territory. Nothing I read said anything about it and no expert I approached knew anything about it. The piano is not just for experts, but the conventional approach to teaching and learning it tends to make it so: understanding is expected to emerge by osmosis from the same extensive practicing that develops "chops." The idea that understanding can be had independently of this, in a form that guides practicing, is unconventional.

What qualifies me, an amateur with no formal musical training, to write about these matters? The lack of training is itself a qualification because the absence of received wisdom about music notation enabled me to see that the emperor has no clothes. Add experience as a university professor developing notations to deal with complexity in another field, computer software, that seemed to me to offer insight by analogy (computer software and piano music both use abstract notations to describe things that are performed on "hardware"). Add long-standing curiosity about the complexity of music notation. Add time to indulge my curiosity, after retiring as a university professor not long after I took up the piano. Add the instincts of a professor to write about it. The quoted opinions of experts suggest the result may be worth checking out.

GUIDE TO READERS

This document began as notes to myself to explain the ideas I was developing. I tried to write it so anyone with no knowledge of music theory and very little experience with the piano could read it — in other words with myself when I started out in mind. It's written to be read sitting in front of a piano keyboard on which to try out the unfamiliar concepts and notations.

The potential audience includes novices at this level, pop and jazz musicians who are not pianists but want to explore harmony on the piano, music teachers interested in a different approach to teaching piano music, and "wannabe" experts interested in understanding music at a higher conceptual level than notes. Chapter 2 develops the basic concepts and notation. Chapter 3 provides some basic examples of pieces of music represented in these terms. Chapter 4 explores the building-block world in depth. Chapter 5 provides examples of pieces that initially puzzled me (the first example is the piece with the footnoted chord progression on the previous page). Chapter 6 provides observations and conclusions. References, acknowledgements and comments from some readers follow. Appendices provide helpful supplementary information.

CHAPTER 2: CONCEPTS & NOTATION

A SIMPLE CONCEPTUAL MODEL

A simple but accurate conceptual model of the available home octaves on the emperor-has-no-clothes piano is provided below: each octave is a horizontal line split into twelve equal parts representing twelve musically equal half tones; successive octaves are a half tone apart, thus vertically aligning all the shared notes represented by the dots.

```
overlapping home octaves      @ . . . . . . . . . . . @
                                @ . . . . . . . . . . . @
                                  @ . . . . . . . . . . . @
                                    and so on
```

The pitch sizes of the half tones increase within the octave to make the pitch of the top note double that of the bottom note. The increasing pitch sizes are represented by equal divisions of the line because they're musically equal. The top and bottom notes are home tonics an octave apart symbolized by **@**. The fundamental difference between the piano and music notation for it is the sharing of intervals between overlapping octaves, symbolized by the vertically aligned dots. The reality is the dots cannot be exactly vertically aligned if the half tones within each octave are exactly musically equal. The piano forces alignment by providing the same piano key for all the vertical dots. The musical ear is relatively insensitive to the resulting slight errors in the pitch sizes of half tones, because half tones are dissonant intervals. Equal temperament tuning minimizes the effect on larger intervals formed of sums of half tones. This must be good enough because the piano has stood the test of time without adjustable piano keys to play the slightly different pitches.

A single piece of music has a single home octave identified by the assignment of the symbolic home tonic **@** to a piano key, but all the possible home octaves are conceptually the same. This simple picture is the jumping off point for PKP. *Everything* in PKP is referred to the single home tonic of a piece, so keeping it fixed in the mind is essential. It's useful while learning or revisiting a piece to put a temporary label (e.g., a small stick-on circle) on the piano key that is the home tonic, to avoid interpreting things relative to the different home tonic of some recently played piece that's still in the mind.

Two kinds of splits of the conceptual home octave shown next bring forward fundamental elements of PKP. The letter **L** labeling the geometric center is one of six letters of PKP's mode-identifying alphabet; other letters and explanations of them will emerge as we go along.

```
                  tonic      pitch center    tonic
                    |              |           |
musical split       @ . . . . . . $ . . . . @     scale frame: stacked fifth & fourth
geometric split     @ . . . . . L . . . . . @     stacked tritones
                                |
                         geometric  center
```

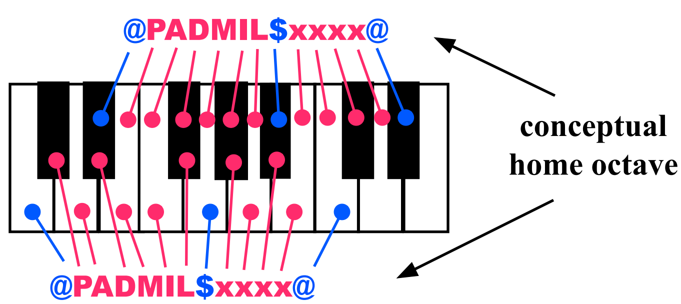
Two kinds of **building-block** intervals are determined by these splits: tritones (one of a kind, size 6 half tones) and fifths and fourths (two of a kind, size 7 and 5 half tones). Tritones are fundamentally dissonant intervals when their notes are sounded together. Fifths and fourths are fundamentally consonant. This small set of building blocks is sufficient for most purposes because smaller and larger intervals emerge as inner or outer intervals of shapes of scales or chords formed from them. The smaller or larger intervals are occasionally needed as independent building-block-like quantities but the building-block notation to come covers this possibility in a simple way that doesn't require explicitly increasing the size of the basic set to include them.

The sounds of tritones, fifths and fourths **sliding** to different keyboard positions and **morphing** into each other as the music moves forward are fundamental to much music. Even with no additional information, this seems plausible because of the visibly fundamental nature of the octave splits that produce them. The notes may be spread out in the music but the essence of the sound is captured by their harmonically equivalent presence within the home octave. The tritone-tritone split morphs into the fifth-fourth split, and vice-versa by altering one note by a half tone. The bottom fifth and the bottom tritone morph into each other in the same way, as do the top fourth and the top tritone.

The **scale frame** includes the pitch center that's shared among most tonic scales (tonic scales without a pitch center exist but they're understood by reference to scales with pitch centers). Knowing the pitch center of a home-octave scale provides little scale-identifying information because almost all primary tonic scales include it. Knowing the geometric center provides a lot of scale information because it's present in only a small number of scales.

THE ALPHABET

The morphing relationship between the two different kinds of building blocks is made explicit by a 6-letter alphabet that identifies building blocks by **anchors**, the positions of their nearest notes above the home tonic. The alphabet is **PADMIL** and its letters identify building blocks going up from the chromatic scale positions identified by them. The distinctive Arial Black font avoids confusion with other uses of the same letters in music notation and theory. Solid letters identify tritones. Fifths or fourths morphed from them are identified by the outlined letters PADMIL. For example, the tritone anchored by **L** may morph into a fourth anchored by $ or a fifth anchored by , leaving the choice to context).



The ambiguity in the identification of fifths and fourths warrants a new term, **fifo**, symbolizing a fifth or fourth with a known anchor but an unspecified size. The names fifth and fourth remain as identifiers of fifos of known sizes. Lumping fifths and fourths together in this way provides deep notational simplification at a higher conceptual level than notes. The terms fifth and fourth refer to the numbers of notes these intervals contain in the classical modes that define the scales of key signatures (5 and 4, respectively). It would be better for our purpose if these terms were "augmented tritone" and “diminished tritone” because this would correspond to their actual keyboard sizes in half tones. As it is, the mismatch between their names and their keyboard sizes must be kept constantly in mind to avoid confusion — a fifth has 7 half tones, not 5; a fourth has 5 half tones, not 4; a major third has 4 half tones.

The surprising result of this way of understanding the conceptual home octave is that any scale with seven or more notes that can be played on the piano is identified by a mode signature provided by a word formed of one or more letters of **PADMIL**. The 6-letter word identifies the chromatic scale. Words formed by pruning letters from this word identify sub-scales of the chromatic scale. The meaning of pruning a letter is morphing a tritone into a scale fifo. Pruning all but one letter leaves the mode signature of a classical mode. A word by itself identifies tritone content of scale. Its mode signature is the word prefixed by **//** standing for “parallel.” Classical modes (also called church modes because of their genesis in early church music) are the scales of key-signature notation.

The word **PADMIL** provides a direct connection to music notation via classical modes. The letters are the first letters of the mode names arranged in the following unconventional order going up the keyboard from a half tone above the tonic to the geometric center of the octave: **P**hrygian, **A**eolian, **D**orian, **M**ixolydian, **I**onian and **L**ydian/**L**ocrian. The corresponding mode signatures are **//P**, **//A**, **//D**, **//M**, **//I** and **//L**. The alphabet order is the order of the anchors of the single tritones of the parallel classical modes going up the keyboard. The letter **L** identifies two modes because they contain the same tritone (in opposite inversions, but the anchor is the same). Knowing one mode means knowing the other (context tells which mode is in play).

The conventional order of the first letters of the mode names is **IDPLMAL**. Anyone with even minimal exposure to the piano knows that playing the seven white keys of the piano starting from different positions yields different sounding relative modes. The conventional order of the modes is of relative modes starting on successive notes of the Ionian mode of tonic C. In this order, **L** appears twice because the two relative modes have different starting notes. The unconventional **PADMIL** order is for parallel modes with the same tonic. In this order, **L** appears only once.

A tritone has only one anchor but may have two possible bass notes in music: one is the anchor and the other is the bass note of its inversion. Inversions are indicated by underlining (e.g., the bass note of the tritone anchored by **L** is the anchor itself, and of tritone **<u>L</u>** is the home tonic **@**). There are no anchors in the top fourth of the scale frame because tritones going up from bass notes in these positions are covered by inversions. This convention makes tritone anchors unambiguous, which, in turn, makes mode signatures formed from them unambiguous.

Considering the alphabet as a circular loop (traversing it in one direction goes off one end to proceed in the same direction from the other end), it may be said that adjacent tritone anchors in the alphabet identify tritones offset by a half tone. This means for tritones **P** and **L** at opposite ends of the alphabet that the bass notes of opposite inversions are a half tone apart (the anchor of tritone **P** is a half tone above the bass note of inverted tritone **<u>L</u>**).

BASIC MODES OF MUSIC

Pentatonic Modes

Pentatonic modes, which contain neither tritones nor half tones, are the foundation scales of music in many cultures worldwide, because anyone with a musical ear can sing tunes from them, harmony is simple, and they generalize simply and directly to more general scales, including classical modes. The book *Modalogy* starts with this premise but stays with the "clothes" of music notation to develop it, thus ending up in a different and much more complex place than PKP.

As illustrated next, parallel pentatonic modes follow from splitting the bottom fifth of the scale frame of the conceptual home octave into unequal parts that determine major or minor tonality, and then splitting the results again to yield 5-note scales with inter-note intervals of whole tones (two half tones) and minor thirds (three half tones, identified by horizontal lines). The splits avoid adjacent minor thirds, which would form tritones. Yellow highlighting identifies symmetric shapes (adjacent whole tones here) that determine the scale in combination with the scale frame. Understanding modes by symmetric shapes is a fundamental feature of PKP.

	@ P A D M I L $ . . . . @
scale frame split	@ . . . x . . $ . . . . @
pentatonic major mode //M	@ . x . x————$. x————@
scale frame split	@ . . x . . . $ . . . . @
pentatonic minor mode //D	@————x . x . $————x . @

Pentatonic major and minor modes formed of all five black keys are easiest to see on the keyboard. The tonic of the all-black-key pentatonic major mode is the black key at the geometric center of the C octave. The tonic of the all-black-key, relative, pentatonic minor mode is the next black key down. A lot of fun can be had, and insight gained, by improvising pentatonic melody and harmony on the black keys. This leads directly to a music-notation-independent understanding of the blues because, as will be shown later, basic 6-note blues modes add one note these modes.

Parallel Classical Modes from Pentatonic Modes

Assuming pentatonic modes are the foundation scales of music, classical modes follow from splitting their minor-third intervals into a half tone and a whole tone. The results for the Ionian and Aeolian modes that are the default major and natural minor modes of key signatures are shown next. The added notes in red text are the single tritones of the classical modes.

	@ **P A D M I L** $ . . . . @
pentatonic major mode //M	@ . x . x————$. x————@
Ionian major mode //I	@ . x . x x . $. x . x @
pentatonic minor mode //D	@————x . x . $————x . @
natural minor mode //A	@ . x x . x . $ x . x . @

The complete set of parallel classical modes constructed this way is summarized next. The six primary modes on the left identified by mode signatures of the form **//X** determine the rest of the table (**X** stands for any alphabet letter). The seventh parallel mode is a "tritone substitute mode" of **//L** (same tritone, all non-tritone notes different); it's a parallel mode because tritone **L** contains the home tonic.

The six **alt-X** modes are all "tritone-substitute modes" of the primary modes (the optional dash is only for readability). Except for **alt-L**, they're not parallel modes of the home tonic because they don't contain it (**alt-X** is actually a relative mode of the transposition of **//X** by a tritone). The **//X** and **alt-X** modes are shown one above the other to bring forward the simplicity of the difference, which is provided by the symmetric shapes highlighted in yellow formed around the tritone. The scales are completed by filling in the scale frame with whole tones.

```
                  @ P A D M I L $ x x x x @
                  -------------------------

Lydian      //L   @ . x . x . L $ . x . x @     major
Locrian     alt-L @ x . x . x L . x . x . @ x   minor

Ionian      //I   @ . x . x I . $ . x . x @     major
            alt-I . x . x . I x . x . x x .     minor

Mixolydian  //M   @ . x . M x . $ . x x . @     major
            alt-M . x . x M . x . x . x x .     minor-major

Dorian      //D   @ . x D . x . $ . x x . @     minor
            alt-D . x . D x . x . x x . x .     minor-major

Aeolian     //A   @ . A x . x . $ x . x . @     minor
            alt-A . x A . x . x . x x . x .     major

Phrygian    //P   @ P . x . x . $ x . x . @     minor
            alt-P . P x . x . x $ . x . x .     major
```

Symmetric shapes formed by and around tritones are a fundamental feature of the PKP way of knowing scales. Symmetry is well known in math and physics to be a cornerstone for understanding complexity, so it's logical that it might do so for music, which is complex by any measure. The only hurdle to overcome is the unfamiliar notation. The notation is symbolic but no math is needed to understand it, only an intuitive understanding of circular and mirror symmetry from everyday life. For a start, the home-octave split into a 2-tritone stack is itself a symmetric shape, with circular symmetry because the inter-note intervals are all the same, just as the radii of a circle are all the same. It's an easy step from there to the yellow-highlighted symmetric shapes in the scales, which have mirror symmetry relative to their geometric centers (not circular symmetry because the inter-note intervals are not all the same).

A Lego-like View of Building Blocks

The building blocks identified by the anchors of classical modes are shown next in Lego-like terms for the Ionian mode. Accumulating the anchors and their opposite ends from all the building blocks yields the scale. This is not to suggest actually constructing a scale this way, only to show that the building blocks fall out of this way of representing scales.

A new notational element is introduced here: the default interpretation of a fifo anchor letter is a fifth; a letter with a strikethrough indicates a fourth (think of the strikethrough as cutting down the size). Strikethroughs are optional but useful; if they're not used, fifo size is left entirely to context (which is useful when getting started on a new piece).

These symbols are simple in text but difficult to produce in handwritten annotations on the written music. The solution for handwritten annotations is different colored pencils, or different outlining of whole symbols by circles for tritones and boxes for fifos.

```
                                @ P A D M I L $ x x x x @
frame fifth & fourth     @      @ . . . . . . $ . . . . .
                         $      . . . . . . . $ . . . . @
core fifth               𝕀      . . . . . I . . . . . . x
core tritone             I      . . . . . I . . . . . x .
core fifth               𝕄      . . . . M . . . . . . x .
-----------------------------------------------------------
secondary fourth         𝕄̶      . . . . M . . . . x . . .
secondary fifth          𝔸      . . A . . . . . . x . . .
secondary fourth         𝔸̶      . . A . . . . x . . . . .
```

Intervals smaller and larger than building blocks are implicit in keyboard shapes formed of combinations of building blocks: they're inner or outer intervals of the combinations. However, it's occasionally useful to bring them in as explicit fifo-like objects in the flow of harmony. This is indicated by putting a fifo anchor symbol in triangular brackets that cancel the normal size interpretation, leaving the interpretation to context (e.g., 〈𝕄̶〉 and 〈𝕄〉 are anchors of intervals of unspecified size, smaller than a fourth for the former, larger than a fifth for the latter). This is sufficient because it's rarely required and the sizes to choose are generally obvious from context.

Every primary classical mode has a 3-part **anchor set** within the lower fifth of the scale frame that distinguishes it from the other primary classical modes (e.g., 𝔸.𝕄**I** for the Ionian mode and **A.**𝕄𝕀 for the Aeolian mode, where the optional dot indicates a skipped alphabet letter). The tritone anchor also provides a fifo anchor that's not listed in the anchor set because it's understood (e.g., 𝕀 for the Ionian mode, 𝔸 for the Aeolian mode). Classical modes are constrained by a rule of no adjacent half tones: the tritone anchor can have only one fifo anchor adjacent to it (e.g., 𝕄 for the Ionian mode), which leaves only one other alphabet letter available as a fifo anchor (e.g., 𝔸 for the Ionian mode) because any other choice would yield adjacent scale half tones (e.g., 𝕃, 𝔻 and ℙ are excluded from the Ionian mode because they would yield adjacent half tones).

The fifo that shares its note with the mode tritone (e.g., 𝕀 of the Ionian mode) alternates between a fourth and a fifth for successive modes in dictionary order. The other scale fifos may be either fifths or fourths. The core fifo anchors identify opposite harmonic roles (e.g., for the Ionian mode, 𝕄 establishes tonality as major and so may characterized as a **resolution anchor**; 𝕀 is offset a half tone away from it and so is automatically a **non-resolution anchor** because of relative dissonance.

Most of the fifos of classical modes are substitutable for each other in harmony because of either harmonic equivalence (e.g., the frame fifth and fourth) or mutual consonance (e.g., all but the mutually dissonant core fifos). This opens the possibility of different combinations of building blocks and different permutations of their notes (following from inversions) being used for essentially the same harmonic function in different places in the same piece. This substitutability makes for harmonic richness but also for notational complexity in music notation, because of the large number and wide variety of different chord symbols that may result. PKP keeps the possibility of harmonic richness but does an end run around the notational complexity.

Pause for a moment think about the simplicity of this. Knowing a parallel mode requires only positioning a tritone anchor anywhere within the lower fifth of the home octave and forming one of two possible symmetric shapes around the tritone. Knowing a mode change boils down to transposing the tritone to another position, and the symmetric shape along with it. Positioning and repositioning the tritone, and forming symmetric shapes around it, is done by eye on the keyboard, without reference to note symbols of any kind. Identifying the mode requires only identifying the bottom tritone note by an alphabet letter. Thinking of the whole scale in terms of note symbols is never required.

Parallel Mode Changes

Parallel mode changes are very simple. One alphabet step up or down in a mode signature alters one note, two steps alters two notes and three steps alters three notes. In each step, one altered note is provided by the tritone (the other tritone note is in both modes). Here are some example mode changes and corresponding note alterations:

- the single step change **//L-//I** alters one note;
- the two-step change **//L-//M** alters two notes;
- the 3-step change **//L-//D** alters three notes;
- the cascaded 2-step changes **//L-//M-//A** alter two notes at each step (the three tritones define a whole tone scale that provides a kind of scaffolding for the changes);
- analogous sequences down or up follow from any starting mode (when changes go off one end of the alphabet, they wrap around to other end, bringing in **alt** modes).

The strong chromaticism of such changes is identified in music notation by accidentals in melody lines and chromatic chords in harmony, without any explicit indication of the origin of the chromaticism, which may be classical modes, non-classical modes, or ornamentation that has no scale implications. Explicit key-signature changes indicate the origin is in classical modes, provided there are no accidentals in the melody and no chords that go outside the key signatures, but much music is written with a single key signature. Even the simplest classical mode changes can be complex in music notation.

"How strange the change from major to minor" — these words from the song *Every Time We Say Goodby* (covered in Chapter 5) express, incidentally, the complex nature of a simple parallel mode change such as **//I-//A** in music notation. Suppose the home tonic is the first black key above C, which is either C# or Db in music notation, depending on context. Then this change is from 5 flats for Db major to 4 sharps for C# minor. The change naturalizes 5 notes and then sharps 4 notes — 9 symbol changes to alter 3 notes! A particularly confusing feature of this change is the different symbols for the minor tonic (C#) and the major tonic (Db) that seem to imply slightly different pitches for the same tonic. They don't — what they actually imply is different pitches for the notes C and D, when used as references for sharps or flats, from the pitches they have as independent notes. The pitch differences are real independently of the piano but not real for the piano, and so overcomplicate written piano music. The piano has stood the test of time without variable pitch piano keys that would enable pianists to play the slight pitch differences. The PKP way of understanding parallel modes does an end run around this misleading complexity.

Chromaticism

Multiple parallel classical modes appearing one after the other bring chromaticism into the domain of classical modes. Parallel modes cover same-mode tonic changes because they're two sides of the same coin. A notation for identifying secondary tonics in mode signatures using a 12-symbol chromatic scale will be introduced later, but the principle is easy to understood without it. The tonic changes indicated below go up into the next octave, which is harmonically equivalent to wrapping around within the home octave. The tonic changes symbolized by the arrows alter no notes. All the note alterations are provided by parallel mode changes within the home octave.

mode change	@ P A D M I L $ x x x x @
//I	x . x . x x . x . x . x x
//A	x . x x . x . x x . x . x
	↑Ionian transposed up a minor third
//A	x . x x . x . x x . x . x
//I	x . x . x x . x . x . x x
	↑Aeolian transposed down a minor third

Transposing the tritone anchor of a classical mode up or down by an in interval in the size range of a half tone to a minor third may be interpreted as either a parallel mode change, or a same-mode tonic change in the opposite direction by a tritone minus the alphabet interval. The next table summarizes this in a notation-free way. The changes in the left column are from an established mode (parallel or alt), identified by a tritone anchor position in the alphabet, to a mode identified by a different position a given number of alphabet steps up (+) or down (-) in the alphabet as a circular loop (steps off one end wrap around to the other end). Steps in the opposite direction add up to a tritone. The yellow highlights identify changes that alter the least notes. This is sufficient to identify the mode signature of a result because changes between primary modes and alt modes alter the most notes.

SAME-MODE CHANGES TO TONICS A TRITONE APART

anchor change	tonic change	altered notes	tonic change	altered notes
+3	-3	3	+3	3
+2	-4	4	+2	2
+1	-5	1	+1	5
0	0	0	± 6	5
-1	+5	1	-1	5
-2	+4	4	-2	2
-3	+3	3	-3	3

Melody lines alone determine whether or not parallel mode changes imply tonic changes because relative modes provide the same notes for harmony (their musical meaning is altered by a tonic change but not the actual notes).

NON-CLASSICAL MODES FROM UNIONS OF BASIC MODES

Non-classical modes of different kinds are common in music (e.g., blues, enriched minor or major) but nothing equivalent to the mode signatures about to be presented exists in music notation. Non-classical modes are represented in music notation by accidentals relative to written key signatures. Accidentals are also used to represent notes of implicit key-signature changes, and to identify ornamental notes that have no scale implications. The problem is, the modes tend to get lost in the written music as distinct musical entities. A unique contribution of PKP is bringing them forward as distinct musical entities by simple mode signatures that define them by construction on the keyboard, without reference to note symbols.

There's often a choice between understanding a piece of music as a succession of classical modes that follow each other over time, or as a much smaller number (often only one) of non-classical modes that use the same tritones. The non-classical option is always simpler when the mode changes in the classical domain are entangled or appear and disappear too quickly.

Non-Classical Modes: Blues

Continuing with the idea that pentatonic modes are the foundation scales of music leads simply and directly to a family of blues scales that are different in kind from classical modes and, as such, provide a first introduction to the nature of the entire domain of non-classical modes. The blues hierarchy shown next is developed bottom up, skipping over the modes between the lines, and understood top-down, including the skipped-over modes as sub-scales.

	@ P A D M I L $ x x x x @	
//DM.L	@ . x x x x x $. x x . @	**blues family scale**
//DM	@ . x x x x . $. x x . @	**pentatonic union**
//M.L	@ . x . x . x $. x x . @	**// mode of melodic minor**
//D..L	@ . ~~x~~ ~~x———x~~ ~~$~~ . x x . @	**// mode of harmonic minor**
//D	@ . x x . x . $. x x . @	**Dorian classical mode**
//M	@ . x . x x . $. x x . @	**Mixolydian classical mode**
//𝔻 + L	@———x . x x $———x . @	**6-note minor blues**
//𝕄 + D	@ . x x x———$. x———@	**6-note major blues**
//𝕄	@ . x . x———$. x———@	**pentatonic major**
//𝔻	@———x . x . $———x . @	**pentatonic minor**

Yellow highlighting identifies characteristic symmetric sequences that make the notes easy to find on the keyboard without thinking in terms of note symbols. The symmetric sequences all follow from the building-block content identified by the mode signatures (which means tritone content for all but the pentatonic modes).

The 8-note **//DM** scale immediately above the top line is the union of the parallel pentatonic major and minor modes at the bottom. The union may be understood to follow from singers untutored in music notation "bending" the tonality-defining note of one mode by a half tone into the note of the other mode that defines the opposite tonality, thus entering the opposite mode. The mode switch yields a sad-happy or happy-sad sound (equating sad with minor and happy with major). This is, to me, the most fundamental characteristic of the blues. The union captures this characteristic in a single scale that's different in kind from classical modes because it has two tritones and four half tones, including three adjacent ones. This is a complex scale in music notation but not in this notation; the simple mode signature determines the complex bits.

The 9-note **//DM.L** family scale at the top follows logically by adding “bent” note **p5** to complete a trio of bent major-scale notes (**p3**, **p5**, **p7**). The mode signature follows from the added note being the anchor of the **L** tritone. Variations determined by signatures **//P.DM**, **//ADM** and **//DMI** sometimes come into play but the master mode is **//DM.L** because it includes, as sub-scales, the two 6-note minor and major blues scales immediately above the bottom line that are generally taught to beginners as “the” blues scales. These scales seem somewhat arbitrary, presented cold, but they follow logically from the master mode by splitting its highlighted half-tone sequence into two parts that determine one-note additions to the pentatonic modes. Turning this around, the master mode is the union of these two scales.

The four sub-scales immediately below the top line are from the major and minor families coming up. That said, they are blues sub-scales in their own right without reference to anything else.

The family of blues modes illustrates the confusing variety of key signatures that may appear for pieces that use non-classical modes. Key signatures for the Mixolydian or Dorian classical modes provide scales that differ by only two notes from the master blues scale, but these are far from the only key signatures seen for blues pieces. A blues key signature may be almost anything, leaving the scale notes to be determined by accidentals. The effect is to make blues seem, misleadingly, an entirely ad hoc musical genre. Accidentals don’t distinguish between notes that determine a scale and ornamental passing notes that have no scale implications. Many low-level details must be sorted out to get a higher level conceptual view. Mode signatures determined by tritone content sort these details out at a high conceptual level.

Non-Classical Modes: Major & Minor Family Scales

In general, combining fundamental modes identified by mode signatures is a good way of generating more general modes. Following the lead of the blues, the union of the default **//I** and **//A** modes of key signatures yields the following enriched major and minor scales.

	@	P	A	D	M	I	L	$	x	x	x	x	@	
//ADMI	@	.	x	x	x	x	.	$	x	x	x	x	@	**//I** + **//A**
//AD.I$^+$	@	.	x	x	.	x	.	$	x	x	+	x	@	**minor family**
//A.MI$^+$	@	.	x	.	x	x	.	$	x	+	x	x	@	**major family**
//AD.I	@	.	x	x	.	x	.	$	x	x	.	x	@	**“bebop” melodic minor**
//A.MI	@	.	x	.	x	x	.	$	x	.	x	x	@	**analogous major mode**
//D.I	@	.	x	x	.	x	.	$	.	x	.	x	@	**melodic minor**
//A..I variations	@	.	x	x	x	x	.	$	x	—	—	x	@	**harmonic minor-major**
	@	.	x	.	x	x	.	$	x	x	.	x	@	**“bebop” major**
	@	.	x	.	x	x	.	$	x	—	—	x	@	**harmonic major**
	@	.	x	x	.	x	.	$	x	—	—	x	@	**harmonic minor**
//D, //A, //M, //I	...													**classical modes**

The result of the union is a 4-tritone, 10-note scale with mode signature **//ADMI** that reduces — by morphing either the **D** or **M** tritone into a scale fifo with the same top note — to a pair of 3-tritone, 9-note modes of minor or major tonality. These modes are identified by **//AD.I**$^+$ or **//A.MI**$^+$, where the plus superscripts indicate the scale includes the top note of the missing tritone identified by the dot in the signature. The only difference between these modes is the tonality in the bottom fifth (highlighted). The modes are two notes short of the chromatic scale but their simple form makes them easy to know on the keyboard. The intricate details of the sub-modes below the line are best understood from examples. The takeaway here is the knowledge that the details are sorted out by a simple mode signature plus context. The melodic minor mode **//D.I** is the Ionian mode with one lowered note in the bottom fifth. The harmonic minor mode identified by ambiguous **//A..I** in a minor context is the melodic minor mode with one lowered note in the top fourth. The harmonic major mode differs from it by one note in the bottom fifth. **//A..I** is ambiguous because the double tritone is circularly symmetric but the ambiguity is generally resolved by context.

The melodic minor, harmonic minor and harmonic major modes are like classical modes in having seven notes and no adjacent half tones. They are the only non-classical modes that have been formally developed in music notation into a set of seven parallel modes that are analogous to the seven classical modes. The book *Modalogy* develops them in conventional terms, with complex results that were part of my motivation for developing PKP. The PKP view of the modes is developed in the next chapter, but the development is not needed to know how to use them in practice because they fall out of family scales.

Other Unions

Unions of classical modes are not restricted to **//A+//I** but unions of modes with less alphabet separation are hardly worth considering as new types of non-classical modes because they generally amount to no more than borrowing a note or two from a nearby parallel classical mode that also appears in the same piece (nearby in both the chromatic scale and the succession of modes).

Two Sides of the Same Coin for Non-Classical Modes

The principle of two sides of the same coin introduced earlier for classical modes also applies to non-classical modes, except there's little use for it. Neither parallel mode changes nor corresponding same-mode tonic changes are desirable in this domain because non-classical modes are already strongly chromatic. In effect, they bundle the chromaticism provided by multiple tritones of multiple parallel modes into single non-classical modes with the same tritone content. The single non-classical modes are identified by readily understandable mode signatures that have no equivalent in music notation.

Some strongly chromatic pieces may be understood either way but a single non-classical mode of extended duration is normally simpler than a fast moving succession of non-classical modes.

CHROMATIC SCALE

Understanding a piece of written music in PKP terms begins with its melody line. Harmony is an add-on left for later. The piano keyboard provides a simple 12-symbol chromatic scale for any tonic, that's sufficient to describe any melody line that can be played on the piano, but music notation provides no simple, single notation for it. We have come a long way without needing an explicit chromatic scale but one is needed for understanding melody lines. I adapted, for this purpose, a chord root notation from Mehegan's jazz piano instruction book. The notation uses Roman Numerals **I-VII** for the seven roots from the master Ionian mode of a tonic, and flatted Roman Numerals for the five chromatic roots in the five whole tone intervals. My adaption, shown next, replaces the Roman Numerals by numbers **1..7** and the flats by prefix **p** standing for "phlat" and meaning "next piano key down."

chromatic scale of the home octave	**1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1**
conceptual home octave	**@ P A D M I L $ x x x x @**

The replacement of the flat symbol avoids the confusion that arises with sharp or mixed sharp-flat scales in music notation. The prefix **p** is not a conventional flat symbol because it only applies to five numbers (**p1** and **p4** are *not* alternate symbols for **7** and **3**), and there are no symbols corresponding to sharps.

The beauty of this chromatic scale notation is it mirrors the look of the C-octave on the piano (the five prefixed symbols are the black keys), giving visibility to the musical functions of the notes in any octave by reference to the C octave. The notation is simple enough to annotate on the staff next to note symbols of a written melody line. The note symbols that have no meaning beyond their position. The symbols of the conceptual home octave are more general than note symbols because they identify building blocks. The cross reference between them enables easy recognition of building blocks in melody lines.

Chords

The genesis of the notation makes obvious that it can be used for chord roots by replacing the numbers by RN symbols.

Secondary Tonics

The scale also provides notation for secondary tonics that may be visited by a piece. A secondary tonic of a relative mode is identified in a mode signature by a suffix of the form **@t**, where **t** is a chromatic-scale symbol for the tonic. For example, the relative mode **//A@p3** is the parallel mode **//I** transposed up a minor third. The notation is indirect because, by definition, secondary tonics have no alphabet of their own.

Melody Lines

The symbolic chromatic scale can be used to represent **skeleton melody lines** that capture how people with musical ears recognize and remember melodies, namely as sequences of pitch intervals going up or down from some starting pitch, independently of rhythm, timing and duration of notes and rests.

REPRESENTATION OF PIECES OF MUSIC

We now have sufficient concepts and notation to represent a piece of music by **skeleton melody line** and **core harmony** represented by an **anchor line** that identifies building blocks in alphabet terms. The combination enable tonic scales of origin of melody and harmony to be inferred in terms of **mode signatures**. This paves the way for some example pieces in the next chapter that give sense of how all this fits together in actual music.

CHAPTER 3: SELECTED EXAMPLES

The best way of learning something new is by studying examples. This begins in this chapter with a sampling of basic pieces that set the stage for digging deeper. Anyone who has followed this so far should be able to understand and play the examples. The objective is not to show what a piece actually "is," or how the composer or arranger might have viewed it, but to arrive at a simple way of understanding it in purely keyboard terms to guide playing it on the piano.

It's useful to think of music as divided into domains identified as basic classical (inherently not chromatic), chromatic classical and non-classical (inherently chromatic). The term "classical" refers to the modes of key signatures, not divisions of music into categories such as classical and pop. The domains determine how chromaticism enters the picture. In the chromatic classical domain, chromaticism comes into play via parallel classical modes, and tonic changes that are the opposite side of the same coin. In the non-classical domain, chromaticism comes into play via scales that are more general than classical modes. Some chromatic pieces may be interpreted as being in either the chromatic classical or the non-classical domain, with the choice depending on the helpfulness of the interpretation. For strongly chromatic pieces, the core harmony is so tritone-intensive that core fifos are often omitted; their inclusion in some examples is sufficient to give a sense of how this works.

Chord symbols are always implied results, never starting points. In shapes that provide rootless voicings of chords, the missing roots are identified by dashes.

In all cases, understanding a piece begins with understanding its melody line. The symbolic chromatic scale is used to represent **skeleton melody lines** that capture how people with musical ears recognize and remember melodies, namely as sequences of pitch intervals going up or down from some starting pitch, independently of rhythm, timing and duration of notes and rests. A skeleton melody line is a sequence of symbols from the chromatic scale with interspersed arrows indicating the direction of the next note up (↗) or down (↘). Colored arrows provide helpful visual separation from the scale symbols, enabling the ups and downs to be easily followed by eye. The pitch intervals between successive notes are determined by the directly visible number of half tones between the scale symbols in the chromatic scale. Understanding the mode of origin of the melody line requires condensing this representation into a set of notes within the home octave.

BASIC CLASSICAL DOMAIN : "HAPPY BIRTHDAY"

A skeleton melody line for this piece is presented next. The melody line is so familiar, it needs no reference. Bar lines provide basic timing. Arrows show melody zig-zags. Asterisks indicate repeated notes, leaving the number of repetitions open (one here). Commas mark ends of phrases. Not here but later, dashes indicate melody gaps of undefined duration that may have accompanying harmony. For concreteness, think of this line as copied from annotations on the staff of an F-major version of the piece.

5*|↗6↘5↗1|↘7, ↘5*|↗6↘5↗2|↘1, ↘5*|↗5↘3↘1|↘7↘6, ↗4*|↘3↘1↗2|↘1

Cross-references for home tonic F are shown next (Appendix B provides a table of cross references for all possible home tonics).

F	-	G	-	A	Bb	-	C	-	D	-	E
1	p2	2	p3	3	4	p5	5	p6	6	p7	7
@	P	A	D	M	I	L	$	.	.	.	.

At a glance, the scale is Ionian, but recognizing a scale in general requires condensing the melody line into a home-octave table of the following form. The two headings enable cross referencing by eye between notes and building blocks identified by alphabet letters. The **x** entries identify melody notes in in any octave of a stack of home octaves. The scale frame is in blue, tritone notes in red. This melody line uses all seven scale notes, so there's no ambiguity.

```
      1 p2 2 p3 3  4 p5 5 p6 6 p7 7  1
      @  P A  D M  I  L $  x x  x x  @
      ________________________________
//▌   x  . x  . x  x  . x  . x  . x  x
```

Integrated Melody and Harmony

Core harmony provided by single building blocks from the above known melody scale is represented by adding an anchor line, above the melody line to correspond with the way chords are shown above the staff in written music. The harmony line and melody line are identified on the left by the symbols ⚓ and ♫, respectively (this is not needed for this simple piece, but is helpful when melody lines are represented by multiple text lines going down the page). From now on, all examples will be presented in this form.

```
⚓     |         ▯ |  ▌         |         ▌ | M         |         ▯ |  ▌        ▯ |         ▌ | M
♫    5*|↗6↘5↗1|↘7,  ↘5*|↗6↘5↗2|↘1,  ↘5*|↗5↘3↘1|↘7↘6,  ↗4*|↘3↘1↗2|↘1
```

The mode tritone is placed in positions that anticipate resolution in the mode. The resolution is postponed in the first and third melody phrases, signified by the tritone harmonizing the last melody note of the phrase. The resolution is immediate in the second and fourth melody phrases, signified by the tritone harmonizing the pre-resolution melody note. Scale fifos are faired into the inter-tritone gaps. They morph into succeeding tritones. A fifth anchored by ▯ (only a fifth on this anchor is possible in this scale) morphs into the selected instances of the mode tritone. A fourth anchored by M harmonizes the resolution tonic (not a fifth to avoid putting a dissonant half tone below the tonic, assumed to be played in the next octave up).

The core harmony established by this anchor line is shown next in Lego-like terms.

```
         1 p2 2 p3 3  4 p5 5 p6 6 p7 7  1
bar      @  . A  . M  I  . $  . x  . x  @
1        .  . .  . .  .  . .  . .  . .  .
2    ▌   .  . .  . .  I  . .  . .  . x  .
3    ▌   .  . .  . .  I  . .  . .  . x  .
4        .  . .  . .  .  . .  . .  . .  .
5        .  . .  . .  .  . .  . .  . .  .
6    ▌   .  . .  . .  I  . .  . .  . x  .
7        .  . .  . .  .  . .  . .  . .  .
     ▌   .  . .  . .  I  . .  . .  . x  .
8        .  . .  . .  .  . .  . .  . .  .
```

```
     1 p2 2 p3 3  4 p5 5 p6 6 p7 7  1
     @  . A  . M  I  . $  . x  . x  @
▯    .  . .  . .  .  I .  . .  . .  . x ↓morph
▌    .  . .  . .  .  I .  . .  . .  x .
▌    .  . .  . .  .  I .  . .  . .  x .  ↓ wobbly slide
M    .  . .  . .  M  . .  . .  x .  . .  ↓ wobbly slide
▯    .  . .  . .  .  I .  . .  . .  . x ↓morph
▌    .  . .  . .  .  I .  . .  . .  x .
▯    .  . .  . .  .  I .  . .  . .  . x ↓morph
▌    .  . .  . .  .  I .  . .  . .  x .  ↓ wobbly slide
M    .  . .  . .  M  . .  . .  x .  . .
```

The tritone “seed” is on the left and its completion on the right. The terms **morph** and **slide** were introduced earlier but the term **wobbly slide** is new. It identifies a combination of a **slide** and a **morph**. The term may be strange but it represents the hand movements exactly: move the hand while adjusting the fingers for the size change. For example, **I-**𝕄 is a contraction of **I-M-**~~𝕄~~ (a tritone slides down a half tone and then shrinks into a fourth); and ~~𝕄~~**-**𝕀 is contraction of ~~𝕄~~**-**𝕄**-**𝕀 (a fourth expands into a fifth which slides up a half tone). This is a useful way of thinking because it represents multi-step anchor lines that may identify mode changes (e.g., **I-**~~𝕄~~ or ~~𝕄~~**-**𝕄**-**𝕀 stays in the Ionian mode, and **I-M-**~~𝕄~~ goes to the parallel Mixolydian mode).

Slides that move off one end of the alphabet wrap around to the other end. For example, **I-A** slides a tritone down a minor third, and **I-<u>A</u>** slides a tritone up a minor third, which amounts to wrapping around to **A** and sliding the result up a tritone (in other words, inverting it). Such simple movements imply mode or tonic changes that can be complex in music notation.

Fifo-only anchors @, 𝕊 and 𝔸 don’t appear in this example but may appear in general in core harmony. A good way of thinking of such appearances is as fifo substitutions to provide more variety. For example, the 𝔸 fifth might be substituted for the ~~𝕄~~ fourth in bar 1 of the example.

Chords

Chords in general will be dealt with in a following chapter but the usual chords implied by this core harmony are easily understood now. The following chord progression is a sequence of seventh chords (4 notes, 2-scale steps apart, from a 7-note classical mode) implied by adding the root line II-V-V-VI-II-V-II-V-I below the core harmony.

IIm7 | V7 | V7 | VIm7 | IIm7 | V7 | IIm7 | V7 | I6

A large number and wide variety of chords with very different chord symbols can be implied by adding different root lines to the same core. Straight dominant seventh chords such as **V7** are but one example of single-tritone chords (see Appendix C for a table of tritone chords of all kinds). Chord symbols define chords but are not a good way of understanding harmonic flow because the same notes (piano keys) are in different positions relative to the roots of different chords.

NON-CLASSICAL DOMAIN: BACKWATER BLUES

This is a simple, 3-chord, 12-bar blues in F, which I learned some years ago in a blues piano workshop at the then Jazz School in Berkeley, as representative of "probably half the blues pieces played by pop and jazz musicians." The only reference I have for it is my course notes.

⚓ | M | D | M | M |
♫ | 5↘4↘p3 ↘1, | ↗5↘4↘p3↘1 | 1 , | — | — |
⚓ | M | D | M | M |
♫ | ↗5↘4↘p3↘1, | ↗5↘4↘p3↘1 | 1 , | — ↗5↗p7↘5|
⚓ | I | D | M | M |
♫ | ↗p7*, ↘5 | ↗1↘p7↘5↘p5↘4↘p3↘ 1 | 1 , | — |

The melody is in the pentatonic minor scale except for bar 10 that adds note **p5** (tritone anchor **L**) to form the 6-note, minor blues scale visited in reverse in bar 10. It's useful to think of the **//DM.L** scale as the governing melody scale because it includes, as sub-scales, not only these minor scales, but also corresponding major scales that can be used in variations, which might include different pairings of melody/harmony tonalities than these. For conceptual uniformity, it's also useful to think of the harmony as from the **//DM.L** scale, except with **I** substituted for **L** to provide a progression of dominant seventh chords on the conventional blues root line **I-IV-I-I, IV-IV-I-I, V-IV-I-I** (the **V**-chord containing tritone **L** is not a dominant seventh chord). This makes a basic blues piece simple for everyone. In either case, the unique appearance of the single tritone announces a "turnaround."

Un-inverted and inverted harmonic cores are shown next, with simple simple voicing lines that satisfy the chords shown (such lines only need to add depth because the tritones provide ample variety). Omitted chord roots are identified by dashes.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
@	.	A	D	M	I	L	$	.	x	x	.	@	
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	x	.	.	I	.	–	.	.	.	+	.	*V7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*

5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
$	.	x	x	.	@	.	A	D	M	I	L	$	
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	.	+	.	.	.	.	.	I	.	$	*V7*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*

All-tritone core harmony is a relatively common feature of blues pieces. An all-tritone core is not only simpler than a mixed tritone/fifo core, it's also musically more definite. Ignoring repetitions, and assuming the governing scale is **//DM.L**, the core tritone sequence of this piece is **M-D-M-D-M-L-D-M**. This is a very definite blues sequence. The order is not as important as the presence of these tritones in some order that fits the melody line. That said, blues cores often include fifos. The higher the ratio of core fifos to core tritones, the weaker the blues sound because fifos are less definite musical elements than tritones (there are twice as many of them, and they're widely shared).

Chord substitution is a favorite trick of jazz musicians, often having the contrary effects of simplifying the keyboard flow of the harmony while complicating its symbolic representation in chord notation. In PKP terms, the basic form of chord substitution boils down to holding the core and changing other notes. For example, raising the bass note a whole tone for the last four bars on the left voices the chord sequence **V7-IV7b13-I9-I9** (among other possibilities). Altering the treble line for the sequence at bottom right to a minor third above the inverted core yields a voicing of "tritone substitute" chords on a root line a tritone away from the original, namely **bII7-VII7-bV7-bV7** (among other possibilities). A tritone substitute chord is a tritone chord with its root a tritone away, containing the same tritones in opposite inversions. Other possibilities have different chord root lines for different types of chords. Make enough substitutions and symbolic chord progressions can quickly become difficult to comprehend for all but experts (e.g., the footnoted example in the opening chapter).

Walking Bass Lines

Walking bass lines, ubiquitous in blues, provide an alternative to chordal harmony. The same anchor symbol identifies a line underneath the melody line as a walking bass line. There's no place to annotate such lines on the written music, but they can be recorded in a separate notebook. This has the benefit of making them shareable among different pieces.

The following borrows, for this piece, a walking bass line from Monk's *Straight No Chaser*. The bass line is on downbeats and the melody notes in blue text are on upbeats. This enables the lines to cue each other, and avoids a few direct dissonances. Arrows are not needed for bass lines because they're not sensitive to the direction of the next note, unlike melody lines.

♫ | 5↘4 ↘p3 ↘1 | ↗5↘4 ↘p3 ↘1 | 1, | |
⚓ | 1 2 p3 3 | 4 p3 2 p2 | 1 3 5 6 | p7 6 5 3

♫ | 5↘4 ↘p3 ↘1| ↗5↘4 ↘p3 ↘1 | 1, | 5↗p7↘5 |
⚓ | 1 2 p3 3 | 1 p3 2 p2 | 1 3 5 6 3 p7 6 p3

♫ |↗p7↘p7 5 |↗1-p7↘5↘p5↘4-↘p3 ↘1 | 1, | |
⚓ 2 3 4 p5 5 6 p7 7 1 2 p3 3 1 2 p3 3

MIXED DOMAINS : “SUMMERTIME”

I learned this piece in D-minor (Aeolian) with this harmony some years ago in a piano comping course given by Susan Muscarella at the then Jazz School in Berkeley. The only reference I have for it is my course notes. The PKP interpretation came later. Minor melody lines in a single classical mode are routinely harmonized from parallel minor classical or non-classical modes, to establish tonality is minor and not relative major. Many chord progressions may be voiced by this core harmony with different root lines, but the root line shown is the one I learned in the course.

```
                  IV          V          IV    I            I    II                  V
⚓|           |  D |         I        | D    | M         |  D  |A                   | AI |
♫| — 5↘p3 |↗5 |5, ↘4↘p3↘4↗5↘p3|↘1↘5 |5, ↗5↘p3 |↗4-4 |4,↘p3↘1↗p3↘1↗p3 |↘2, |

                                        II                     pVII      V   I
⚓                                      | A                  |   A      AI | $ |
♫ | ——————————repeat above——————————— |5↗p7↘5↗p7↗1↗p3, |↗5↘4↘p3 — |↘1, |
```

This would be identified as “ornamented basic classical” except the ornamentation is visibly from the non-classical minor family scale with mode signature **//AD.I+**. For improvisation, the whole piece may be understood as from that scale, because the melody scale is a sub-scale.

The 6-note melody line with a single half tone implies a minor classical mode that could be either Aeolian or Dorian, but the visibly obvious choice in this context is Aeolian.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
6 melody notes	x	.	x	x	.	x	.	x	.	.	x	.	x	
Dorian //D	x	.	x	x	.	x	.	x	.	x	x	.	x	
Aeolian //A	x	.	x	x	.	x	.	x	x	.	x	.	x	☜

Shown next is an example set of harmony shapes for bars 1-8 plus bar 16, formed by adding voicing notes to the core. These shapes voice the chords on the right (omitted chord roots are identified by dashes).

shape	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	a voicing of	
	@	.	A	D	.	I	.	$	x	x	.	x	@	.	A	D		
D5	.	.	.	D	.	–	.	.	.	x	.	.	.	.	x	.	*IV-7b5*	
I4	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	x	*V-7#5*	
D5	.	.	.	D	.	–	.	.	.	x	.	.	.	.	x	.	*IV-13*	
M5	.	.	.	.	M	.	.	.	.	.	x	.	–	.	.	x	*I-7#9*	ornamental
D5	.	.	.	D	.	.	.	.	.	x	.	.	–	.	x	.	*I-m6(9)*	
A4	.	.	A	.	.	.	.	.	x	.	.	.	x	.	.	.	*II-m7b5*	
AI	.	.	A	.	.	I	.	–	x	.	.	x	.	.	.	.	*V-7b9*	
4$	.	.	.	x	.	.	.	$	.	.	.	.	x	.	.	.	*I-m*	final resolution

Numeric suffixes and one numeric prefix on the left show the numbers of half tones the voicing notes are above or below the core (they're not shown in the anchor line for simplicity of presentation, but could be).

As summarized next, there's a strong contrast between the simplicity of these shapes and the complexity of the chord symbols. The adjustment of chord symbols to fit scales or places in scales they don't quite fit is like "banging square pegs into round holes" — the results tend to be messy.

chord scale notes (bars 1-8)	**piano keys**
root of ***I-m6(9)***, "5" of ***IV-13***, "7" of ***II-m7***	**1**
root of ***II-m7b5***, "9" of ***I-m6(9)***, "13" of ***IV-13***	**2**
"7" of ***IV-7***, "#5" of ***V***, "#9" of ***I-7#9***	**p3**
"b5" of ***II***, "b9" of ***V-7b9***	**p6**
"b5" of ***IV-7b5***	**7**

CHROMATIC CLASSICAL DOMAIN : “I GOT RHYTHM”

This Gershwin piece is the origin of widely copied chord changes called “Rhythm Changes” by jazz musicians. My source for it is *The Standards Real Book*, Sher Music (2000), p191. The melody line is Ionian of the home tonic (Bb) except for one highlighted “outside” note that isn’t a passing note because it marks an important melody “turnaround” instead of only filling in a same-direction scale sequence. This single note is the trigger for the mode/tonic changes in these bars. Before that, the “outside” harmony tritones are purely ornamental.

⚓	| | ▯ **I** | | ▯ **I** | ╪ **M** | **D** **A** | 𝕄 **I** | 𝕄 **I** |
♫	**| — 5↗6 |↗1↗2, | 2↘1 | 6↘5, |↗5↗6 |↗1↗2, ↗4↘2 |↗3*↘2↗3↘2 | ↘1 — |**
⚓	| 𝔸 | **A** | 𝔸 | **P** | $ | **L** | ▯ | **I** |
♫	**| 3 3 | 3↗p5, | 3 3 | 3↘6, | 2 2 | 2↗3, | ↘2 2| ↘5, |**

The melody/harmony modes are summarized next. Notes highlighted in grey are the Ionian tonics going down by fifths (**6-2-5-1**) implied by the mode sequence, in which one scale note changes at each step.

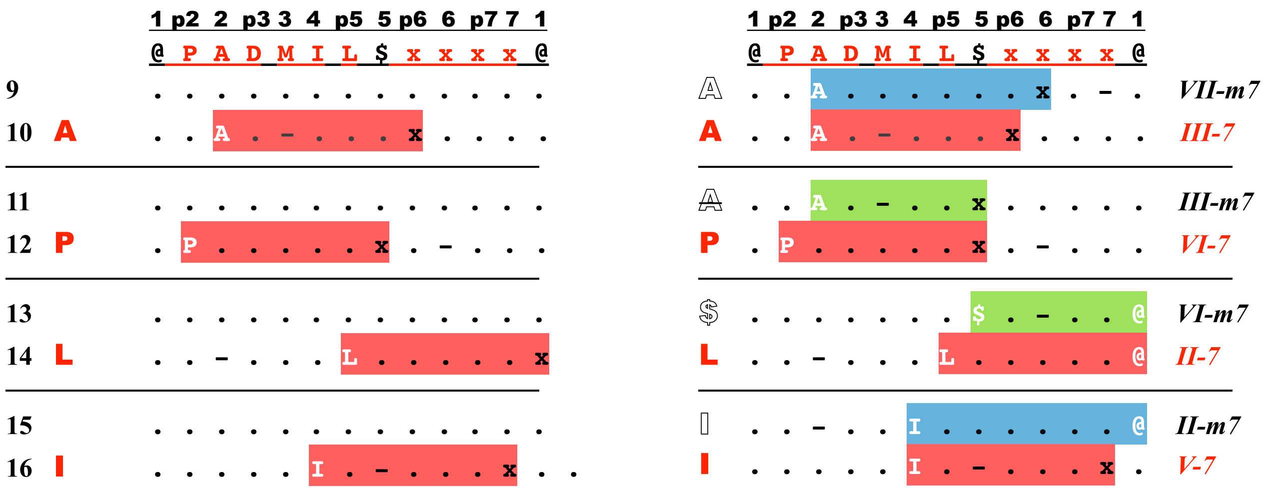
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	P	A	D	M	I	L	$	x	x	x	x	@
bars 9-10	**altA**	.	x	x	.	x	.	x	.	x	x	.	x	.
bars 11-12	**altP**	.	x	x	.	x	.	x	x	.	x	.	x	.
bars 13-14	**//L**	x	.	x	.	x	.	x	x	.	x	.	x	x
bar 15-16	**//I**	x	.	x	.	x	x	.	x	.	x	.	x	x

Skipping bars 1-3, here follows the harmony for bars 4-8, seed tritones on the left, full core and implied chords on the right. The **M-D-A** tritone sequence is ornamental. The fifos on the right are from the ***//I*** mode. The chords follow from the root line (omitted roots in the building blocks are identified by dashes).

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		@	P	A	D	M	I	L	$	x	x	x	x	@
4		.	.	.	.	.	.	.	.	.	.	.	.	.
	I	.	.	.	.	.	I	.	–	.	.	.	x	.
5		.	.	.	.	.	.	.	.	.	.	.	.	.
	M	–	.	.	.	M	.	.	.	.	.	x	.	.
6	**D**	.	.	.	D	.	–	.	.	.	x	.	.	.
	A	.	.	A	.	.	.	.	.	x	.	–	.	.
7		.	.	.	.	.	.	.	.	.	.	.	.	.
	I	.	.	.	.	.	I	.	–	.	.	.	x	.
8		.	.	.	.	.	.	.	.	.	.	.	.	.
	I	.	.	.	.	.	I	.	–	.	.	.	x	.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
▯	.	.	–	.	.	I	.	.	.	.	.	.	x	*II-m7*
I	.	.	.	.	.	I	.	–	.	.	.	x	.	*V-7*
╪	.	.	.	.	.	I	.	–	.	.	x	.	.	*V-m7*
M	–	.	.	.	M	.	.	.	.	.	x	.	.	*I-7*
D	.	.	.	D	.	–	.	.	.	x	.	.	.	*IV-7*
A	.	.	A	.	.	.	.	.	x	.	–	.	.	*pVII-7*
𝕄	–	.	.	.	M	.	.	.	.	x	.	.	.	*I-6*
I	.	.	.	.	.	I	.	–	.	.	.	x	.	*V-7*
𝕄	–	.	.	.	M	.	.	.	.	x	.	.	.	*I-6*
I	.	.	.	.	.	I	.	–	.	.	.	x	.	*V-7*

Here follows the harmony for bars 9-16 in which mode changes occur. The note changes of one per successive mode away from **//I** are cumulative. This means the note differences of **altA** (bars 9-10), **altP** (bars 11-12) and **//L** (bars 13-14) from **//I** (bars 15-16) are, respectively, three notes (**p2**, **p5, p6** replace **1**, **4**, **5**), two notes (**p2**, **p5** replace **1**, **4**) and one note (**p5** replaces **4**). This excludes the replaced **//I** notes as harmony notes for that mode. The exclusions decide the core fifos that can be morphed downward into the tritones, without thinking about chord symbols. The chords on the right are results of assigning the root line | VII III | III VI |VI II | II V | to the resulting core.



This is very simple seen this way, but can be very complex if — as is standard practice — you think in terms of degree numbers in chord scales relative to different key signatures of origin of the tritones. The four tritones of these bars originate in four different key signatures, three of them implied by accidentals relative to one of them. The home tonic is Bb so the tonic change sequence **6-2-5-1** is G-C-F-Bb, for which Ionian mode key signatures are 1 sharp, empty, 1 flat and 2 flats. This is a relatively easy combination of key signatures but other combinations for other home tonics can be much more difficult (see Appendix B for key signatures for different tonics).

ORNAMENTED BASIC CLASSICAL : "OVER THE RAINBOW"

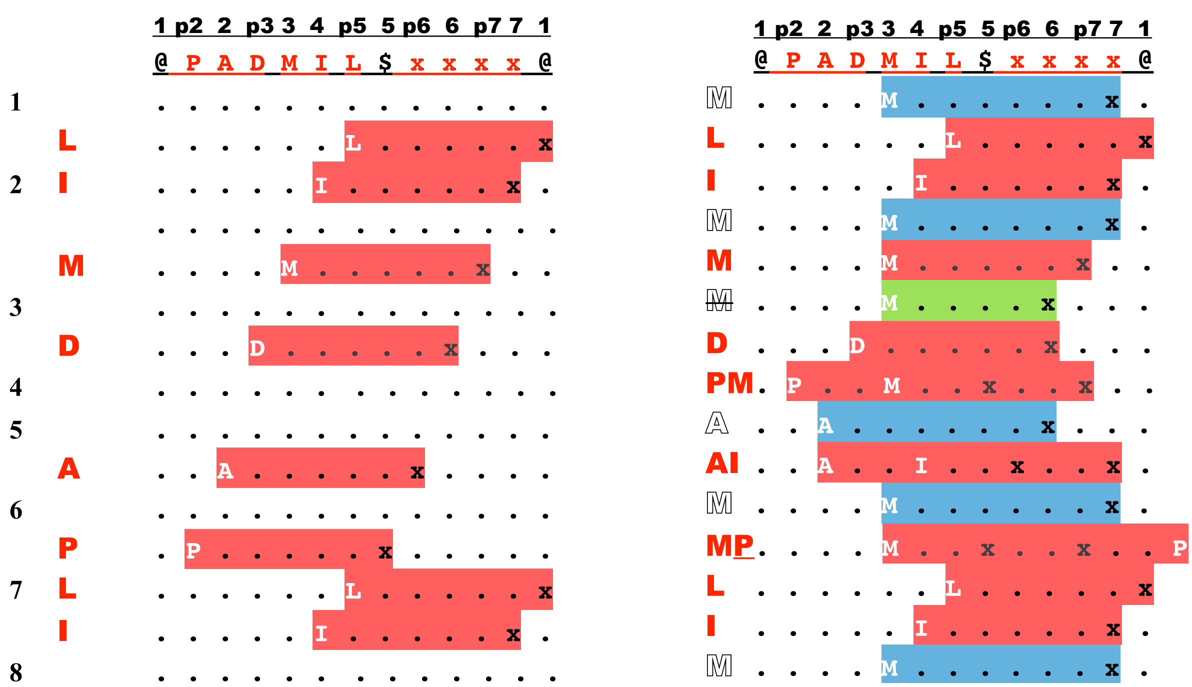
This is an exercise in creating ornamental harmony from scratch without even thinking about harmony scales or chords. The example is the first part of the melody line of *Over the Rainbow*, which is so well known it doesn't need a reference. The melody line is straight Ionian, trending downwards in zig-zags over an octave range. For concreteness, assume the home tonic is Eb.

The downward trending melody line is given a downward trending tritone anchor line consisting of the alphabet in reverse order starting on **L** and wrapping around to **I**: **L-I-M-D-A-P-L-I**. This is in the spirit of rhythm changes if not exactly a variation of it. The tritones are spread out over the melody line and positioned for consonance with it as follows.

⚓ | L | I M | D | | A | P | L I | |

♫ | 1↗1, |↘7↘5↗6↗7↗1, |↘1↗6 |↘5, |↘6↗4, |↘3↘1↗2↗3↗4, |↘2↘7↗1↗2↗3 |↘1, |

The tritone "seed" on the left below provides a framework for adding core fifos and some tritone variations on the right. The double tritones provide some elegant symmetric relationships between successive shapes (e.g., **PM-**𝔸 shrinks the outer notes of the double tritone inwards a half tone; and 𝔸-**AI-**𝕄 moves a fifth aligned with the bottom note of **AI** to a fifth aligned with the top note)



The result is representative of many pieces with simple melody lines from classical modes and rich ornamental harmony. As a general rule, harmony can't be trusted to indicate melody modes.

⚓ | 𝕄 L | I 𝕄 M | ~~𝕄~~ D | PM | 𝔸 AI | 𝕄 MP | L I | 𝕄 |

♫ | 1↗1, |↘7↘5↗6↗7↗1, |↘1↗6 |↘5, |↘6↗4 — |↘3↘1↗2↗3↗4, |↘2↘7↗1↗2↗3 |↘1, |

CHAPTER 4 : THE BUILDING-BLOCK WORLD

This chapter delves into the details of using building blocks to identify tonic scales and to form shapes that voice chords. There are no new concepts here, only development of ones already presented, so it's possible to skim the chapter to see what's in it, as a preparation for knowing where to look for details when needed, and then to go on to the more advanced examples in the next chapter.

TONIC SCALES FROM BUILDING-BLOCK CONTENT

The following short dictionary mode signatures for tonic scales summarizes in half a page all the scales and more in scale dictionaries such as *The Source*. The mode signatures on the left identify scales by tritone content.

signature	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	scale type
PADMIL	x P A D M I L x x x x x x	chromatic (12 notes)
//P.DM.L	. P . D M . L x . x x . x	diminished (8-notes, min-maj)
//AD.IL	. . A D . I L . x x . x x	diminished (8 notes, min)
//PA.MI	. P A . M I . x x . x x .	diminished (8 notes, maj)
//ADMI	. . A D M I———-x x x x .	no name (8 notes, min-maj)
//A.M.L	. . A . M . L . x . x . x	whole tone (6 notes, maj)
//P.D.I	. P . D . I . x . x . x .	whole tone (6 notes, min)
==========	==========	==========
//DM.L	@ . A D M I L $. x x . @	min-maj blues family (9 notes)
//DMI	@ . A D M I . $. x x x @	variation
//ADM	@ . A D M I . $ x x x . @	variation
//P.DM	@ P A D M I . $. x x . @	variation
//AD.I+	@ . A D . I . $ x x + x @	minor family (9 notes)
//A.MI+	@ . A . M I . $ x + x x @	major family (9 notes)
//DM	@ . A D M I . $. x x . @	pentatonic union (8 notes), a basic blues scale
//A..I	@ . A D M I . $ x———x @	harmonic minor-major (8 notes) (master of //**PM,AI,DL,MP,I<u>A</u>,L<u>D</u>**)
//D.I	@ . A D . I . $. x . x @	melodic minor (7 notes) (master of //**PD,AM,DI,ML,I<u>P</u>,L<u>A</u>**)
//I	@ . A . M I . $. x . x @	Ionian (7 notes) (master of //**P,A,D,M,I,L**)
//M	@ . A . M———$. x———@	pentatonic major
//D	@———D . I . $———x . @	pentatonic minor

The scales above double line are atonal, with symmetric shapes (same interval sequence going up and down). The scales below it are tonal, with asymmetric shapes. These scales are completed by populating the anchor sets in the lower fifth of the scale frame with fifo-only anchors. The letters **ADMI** are fundamental to these anchor sets, with different omissions of letters or different identifications of them as fifo-only anchors determining different scales. Minor-third intervals of scales are shown as solid lines to make them stand out to the eye. These are by no means the only possible scales but the dictionary is easily extended if anyone sees a need.

The focus of PKP is tonal music but atonal scales are included because shared letters of their defining words make them visible structural parents of scales lower down in the dictionary; and also because they may be used ornamentally in tonal music. Atonal scales have no minor, major or minor-major tonality by themselves but representing them by words that place them in the context of the home tonic gives them the relative tonality shown on the right (the presence of **D**, **M** or **DM** identifies minor, major or minor-major tonality).

There are no 5-letter words because the implied scales would be too close to the chromatic scale to be usefully distinguished from it. As scale identifiers, single-letter words are reserved for scales with seven or more notes, which includes classical modes but not 6-note blues scales containing single tritones that are understood as sub-scales of higher level blues scales.

The zone within the dictionary ranging from 4-letter words down to 2-letter words that identify parallel modes is complex in music notation. Scales must be understood as enumerated notes relative to one of more than twelve possible different key signatures. The role of classical modes as the defining scales of key signatures makes music from scales that differ markedly from them misleadingly complex. The dictionary makes this zone simple by pushing note symbols and interval inversions down to a lower conceptual level. The three scales in the middle formed around three asymmetric triple tritones provide important families of tonal scales.

SCALE FAMILIES

Scale families provide a simple, unified view of a very large number of scales of music notation of different kinds in different key signatures that are, in aggregate, overwhelmingly complex for all but experts. These scales are built up from a foundation of parallel pentatonic minor and major modes, the scales of simple "folk music" in many cultures worldwide. The classical modes that provide the alphabet were developed in the previous chapter from these modes by splitting their minor-third intervals. Scales beyond these follow from forming unions of parallel modes of opposite tonalities.

Blues Family

The union of the pentatonic major and minor modes with an added note that splits the octave into geometric halves yields a 9-note blues master scale **//DM.L**, of which **//DMI**, **//ADM**, **//P.DM** are variations.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
//𝕄 pentatonic major	@	.	𝔸	.	𝕄	—	—	$	.	x	—	—	@
//𝔻 pentatonic minor	@	—	—	𝔻	.	𝕀	.	$	—	—	x	.	@
//DM = **//𝕄** + **//𝔻**	@	.	𝔸	D	M	𝕀	.	$	.	x	x	.	@
//DM + **L** **adds geometric octave split**	x	.	.	.	.	.	x	.	.	.	.	.	x
//DM.L **master scale**	x	.	𝔸	D	M	𝕀	L	$	.	x	x	.	x

The 8-note union is a natural consequence of "bending" the tonality-establishing notes of simple pentatonic music downwards to give a major piece a sad twist, or upwards to give minor piece a happy twist. The bent notes in the lower fifth join with pentatonic notes in the upper fourth to form two tritones. Also bending the pitch center adds the center note of the geometric octave split seen earlier, which forms a tritone with the tonic. The result is a 9-note scale with three tritones and five adjacent half tones that differs markedly from the classical modes that are the scales of key signatures, which have seven notes, one tritone and no adjacent half tones. This explains why blues pieces are often complex in music notation (e.g., the footnoted chord progression in the opening chapter).

I learned the 9-note scale in conventional terms some years ago in a blues piano workshop at the then Jazz School in Berkeley, but this intuitively simple way of understanding and remembering it is my own invention and appears to be novel. Pianists and music teachers I have asked don't seem to know about it. Books I have consulted don't say anything about it.

The three tritones of master **//DM.L** determine six notes, the scale frame adds one note (the **L** tritone already includes the tonic), and the other two non-tritone notes are from the foundation pentatonic scales. This 9-note scale provides many sub-scales that are stalwarts of blues pieces, shown next listed in order of decreasing tritone content.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
sub-scales of //**DM.L**	x	.	𝔸	D	M	𝕀	L	$	.	x	x	.	x	
//**DM** (8 notes)	@	.	𝔸	D	M	𝕀	.	$	.	x	x	.	@	//𝕄 + //𝔻
//**D..L** (7 notes)	@	.	𝔸	D	.	𝕀	L	$	.	x	x	.	@	// mode of harmonic minor
//**M.L** (7 notes)	x	.	𝔸	.	M	𝕀	L	$	.	x	x	.	x	// mode of melodic minor
//**D** (7 notes)	@	.	𝔸	D	.	𝕀	.	$	.	x	x	.	@	Dorian
//**M** (7 notes)	@	.	𝔸	.	M	𝕀	.	$	.	x	x	.	@	Mixolydian
//𝕄 + **D** (6 notes)	@	—	—	𝔻	.	𝕀	L	$	—	—	x	.	x	major blues
//𝔻 + **L** (6 notes)	@	.	𝔸	D	𝕄	—	—	$	.	x	—	—	@	minor blues
//𝕄 (5 notes)	@	.	𝔸	.	𝕄	—	—	$	.	x	—	—	@	pentatonic major
//𝔻 (5 notes)	@	—	—	𝔻	.	𝕀	.	$	—	—	x	.	@	pentatonic minor

A contrasting top-down view to this bottom-up view is provided by the following hierarchy of scales and sub-scales starting from parent scale **P.DM.L** (an atonal scale called "diminished").

P.DM.L

|| morph

DM.L

/ | \ sub

DM M.L D..L

\ | / sub

D, M

This scale morphs into the **DM.L** scale (the **P** tritone morphs into a fifo while holding its upper note, the pitch center, fixed). The morphing breaks the symmetry of the diminished scale, yielding an asymmetric tonal scale that contains the other scales as sub-scales that follow from pruning notes.

The words **M.L** and **D..L** determine parallel "melodic" and "harmonic" modes, (described later) that share with classical modes the properties of seven notes and no adjacent half tones.

Conspicuously missing are tritones **A** and **I** from the classical modes conventionally regarded as the default minor and major modes of key signatures. Fifo anchors 𝔸 and 𝕀 identify in-scale fifos but tritones anchored by **A** and **I** are ornamental extras.

Blues has high tritone content but it also has high fifo content (five possible fifo anchors from the **DM.L** scale). A strong blues sound follows from core harmony that has correspondingly high tritone content (for example, the footnoted chord progression of *Goodbye Porkpie Hat* in the opening chapter). Weakening the core tritone content in favor of in-scale fifos weakens the blues sound.

Minor and Major Families

The union of the parallel Aeolian and Ionian modes forms a 10-note scale of mixed minor-major tonality determined by the word **ADMI** that is a master scale of family scales of major and minor tonality. This scale reduces to 9-note minor or major family scales by morphing the **M** or **D** tritone into a fifo with the same top note. The morphed scales are identified by the augmented words **AD.I⁺** or **A.MI⁺** where the plus superscript indicates the top note of the missing tritone is retained. The result is both interesting and simple: an all-half-tone top end for both scales, with bottom ends of different tonalities.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
Ionian mode	**//A**	@	.	𝔸	.	𝕄	I	.	$	.	o	.	x	@	**foundation scales**
Aeolian mode	**//I**	@	.	A	𝔻	.	𝕀	.	$	x	.	o	.	@	
10-note union	**//ADMI**	@	.	A	D	M	I	.	$	x	x	x	x	@	
minor family scale	**//AD.I⁺**	@	.	A	D	.	I	.	$	x	x	+	x	@	**morph major tritone**
major family scale	**//A.MI⁺**	@	.	A	.	M	I	.	$	x	+	x	x	@	**morph minor tritone**

As for the blues, the 9-note scales provide sub-scales shown next that are stalwarts of minor and major music.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	
sub-scales of AD.I⁺		@	.	A	D	.	I	.	$	x	x	+	x	@
AD.I	**bebop melodic minor**	@	.	A	D	.	I	.	$	x	x	.	x	@
//A..I	**harmonic minor**	@	.	A	𝔻	.	I	.	$	x	.	.	x	@
//D.I	**melodic minor**	@	.	𝔸	D	.	I	.	$	.	x	.	x	@
//D	**Dorian**	@	.	𝔸	D	.	𝕀	.	$	.	x	+	.	@
//A	**Aeolian**	@	.	A	𝔻	.	𝕀	.	$	x	.	+	.	@

As shown next, the minor and major hierarchies are slightly more complex than the **//DM.L** blues hierarchy because **ADMI** is not the only parent atonal scale, and the classical modes at the bottom are morphs of the modes above them, not sub-scales (they're sub-scales of **AD.I⁺** and **A.MI⁺**).

The double tritones **D.I** and **A..I** on the left, and **A.M** and **A..I** on the right, determine parallel "melodic" and "harmonic" modes that share with classical modes the properties of seven notes and no adjacent half tones.

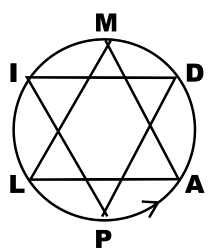
ADMI, AD.IL	**ADMI, PA.MI**
‖ morph	‖ morph
AD.I⁺	**A.MI⁺**
| sub	| sub
AD.I	**A.MI**
/ \ sub	/ \ sub
D.I **A..I**	**A.M** **A..I**
‖ ‖ morph	‖ ‖ morph
D **A**	**M** **I**

PARALLEL "MELODIC" AND "HARMONIC" MODES

The concept of parallel modes applies to any scale — they're rotated transpositions of a master scale — but its systematic development in music notation is restricted to single- and double-tritone modes identified by the terms "melodic" and "harmonic" that share with classical modes the properties of seven notes and no adjacent half tones. In PKP, the parallel modes are determined by transpositions of a master word within within the alphabet as a circular loop (meaning wrapping the letters around to the other end of the alphabet). The transpositions may be understood graphically as shown next (spacer dots are omitted for brevity).

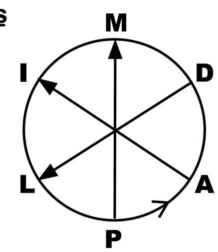
melodic minor modes

PD	~~**alt PD**~~
AM	~~**alt AM**~~
DI	~~**alt DI**~~
~~**IP**~~	~~**alt IP**~~
ML	**alt ML**
LA	**alt LA**



harmonic minor-major modes

PM	~~**alt PM**~~
AI	~~**alt AI**~~
DL	**alt DL**
MP	~~**alt MP**~~
IA	~~**alt IA**~~
LD	**alt LD**



→ indicates letter order for primary modes (secondary modes are opposite)

crossed-out modes don't contain the home tonic but may be used ornamentally

The melodic and harmonic modes are different in kind because each of the words is unique for the former and half of the words are repetitions of the other half in the opposite order for the latter. This is because the double tritones of the latter are circularly symmetric (all inter-note intervals are minor thirds in any inversion). The harmonic modes can be dauntingly complex in conventional terms (Appendix D) because the irregularity of the master mode makes rotated transpositions doubly irregular. The irregularity can be finessed by making the master mode mixed minor-major, leaving the choice of a minor or major master to context. The choice is a minor detail, except for the master mode itself.

The three words **A.M**, **D.I** and **M.L** that fall out of the earlier scale hierarchies determine four parallel modes of the melodic minor mode identified by **D.I**. The three words determine four modes because any word containing **L** determines a primary parallel mode and an **alt** mode with all non-tritone notes different.

The two words **A..I** and **D..L** that also fall out of the foregoing scale hierarchies determine six possible parallel modes of a harmonic minor-major master scale (two modes for the former and modes for the latter).

Details follow for information, but most of the modes fall directly out of blues, minor or major family scales.

Parallel "Melodic" Modes

The master mode is conventionally understood to be the melodic minor identified here by **D.I** but but **A.M** is logically just as valid. In any case, a simple way of knowing them is as morphed whole tone scales. Whole tone scales are determined by **A.M.L** and **P.D.I** are the simplest keyboard shapes possible (5 stacked whole tones). The **A.M**, **M.L** and **L.A** modes are morphed from the **A.M.L** scale; the **P.D**, **D.I** and **I.P** modes are morphed from to **P.D.I** scale. The morphs split one of the whole tones into two half tones. Neither of the **I.P** modes contains the home tonic, so one is called **alt** and the other **altalt**. Here follows a summary of the parallel modes that highlights the splits in the whole-tone scales. The bottom note of the split is the starting note of a transposed melodic minor scale.

mode	source		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	tonality determiners highlighted
alt-LA		AML	x	.	A	D	.	I	L	.	x	.	x	.	x	
//LA		AML	x	.	A	.	M	.	L	.	x	x	.	x	x	
altalt-IP	PDI		.	P	.	D	.	I	.	x	x	.	x	x	.	pseudo-parallel mode (Phrygian-flat-one)
alt-IP	PDI		.	P	A	.	M	I	.	x	.	x	.	x	.	pseudo-parallel mode (Ionian-sharp-one)
alt-ML		AML	x	P	.	D	M	.	L	.	x	.	x	.	x	
//ML		AML	x	.	A	.	M	.	L	x	.	x	x	.	x	
//DI	PDI		@	.	A	D	.	I	.	$	.	x	.	x	@	
//AM		AML	@	.	A	.	M	I	.	$	x	.	x	.	@	
//PD	PDI		@	P	.	D	M	.	L	x	.	x	.	x	@	

The **//IP** mode marked by ✖ isn't a proper parallel mode because it doesn't include the tonic but it's included as a pseudo-parallel mode because it's so close to being one (it's the Ionian mode with the tonic raised a half tone). The **//IP** pseudo mode has major tonality and is partnered by an **alt-IP** mode of minor tonality with all non-tritone notes different.

Parallel "Harmonic" Modes

Making the minor-major mode the master reduces fourteen parallel modes that are often intricate and difficult to comprehend or remember, into the eight simple parallel modes summarized next.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
//DL	x	.	A	D	—	—	L	$	.	x	o	o	x	//LD	x	—	—	D	M	.	L	o	o	x	.	o	x
alt-DL	x	P	.	D	M	o	L	.	o	x	—	—	x	alt-LD	x	P	A	D	.	I	L	—	—	x	o	.	x
//AI	@	.	A	D	M	I	.	$	x	—	—	x	@	//IA	@	P	A	.	M	I	—	—	x	o	.	x	@
//PM	@	P	—	—	M	o	.	x	o	o	x	.	@	//MP	@	P	.	D	M	—	—	x	o	.	x	o	@

Examples of **//DL** and **alt-DL** modes from the blues are shown next (the examples of conventional names for them are from Appendix D).

```
        1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1          1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
DM.L    x  .  𝔸  D  M  𝕀  L  $  .  x  x  .  x     DM.L    x  .  𝔸  D  M  𝕀  L  $  .  x  x  .  x
//DL    x  .  𝔸  D————————L  $  .  x  o  .  x     alt-DL  x  ℙ  .  D  𝕄  .  L  .  o  x————————x
   (e.g., “Romanian”)                           (e.g., “Leading Tone Minor Diminished”)
```

CHORDS FROM BUILDING BLOCKS

When piano music is fully written out on the grand staff, the flow of the music is directly visible. The problem is the amount of complex detail is overwhelming for all but experts. When music is separated into melody on a staff and harmony represented by chord symbols above the staff, only the flow of melody is visible because chords may be voiced in different ways to give different harmonic flows that complement the melody line in different ways.

A piano teacher once told me to learn chords by picturing the chord symbols on the keyboard. I bought a chord symbol wall chart that did exactly this. As I looked at it while trying to learn pieces with these chords, I began to see it as hiding simple keyboard shapes that move in simple ways behind a facade of misleading complexity. This led me to see voicings of chord progressions as built up from harmonic cores without reference to chord symbols beyond identifying tritones. Knowing the scales of origin of the harmony from mode signatures provided by the tritones makes transforming cores into shapes to voice chords simple.

The seventh chords from the Ionian mode shown next provide the basic symbols of chord notation. Their core building blocks, shown in color, are analogous to “guide intervals” used in standard chord voicing practice. In this practice, guide intervals are usually the 3rd-7th intervals of chord scales. These are the core building blocks of chords I, II and V here, but not of the others. The core building block of a chord in PKP is the most unique building block of the chord in the tonic scale. This is always a tritone for chords containing a tritone. Otherwise it’s one of the two fifos of the scale anchored a half tone apart (e.g., fifths anchored by 𝕀 and 𝕄 for the Ionian mode). Think of the bottom or top segments shown in grey as the visible parts of enrichment fifos, of sizes to be determined, snapped onto the core building blocks underneath them at the anchor points (the fifos are all fifths for basic seventh chords going up from these roots, but these are not the only possibilities).

```
                             1  p2 2  p3 3  4  p5 5  p6 6  p7 7  1  p2 2  p3 3  4  p5 5  p6 6
chord     shape              @  .  𝔸  .  𝕄  I  .  $  .  x  .  x  @  .  𝔸  .  𝕄  I  .  $  .  x
I-M7      434                @  .  .  .  M  .  .  o  .  .  .  x  .  .  .  .  .  .  .  .  .  .
II-m7     —  343             .  .  x  .  .  I  .  .  .  o  .  .  x  .  .  .  .  .  .  .  .  .
III-m7    —  343             .  .  .  .  M  .  .  o  .  .  .  x  .  .  x  .  .  .  .  .  .  .
IV-M7     434 —              .  .  .  .  .  I  .  .  .  o  .  .  x  .  .  .  x  .  .  .  .  .
V-7       —  —  433          .  .  .  .  .  .  .  @  .  .  .  x  .  .  o  .  .  I  .  .  .  .
VI-m7     —  343             .  .  .  .  .  .  .  .  .  x  .  .  o  .  .  .  M  .  .  x  .  .
VII-m7b5  —  —  —  334       .  .  .  .  .  .  .  .  .  .  .  x  .  .  o  .  .  I  .  .  .  x
```

The interval stacks in the “shape” column include only minor thirds (symbolized by **3** , standing for three half tones, as before) and major thirds (symbolized by **4**). For chords, this is a variation of a standard notation called "figured bass notation" (Appendix B), in which different numbers represent counts of scale steps between chord notes).

Omitting the inner notes of the core building blocks reduces the chords to a core building block with a major or minor third above or below it, with their sizes determined by scale position (for example, for the first chord, the "third" below the core fifth can only be major). The resulting 3-note shapes are thin voicings of the chords that are often sufficient in the context of a piece of music. The missing inner notes are likely to be implied by most contexts because they're the tonic or pitch center of the home octave, or notes a whole tone above them.

This yields at one stroke a way of voicing seventh chords of classical modes starting from core building blocks (add a major or minor third from the scale above or below a core building block), and a way of notating the voicings (add the size number as a suffix or prefix to the anchor).

Chords From Core Building Blocks

Putting all the core building blocks of the chords in the bottom octave, and rearranging the root order, provides the following revealing picture within the home octave.

```
           1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3 3 4 p5 5
shape      @ .  A .  M I .  $ .  x .  x @ .  A .  M I .  $    chord
▯4         . .  . .  . I .  . .  O .  . x .  . .  x . .  .    IV-M7
▌3         . .  . .  . I .  . .  O .  x . .  x .  . . .  .    VII-m7b5
M3         . .  . .  M . .  O .  . .  x . .  x .  . . .  .    III-m7
M3         . .  . .  x . .  O .  x .  . x .  . .  . . .  .    VI-m7
3▯         . .  x .  . I .  . .  O .  . x .  . .  . . .  .    II-m7
3▌         . .  x .  . I .  O .  . .  x . .  . .  . . .  .    V-7
4M         x .  . .  M . .  O .  . .  x . .  . .  . . .  .    I-M7
```

Transforming this so the chords go up from the roots yields the difficult-to-play result shown next. This is difficult to play because all the fingers must be lifted and moved by large jumps, while also adjusting them for "wobbles."

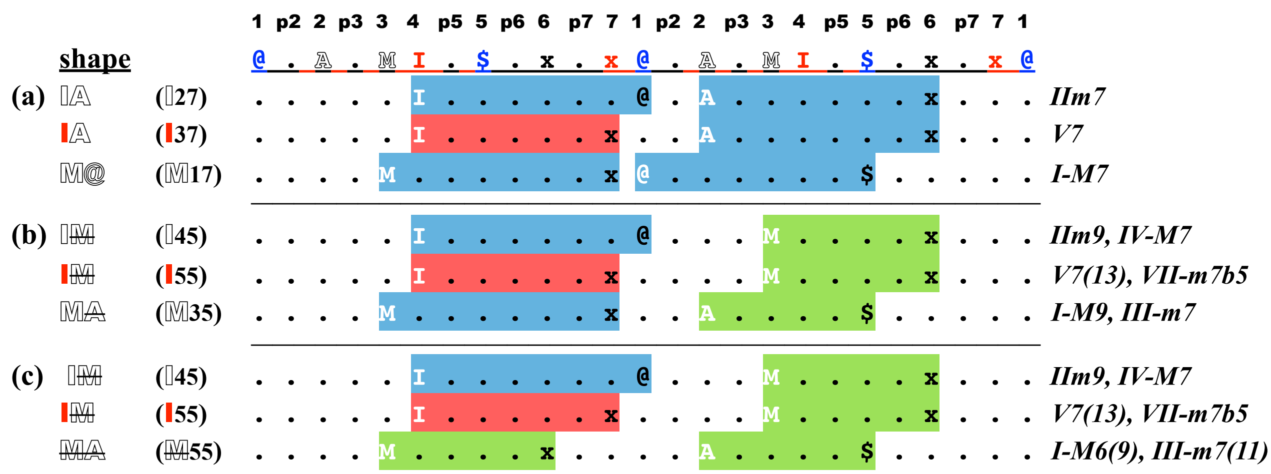
```
           1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3 3 4 p5 5 p6 6
shape      @ .  A .  M I .  $ .  x .  x @ .  A .  M I .  $ .  x    chord
▯4         . .  . .  . I .  . .  O .  . x .  . .  x . .  . .  .    IV-M7
▌4         . .  . .  . . .  . .  . .  x . .  O .  . I .  . .  x    VII-m7b5
M3         . .  . .  M . .  O .  . .  x . .  x .  . . .  . .  .    III-m7
M3         . .  . .  . . .  . .  x .  . O .  . .  M . .  x .  .    VI-m7
3▯         . .  x .  . I .  . .  O .  . O .  . .  . . .  . .  .    II-m7
4▌         . .  . .  . . .  x .  . .  x . .  O .  . I .  . .  .    V-7
4M         @ .  . .  M . .  O .  . .  x . .  . .  . . .  . .  .    I-M7
```

Such shape progressions may be understood as sliding a "scale shape" to different positions in the same scale. The scale shape of a seventh chord from a classical mode has two scale steps between successive notes, but many other scale shapes are possible. Scale shapes are conceptually simple but have the disadvantage of requiring independent knowledge of the scale. Keyboard shapes determine scales. What's more, they enable starting out as one must continue, by working in terms of wobbly slides with different inter-note intervals on the keyboard (two scale steps in a classical mode may be a major or a minor third).

There's obvious benefit in learning the easier-to-play morphed version first and then switching to the jumpy version, if desired, by inverting selected building blocks (which is always easy when building blocks are known by their anchors).

Open Voicings

The following 4-note open voicings of the original chords follow from putting the different building blocks in different adjacent octaves. These shapes can be represented by pairs of anchors that are equivalent to the anchor-with-numeric-offsets notation.



Example (a) is directly from the original. Examples (b) and (c) are simple variations that provide voicings of a variety of complex-looking chord symbols. This vividly illustrates the banging-square-pegs-into-round-holes nature of chord symbols. Simple variations in the building blocks yield complex variations in the chord symbols because of having to adjust them to shapes they don't quite fit. See Appendix C for more on chords.

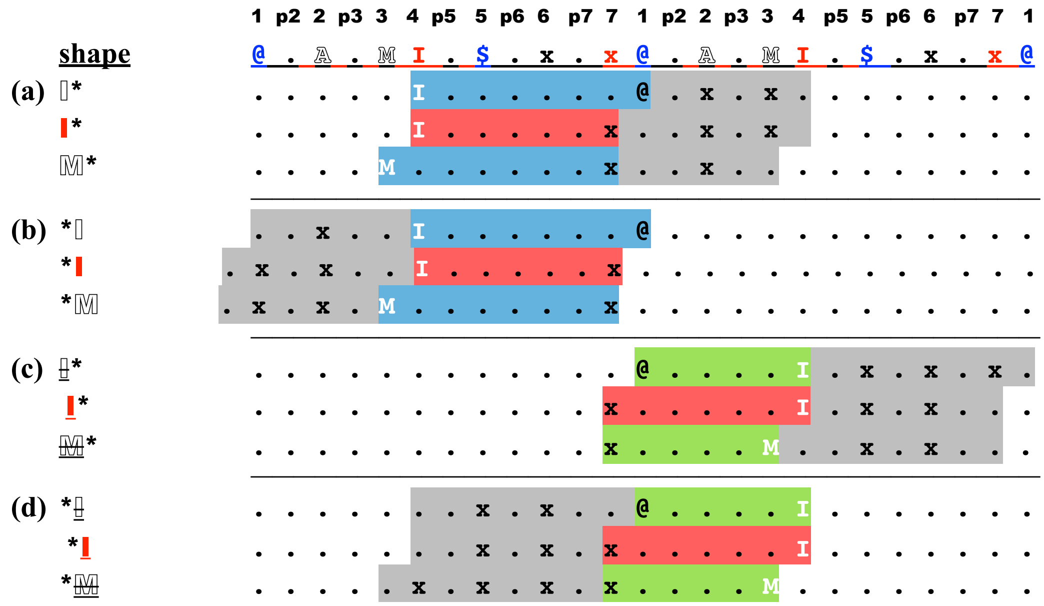
Example (c) is particularly simple: move the middle note down a scale step and then move all notes down a scale step. The shapes in the last two steps are "all fourths" (counting the tritone as an augmented fourth). These are far from the only possible variations but they give a good sense of the often deceptively complex nature of chord progressions.

The double-anchor notation covers the 3-note shapes described earlier: simply omit the top note of the upper building block. That said, it's simpler to work with core building blocks and offsets from them if 3-note shapes are satisfactory.

Octave Shapes

Octave shapes provide a simple way transforming thin core harmony into 3-note shapes with all notes different.

There are four different ways of forming octave shapes from a core. Start with the core un-inverted (a, b) or inverted (c, d) and add a voicing note to complete an octave, on the side indicated by the



asterisk prefix or suffix. Then move the voicing note to a selected scale note closer to the core (the likeliest choices are marked by x in the grey-shaded areas) to form a 3-note shape that goes well with the melody line.

The octave shapes of (a) and (d) or of (b) and (c) have the same notes an octave apart and so are harmonically equivalent. The difference is in the final, less-than-octave shapes. The asterisks could be replaced by numeric offsets as illustrated earlier (e.g., ▌* of (a) could become ▌3 or ▌5) but this is often overly precise because choices are few and tend to be obvious from context.

Playing shapes of this kind is a standard keyboard exercise for beginners but the ability to write them down in this simple way as shapes that voice chords without using chord or note symbols is novel. Voicings of a wide variety of chord progressions with the same core are easily created this way.

This picture is for single-tritone classical modes but the concept is the same for multi-tritone modes. The details are different because multiple tritones provide a more definite core that requires fewer core fifos to complete (sometimes none). The tritones overlap, which suggests, going forward, voicing them on opposite sides for smooth flow.

DIFFERENT WAYS OF PLAYING HARMONY

Chords that seem very different from their chord symbols may be hardly different at all in the flow of harmony on the keyboard. A dominant seventh chord such as *V-7* and a half-diminished seventh chord such as *V-m7b5* (or a *II-m6* chord with the same notes) differ only in one note that amounts to using an overlapped fifth or fourth to complete the chord. The alteration of the chord symbol is like banging a square peg into a round hole — the result is messy. This example is from the same classical mode. Things only get messier with more general scales.

Early in my musical adventure, I learned about simple "open" voicings of complex chords in a "piano comping" course given by jazz pianist Susan Muscarella at the Jazz School in Berkeley (now the Jazz Institute). The voicings were based on chord progressions that were complex to begin with, making figuring out the voicings in terms of reorganized degree numbers a complex process with difficult-to-remember results.

At the time, jazz pianist Taylor Eigsti was an artist in residence at the Jazz school who, I heard, was teaching a chord-symbol-free way of creating voicings based on representing chords as “scale shapes” determined by counts of scale steps between adjacent chord notes. This resonated with my developing ideas about building blocks, so I contacted him to learn about his method. I learned that he recommends, to beginning jazz piano students, a practicing regime of moving scale shapes of chords to different positions in and between classical modes, without reference to chord symbols. The objective is developing the instinctive moves required of jazz pianists. Although the scale shapes are fixed, the keyboard shapes are “wobbly,” requiring adjustment of the fingers to play. This combined with lifting all the fingers off the keyboard and moving them by often largish intervals to other keyboard positions makes this style of playing difficult. The method has no notation and is learned only from exercises.

I thought there must be a simple way of notating these approaches in common terms. After much experimentation, I found PKP. The big difference from what I learned from Susan Muscarella is core-based representations of chords that are independent of chord symbols (apart from identifying tritones from the chord table in Appendix C).

The big difference from Taylor Eigsti’s method is a notation for “wobbly” shapes on the keyboard. Given that these shapes are the general case, it seems useful to start out thinking in their terms, using a notation that represents them.

The examples so far have all been from the Ionian mode but the concepts are general. The general case is actually simpler because more tritones identify more scale notes.

CHAPTER 5: ADVANCED EXAMPLES

This chapter explores a smorgasbord of example pieces that I found difficult to understand when I first encountered them in music notation. The idea of a smorgasbord is to offer something for everyone. Digging into one or two selected pieces in is sufficient to internalize the concepts; the remaining pieces are available for interest. The harmony isn't itself novel, the only novelty is in the unconventional representation. The first example is the blues by Mingus with the footnoted complex chord progression presented in the opening chapter. The second example is a classical piece by Debussy that uses the same blues scale. All the examples are independent of each other and so may be approached in any order.

The examples are intended to be understood by playing them, not just looking at pages, and so should be approached in front of a piano on which they can be tried. Start with the melody line and the tritone core to establish basic flow. Play the tritone core as octave shapes first. Then play the actual shapes shown, including interpolated fifo shapes. The simplicity of finding flow "bottom up" from core harmony is in stark contrast to the complexity of finding it "top down" from chord symbols.

NON-CLASSICAL DOMAIN : "GOODBYE PORKPIE HAT"

This blues in Eb is a poster child for chromatic music that's difficult on the page for all but experts (perhaps even for them) and yet is very simple on the piano. My source for this piece is *Mingus Fakebook*, Hal Leonard (1991). Trying to learn this piece from this source was one of the stimuli that sent me down the path to PKP. The origin of the complexity (illustrated by a footnote in Chapter 1) is twofold: an imaginative chord root line by a creative bassist; and the addition of blues melody notes to chord symbols defined by classical modes (to give an indication of the melody scale not needed with PKP).

The simplicity on the piano begins with a simple melody line that's mostly in either the all-black-key pentatonic minor mode of the home tonic (**1=**Eb) or its 6-note minor blues extension that adds one white key (**p5=**A); the few deviations from these scales in bars 6-7 are highlighted in yellow (the **7.p2** notation means the first note is "crushed" into the second note on the same beat).

The melodic simplicity continues with smoothly flowing core harmony that's mostly tritones or double tritones from the blues family scale **//DM.L**. The triangular brackets around a fifo anchor means the size is to left open, to be defined by the flow. The symbol **LL** identifies octave shapes centered on the tritone anchor, that announce and terminate the ornamental segment in bars 6-7. The next page provides a Lego-like view of the harmony.

⚓ | **M** **L** | ⟨▯⟩ **LM** | ⟨▯⟩ **L** | ⟨▯⟩ **M** |

♫ **5|↗1↗p3↘1↗p3↗4↘p3, ↘1↘p7|↗1↗p3↘1↘p7↗1, ↘5|↗1↗p3↘1↗p3↗p5↘p3, ↘1↘p7|↗1↗p3↘1↘p7↗1, 1|**

A

⚓ | ⟨▯⟩ **LL** | **AI** | **PD** | **LL** ⟨▯⟩ |

♫ **|↗5↗p7↘1↗p3↗4↘p3, ↘p7↗p3|↗5↗1↘1↗p3↗p7↘5↘p7↘p6, |↗p3↗p5↘7.p2↘7↗p5↘3|↗p3↘1, ↗p5↘4|**

⚓ | **M D** | **I** **L** | **M L** | **P** **LM** |

♫ **|↘p3↘1, |↗4↘p3↘1↘p7↗p5↘4↘p3↘1|↘p3↘1, |5↘4↘p3↘1↘p7↘5|**

The following Lego-like view of the harmony adds voicing notes. Asterisks mark the side on which to place them, leaving the actual notes open (the asterisks are not shown in the anchor line for presentation simplicity). This is very easy to play — mostly hold two notes and move the other.

The complex written chord progression voiced by the these shapes is shown on the right (omitted roots are shown by dashes in the shapes). Any omitted notes are picked up in the flow. The chord root line can provide a bass line underneath if desired, but the melody line with this harmony conveys the haunting sound of the piece very effectively without it.

```
                    1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
                    @ .  A D  M I L  $ .  x x  . @
1   *M              @ . . . M . . o . . x . .        I-7#9
    L*              @ . . . . . L . - . x . .        pVI-9(13)
2   ⟨▯⟩             @ - . . . I . . . . x . .        pII-M9
    LM              @ . . . M . L . . . x . .        pV-7#11
3   ⟨▯⟩             @ . - . . I . . . . x . . .      II-9sus
    L*              @ . . . . . L . - . x . .        pVI-9(13)
4   ⟨▯⟩             @ . - . . I . . . . x . .        II-7sus
    *M              @ . . . M . . . . . x . .        I-7
5   ⟨▯⟩             @ . . . . I . . . . x . .        IV-m11
    LL              @ . . . . . L . - . . . x        pVI-7(13)
6   AI              . . A . . I . . x . . x .        II-m7b5 + V-7#5#9
7   PD              . P . D . . . $ . x . . .        VI-13#11 + II-7(13)
8   LL              @ . . . . . L . - . . . x        pVI-7
    ⟨▯⟩             @ - . . . I . . . . x . .        pII-M7
9   *M              @ . . . M . - . . . x . .        pV-7(13)
    *D              @ . . D . - . . . x . . .        IV-7
10  *I              @ . . . . I . - . . . x .        V-7
    L*              @ . - . . . L . . . x . .        II-7
11  *M              @ . . . M . . . . . x . .        I-7#9
    L*              @ . . . . . L . - . x . .        bVI-7
12  P*              . P . . . . . $ . . . x .        pII-M7#11
    LM              @ . . . M . L . . . x . .        bV-7#11
```

NON-CLASSICAL DOMAIN : "PRELUDE TO AN AFTERNOON OF A FAUN"

This and the previous piece make strange bedfellows. The thing they share is use of the same blues scale. I was motivated to investigate this piece by things said about it in an article about Debussy in the New Yorker: *Beauty in the Void,* Alex Ross, The New Yorker, Oct. 29, 2018. My source for the written music is the *Classical Fake Book*, 2nd Edition, Hal Leonard (2013), page 222. The key signature is 5 sharps and the home tonic is C#, identified by the closing note of the melody line. It's in PKP's "non-classical domain" because of the use of the blues scale for melody.

In the skeleton melody line, the double arrow at the end of A means jump two octaves. The notes highlighted in yellow are ornamental passing notes because they're in the middle of scale sequences going in the same direction, which can be included or skipped without materially affecting the sound (if skipped, hold the previous note or anticipate the next one to preserve the timing).

A ♫ |**1 —↘p7↘6↘p6↘5↘p5↗5↗6↗p7↗7**|↗**1 —↘p7↘6↘p6↘5↘p5↗5↗6↗p7↗7**|
♫ |↗**1**↗**2**↗**5**↘**p3**↘**5**↘**p7**|**p7-p7**↗**1**↘**6**↗↗**5**|
B ⚓ | ℙ **PM** | ℙ **PM** |
♫ |**1 —↘p7↘6↘p6↘5↘p5↗5↗6↗p7↗7**|↗**1 —↘p7↘6↘p6↘5↘p5↗5↗6↗p7↗7**|
⚓ | **AI** **M** 𝔻 | **AI** | 𝔸 𝕄 𝕀 |
♫ |↗**1**↗**2**↗**5**↘**p3**↘**5**↘**p7**| — **p7**↗**1**↘**6**| — ↗**1**↗**2**↘**7** —↘**6**↗**3**|
C ⚓ | 𝔻 **M**| ℙ 𝔻 | ℙ **AI** |
♫ |↘**5**↗**6**|↗**p7** —↘**p6**↘**3**↗**5**↗**p7**↗**1**|↘**6**↘**5**↘**3**↗**5**↗**p7**↗**1**|
⚓ | | 𝔻 |
♫ |↘**5**↗**p7**↗**1**↘**5**↗**p7**↗**1**↘**5**↗**p7**↗**1**|↘**5**↗**p7**↗**1**↘**5**↗**p7**↗**7**|↗**1** —|

Condensing all the melody notes into the home octave reveals the **//DM.L** scale. The repetitive, sparse melody line at the end of C is from the parallel pentatonic minor mode that's a sub-mode of it.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
//DM.L	@	.	x	x	x	x	x	$	.	x	x	.	@	**blues family scale**
									+			+		**passing notes**
//𝔻	@	.	.	x	.	x	.	$	.	.	x	.	@	**pentatonic minor**

There's no harmony in A. The harmony elsewhere is only for blues-scale notes, never passing notes. Striking features of the harmony are its similarity in function to a blues: it punctuates points where the melody changes direction (e.g., ℙ and **PM** in B); the elements that provide dissonance relative to the melody line are often outside the blues scale (e.g., **AI**), and are therefore ornamental (no scale implications); tritones establish the flow and fifos are faired in between them.

The simple harmony is shown next. Harmonized melody notes **1-p3-p5-5-6-p7-1** are shown on the right to provide context.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	.	A	D	.	I	.	$	x	x	.	x	@	
B 1	P	.	x	.	.	.	.	.	.	x	.	x	.	.	1
	PM	.	x	.	.	x	.	.	x	.	.	x	.	.	p5
2	P	.	x	.	.	.	.	.	.	x	.	x	.	.	1
	PM	.	x	.	.	x	.	.	x	.	.	x	.	.	p5
3	AI	.	.	x	.	.	x	.	.	x	.	.	x	.	1
	M	.	.	.	.	x	.	.	.	.	.	x	.	x	5
	D	.	.	.	x	.	.	.	.	.	.	x	.	x	p3
4	AI	.	.	x	.	.	x	.	.	x	.	.	x	.	p7
5	A+	.	.	x	.	.	.	.	.	.	.	x	.	.	6
	M+	.	.	.	.	x	.	.	.	.	.	.	.	x	2
	I	.	.	.	x	.	x	.	.	.	.	.	.	x	6
C 1	D	.	.	.	x	.	.	.	.	.	.	x	.	.	5
	M	.	.	x	.	x	.	.	.	.	.	x	.	.	6
2	P+	.	x	.	.	.	.	.	.	.	x	.	.	.	p7
	D	.	x	.	x	.	.	.	.	.	.	x	.	.	6
3	P+	.	x	.	.	.	.	.	.	.	x	.	.	.	6
	AI	.	.	x	.	.	x	.	.	x	.	.	x		5
4	-	.	.	.	.	.	.	.	.	.	.	.	.	.	
5	-	.	.	.	.	.	.	.	.	.	.	.	.	.	
6	D	.	.	.	x	.	.	.	.	.	.	x	.	.	1

CHROMATIC CLASSICAL DOMAIN: “LAURA”

My source for this piece in C is *The Jazz Book*, John Brimell, CPP/Belwin, 1989, p24. The book is sub-titled *Today's Easy Adult Piano*, but the piece is easy in music notation only in the sense that the page is not black with notes. The changes involve multiple implicit key signatures. Some of the melody lines are ambiguous, requiring some creative guessing to interpret.

(a) ⚓ | | LD | | |

♫ | 7* | — ↘p7↗7↘p7↘p5↗5 | ↗6*↘3 — | — | ...//L

(b) ⚓ | | PM | | D |

♫ | ↗6*| — ↘p6↗6↘p6↘3↗4 | ↗5 — | — | ...//M

(c) ⚓ | AI | I | | | D DL |

♫ | — ↘p3↗4 | — ↘p3↗4↘p3↗4 | ↗5*↘2 — | — ↗p3↗5↗p7 |↗p3*↘p6 — | ...//A

(d) ⚓ | LA | L | AI |

♫ | — ↗6 | ↗2 — | — ↗3↘1↗2 — | ... //𝕄

(e) ⚓ | | AI | L | |

♫ | — ↗4↗5 | — ↘3↗4↗p6↗7 | ↗2↘1↗2 — | — ↘7↗1| ...//AI

(f) ⚓ | DL AI | AI | | |

♫ | ↗p3 p3 | — ↗3↘p3↘2↘6↗7 | ↗1 — | — | ...//I

When I first encountered this piece, I struggled with trying to understand how the harmony implied the melody and finally realized I had it backwards. The melody comes first and the harmony is an add-on. The melody interpretation is shown next.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
(a) ambiguous melody	.	.	.	.	x	.	x	x	.	x	x	x	.	
scale interpreted as //L	@	.	x	.	x	.	x	$	.	x	+	x	@	//L@5 is //I transposed up a fifth
(b) ambiguous melody	.	.	.	.	x	x	.	x	x	x	.	.	.	
scale interpreted as //M	@	.	x	.	x	x	.	$	+	x	x	.	@	//M@4 is //I transposed up a fourth
(c) melody is //A	@	.	x	x	.	x	.	$	x	.	x	.	@	//A@p3 is //I transposed up a min. third
(d) ambiguous melody	x	.	x	.	x	.	.	.	.	x	.	.	x	
scale interpreted as //𝕄	@	.	x	.	x	.	.	$	.	x	.	.	@	pentatonic major goes to (a) or (e)
(e) melody is major //AI	@	.	x	.	x	x	.	$	x	.	.	x	@	harmonic major
(f) ambiguous melody	@	.	x	x	x	.	.	.	.	x	.	x	@	
scale interpreted as //I	@	.	x	+	x	x	.	$	.	x	.	x	@	//I

Without even thinking about specific scales, the visible transposition down a whole tone of the first seven melody notes of (a) to the first seven notes of (b) suggests a very simple pattern of same-mode transpositions down by whole tones extending from (a) to (c) (highlighted in grey). The home-tonic mode of (c) is unambiguously Aeolian **//A**, suggesting home-tonic modes **//L-//M-//A** for (a)-(b)-(c), with tritone anchors going down by whole tones. The changes provide same-mode (Ionian) tonic changes **5-4-p3** down by whole tones. The highlighted passing notes are beautifully consistent:

they're a minor third up from these Ionian tonics; in the case of (a) and (b), they're borrowed from the next mode in the succession. Understanding these Ionian tonic changes is conceptually useful, but the parallel mode changes provide all the notes.

The harmony sounds good but is only loosely related to the melody modes in which it appears. The harmony is dominated by nine instances of the three double tritones **DL**, **PM** and **AI**. These are the only three diminished seventh chords that exist: the four possible inversions select different notes as roots but the mirror symmetry of the shapes gives them the same form in any inversion (a stack of three minor thirds), and context decides the inversion. I read somewhere that these are chords of last resort — when you're unsure about an appropriate chord, use one of them.

The double tritones may be understood as cueing the melody modes as follows.

(a) **L** of **DL** cues **//L**

(b) **M** of **ML** cues **//M**

(c) **A** of **AI** cues **//A**

(d) the mixed harmony is purely ornamental

(e) **AI** cues major **//AI**

(f) **I** of **AI** cues **//I**

CHROMATIC CLASSICAL DOMAIN : “BODY AND SOUL”

This piece is strongly chromatic in a way that’s particularly complex in music notation. My source for the written music is *The Ultimate Jazz Fakebook*, Hal-Leonard (1988), p67. Three successive sections have three different key signatures: A 5 flats (Db-Ionian); B 2 sharps (D-Ionian); C 1 flat (D-Aeolian, but actually Dorian due to a natural in the written melody line). Ionian and Dorian of tonic D are nearby parallel modes. Ionian of Db is a distant mode with many notes different, one of which is the tonic Db, which doesn’t appear as melody note in either of the other two sections. It could be argued that there are two different home tonics, namely Db and D. However, that’s not the philosophy of PKP: a single home tonic enables harmony of the different sections to be understood in the same terms.

D is a good choice for the home tonic because it makes B and C that end the piece on the page visibly simple. Given D is **1**, Db of A is **7**, making its melody line look less simple, but there’s good news. The Ionian mode of tonic **7** is **//I** of the home tonic transposed down a half tone, which is simple in chromatic-scale terms: every note moves down a half tone (or up a half tone in playing order). Keep in mind that this is not the same as the relative mode **//I@7**, which is Locrian (the transposed Ionian mode is actually **altM@7**).

The simple melody line is from a succession of three classical modes identified on the right.

A ⚓ (L) M (A) (AI)

♫ | p2↗p3↘p2↗p3↘p2|↗p6 p6 — , |↘p3↗p6↘p3↗p6↘p3| ↗p2↘7↘p7↘p6, |

⚓ (P) M M (AI) L

♫ |↗7↘p6↘3↘7|↗p3↘p2, |↘7↗p2↗p3↗p5 p5↗p6↘2 |↘7— |↘7— | ...altM@7

B ⚓ (A) I

♫ |↗1↗2↗3↗5 5 5|↗1 1↘3↗5 5↘4|↘3 3↘1↗2↘7↘5|5 — | ...//I

C ⚓ D (PM) D D (A P L LA)

♫ |↗1↗2↗p3↗5 5 5|↗2 2↘6↗1 1↘5|↗p7 p7↘5↗6 6↘4|↘2↘p2↘1↗p6| ...//D

The elements of harmony in parentheses are ornamental. When I first encountered this piece, I struggled with trying to understand how this harmony implied the melody and finally realized I had it backwards. The un-parenthesized parts of the harmony follow from the melody in an analogous manner to *Happy Birthday*. The whole tone scale **A.M.L** provides many of the ornamental elements of in A. The double tritones **AI** and **PM** are diminished seventh chords that originate in no classic mode, but are often used for harmonic ornamentation in classical-mode domains (only three of these chords with different notes exist).

CHROMATIC CLASSICAL DOMAIN : “GIANT STEPS”

My source for Coltrane’s jazz classic is *The Real Book*, 6th Edition, Hal-Leonard. It cycles rapidly through 3 Ionian modes of tonics G, Eb and B that are a major third apart and differ by 4 notes. The F# melody line is a sequence of short segments from these modes that combine into 9-note scale that’s very far from a classical mode and too special to be of general use. That said, the piece is easily understood in terms of classical modes relative to the F# home tonic, as shown next. All the notes identified by this view are as written. The highlighted melody segments identify two parallel classical modes that determine the melody line. The only tritone of the three that’s actually present in the melody line is **M** (tritones **D** and **I** are also present incidentally, not as elements of a generally useful mode signature).

⚓ | ~~M~~ L | I A | P | L L | I A | P M | ~~M~~ |
🎵 | 1↘p6 | ↘4↘p2↗3 | 3, | ↘4↘p3, | ↗p6↘3 | ↘p2↘6↗1 | 1, |
⚓ | A | P | L | I | M | ~~M~~ | A | P | M |
🎵 | ↗p2↘7↗3 | 3, | ↗4↘p3↗p6 | p6, | ↗6*↗1 | 1, | ↗p2*↗3 | 3, | ↘1* |

Playing the melody requires tracking 2 modes, identified by highlighting, through 6 relatively infrequent changes.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
alt-L	x	x	.	x	.	x	x	.	x	.	x	.	x	**provides melody segments**
alt-A	.	x	x	.	x	.	x	.	x	x	.	x	.	**provides melody segments**
//M	x	.	x	.	x	x	.	x	.	x	x	.	x	**provides tonic & major tonality**
melody scale	x	x	.	x	x	x	.	.	x	x	x	x	x	**composite of melody segments**

The simple tritone progression shown next for bars 1-7 provides a framework that determines fifos morphed into or from the tritones, without thinking about the harmony scales or the chords from them on the right. Arrows on the left mark the tritones that identify the melody modes.

			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		
			@	P	A	D	M	I	L	$	x	x	x	x	@		
	1	~~M~~	.	.	.	.	M	–	.	.	.	x	.	.	.	*BM7*	*(IV)*
–>		L	.	.	.	.	.	.	L	.	–	.	.	.	x	*D7*	*(pVI)*
	2	I	.	–	.	.	.	I	.	.	.	.	.	.	x	*GM7*	*(pII)*
		A	.	.	A	.	–	.	.	.	x	.	.	.	.	*Bb7*	*(III)*
	3	P	.	P	.	.	.	.	.	.	x	–	.	.	.	*EbM7*	*(VI)*
	4	L	.	x	.	–	.	.	L	.	.	.	.	.	.	*Am7*	*(pIII)*
–>		L	x	.	.	.	.	.	L	.	–	.	.	.	.	*D7*	*(pVI)*
	5	I	x	–	.	.	.	I	.	.	.	.	.	.	.	*GM7*	*(pII)*
–>		A	.	.	A	.	–	.	.	.	x	.	.	.	.	*Bb7*	*(III)*
	6	P	.	P	.	.	.	.	.	.	x	–	.	.	.	*EbM7*	*(VI)*
		M	–	.	.	.	M	.	.	.	.	.	x	.	.	*F#7*	*(I)*
	7	~~M~~	.	.	.	.	M	–	.	.	.	x	.	.	.	*BM7*	*(IV)*

The piece offers many scales on which to base improvisations, for example: the whole tone scale defined by the tritone cluster **AML**; the **//M** mode; the blues scale identified by **//DM.L**, and its sub-scales.

NON-CLASSICAL DOMAIN: WHEN SUNNY GETS BLUE

My source for this piece is sheet music by Jack Segal and Marvin Fisher from Hal-Leonard (1956), numbered HL00351105. The melody line from this source is a **//DM.L** blues in G, with a 4-bar section in the bridge that visits **//L**. The harmony shown below is a modification of the written harmony, which is a mix of fifo and tritone chords that sound fine but are confusingly irregular. Ornamental touches are highlighted in yellow — the ones in the melody line are from the written music and the ones in the harmony are part of the reharmonization.

⚓ | **M** **D** | **L** **D** | **M** **D** **L** | **I** |
♫ 4 ||: ↗2↘1↗2↘4, — 4 | ↗4↘p3↘p7↘p5↗4↘p3 | p3, ↘2↘1↘p7↘6, ↘5↗1 | ↘6, — |
⚓ | **M** **D** **A** | **P** **L** **I** |
♫ | ↘5↗p7↗1↘p7, ↗p2↘p7↗1↘p7, | ↘4↗p7↗1↘p7, ↗p2↘p7↗1↘p7, |
⚓ | **M** **D** | **L** **M** **D** | **M** **L**
♫ | ↘5↗p7↗1↘p7, ↗4 — ↘p3↘p7 | ↗2 — — —, ↘4 :|| ↗2 — |

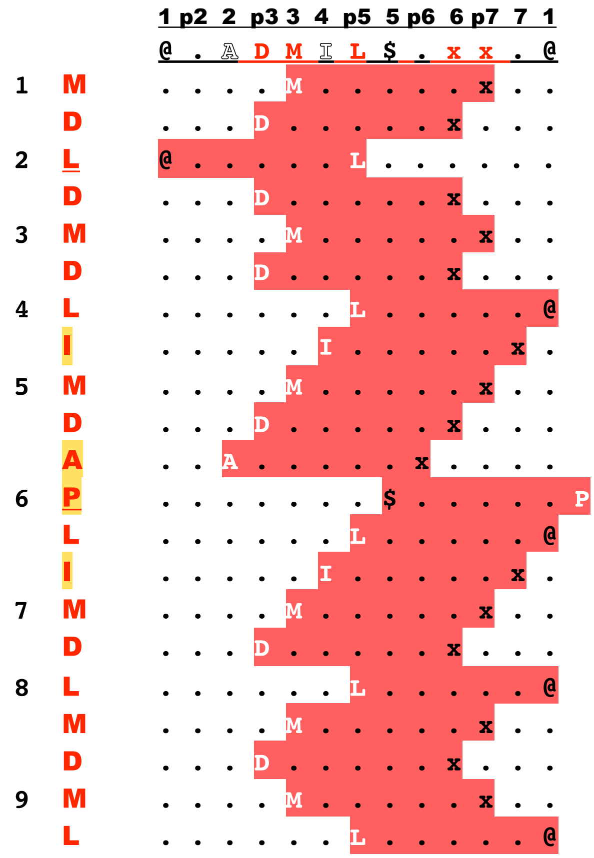
⚓ | $ ~~$~~ | **L** ~~L~~ | $ $ ~~$~~ | **L** ~~L~~ |
♫ | ↘7↗1↘7↗1, ↗2↗3 | ↗p5 ↘2*, 2↘7*, | 7↘5↗6, ↗7↗1 | ↗2 — | **...//L**
⚓ | ~~I~~ **M** | ~~M~~ A | ~~A~~ | **L** **M** **D** |
♫ | p7↗1↘p7↗1↗2 — ↗p3 | ↗3↘1*, 1↘6*, | 6↘4↗5 ↗6↗p7 | ↗1 — —, ↘4 |

repeat bars 1-6, followed by this ending, repeated twice

⚓ | D **D** | ~~A~~ **D** ~~A~~ **D** |
♫ | ↗2↗4↗5↘4, ↗p6↘4↘p2 | ↘1 — | 1 — |

Play the harmony in the first instance as octave shapes by doubling the treble line of the core an octave down. The 5-tritone runs down by half tones in bars 4-5 and 6-7 are very easy to play and sound good with the melody line. They include ornamental tritones **I**, **A** and **P** that are easily morphed into in-scale fifos, if desired.

The Lego-like harmony is shown next. The harmony of the first four bars of the bridge is in the **//L** mode (Lydian). The final four bars of the bridge return to the original blues scale. The transition between them in bars 13-14 is marked by the distinctive sound of a fourth-fourth slide down a half tone.


1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
@ . A D M I L $. x x . @
1 M
D
2 L
D
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4 L
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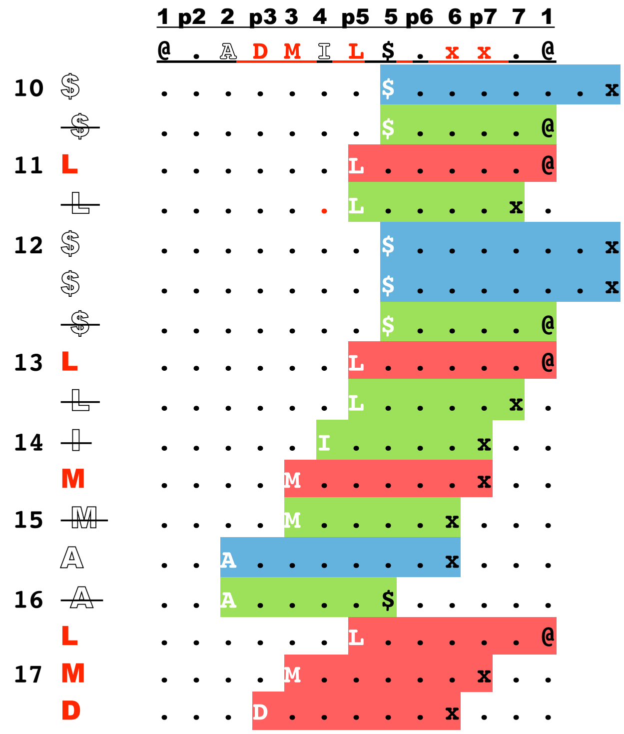
5 M
D
A
6 P
L
I
7 M
D
8 L
M
D
9 M
L


1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
@ . A D M I L $. x x . @
10 $
$
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L
12 $
$
$
13 L
L
14 I
M
15 M
A
16 A
L
17 M
D

NON-CLASSICAL DOMAIN : “ALL OF ME”

I learned this version of *All of Me* in C (along with *No Greater Love* in Bb coming up) in Susan Muscarella’s piano comping course some years ago at the then Jazz School in Berkeley (now the Jazz Institute), as an example of using 4-note “open” voicings of extended and altered seventh chords to accompany soloists. The chord progression is strongly chromatic and therefore complex in chord symbol terms. The open voicings rearrange the chord notes and sometimes omit the roots. The standard way of explaining such voicings is by rearranged degree numbers of chord scales, in which the same notes from adjacent chords have different degree numbers relative to different roots that are sometimes omitted. Such representations are indirect relative to the keyboard, difficult to comprehend as a whole, or remember, and difficult to annotate on the written music because they require too many symbols. This complexity goes away when the shapes are understood in terms of core building blocks.

The skeleton melody line and harmony anchor line are shown next. Trying to understand this as a succession of classical modes bogs down in complexity. The simplest way of remembering it is this: melody and harmony are both from the 9-note, major family mode **A.MI⁺** (**//I** plus notes **p6** and **p7**), with the passing ornamentation shown highlighted in yellow. The two appearances of minor note **p3** in the melody line are passing notes because omitting them has no substantive effect on the sound. The appearances of **P** and **L** in the harmony are ornamental substitutes for **A** and **I** because substituting the latter for them sounds fine.

⚓ | M I | M | A M | | P | P M | P | $ |

♫ | 1↘5↘3 — | 3 , ↗1↗2↘1 | ↘7↘p6↘3 — | 3 , | ↗6↘5↘3 — | 3 , ↘p3↗3↗p7↘6 | ↘5 ↘4|4 , |

⚓ | A | A | A | A | $ | L | ▯ | I |

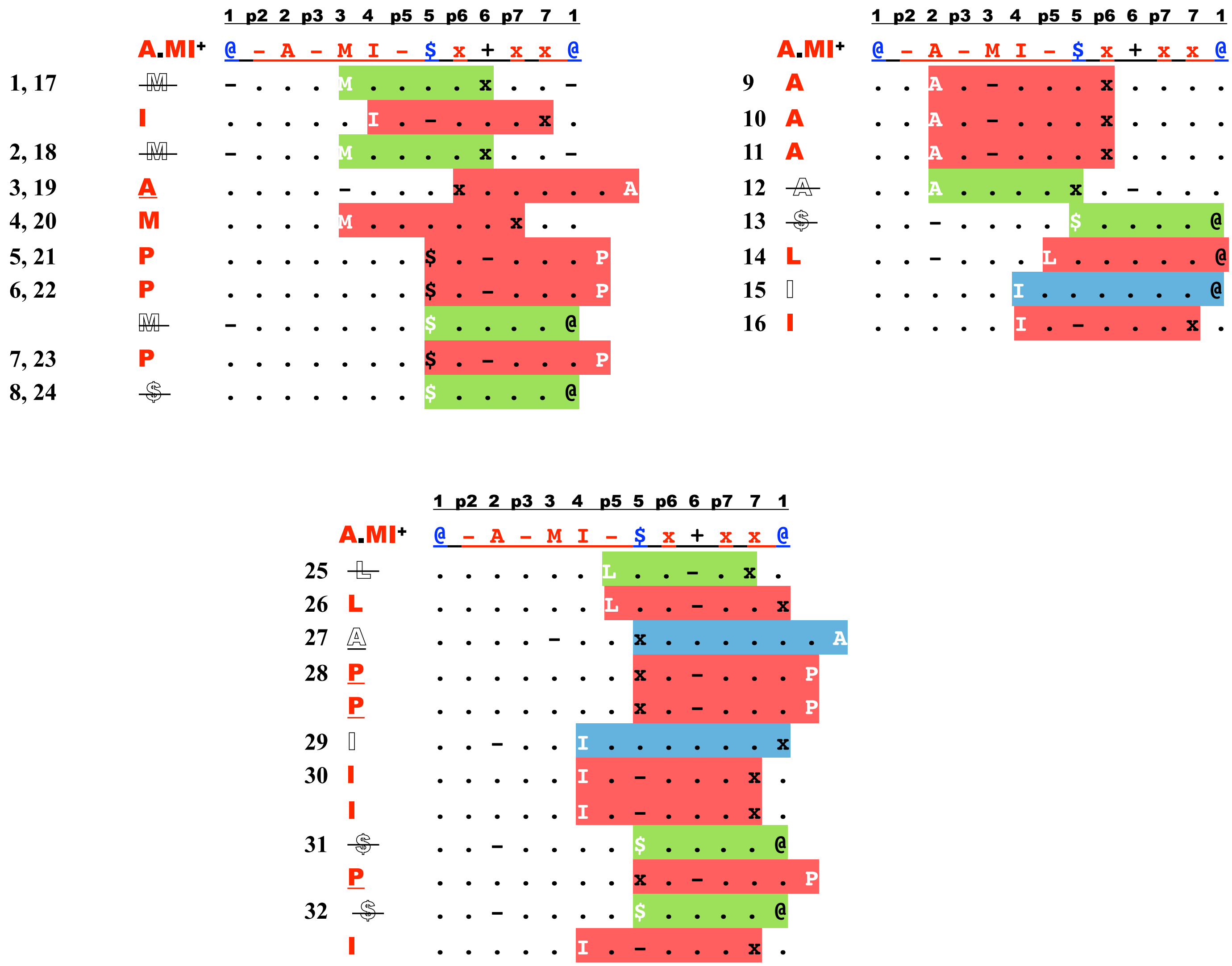
♫ | ↘3↘p3↘2— | 2, ↗3↗p6↗7 | ↗2 — ↘1 — | 1 , |↘7↘p7↘6 — | 6, 6↗2↘7 |↘6 — |↗7, |

repeat first 8 bars and then finish with this

⚓ | L | L | A | P P | ▯ | I I | $ P | $ I |

♫ |↗2—↘1↘7 | ↗2 , ↘1 |↘7 — ↘3↗5 |↗7 , ↘6 |↗1 — ↘6↗1 | ↗3 3 | ↘1 — | — — |

The tritone core provides a framework for fairing in core fifos without reference to anything else. The next picture shows what this looks like in terms of successive building blocks on the keyboard. This intended to represent a picture in the mind’s eye implied by the above, not something to be written down. Play this harmony in the first instance as octave shapes formed by doubling the treble line of the core an octave down. The only exception is bars 3-4 in which the tritones establish outer notes a minor seventh apart that may be held for both, only moving the middle note.



Four-note "open" voicings developed from the same core are shown next. This is intended to represent a picture in the mind's eye implied by the notation in the below left column (which would be annotated above the staff). The shapes are voicings of the chords shown on the right. Chords on roots I, II and V are from the Ionian mode. The other chords (highlighted in yellow) provide visibly simple segues between the Ionian voicings. The chord symbols are all satisfied serially in the flow. This is remarkably simple, conceptually, compared to the standard way of understanding the same thing in terms of degree numbers of chord scales. The four notes are difficult to play with the left hand, but easy-to-play 3-note voicings of the same chords follow from omitting the top notes; the top notes can be added by the left hand under the melody line.

Bars 1-8

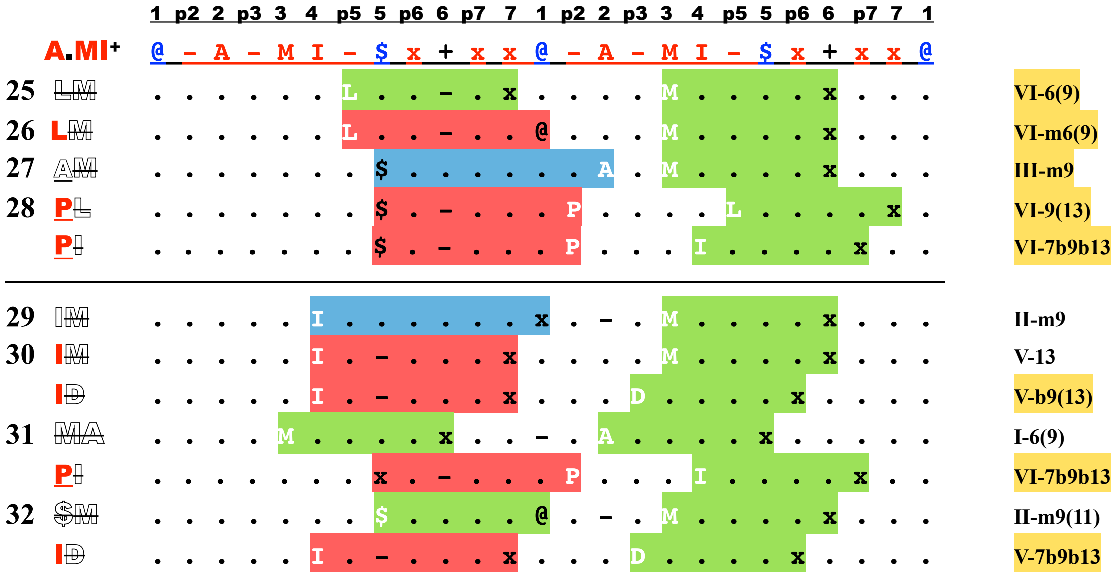
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	A.MI+	@	–	A	–	M	I	–	$	x	+	x	x	@	–	A	–	M	I	–	$	x	+	x	x	@	
1	MA	.	.	.	.	M	.	.	.	.	x	.	.	–	.	A	.	.	.	.	$	.	.	.	.	.	I-6(9)
	IM	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	.	M	.	.	.	.	x	.	.	.	V-9(13)
2	MA	.	.	.	.	M	.	.	.	.	x	.	.	–	.	A	.	.	.	.	$	.	.	.	.	.	I-6(9)
3	LA	.	.	.	.	–	.	L	.	.	.	.	x	.	.	A	.	.	.	.	.	x	.	.	.	.	III-9
4	MA	.	.	.	.	M	.	.	.	.	.	x	.	.	.	A	.	.	.	.	.	x	.	.	.	.	III-9b5
5	PL	.	.	.	.	.	.	.	x	.	–	.	.	.	P	.	.	–	.	L	.	.	.	.	x	.	VI-9(13)
6	P⟨I⟩	.	.	.	.	.	.	.	x	.	–	.	.	.	P	.	.	.	I	.	.	.	x	.	.	.	VI-7b13
7	$M	.	.	.	.	.	.	.	$	.	.	.	.	@	.	–	.	M	.	.	.	.	x	.	.	.	II-m9
	P⟨I⟩	.	.	.	.	.	.	.	x	.	–	.	.	.	P	.	.	.	I	.	.	.	x	.	.	.	VI-7b13
8	$M	.	.	.	.	.	.	.	$	.	.	.	.	@	.	–	.	M	.	.	.	.	x	.	.	.	II-m9

Bars 9-16

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	A.MI+	@	–	A	–	M	I	–	$	x	+	x	x	@	–	A	–	M	I	–	$	x	+	x	x	@	
9	AM	.	.	.	.	.	.	.	.	x	.	.	.	.	.	A	.	M	.	.	.	.	x	.	.	.	III-7
10	AI	.	.	.	.	.	.	.	.	x	.	.	.	.	.	A	.	.	I	.	.	.	.	.	x	.	III-7b9
11	$M	.	.	.	.	.	.	.	$	.	–	.	.	@	.	.	.	M	.	.	.	.	x	.	.	.	VI-m7
	AI	.	.	.	.	.	.	.	.	x	.	.	.	.	.	A	.	.	I	.	.	.	.	.	x	.	III-7b9
12	$M	.	.	.	.	.	.	.	$	.	–	.	.	@	.	.	.	M	.	.	.	.	x	.	.	.	VI-m7
13	$M	.	.	.	.	.	.	.	$	.	.	.	.	@	.	–	.	M	.	.	.	.	x	.	.	.	II-9sus
14	LM	.	.	.	.	.	.	L	.	.	.	.	.	@	.	–	.	M	.	.	.	.	x	.	.	.	II-9
15	IM	.	.	.	.	.	I	.	.	.	.	.	.	@	.	–	.	M	.	.	.	.	x	.	.	.	II-m9
16	IM	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	.	M	.	.	.	.	x	.	.	.	V-9
	ID	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	D	.	.	.	.	x	.	.	.		V-7b9b13

Bar 17-24 - repeat bars 1-8

Bars 25-32



Understanding how the shapes satisfy the chord symbols requires knowing the relationship between the numeric suffixes of the chords and the chromatic scale. I explain this only to make clear that the shapes satisfy the written chords, not to suggest figuring the shapes out this way. In bars 3-4, chord suffix “9” is chromatic scale note **p5** and chord suffix “b5” is chromatic scale note **p7**. The 3-note voicings in bars 3-4 don’t include **p5** but it’s included in bar 5, thus satisfying the bar-4 chord in the flow. The bar-4 shape jumps up a minor third in bar 5, with a wobble that puts a fourth on top instead of a major third. The wobble provides degree “13” of the chord. It also provides degree “9” of the bar-4 chord, thus satisfying two chord symbols at one stroke.

NON-CLASSICAL DOMAIN : "NO GREATER LOVE"

I learned this piece in Bb major in Susan Muscarella's piano comping course at the then Jazz School in Berkeley. The melody mode is major-family **//A.MI⁺** throughout, with two passing notes highlighted in yellow. The strongly chromatic harmony is in this mode, with the addition of ornamental tritones **P**, **D** and **L** that don't change the melody mode (this includes the altered harmony pattern in the bridge). The numeric suffixes in the anchor line identify voicing notes by the number of half tones they are above the core (i.e., above the anchored building block).

♫ 7↗1↗2 |

⚓ | ~~𝕄~~5 | D5 | A4 | P4 |

♫ | ↘1↘7↘6↘3 | ↗5↘p5↘4↘1 | ↗3 | 3 3↗7↘p7 |

⚓ | I5 | 𝕄5 | 𝕀5 | I5 |

♫ | ↘6 | 6 6↗3↘p3 | ↘2 | 2 , ↘7↗1↗2 |

repeat bars 1-4

⚓ | L4 | 𝕀4 I5 | ~~𝕄~~5 | D5 |

♫ | ↘6, 6↗3↘2 | ↘1 — ↗2 — | ↘1 , | — |

⚓ | A..I | $3 | I..A | $3 |

♫ |↗3↗p6↗7↗2 | ↘1, | ↘p6↗7↗2↗4 | ↘3, |

⚓ | I..A | $4 | L4 | I3 |

♫ | ↘3↗p6↗7↗2 | ↘1↘7↘6↘3 |↘2, — | — ↗7↗1↗2 |

---

repeat bars 1-4

⚓ | L4 | 𝕀4 I5 | ~~𝕄~~5 | 𝕀4 I5 |

♫ | ↘6*↗3↘2 |↘1—↗2— |↘1— , | — — |

Here's how to picture the harmony on the keyboard.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	
	A.MI+	@	.	A	.	M	I	.	$	x	+	x	x	@	.	A	.	M	
1	$\cancel{M}_5$	.	.	.	.	M	.	.	.	.	x	.	.	.	.	x	.	.	
2	D_5	.	.	.	D	.	.	.	.	.	x	.	.	.	.	x	.	.	
3	A_4	.	.	A	.	.	.	.	.	x	.	.	.	x	.	.	.	.	
4	P_4	.	P	.	.	.	.	.	x	.	.	.	x	.	.	.	.	.	
5	I_5	.	.	.	.	.	I	.	.	.	.	.	x	.	.	.	.	x	
6	M_5	.	.	.	.	M	.	.	.	.	.	.	x	.	.	.	.	x	
7	$\square_5$	.	.	.	.	.	I	.	.	.	.	.	.	x	.	.	.	.	x
8	I_5	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	.	x	
9-12 repeat 1-4																			
13	L_4	.	.	.	.	.	.	L	.	.	.	.	.	x	.	.	.	x	
14	$\square_4$	.	.	.	.	.	I	.	.	.	.	.	.	x	.	.	.	x	
	I_5	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	.	x	
15	$\cancel{M}_5$	.	.	.	.	M	.	.	.	.	x	.	.	.	.	x	.	.	
16	D_5	.	.	.	D	.	.	.	.	.	x	.	.	.	.	x	.	.	
17	AI	.	.	A	.	.	I	.	.	x	.	.	x	.	.	.	.	.	
18	$\cancel{\$}_3$	.	.	.	.	.	.	.	$	.	.	.	.	x	.	.	x	.	
19	IA	.	.	.	.	.	I	.	.	x	.	.	x	.	.	A	.	.	
20	$\cancel{\$}_3$	.	.	.	.	.	.	.	$	.	.	.	.	x	.	.	x	.	
21	AI	.	.	A	.	.	I	.	.	x	.	.	x	.	.	.	.	.	
22	$\cancel{\$}_4$	.	.	.	.	.	.	.	$	.	.	.	.	x	.	.	.	x	
23	L_4	.	.	.	.	.	.	L	.	.	.	.	.	x	.	.	.	x	
24	I_5	.	.	.	.	.	I	.	.	.	.	.	x	.	.	.	.	x	
25-28 repeat 1-4																			
29	L_4	.	.	.	.	.	.	L	.	.	.	.	.	x	.	.	.	x	
30	$\square_4$	.	.	.	.	.	I	.	.	.	.	.	.	x	.	.	.	x	
	I_5	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	.	x	
31	$\cancel{M}_5$	.	.	.	.	M	.	.	.	.	x	.	.	.	.	x	.	.	
32	$\square_4$	.	.	.	.	.	I	.	.	.	.	.	.	x	.	.	.	x	
	I_5	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	.	x	

A walking-bass-line version presented in class is translated here into chromatic scale notation. Highlighted out-of-context notes at the ends of bars are passing notes to in-context notes in the next bar. Directional arrows are not needed in the bass line because vertical positioning of the notes is determined by the melody line. The bass line rhythm is steady 4/4.

♫ 7↗1↗2 |

♫ | ↘1↘7↘6↘3 | ↗5↘p5↘4↘1 | ↗3 , — |— 3↗7↘p7 |

⚓ 1 5 1 3   4 1 4 7   p7 4 p7 4   6 3 6 p3

♫ | ↘6 , — |— 6↗3↘p3 | ↘2 , — |— ↘7↗1↗2 |

⚓ 2 6 2 p3   2 6 2 p2   2 6 2 p6   5 2 5 p2

<u>repeat bars 1-4 …</u>

♫ | ↘6, 6↗3↘2 | ↘1 ↗2 — | ↘1, — |— |

⚓ 2 6 2 p2   2 p6 5 7   1 3 4 p2   5 7 1 p6

♫ | 3↗p6↗7↗2 |↘1, — | ↘p6↗7↗2↗4 | ↘3, — |

⚓ 7 4 3 p6   6 p6 6 p7   7 4 3 p7   6 3 6 p7

♫ | ↘3↗p6↗7↗2 |1↘7↘6↘3 |↘2 , — |— ↗7↗1↗2 |

⚓ 7 4 3 p6   6 p6 6 p3   2 2 6 p7   5 7 2 7

<u>repeat bars 1-4 …</u>

♫ | ↘6*↗3↘2 |↘1—↗2— |↘1, — |— |

⚓ return to chordal harmony

CHROMATIC CLASSICAL DOMAIN : "ALL THE THINGS YOU ARE"

I learned this strikingly beautiful piece early in my musical adventure, and found it easy to play but difficult to understand. The source is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). The key signature is 4 flats and the home tonic is Ab, which identifies classical mode **//I** (Ionian major). The piece starts and ends in this mode but, in between, moves back and forth between major and minor of the home tonic in ways I initially found puzzling.

The skeleton melody line and and associated core harmony for bars 1-26 where all the changes occur are shown below.

(a) ⚓ | $ | I | I | M | M |
♫ | **1** | ↗4↘1 | ↘7* | 7↗3 ↘7 | ↘6*| ... **//I** : Ionian

(b) ⚓ | D | D | |
♫ | 6↗p3↘6 | ↘p6 | — , | ... **altD@p6** : Phrygian

(c) ⚓ | A | $ | L | L | M | P PM |
♫ | ↘5 | ↗1↘5 | ↘p5* | p5↗7↘p5 | ↘3* | 3↗p5↗5↘p5↘3 | ... **//L@5** : Ionian

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(d) ⚓ | L | M | M | D | D |
♫ |↘p3, ↘p5↗7↗p5 | p5↘3* | — ↘5↗p6↗3| ↘p3 | — |
⚓ | M | |
♫ | — ↘p5↗7↗p3 | p3↘p2* | ... **altM@p3** : Phrygian

(e) ⚓ | P | @ | A |
♫ | — ↘2↗p3↗p2 | ↘1* | — | ... transition

(f) ⚓ | | I | ...
♫ | **1** | ↗4↘1 |↘7* ... ... **//I** : Ionian

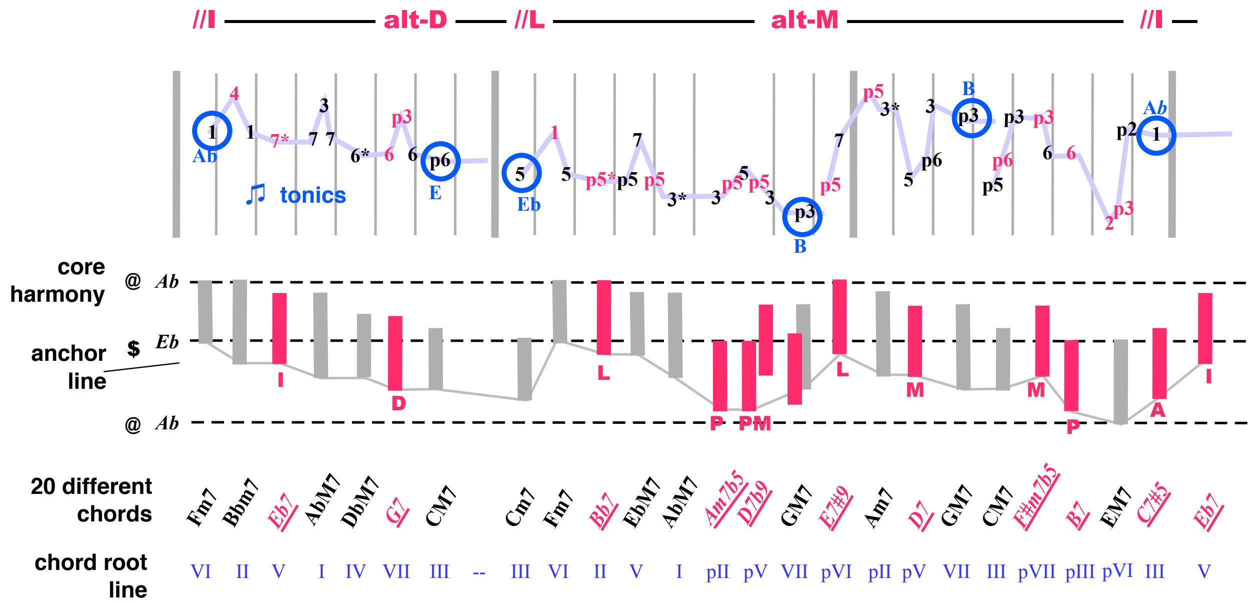
The melody notes transposed into the home octave provide the home-tonic modes.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
(a) **//I**	x	.	x	.	x	x	.	x	.	x	.	x	x	Ionian of tonic **1**
(b) **altD@p6**	.	x	.	x	x	.	x	.	x	x	.	x	.	Phrygian of tonic **p6**
(c) **//L@5**	x	.	x	.	x	.	x	x	.	x	.	x	x	Ionian of tonic **5**
(d) **altM@p3**	.	x	.	x	x	.	x	.	x	.	x	x	.	Phrygian of tonic **p3**
(e)	x	x	x	x	.	.	.	.	.	.	.	.	x	transition
(f) **//I**	x	.	x	.	x	x	.	x	.	x	.	x	x	Ionian of tonic **1**

The melody line alternates between Ionian and Phrygian modes of the tonic sequence

1↘p6↘5↘p3↘1 highlighted in blue (in letter notes this is Ab-E-Eb-B-Ab). The tritone change **I↗L** and the tonic change **1↘5** identify an Ionian-Ionian change. The tritone change **D↗M** and the tonic change **p6↘p3** identify a Phrygian-Phrygian change. The parallel and alt modes of the home tonic are relative modes of these (the alt modes follow from the absence of the home tonic from the two Phrygian modes). The strong presence of the Phrygian mode may seem strange but is not unusual in strongly chromatic music.

The following overview sketch contrasts the conceptual and actual simplicity of the piece with the complexity of the written music. The chord progression begins with a simple Ionian root line that starts to be misleading at bar 7 and continues to be so until bar 13. Being misleading stops after that but complexity continues. The transition back to the home tonic is particularly complex.



NON-CLASSICAL DOMAIN : “CHELSEA BRIDGE”

The source for this hauntingly beautiful Strayhorn piece is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). There are two key signatures but only one home tonic: a key signature of 5 flats and tonic Db identify the opening Ionian mode; a key signature of 4 sharps and tonic C# identify a following parallel Aeolian mode. The same black key is the home tonic of both (Db/C#). The notational complexity that follows from this has been discussed elsewhere in these pages. This piece multiplies the complexity by returning to the original Ionian mode at the end via naturals relative to the 4-sharps key signature. Between the parallel classical modes are chromatic passages that multiply the notational complexity once more. The result is a confusing muddle of many more than twelve note symbols relative to the two written key signatures. The piece is musically sophisticated by any measure, but this notational complexity is way out of proportion to the sophistication.

I first learned this piece by rote with great difficulty from the written music, but thought that anything that sounds so good must have simple musical logic behind it. It does, as shown below: the basic structure is major-minor-major where the major and minor sections open with **//I** and **//A** and then branch out into non-classical modes of the same tonality, and the concluding major section is **//I**. Yellow highlighting identifies ornamental elements relative to identified scales. Parallel modes of the melodic minor determine melody and harmony scales in bars 2-5. All the scales are shown on the next page.

⚓ | M D A P | ... ornamental
🎵 ||: — 6↗7, ↗1↗2, ↗3↗4↗5 | ... //I

⚓ | L.A | M.L | L.A M.L | P | ... determines // modes
🎵 | ↗p6—, ↗6↘p5 | p5, ↗5↗p6 | p6, ↗6↘p5—, ↗5↘p2 | p2 | ... moves between modes

1______________ 2________________

⚓ | ▯ | I | 𝕄 | ~~𝕄~~ D A | ... determines //I
🎵 | ↗3 ↘1—, | ↗3↗5—, | ↘3↘1— :|| — 5↗p6 | ... moves in //I

⚓ | A | | A | ... //A
🎵 | ↗p7* | p7↘1↗p3↗5↗p7↗2 , ↗p3 | ↘2↘p7↗5↘p3↘1 ↘7↗p5↘2 | ... //A

⚓ | P | I ▯ | |) ... altalt-IP
🎵 | ↘p2, — ↘p3 ↗4 | ↗5↗p6↗p3— ↘p2↘7 | ↘p7 ↗p2—, ↘7↘p7 |)
⚓ 𝕄
🎵 | ↘6, ↘p5↗6↗p2, ↘6 ↗p2 | ... ornamental

⚓ M.L D A P ... ornamental
🎵 | ↗p5, ↘6↗7, ↗1↗2, ↗3↗4↗5 | ... //I

The only complicated part from a playing perspective is bars 2-5. In these bars, the harmony double tritones may be implied by playing their outer notes as augmented fifths moving down and up by whole tones, as suggested by the following picture. Playing these augmented fifths with the sparse melody line is very easy, and sounds good.

		p7	7	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
		x	x	@	P	A	D	M	I	L	$	x	x	x	x	@
bars 2, 4	L.A			@	.	x	.	.	.	x	.	x	.	.	.	.
bars 3, 5	M.L	x	.	@	.	.	.	x	.	x	.	.	.	.	.	.

Here are the melody scales. In bars 2-9, the various parallel modes, all with major tonality, are determined jointly by the melody and harmony. In bars 2-5, the melody line sequences **p6-6-p5-5** and **p5-5-p6-6** move back and forth between the **LA** and **ML** modes. In bars 6-9, the sparse melody line exercises the major triad of the **//I** mode identified by the harmony. In bars 13-15, the **IP** minor mode follows the **//A** minor mode like the several major modes follow the opening **//I** major mode.

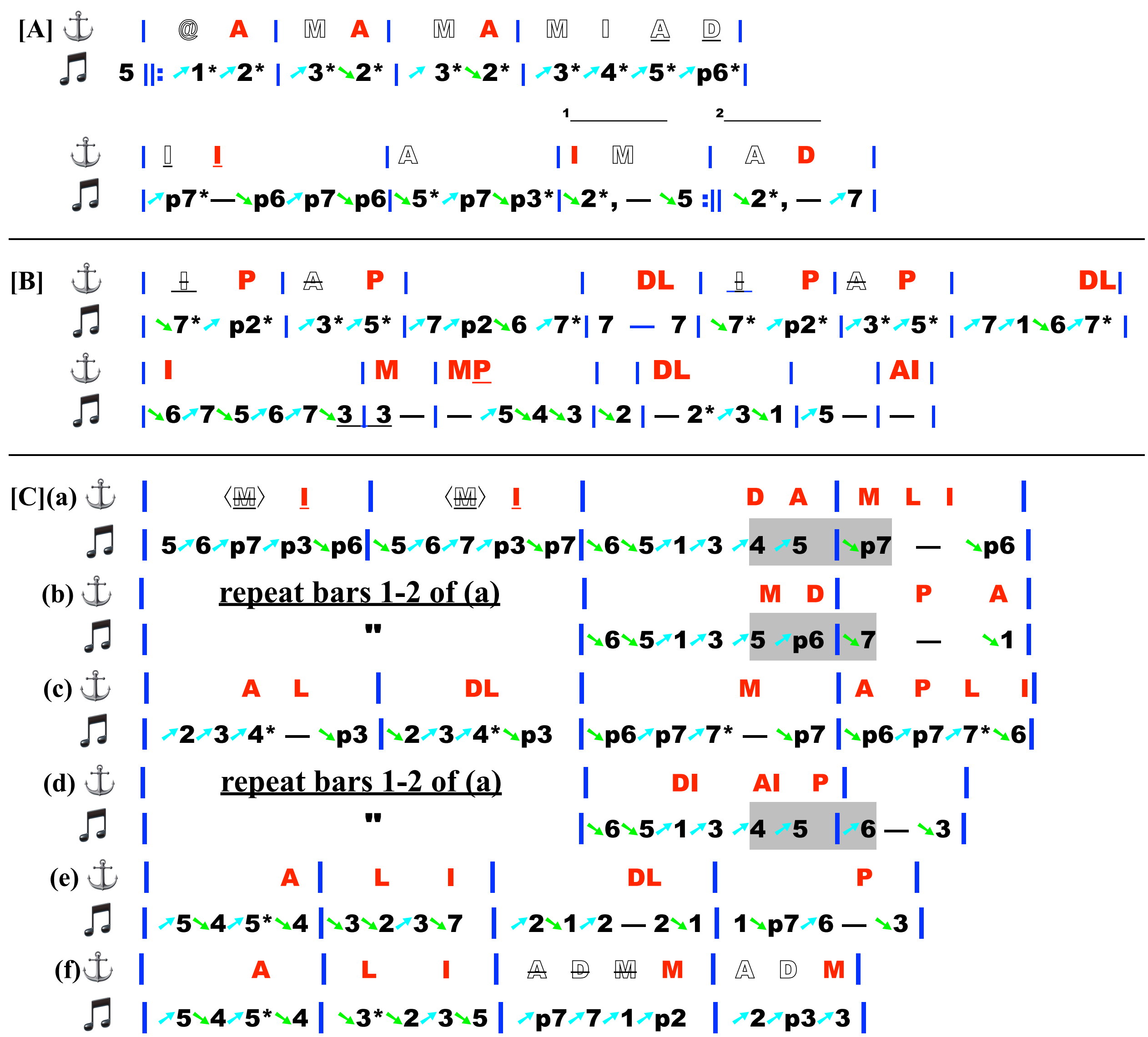
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	A	D	M	I	L	$	x	x	x	x	@	
bar 1	x	.	x	.	x	x	.	x	.	x	.	x	x	**//I**
bars 2, 4	x	.	x	.	x	.	x	.	x	x	.	x	x	**//LA**) note differences highlighted
bars 3, 5	x	.	x	.	x	.	x	x	.	x	x	.	x	**//ML**)
bars 6-9	x	.	x	.	x	x	.	x	.	x	.	x	x	**//I**
bars 10-12	x	.	x	x	.	x		x	x	.	x	.	x	**//A**
bars 13-15	.	x	.	x	.	x	.	x	x	.	x	x	.	**altalt-IP** (Phrygian flat one)
bars 16	.	x	.	.	.	.	x	.	.	x	.	.	.	ornamental transition
bar 17	x	.	x	.	x	x	.	x	.	x	.	x	x	**//I**

The melody and harmony have an elegant simplicity in these terms that is not difficult to understand, remember or play.

NON-CLASSICAL DOMAIN : "LUSH LIFE"

This beautiful Strayhorn piece is melodically and harmonically rich, and challenging to play from the written music because there are often two or more chords per bar, many of them chromatic relative to the written key signature of five flats (major tonic Db). The piece is so chromatic that there is more than one way of understanding it. It can be seen as daisy-chained, short segments of successive parallel classical modes, but this quickly bogs down in complexity.

The view of it here is in terms of two non-classical parallel modes of the home tonic for both melody and harmony: one is the 10-note **//ADMI** minor-major mode in the verse [A] and chorus [C]; the other is the **alt-IP** mode (Ionian sharp one) in the bridge [B].



These modes lead naturally to short, easy-to-remember segments in classical and other modes that follow from the flow. For example, **alt-IP** morphs into **//I** halfway through [B] by altering one note. From time to time, the harmony of the different sections includes **P** and **L** as substitutes for **A** and **I**, but thinking of these as ornamental relative to the tonal scale **//ADMI** keeps things conceptually simple. The grey shading in [C] highlights differences.

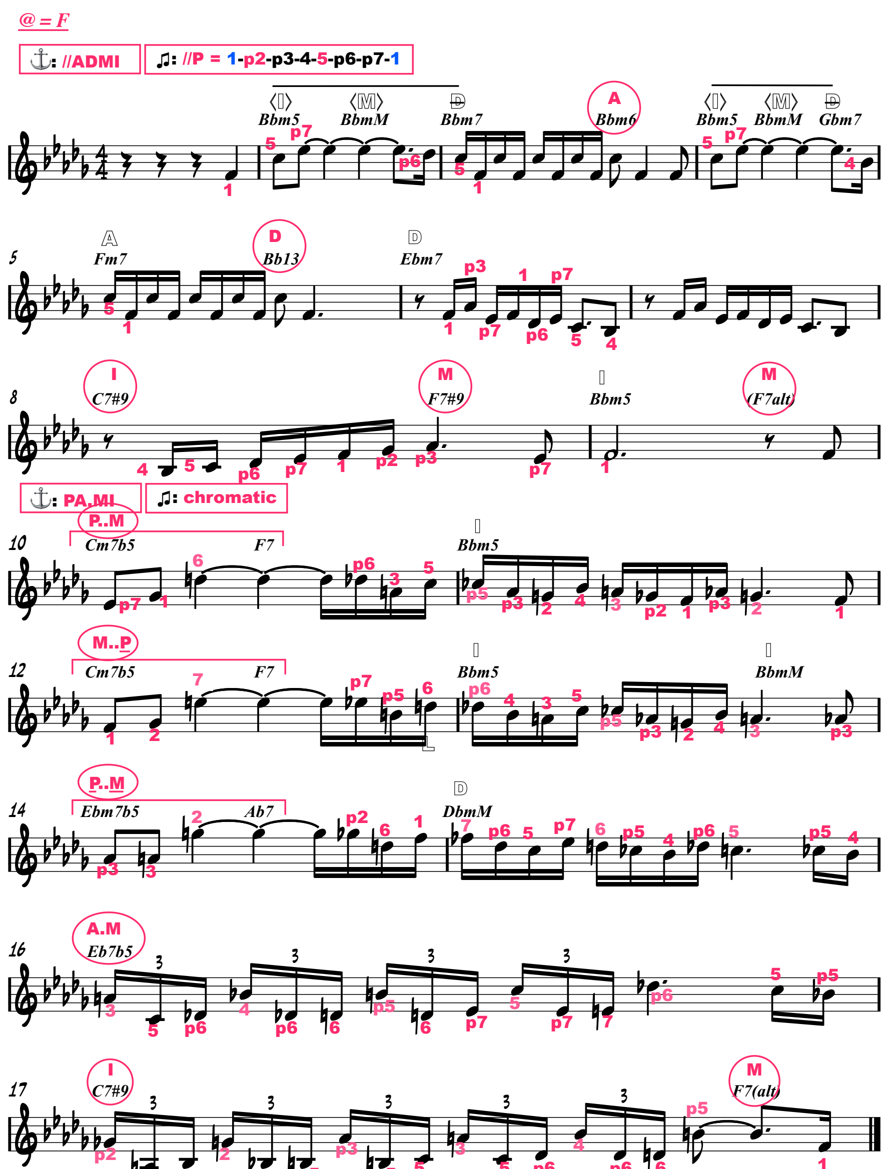
There are no voicing extensions because the intent is to play octave shapes for single tritones and for fifos, which adds depth without adding new notes (the piece is so melodically and harmonically rich that the voicings don't have to add variety). Harmony fifos are mostly omitted for simplicity, except for characteristic sequences of them in [A] and the ending bars of [C].

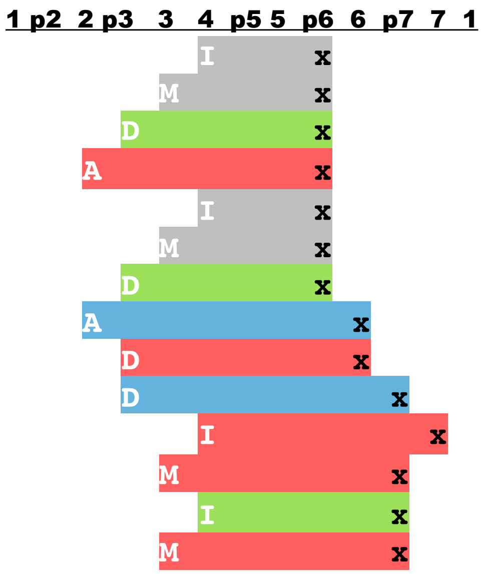
The double tritones **PM**, **AI** and **DL** that appear in many places in the harmony are diminished seventh chords, commonly used ornamentally in any context in which they sound good. The double tritones have circular symmetry (all intervals are minor thirds), so only three of them with different notes exist. They can be played as four notes but they can also be voiced with three notes (a tritone with a minor third on top, or the opposite) or even with two notes (outer notes a major sixth apart), when the context implies the other note(s). A sequence of two or three of them, often used as an ornamental transition between modes, can be played as as sequence of major sixths anchored by any three of six tritone anchors (e.g., **P-A-D** going up and **L-I-M** going down, to name just two possibilities — others require inversions of individual tritones).

The chromatic melody-line run in the final two bars is a signature of the piece.

MIXED DOMAIN : "THE PEACOCKS"

This wraps up the chapter with an example in annotated music notation. This strongly chromatic piece is one of the most haunting jazz pieces I have heard or learned to play. It sounds so "right" as written that jazz improvisations rarely stray far from it.

The home tonic is determined to be F by the final notes of bars 9 and 17. In bars 1-9, the combination of the 5 flats key signature, the home tonic and the lack of accidentals in the melody line identifies the melody scale as Phrygian (mode signature **//P**). The melody line in bar 8 runs up through all the notes of this scale starting a fourth up (fifth down) from the tonic. The contrasting harmony is from the non-classical domain. The over-lines in bars 2 and 4 indicate held treble notes. All the annotations can be handwritten except for the fifo anchors in outline text form, which can be written in plain text outlined by boxes.

Harmony in bars 2-9 contrasts strongly with the melody. The core harmony in these bars, shown next, is visibly from the atonal **ADMI** scale (which is minor-major relative to the home tonic). The anchors of the grey-shaded minor and major thirds are specified by angle brackets because these intervals are not in the building-block set, but the choices shown are obvious ones from context, sound exactly right to ear, and voice the written chords in context (their sizes could be indicated by numeric suffixes 3 and 4, but these sizes are obvious from context).



The melody of the second section consists mainly of individually simple, descending patterns from the full chromatic scale that are prompted by the harmony.

CHAPTER 6: OBSERVATIONS & CONCLUSIONS

Music notation is here to stay and must be lived with, but this this doesn't preclude alternate views of piano music from a different perspective. I looked for an alternate view of harmony and found one based on intervals.

The central role of tritones in PKP tends to seem inappropriate to people savvy in music notation, for several reasons. In general, it goes against received musical wisdom. More specifically, tritones cannot even be written down in music notation until a key signature is established, so imagining them as identifiers of scales is difficult. Anything involving multiple tritones is complex in music notation because it implicitly brings in other key signatures — the more tritones, the more complexity. The very concept of fifos is unconventional and even more so is the concept that they can be treated as morphed tritones, understood from context. The payoff is insight into deep structure that is difficult to see in music notation. Appreciating these things requires stepping outside the box of music notation, at least momentarily.

Thinking in notes and thinking in intervals *is* different in kind. Many experts who have learned the hard way to think in notes apparently find thinking in intervals too alien to contemplate. Comments from them have run the gamut from "obviously wrong" to "wrong headed" to "naively simplistic" to "overwhelmingly complex." One expert said I had "found a tritone hammer and saw everything as a nail" as if the very existence of tritones as useful size tools was unthinkable. The book *Modalogy*, devotes 50 or so complex pages to parallel modes without ever mentioning the possibility that tritones might simplify things by providing defining notes.

Because I started out seeing intervals and notes as dual views of the same thing, I came to find it natural to switch back and forth. I have made my best effort in this document to convey the simplicity of this. I believe, based on my personal experience as an adult beginner, that any beginner could easily learn to think this way.

I know from personal experience that PKP helps with learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. It turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

The jumpy chord root lines in many of the examples demonstrate that referring chords to constantly moving roots creates misleading complexity. This complexity is both conceptual (the underlying, smoothly flowing core harmony is obscured) and actual (playing the jumpy harmony is technically difficult because all the fingers must be lifted and moved by large intervals). It's easier to learn the core harmony and introduce the jumps later, if desired, via inversions, which are always easy on the keyboard, given an anchor line.

GENERAL BENEFITS

PKP annotations above the staff tell you many things. Combined with context provided by the melody line, they tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not right at any point by identifying tonic scales in play in a very immediate way. They tell you about transitions between major and minor tonality and between different tonics. The provide a basis for enrichment and improvisation.

Scale, modes and chords are seen as part of a continuum expressed in the same terms. This seems to me to be a very powerful benefit, compared to seeing everything beyond classical modes as special cases identified by accidentals (which includes chromatic chords with suffixes that imply accidentals). The representations of scales and chords in terms of tritone clusters are unfamiliar to expert pianists but the interval stacks that they imply on the keyboard are completely familiar.

With PKP, playing music is guided by note-based symbology on the music page (one dual view) plus PKP annotations above the staff (the other dual view) that suggest core harmony and make changing tonic scales and tonics directly visible to the eye in terms of changing dictionary words.

A scale dictionary of less than a page covers much ground. The PKP dictionary provides a novel, high-level, conceptual view of tonic scales that's easier to hold in the mind's eye than the very much larger number of spelled out tonic scales in music notation. The dictionary covers 12x39=468 scales of 12 different tonics in music notation (more than is covered by scale dictionaries such as *The Source*). Words of 1-4 letters from the 6-letter alphabet identify, by tritone content, 39 tonic scales/modes with 7 or more notes, 1-4 tritones, and no intervals larger than a minor third. The count of 39 scales/modes includes 11 single scales covered by single words and 4x7=28 parallel modes covered by transpositions of 1-2 letter master words (only the master mode is identified in the dictionary). A selected home tonic anchors all of this to the keyboard.

The words are analogous to the words of biological DNA in the sense that they identify deep structure without spelling out the details. Words of biological DNA are "expressed" as proteins, the building blocks of life. **PADMIL** words are "expressed" as building blocks of scales and harmony. Biological DNA is "extracted" from biological samples and interpreted by sophisticated machines called "sequencers." **PADMIL** scales and building blocks are "extracted" and "sequenced" from musical lines by sophisticated machines called human eyes. Biological DNA is "inherited" from parents. **PADMIL** words are "inherited" from general knowledge about them encoded in the dictionary.

Symmetry breaking in the scale dictionary and symmetry breaking in core harmony are two sides of the same coin. In the dictionary, it yields scales with fewer tritones. In harmony, it releases the tension of dissonance to produce consonance. Seeing the simplicity of this requires forming mental images of it relative to the schematic tonic octave that are independent of specific assignments of the octave to the keyboard.

Understanding harmony is possible independently of practicing. Keyboard symmetries centering around tritones enable high-level representations of harmony that are simple in their own terms and deep in harmonic meaning in any terms.

Music is freed from its key-signature straightjacket. Harmony scales are understood at a glance from dictionary words aggregated by eye, instead of having to be figured out or remembered from key signatures, accidentals and chromatic chord symbols. Tonic changes are no more difficult than any other changes.

Anchor lines that define harmonic cores replace chord root lines as the fundamental musical lines of harmony. Chord roots are visibly secondary elements in the flow of the music because inversions move them into the body of the chord, where they become no more than another chord note that may be omitted if implied by context.

Sight reading anchor lines provides deep context. Anchor lines tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not by identifying tonic scales in play. They tell you about transitions between major and minor tonality and between different tonics.

Multiple explicit or implicit key signatures become a non-issue. This is true whether the key signatures imply tonic changes or only scale changes for the same tonic. Tonic changes amount to no more than moving (conceptually) a tonic pointer within the home octave and reinterpreting the alphabet letters relative to it, without changing any notation.

Chord symbols become a non-issue. Chord complexities that result from banging square pegs (chord symbols) into round holes (places in scales where they don't quite fit) are misleading. Chords described by chord symbols are, on the keyboard, almost universally composed of combinations of PKP's two kinds of building blocks, of which only tritones are always core. A melody line plus an anchor line above the staff imply a voicing of an unidentified chord progression. Specific chords are identified by assigning roots.

Learning blues prepares you for anything music notation can throw at you. Blues is an example of "folk music" that evolved independently of music notation and therefore tends to be regarded as naive when measured against its depth and complexity. This condescending view is unwarranted. Blues is harmonically sophisticated in a highly organized way that makes it, to my mind, a better starting point than classical modes of key signatures for understanding many forms of music.

Sharps and flats are not needed. Building blocks are the same number of half tones no matter how they are notated. This enables PKP to dispense with sharps and flats without any loss of musical accuracy. Expert pianists tend to think they are still needed to identify context because they have learned to use them for that, but note positions relative to tonics are sufficient.

The simple flow of harmony in harmonic cores, in which building blocks slide or morph into other building blocks, is universal. Nothing constrains harmony to be played this way but the flow is so natural and easy to play that much is to be gained by learning pieces this way and then transitioning to other playing styles if desired.

Tritone-intensive harmonic cores of much music provide so much harmonic variety that voicing them requires only adding depth. Depth is easily added by doubling the treble line of the harmonic core an octave down, or the bass line an octave up, and then making adjustments to move points on the line closer to the core, if desired.

No more piano wizardry is required to have fun playing these kinds of music than the ability to sight-read melody lines in music notation. Anyone with eyes to see intervals on the keyboard can infer harmonic cores from anchor lines annotated above the staff.

The simple way harmony is represented provides great freedom in choosing how to play it. Playing styles may range between the extremes of smoothly flowing harmony within the octave of the home tonic of piece to jumpy harmony in which all harmony notes go up from a chord root line. Improvisation follows naturally and is fun even if not expertly done.

The simplification provided by the alphabet is different in kind from simplifying chord symbols. Any sophistication in the harmony is in the tonic scales of origin of the chords.

Mode signatures provide an above-the-staff markup notation for the concept of "modal jazz" which is otherwise a concept without a notation.

PKP annotations above the staff are not always necessary. With experience, straightforward chord progressions may be sight-read in PKP terms.

SOME REFERENCES

1. Barta, *The Source: The Dictionary of Contemporary and Traditional Scales*, Hal Leonard (1995), for helping me to be sure I was not missing important scales.
2. Mehegan, *Jazz Improvisation 1: Tonal and Rhythmic Principles*, Watson-Guptil (1984), for Roman-numeral chord notation, and for teaching me (unintentionally) that it is not a solution for complex chromatic chord progressions, but a problem if pushed beyond its basic function of specifying chord root lines.
3. Eskelin, *Lies My Music Teacher Told Me*, Stage Three Publishing (1994) for insight into the nature of scales and musical "perfection," and for encouraging me to think outside the box.
4. Dmitri Tymoczko, *A Geometry of Music* (2011) for stimulating discussions of how to think about music from different angles.
5. Mark Levine, *The Jazz Theory Book*, Sher Music Co. (1995) for providing examples of well known jazz scales and harmonic forms using them in conventional notation against which to verify PKP coverage.
6. George Russell, *The Lydian Chromatic Concept of Tonal Organization*, http://www.georgerussell.com/lc.html, for making me aware that PKP covers the concept, because nothing is changed by replacing the Ionian mode by the Lydian mode as the default reference major mode for any piece of music.
7. Edward Frenkel, *Love and Math: The Heart of Hidden Reality*, Perseus (2013) (on Kindle), for many insights into the usefulness of symmetry.
8. Jeff Brent with Schell Barkley, *Modalogy — Scales, Modes & Chords: the Primordial Building Blocks of Music,* Hal Leonard (2011), for the most comprehensive treatment I have found of this subject. The result is painstakingly accurate but overwhelmingly complex because it sticks to representations based on sharps and flats that follow from the misalignment of pitches of notes of overlapping octaves.
9. Ross W. Duffin, *How Equal Temperament Tuning Ruined Harmony (and Why You Should Care)*, W.W. Norton (2007), for an understanding of the piano's deviation from musical perfection.
10. Stephon Alexandar, *The Jazz of Physics: The Secret Link Between Music and The Structure of the Universe*, Basic Books (2016). This amazing book resonates strongly with my own ideas. The author is a jazz musician and physicist who speaks of doing physics as informing the playing of jazz and playing jazz as informing the doing of physics. Both require rising above formalisms to form conceptual representations, with symmetries highlighted as particularly important.

ACKNOWLEDGMENTS

I did not take this musical journey alone. I received comments and help from many people over the decade or so the ideas were germinating and consolidating.

Music theorist Paul Steinbeck encouraged me to continue writing at a time when I was becoming discouraged about finding a way of bringing my ideas before the music community. Jazz pianist, teacher and composer Taylor Eigsti has been an inspiration to me. Although I have never been a piano student of his, I have learned much from him in sporadic discussions in person and by email. I am deeply grateful for his willingness to take time away from a busy schedule to engage in these discussions. A short series of piano lessons from SF jazz pianist Michael Parsons helped me to see more clearly the relationship between my ideas and standard jazz-piano practice.

Thanks to SMT (Society for Music Theory) members Charise Hastings, Peter Shultz and Neil Newton for insightful email comments and encouragement following announcements of my website on an SMT mailing list. Thanks to music professor Robert Rawlins for email encouragement, and for helpful examples and comments. Aaron Blumenfeld and Susan Muscarella gave helpful courses at the Jazz School in Berkeley (now the Jazz Institute) that provided many examples to chew on (including, from Susan Muscarella's course, some interesting variations on *Summertime* in D minor that helped me understand ornamental scales). Amateur pianist and Jazz School Board Chair Susan Brand, and concert pianist and music entrepeneur Robert Taub, provided early encouragement.

My piano teacher in San Francisco for several years, Ken Fishler, provided inspiration and how-to information on chord voicings, while patiently tolerating and responding to my question-everything approach. My first piano teacher in Ottawa, Canada, Sally Robinson, started me off right as an adult beginner by helping me to learn favorite harmonically sophisticated pieces by following her fingers on the keyboard without understanding anything about what I was doing except that it sounded right — being able to play these pieces, even if clumsily and by rote, enabled understanding to seep in gradually. I think that without this particular way of starting out — fingers on the keyboard first, written music later — I would not have started thinking about keyboard harmony in the way I did. It forced to my attention the large gap between simplicity on the keyboard and complexity of the full notation that represents it.

I wore out the patience of many music professionals by bombarding them with unsolicited email requests for comments on my ideas. I would like to thank four in particular, who politely responded to numerous emails in spite of being uncomfortable with my ideas: Harry Likas, Dmitri Tymoczko, Jeff Brent and Daniel Glover. Their criticisms helped me change my explanations without changing my mind.

Thanks to friends Marva Black, Mike Budde, Peter Marchant and Selinda Spugies for helpful insights on aspects of this material that helped me to understand how to get the ideas across better. Thanks to singer extraordinaire and friend Lorna Kollmeyer for helping me figure out how to organize piano accompaniment in guitar-strumming style.

My wife Sheila's accurate musical ear helps me know what does and does not sound good on the piano, in addition to making my life generally interesting. My grandsons Joshua and Ethan Feiber provided encouragement and comments; Joshua set up the website www.pianotheoryman.com as birthday gift; Ethan, who learned very young to play the piano impressively well by ear, has experimented with the notation and found it helpful for approaching written music.

SOME COMMENTS FROM READERS

The following comments provide a kind of history of the development of PKP (the dates on the left identify when the commenters read different drafts of this material). The unconventionality of putting tritones on center stage has tended to get in the way of expert musicians accepting the ideas at all, let alone seeing them as simple, which has not been helped by the fact that it took me a long time to find the simple way of explaining the ideas that appears in this document. I continue to think the ideas are important for learning the piano in a less restrictive manner than is conventional but have not yet been able to find an audience. At the very least, the more recent comments show that the PKP method is more than just a fantasy of an over-enthusiastic amateur.

(2016) Taylor Eigsti more recently than below. "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

(2011) Paul Steinbeck. Assistant Professor of Music Theory; Washington University, St. Louis

"The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music."

(2009) Robert Rawlins. University Music Department Chair (Rowan University); jazz musician; teacher; author of several books on jazz

"I became aware of Raymond Buhr's novel method for analyzing and voicing chromatic chord progressions in 2008 through a draft of a paper he wrote on the subject. I have kept up to date on developments of the method and we have had many email exchanges discussing issues of interpretation and application. I am a member of his intended target audience—a jazz musician who is not a pianist who needs to work out harmonic patterns on the piano from time to time. I am also a music teacher who has actually tried out aspects of his method on students. I can vouch from personal experience for his method's helpfulness in dealing with complex chromatic chord progressions."

(2009) Susan Brand. Board Chair, The Jazz School in Berkeley; amateur pianist

"When Raymond Buhr consulted me about his theory of chromatic chord progressions, I was immediately struck by his ability to analyze and attempt to simplify this complex subject. Mr. Buhr brings a unique perspective and a great deal of enthusiasm, depth of understanding and originality of viewpoint to the subject. Over the years I have watched the continuous work that he has put into editing and rethinking his work. He has had ongoing consultations with many knowledgeable musicians/ teachers/editors and all have contributed to the development of the method described in this book. His ideas offer a way of understanding musical theory that will add greatly to the field and will be extremely helpful to musicians and music educators."

(2008) Taylor Eigsti. Jazz pianist, composer, teacher, former Artist in Residence at The Jazz School in Berkeley

"Through the brilliant lens of an engineer, Raymond Buhr has laid out an analysis of harmony that is a unique and complex look at the right-brain from the left-brain's perspective."

APPENDIX A: UNCONVENTIONAL ELEMENTS

TERMINOLOGY & NOTATION

- **anchor:** identifies a building block by the position of its bottom end relative to the home tonic
- **anchor set:** set of tritone and fifo anchors that define a scale by construction
- **anchor line:** anchor sequence written above the staff (outlined by circles for tritones & boxes for fifos)
- **alphabet: PADMIL** identifies anchors by the first letters of the names of classical modes
- **building block:** tritones or fifos (fifths or fourths) anchored at alphabet positions
    - tritone or fifo anchors identified, respectively, by **PADMIL** or PADMIL
    - default direction is up from anchor; underline indicates down
    - tritone is same size either way; default fifo size is fifth, strikethrough indicates fourth
    - uncommitted fifo anchors are represented by fifo anchors in angle brackets
- **chromatic scale: 1-p2-2-p3-3-4-p5-5-p6-6-p7-7-1**
- **context:** provided by mode signatures for melody and harmony plus flow of both
- **core:** sequence of building blocks of harmony identified by an anchor line
- **family:** a set of sub-scales of a family scale defined by a single mode signature
- **fifo:** fifth or fourth that are opposite inversions (add up to an octave)
- **flow:** formed by morphs and slides of building blocks
- **frame:** defined by the tonic **@** and pitch center **$** of a tonic octave (plus sometimes by tonality)
- **mode signature:** word identifying a tritone cluster and implying an anchor set via the scale dictionary
- **pattern:** organized arrangement of intervals on the keyboard or over time
- **pitch center:** note identified by a fifth/fourth octave split, symbolized by **$**
- **morph:** small change in the size of a building block while holding one end fixed
- **outside:** not in a given tonic scale (as distinct from "chromatic" meaning not in a key-signature scale)
- **shape:** combination of building blocks
- **slide:** size-preserving movement of a building block
    - **wobbly slide:** combined morph and slide
- **symmetry:** same keyboard-interval sequence going up and down a shape
- **phlat:** prefix **p** identifying chromatic-scale notes in the whole tone gaps of the major scale of a tonic
- **tonic pointer:** suffix of form **@t** attached to an anchor symbol to indicate a secondary reference tonic
- **word:** set of alphabet letters with optional dots indicating skipped letters

ABOUT THE ALPHABET

A special, boldfaced font for the **PADMIL** alphabet distinguishes these uses of the letters from other uses in music (note symbols "A" and "D", RN symbol "I"). Letters **A** and **D** cannot be confused with letter notes because they identify adjacent piano keys; the different letters are never mixed together in the use of PKP. That said, when the music includes letter notes "A" or "D" avoiding confusion requires effort. I have found the effort worthwhile. I thought of substituting the Greek letters epsilon (**Є**) and delta (**Δ**) for **A** and **D** and the English letter **Y** for **I**, to yield **PЄΔMYL**, pronounced almost the same and therefore easy to remember as having the same meaning (epsilon represents "Ae" of Aeolian). However, the mix of different alphabets introduces a different kind of confusion. I have not been able to think of anything better than **PADMIL**.

APPENDIX B: ABOUT SCALES

KEY-SIGNATURE SCALES

The following summary of the standard interpretation of the major and natural minor tonic scales of key signatures (Ionian and Aeolian modes) is a helpful reference. It shows the sharped notes of sharp scales in red and the flatted notes of flat scales in blue. Although there are only five black piano keys, scales with six flats or sharps exist because of the scale-spelling rule that the same letter note cannot appear on both sides of a half-tone interval. For example, note B is C*b* in a 6-flats scale.

Major (Ionian) and relative minor (Aeolian) scales are shown on the same line because they have the same key signature. Parallel minor scales (diagonal lines) have the same tonic, which does not necessarily mean the same note symbol because the same piano key may be represented by different note symbols (e.g., the parallel D*b* major and C# sharp minor scales, which have the same black-key tonic on the piano, have different note symbols for it).

Ionian	rel. Aeolian	key sig.	Ionian scale
C	A	empty	C-D-E-F-G-A-B-C
F	D	1 *b*	F-G-A-B*b*-C-D-E-F
B*b*	G	2 *b*	B*b*-C-D-E*b*-F-G-A-B*b*
E*b*	C	3 *b*	E*b*-F-G-A*b*-B*b*-C-D-E*b*
A*b*	F	4 *b*	A*b*-B*b*-C-D*b*-E*b*-F-G-A*b*
D*b*	B*b*	5 *b*	D*b*-E*b*-F-G*b*-A*b*-B*b*-C-D*b*
G*b*	E*b*	6 *b*	G*b*-A*b*-B*b*-C*b*-D*b*-E*b*-F-G*b*
F#	D#	6 #	F#-G#-A#-B-C#-D#-E#-F#
B	G#	5 #	B-C#-D#-E-F#-G#-A#-B
E	C#	4 #	E-F#-G#-A-B-C#-D#-E
A	F#	3 #	A-B-C#-D-E-F#-G#-A
D	B	2 #	D-E-F#-G-A-B-C#-D
G	E	1 #	G-A-B-C-D-E-F#-G
C	A	empty	C-D-E-F-G-A-B-C

Ionian tonics going down by fifths

switch from flat to sharp keys

parallel (same tonic) Ionian & Aeolian modes
- - - - same tonic symbols
—— different tonic symbols

Avoiding multiple written key signatures by using accidentals to identify implicit key changes replaces one kind of complexity by another because accidentals have many more uses than this (e.g., ornamental passing notes, or identifying scales that originate in no key signature). Sorting out the possibilities requires considerable head-scratching at a very low conceptual level.

Key-signature-based music notation has stood the test of time and is here to stay because of the huge legacy of music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation.

CHROMATIC SCALE

The chromatic scale represents an octave on the piano keyboard as notes determined by twelve half tones, and overlapped octaves as sharing the same notes. The fundamental two modes of music notation, namely Ionian and Aeolian, are represented as follows (tonics underlined).

//A	**Aeolian**	**1-2-p3-4-5-p6-p7**
//A@p3	**relative Ionian**	**1-2-p3-4-5-p6-p7 —> p3-4-5-p6-p7-1-2**
//I	**Ionian**	**1-2-3-4-5-6-7**
//I@6	**relative Aeolian**	**1-2-3-4-5-6-7 —> 6-7-1-2-3-4-5**

Experts develop rules of thumb about sharps and flats as indicators of what comes next and see this way of representing scales as omitting these clues. The clues are present in a different way in the positions of the scale notes relative to a home tonic and to each other.

CROSS REFERENCES

The following table summarizes the relationship between PKP notation and the note symbols of music notation. This is the simplest case. More generally, naturals may enter the note-symbol picture, and white pianos may be represented by sharps or flats (e.g., B=Cb, C=B#, F=E#, E=Fb).

1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**
@	**P**	**A**	**D**	**M**	**I**	**L**	**$**	**.**	**.**	**.**	**.**
C	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**
C#/Db	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**
D	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**
D#/Eb	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**
E	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**
F	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**
F#/Gb	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**
G	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**
G#/Ab	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**
A	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**
A#/Bb	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**
B	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**

SCALES AS INTERVAL STACKS

The representation of scales as interval stacks is an adaption of a notation called “Figured Bass Notation” (Wikipedia) for identifying harmony by annotating bass notes on a staff with number stacks that represent counts of scale steps going up from the bass notes. The adaption replaces counts of scale steps with counts of half tones, and lists the stacks horizontally instead of vertically. For representing scales, the numbers are restricted to **1** (half tone), **2** (whole tone) and **3** (minor third). Any scale may be spelled out as a stack of these numbers that adds up to twelve. For example, the Ionian mode is identified by **|2212|221|** and the Aeolian mode by **|2122|122|**, where the vertical lines indicate the scale frame. The Ionian stack also identifies inter-note interval sequence of the singer’s solfege scale **do-re-mi-fa-so-la-ti-do**. The interval stacks are the same for parallel or relative modes, which aids clear thinking about the nature of modes and the relationships between them. For example, the interval stack of the Aeolian mode is visibly a rotation of that of the Ionian mode.

APPENDIX C: ABOUT CHORDS

Chords identified by chord symbols of any kind boil down to combinations of the two kinds of building blocks. Here follow some examples. Two fifos (fifths or fourths) form major-7, minor-7 or major-6 chords. One fifo and one tritone form dominant-7, half-diminished-7 (a.k.a. minor-7-♭5) or minor-6 chords. Three fifos or two fifos and a tritone form 9th, 11th and 13th extensions of these chords. When tonic scales depart from key signatures, the same kinds of building blocks are available from the scales but now more than one tritone is available. For example, two tritones form diminished-7 or dominant-7-♭5 chords. Two tritones and one fifo (that may share a note with a tritone) form variations of other chord types such as dominant-7♭9. The exceptions prove the rule. An augmented fifth (8 half tones or two major thirds) that are "building blocks" of minor-major-7 chords are combinations of scale fifos or tritones with their inner notes omitted). Simpler triad chords are, structurally, truncated combinations of combinations of two building blocks.

ANCHOR LETTERS FROM TRITONE CHORDS

As described in the body, tritones are the agents of context change. They are directly visible in melody lines if you see them in terms of keyboard intervals between successive notes. They may be extracted from chord progressions using the following table of tritone chords. In either case, they're represented by anchor letters from the **PADMIL** alphabet that identify the position of their nearest notes above the home tonic.

The following table of tritone chords summarizes the simplest examples of chords with tritones in different positions. Only variations that alter tritone content are included. For example, R7, R9, R7#9, R13 and R9(13) are all variations of R7 with the same tritone content and so are all represented in the table by R7 (variations are left to context). Diminished seventh chords (dim7) have no counterpart in classical modes. Sus chords are not shown because they have no tritones. They are typically V-7♯3 chords. A Vsus-V-I progression is a substitute for a II-V-I progression in which one note is altered between the first two steps.

offset of tritone bass note above root R	suffixes on root symbol R									
	7	7♯9(13)	9♭13 (or ♯5)	7♭9	7♭5 (or ♯4 or ♯11)	dim7 (or o7)	m7♭5	m6	M7(11)	♭9	M7♯11 (or ♭5)
fourth									x		
major third	x	x	x	x	x						
minor third		x				x		x			
whole tone			x								
half tone				x						x	
0					x	x	x				x

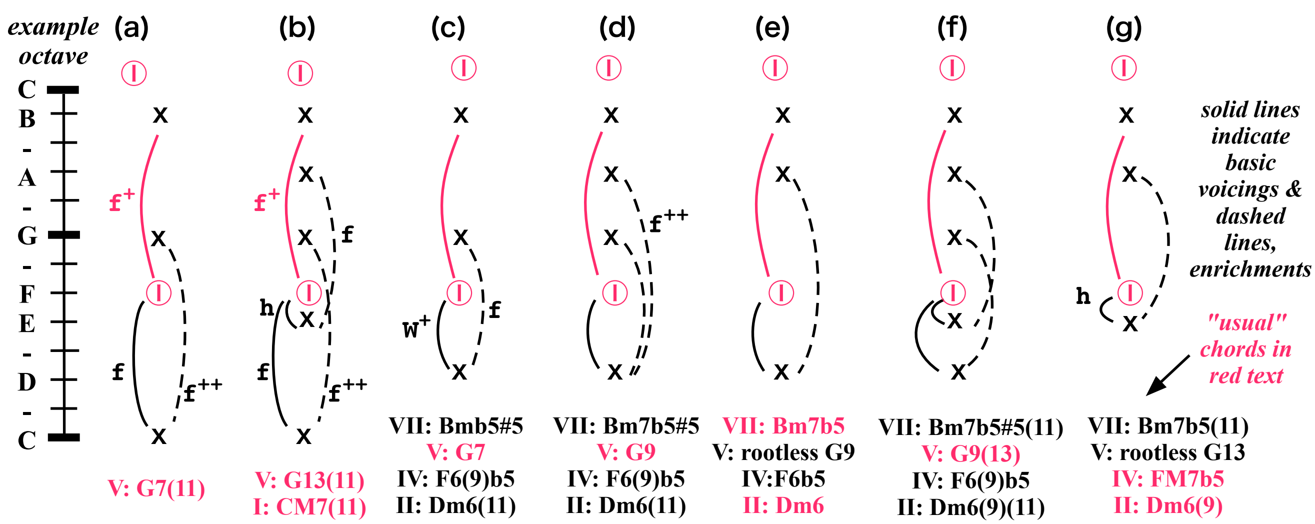
Inverting the tritone of a chord into the home octave (if it isn't already there) determines its PKP anchor. For example, The tritone of a V-7 chord is partially outside the home octave but its inversion is within it. The bass note of its inversion is a fourth above the home tonic which makes its anchor **I**.

Double tritones are the normal upper limit for tritone content of chords generally encountered above the staff in fake books, lead sheets and sheet music.

MISLEADINGLY COMPLEX CHORD SYMBOLS

Misleadingly complex chord symbols follow from "banging square pegs into round holes" (fitting basic chord symbols into places in scales where they don't quite fit). This is illustrated next by the startling variety of single-tritone chords from the Ionian mode that may be voiced by the single tritone plus a carefully selected scale note below or above the tritone (or both). The symbols $\mathbf{f}$, $\mathbf{f}^+$ and $\mathbf{f}^{++}$ stand for fourth, tritone (augmented fourth) and fifth (this was prior to my decision to use numeric suffixes or prefixes for these intervals of keyboard shapes).

Enrichments by overlapped fifos are important when the chords are played by themselves but the notes they add are visibly fundamental scale notes (the pitch center and a whole tone above it) in the context of the flow of a chord progression, which the ear tends to hear even if the notes aren't sounded. A bass note a half tone below the tritone is equivalent to a fourth above it (same note), yielding the interval stack $\mathbf{f}$-$\mathbf{f}^+$. The result is an "all fourths" shape. Seventh or sixth chords voiced by all fourths shapes have extended or altered chord symbols. This is in contrast to the "all thirds" shapes of seventh chords identified by basic chord symbols.



The figure and these examples are specific but the concepts are general. Different voicings of different tritone chords may created in a very simple way by specifying a bass or treble note that implies visibly obvious enrichment fifos from the tonic scale. The tonic scale is understood from tritone content. The enrichments are without reference to chord symbols; they're usually so obvious that no extra notation is needed.

Well formed voicings of chord progressions follow from morphing tritones into fifos. For example, for tonic C, the fifo-tritone sequences in (a) and (c) voice chords as follows:

- (a) $\mathbf{f}$-$\mathbf{f}^+$ identifies a 3-note voicing of Dm7(11)-G7 or G7♯3-G7, both equivalent in the flow to Dm7-G7.
- (c) a different $\mathbf{f}$-$\mathbf{f}^+$ is a different 3-note voicing of Dm7(11)-G7 or G7♯3-G7, also equivalent in

the flow to Dm7-G7.

FIGURED BASS NOTATION; EXTENDED CHORDS

Figured bass notation provides a simple representation of chords from the the highly regular scales of classical modes (7 notes, no adjacent half tones). It represents chords by stacks of numbers going up from a bass note, in which each number is a count of the scale steps to the next note up. The stacks are annotated on a staff as literal stacks of numbers (vertical lists). I write them here as horizontal lists separated by plus signs. For classical modes, a major or minor third is always two scale steps and whole tone is always one scale step (e.g., a seventh chord is **2+2+2** and a sixth chord is **2+2+1**).

Figured bass notation can be helpful for understanding extended chords of classical modes that follow from splitting chord steps of size **2** into **1+1** sequences and then moving the added note up an octave. The **+4** on top of two of the extended chords on the right is a consequence of avoiding **1+1+1** sequences in the in-place forms. The corresponding chord symbols have degree-number suffixes 6, 9, 11 or 13 added to basic symbols to identify the extensions. Extension suffixes altered by sharps or flats are needed to represent chords from scales that are not classical modes.

chord type	(2) = split	in place	extended
seventh	2+2+2	2+2+2	–
ninth	(2)+2+2	1+1+2+2	2+2+2+2
eleventh (seventh+11th)	2+(2)+2	2+1+1+2	2+2+2+2+4
thirteenth (ninth+13th)	(2)+2+(2)	1+1+2+1+1	2+2+2+2+2+4

APPENDIX D: ABOUT PARALLEL MODES

MODES FROM MODALOGY

The non-classical parallel modes as presented in the book *Modalogy* are summarized next, except with notes in numeric-chromatic-scale notation, instead of in the RN (Roman Numeral) notation with sharps and flats used in *Modalogy*. Corresponding PKP mode signatures are shown on the left.

The purpose is twofold. One is to verify the PKP view of modes relative to *Modalogy*. The other is to illustrate the complexity of the conventional representations. For example, the tritone anchored by **D** that is a component of many of the minor modes is understood as **p3-6** in the symbolic chromatic scale, and that's it. Inversions in different places in harmony are left to context. In *Modalogy*, tritone anchor **p3** is ♭III or ♯II and tritone anchor **6** is VI or 𝄫VII, and that's only the anchors.

The alphabet soup of names and the intricate details of the scales makes them very difficult to distinguish functionally, or to remember as enumerated scales. Seeing the possibility that tritones might be fundamental scale-defining objects for these modes is effectively impossible in these terms. This impossibility is demonstrated by a discussion in *Modalogy* of defining and non-defining notes of the many and various modes that never mentions tritones.

The simple, unique mode signatures of PKP are novel. Green highlighting brings forward to the eye that some modes are of opposite tonality to the master mode, or of mixed tonality.

Parallel Modes of the Melodic Minor |WhWW|WWh|

alt-LA	**1-2-p3-4-p5-p6-p7-1**	**Aeolian Diminished (Aeolian-♭5)**
LA	**1-2-3-p5-p6-6-7-1**	**Lydian Augmented (Lydian-♯5)**
alt-ML	**1-p2-p3-3-p5-p6-p7-1**	**Jazz Altered (Locrian-♭4)**
ML	**1-2-3-p5-5-6-p7-1**	**Lydian Dominant (Lydian-♭7)**
DI	**1-2-p3 -4-5-6-7-1**	**Melodic, or Jazz, Minor (Ionian-♭3)**
AM	**1-2-3-4-5-p6-p7-1**	**Jazz Mixolydian (Mixolydian-♭6)**
PD	**1-p2-p3-4-5-6-p7-1**	**Jazz Phrygian (Phrygian-♯6)**

Parallel Modes of the Harmonic Minor |WhWW|hW$^+$h|

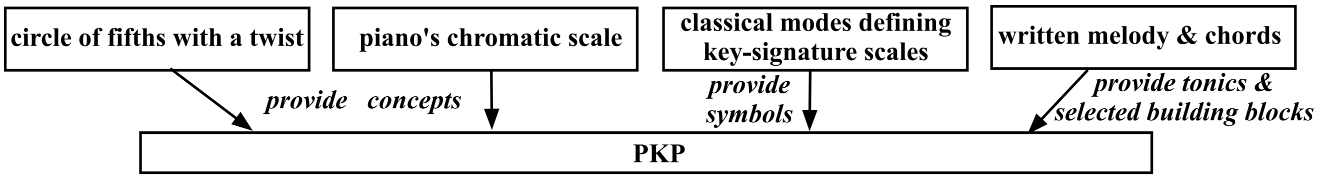
DL	**1-2-p3-p5-5-6-p7-1**	**Romanian, Dorian ♯4, Mishebarakh**
alt-DL	**1-p2-p3-3-p5-p6-6-1**	**Leading Tone Minor Diminished, Super Locrian 𝄫7**
LD	**1-p3-3-p5-5-6-7-1**	**Lydian Blues Major, Lydian ♯2**
alt-LD	**1-p2-p3-4-p5-6-p7-1**	**Jazz Phrygian Diminished**
AI	**1-2-p3-4-5-p6-7-1**	**Harmonic Minor, Aeolian ♮7, Jazz Minor ♭6, Mohammedan**
IA	**1-2-3-4-p6-6-7-1**	**Ionian Augmented**
PM	**1-p2-3-4-5-p6-p7-1**	**Phrygian Dominant**

Parallel Modes of the Harmonic Major |WWhW|hW$^+$h|

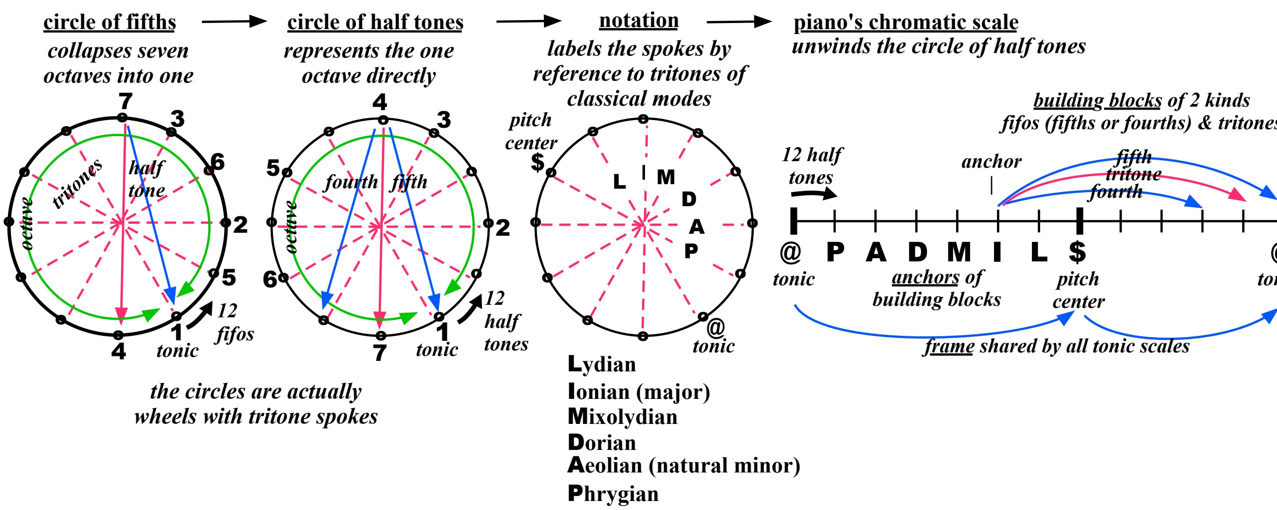
DL	**1-p3-3-p5-p6-6-7-1**	**Lydian Blues Augmented, Lydian Augmented ♯2**
alt-DL	**1-2-p3-4-p5-6-p7-1**	**Jazz Minor ♯4, Lydian Diminished**
LD	**1-2-p3-p5-5-6-7-1**	**Lydian Melodic Minor, Lydian ♭3**
alt-LD	**1-p2-p3-4-p5-p6-6-1**	**Leading Tone Major Diminished, Locrian 𝄫7, Locrian Diminished-7**
AI	**1-2-3-4-5-p6-7-1**	**Harmonic Major**
PM	**1-p2-3-4-5-6-p7-1**	**Jazz Phrygian Dominant, Mixolydian ♭2**
MP	**1-p2-p3-3-5-p6-p7-1**	**Altered Phrygian Dominant, Phrygian ♭4, Superlocrian ♮5, Superphrygian**

APPENDIX E: ABOUT SYMMETRY & SYMMETRY-BREAKING

The elements of PKP are summarized below in a way that provides a link between the concept of symmetry breaking in these pages and in the book *The Jazz of Physics*.



The circle of fifths shown below left presents the basic elements of tonic scales and chord progressions from them in one simple picture. The points going counterclockwise around the circle are 12 notes a fifth apart (7 half tones) across 7 keyboard octaves (12 fifths x 7 half tones = 7 octaves x 12 half tones). The circle is manifestly a wheel with tritone spokes. The notes of the major scale are the numbered points around the half circle delimited by the highlighted tritone spoke (6 half tones).



The circle visibly establishes two kinds of Lego™-like **building blocks** as fundamental elements of music, namely fifths or fourths referred to jointly as **fifos** for simplicity, and tritones. The circle as labeled identifies fifths going up the keyboard through 7 octaves or fourths going down the keyboard through 5 octaves. For the major scale, it identifies a basic chord root line going down by five fifths (3½ octaves) or five fourths (2½ octaves) to the tonic (equivalent to progressively zig-zagging down a fifth and up a fourth within one octave).

The pictorial geometry of the circle of fifths and the musical geometry of its interpretation are different in a way that is counterintuitive (different numbers of octaves going around the circle for fifths and fourths, very different musical sizes of lines of almost the same pictorial length that cut across the circle). This becomes confusing for more general chord progressions of this and other scales.

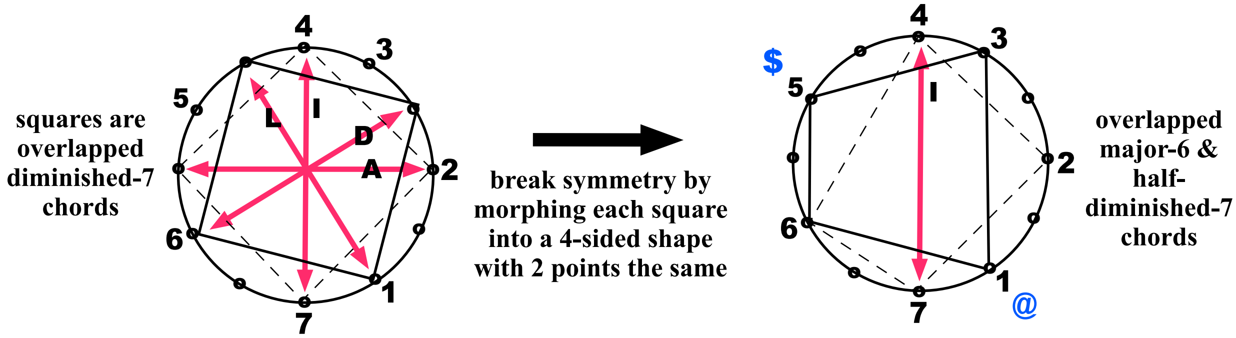
The circle of half tones rearranges the spokes of the circle to put the labeled points around it in scale order. The result is an intuitively natural **@1** between pictorial geometry and musical geometry. Around the circle remains an octave and across any spoke remains a tritone. The half tones across the circle now go around it and the fifos around the circle now go across it.

The end points of all the spokes of either circle visibly identify all the points around it, suggesting a notation that labels spokes (building blocks) instead of points (notes). This simple observation is the twist that leads to PKP's unique combination of simplicity and depth. The spoke labels, called **anchors**, directly identify tritones but also identify fifos. In the chromatic scale that unwinds from the circle, each anchor letter identifies a tritone and two fifos morphed from it. The letters identify relative positions of the anchors in the chromatic scale, and outlining them differently in annotations above the staff distinguishes the different kinds of building blocks (circles for tritones and squares for fifos, e.g., I and $\boxed{\text{I}}$). Tritone anchors are unambiguous independently of context and fifo anchors are ambiguous but the ambiguity is generally resolved by context. No anchors are needed in the top pitch half of the octave because inversions cover them.

The letters of the **PADMIL** alphabet mark the relative positions of the anchors of the single tritones of classical modes, but this is only to provide a correspondence between PKP notation and music notation. It does not bind the letters to the classical modes, or to tritones. The letters are in a special boldfaced font to distinguish them from other uses of some of them in music notation/theory (there is no actual possibility of confusion because the different notations are never mixed together).

SYMMETRY BREAKING IN THE CIRCLE OF FIFTHS

According to *The Jazz of Physics*, symmetry-breaking is a deep feature of how both music and the universe work. Here's a view of breaking the symmetry of a diminished scale to yield a major scale that corresponds to an example on page 66 of the book (in the book, the dotted square and its transformation are not shown and no mention is made of tritones). This kind of thing is fascinating to anyone with a mathematical bent but probably not anyone else.



The symmetry-breaking is shown next in progressive steps that morph tritones into fifos.