Algebra 2 Final Exam Study Guide

Imaginary Numbers $\rightarrow i^2 = -1, \quad i^3 = -1, \quad i^4 = i$

01 $(5-2i)(6+4i)$

$30 + 20i - 12i - 8i^2$

$30 + 8i - 8(-1)$

$30 + 8i + 8$

$38 + 8i$

02 $(3+3i)(4+6i)$

03 $(2-4i)(7-3i)$

04 $(2+i)(8-3i)$

05 $(3-2i)(3+2i)$

Vertex $\rightarrow \dfrac{-b}{2a} = x$ then substitute x-valueinto the equation to get y. Answer= (x, y)

06 $2x^2 + 8x - 20$

$x = \dfrac{-b}{2a} = \dfrac{-(8)}{2(2)} = \dfrac{-8}{4} = -2$

$y = 2(-2)^2 + 8(-2) - 20$

$= 2(4) + 8(-2) - 20$

$= 8 + {}^{-}16 - 20$

$= -28$

The vertex is located at (-2,-28)

07 $4x^2 + 24x - 1$

08 $12x^2 + 24x - 10$

Rational Roots $\rightarrow \dfrac{p}{q}$

09 The possible roots of $2x^2 + 8x - 20$

$P: -20 \rightarrow \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$Q: \quad 2 \rightarrow \pm1, \pm2$

$\dfrac{p}{q}: \pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm\dfrac{5}{2}$

10 $4x^2 + 24x - 1$

11 $5x^2 + 24x - 3$

Solving Inequalities → Solving using factoring, then draw the number line

12 $5x^2 + 13x \geq -6$

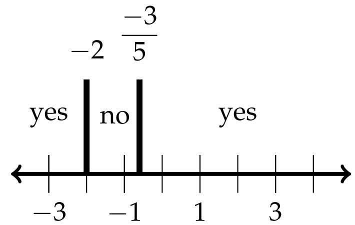
$$\begin{aligned} 5x^2 + 13x \quad &\geq -6 \\ +6 &\geq +6 \\ \hline 5x^2 + 13x + 6 &\geq 0 \end{aligned}$$

$$(\overset{1}{\cancel{5}}x + \overset{2}{\cancel{10}})(5x+3) \geq 0$$

$$(x+2)(5x+3) \geq 0$$

$x + 2 = 0 \rightarrow x = -2$

$5x + 3 = 0 \rightarrow x = \frac{-3}{5}$



Solution: $\boxed{x \leq -2 \text{ or } x \geq \frac{-3}{5}}$

13 $-3x^2 + 2x \geq -1$

(Be careful of the negative start)

Multiplying Rational Functions

14 $\dfrac{x^2 - 16}{4x^2 + 40x} \cdot \dfrac{x^2 + 12x + 20}{x^2 + 6x + 8}$

*Factor everything

$x^2 - 16 = (x-4)(x+4)$	Difference of Squares
$4x^2 + 40x = 4x(x+10)$	GCF
$x^2 + 12x + 20 = (x+10)(x+2)$	Diamond Method
$x^2 + 6x + 8 = (x+4)(x+2)$	Diamond Method

$$\frac{(x-4)(x+4)}{4x(x+10)} \cdot \frac{(x+10)(x+2)}{(x+4)(x+2)}$$

$$\Downarrow$$

$$\frac{(x-4)\cancel{(x+4)}}{4x\cancel{(x+10)}} \cdot \frac{\cancel{(x+10)}\cancel{(x+2)}}{\cancel{(x+4)}\cancel{(x+2)}}$$

$$\Downarrow$$

$$\boxed{\frac{x-4}{4x}}$$

15 $\dfrac{x^2 - 25}{5x^3 + 2 - x^2} \cdot \dfrac{(x + 8x + 16)}{x^2 - 9}$

Simplifying

16 $\frac{9x^2y}{4x} \cdot \frac{16xy}{18y}$

$\frac{9xxy}{4x} \cdot \frac{16xy}{81y}$ Expand Everything

$\frac{9\cancel{x}x\cancel{y}}{4\cancel{x}} \cdot \frac{16xy}{81\cancel{y}}$ Cancel variables

$\frac{9x}{4} \cdot \frac{16xy}{81}$ Clean-up answer

$\frac{\cancel{9}x}{\cancel{4}} \cdot \frac{\cancel{16}^{4}xy}{\cancel{81}^{9}}$ Reduce numbers

$\boxed{\frac{4x^2}{9}}$ Answer

17 $\frac{21x^2y}{6x} \cdot \frac{12xy}{7y}$

18 $\frac{9x^2y}{25x} \cdot \frac{5xy}{3y}$

Solving Rational Equations→ Factor, Restrict, Solve, Check Restrictions

19 $\frac{x-3}{x-1} = \frac{x-6}{x+2}$

$x \neq 1, -2$ Restrictions

$(x-3)(x+2) = (x-6)(x-1)$ Cross Multiply

$x^2 - x - 6 = x^2 - 7x + 6$ Multiply

$\Downarrow$

$\cancel{x^2} - x - 6 = \cancel{x^2} - 7x + 6$ Same thing both sides

$-x - 6 = -7x + 6$

$+6 \quad +6$ (Add to both sides)

$-x = -7x + 12$

$+7x = +7x$ (Add to both sides)

$6x = 12$

$\frac{6x}{6} = \frac{12}{6}$ (Divide by 6 both sides)

$x = 2$

Check the restrictions to make sure that the answer is okay. In this case, it works out so $\boxed{x = 2}$.

20 $\frac{x-2}{x-4} = \frac{x-1}{x+3}$

Asymptotes

Horizontal: Look at top and bottom degrees
Vertical: Restrict the domain (only look at the bottom)

21 $\frac{x+2}{x(x+3)}$

Horizontal:
The top has a degree of 1.
The bottom has a degree of 2.

This means the horizontal asymptote is the x-axis.

Vertical:
The bottom has x and $(x+3)$ in the bottom.
Each of those cannot equal zero so $x \neq 0$.
For the other one $x+3 \neq 0 \Rightarrow x \neq -3$.

Answer:
Since $x \neq 0$ and $x \neq -3$, this means there are vertical asymptotes at $x = 0$ and $x = -3$

22 $\frac{x^2+9}{x(x+2)}$

23 $\frac{4}{x^2-x-6}$

24 $\frac{1}{x^2-7x-10}$

25 $\frac{(x+2)(x+3)}{(x+3)}$

Sequences/Series → Arithmetic, Geometric, Convergent, Divergent

26 Notes:

$a_n = a_1 + d(n-1)$	Arithmetic
$a_n = a_1(r)^{n-1}$	Geometric
$S_n = \frac{a_n(r) - a_1}{r-1}$	Geometric
$S_n = \frac{(a_1 + a_n)n}{2}$	Arithmetic
$T = \frac{a_1}{1-r}$	Convergent

27 Find the sum of the first 39 multiples of 3.

28 Complete problems 22, 24, and 26 on the next page.

(21) Simplify the expression. $(6+5i)(-2-2i)$.

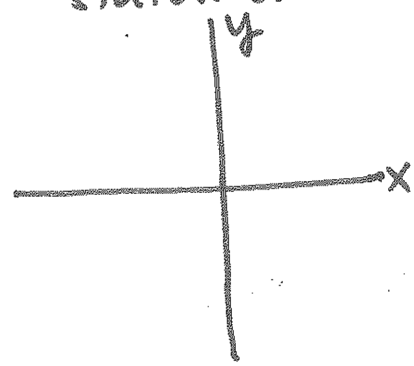
(22) Which is a divergent series?

ⓐ $\sum_{k=1}^{\infty} 3n-4$ ⓑ $\sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k$

ⓒ $2\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k$ ⓓ $\sum_{k=1}^{\infty} \left(\frac{5}{4}\right)^k$

(23) Graph & Identify true statements about

$$\frac{5}{x^2-4x+4}$$



ⓐ Domain is $\mathbb{R}$ except 5.

ⓑ Range is $\mathbb{R}$ except 0 and all negative values

ⓒ The function has 1 vertical asymptote.

ⓓ (0,0) is a point on the graph

ⓔ The graph exists on the first two quadrants

ⓕ The y-intercept is above the x-axis.

(24) Which of the following series are convergent?

(a) 100, 50, 25, ...

(b) 2, 4, 8, ...

(c) $\sum_{n=1}^{\infty} \frac{1}{3^n}$

(d) 2, 2.5, 3, 3.5, . . .

(e) $\sum_{n=1}^{4} \left(\frac{1}{3}\right)^n$

(f) $\sum_{n=1}^{\infty} \frac{5}{7}n$

(25) Solve. $\frac{x-1}{x-3} = \frac{x-2}{x-4}$

(26) Use sigma notation to rewrite the finite series and the compute

-2, -1, 0, 1, 2, 3, 4

$S_7 =$ ________

(27) Multiply and list restrictions

$\frac{5x^3y}{4x} \cdot \frac{8xy}{10y}$

(28) Add the rationals and list restrictions

$\frac{3x}{9x^2+6x+1} + \frac{4x}{3x+1}$