THE EMPEROR HAS NO CLOTHES: MUSIC AS IT ACTUALLY IS ON THE PIANO KEYBOARD

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I approached the piano as an adult beginner interested in learning to play jazz, which eventually led me to wonder how jazz pianists can improvise in terms of a music notation that seemed to me to be misleadingly complex for the piano. In the words of the naive child in the fairy tale, "the emperor has no clothes," meaning the piano, seen directly without the obscuring "clothes" of music notation, is much simpler than music notation for it. Music notation is misleading in seeming simple for pieces in easy key signatures that don't depart from the key signature, and gradually increasing in complexity as pieces become more "interesting," to the point of becoming unapproachable by all but experts.[1] The piano is not just for experts, but this sadly limits access to it by others because the increasing complexity is misleading. I had played trumpet in school bands as a youth, and so knew enough music notation to read single melody lines, but became curious about why lines starting from different pitches that were audibly the same line were represented so differently. I had learned the interval-based "solfege" scale **`do-re-mi-fa-so-la-ti-do`** in music classes and wondered why something like it wasn't the basis of music notation. I never had the time to give it much thought until I took up the piano an adult beginner, looking for a hobby for my looming retirement. After a period of piano lessons, the increasing complexity of pieces I was encountering revived my curiosity about music notation. When I retired, I had the time to indulge my curiosity. This document is the result.

I "reverse engineered" pieces of written music I was learning, to understand them in terms of **keyboard shapes**, which the piano is uniquely equipped to present to the eye in terms of intervals measured in half tones, the pitch intervals between adjacent piano keys. I learned that chord symbols, which should help with the complexity problem, are actually part of it.[2] Making them part of the solution requires seeing them as shapes formed from combining or splitting a small number of Lego-like **building blocks** that are the same anywhere on the keyboard, in terms of sizes measured in half tones. The result is a "no-clothes" notation I call **PKP**, standing for **P**icturing **K**eyboard **P**atterns. The basic elements of PKP are hidden in plain sight on the keyboard and in standard practice for voicing chord progressions of music notation. PKP bundles these elements into a simple notation that may be annotated on the written music to guide understanding and playing, and can also be written down separately as a playable shorthand notation for pieces of music. PKP is not a replacement for music notation but a lightweight complement to it. Its unique combination of simplicity and depth are either a serendipitous side effect of the organization of the piano keyboard or a fundamental property of music

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[1] An example of misleading complexity is provided by one of the simplest and most distinctive changes in music to the ear, namely a change from major to minor tonality of the same tonic. The example is for the tonic provided by the black piano key immediately above C, variously known as C# or Db. The change is from 5-flats of Db major to 4-sharps of C# minor (Appendix B provides a summary of key-signature scales). The change naturalizes 5 notes and then sharps 4 notes — 9 symbol changes to move 3 notes down a half tone! The different symbols for the major tonic Db and the minor tonic C# seem, misleadingly, to imply slightly different pitches for the home tonic. What they actually imply is slightly different pitches for the notes C and D when used as references for sharps or flats. These different pitches don't exist on the piano.

[2] The following chord progression for the haunting E*b* blues *Goodbye Porkpie Hat* written by Mingus as a tribute to Lester Young is a "poster child" for misleading chord complexity. The progression is E*b*7#9—B9(13)—EM9—A7#11—D*b*9sus—B9(13)—D*b*7sus—E*b*7—A*b*m11—B7(13)—Fm7*b*5—B*b*7#5#9—C13#11—F7(13)—B7—EM7—A7(13)—A*b*7—B*b*7—D*b*7—E*b*7♯9—B7—EM7#11—A7#11 (Appendix C provides a summary of chord symbols). There are 24 chords, an average of 2 per bar for 12 bars, only 5 of which (highlighted in blue) don't contain tritones. The tritone content is the basis of deep structure of startling simplicity that is the subject of this document (Chapter 4 presents this piece as an example).

that's obscured by music notation. Either way, they provide contextual cues to help in learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. This turns the conventional relationship between practicing and understanding on its head: understanding guides practicing instead of only emerging from it.

A 6-letter, DNA-like **alphabet** identifies the position and size of building blocks. Letters identify single building blocks and words identify shapes that form scales and chords. The shapes are determined relative to a universal **home octave** that can be positioned anywhere on the keyboard by assigning a piano key to be the **home tonic**. Thus, pieces of tonal music are understood in terms of **parallel modes** of their home tonic. This is in contrast to the key signatures of music notation that identify relative modes and put parallel modes in different key signatures. This difference is responsible for PKPs unique combination of simplicity and depth.

PKP and music notation provide **dual views** of music based on intervals vs. notes. Duality is a well known way of dealing with complexity in math and physic, so its helpfulness for dealing with the complexity of music notation should not come as a surprise. Another well known way of dealing with complexity is finding simple **symmetries** in a dual domain that defeat complexity in the original domain. The most surprising aspect of PKP is the central role played by symmetries based on tritones in the interval domain defeating complexity in the note domain. Tritones are building blocks half a keyboard octave in size (6 half tones). No one savvy in music notation would suspect them to be fundamental because anything involving more than one tritone is complex in music notation. The other building blocks are fifth and fourths a half tone larger or smaller than tritones. Fifths and fourths are the same kind of building block (consonant, invert into each other) warranting a new term for the kind, namely **fifo** (standing for fifth or fourth). The small difference in size between tritones and fifos (a half tone) and the large difference in sound (dissonance vs. consonance) makes these building blocks fundamental to both the structure and the sound of music. Simply put, **tritones are simple, fifos are understood as morphed tritones (the reason for the new term), and shapes formed by combining or splitting tritones and fifos are the fundamental shapes of music, namely scales and chords**. There are nuances but this is the substance.

I had many questions about music notation but one stands out: Given that scales are determined by key signatures, how can altering a particular note by a half tone not only tell the ear that the scale has changed but also what the new scale is? PKP provide an answer that goes to the heart of the enterprise, explained in Chapter 2.

In the process of developing these ideas, I searched for signs of them in the literature, and reached out to experts for the same thing, and found nothing equivalent. Closest were some loosely related ideas about symmetry in the book *The Jazz of Physics*, and about chords as "scale shapes" from jazz pianist Taylor Eigsti. The relationship of PKP to these ideas is explained in the coming chapters.

I have been asked: If the ideas are so good, why has no enterprising young pianist discovered them? The answer has several parts. So much time is required to master both the piano as an instrument, and music notation for it, that any young person who aspires to mastery has no time for anything else. In the process, music notation becomes embedded in the mind as the only way of thinking about music. The thinking "outside the box" that produced PKP brought in many elements from outside music that I happened to know about in different ways. Training in math and physics introduced me to the concepts of duality and symmetry. Research experience in software design — as a professor in a university department of “systems and computer engineering” — helped me to think about cooperating streams of melody and harmony on the piano as like cooperating sequential processes in software, leading to many useful insights. The final ingredients were a strong sense of other possibilities, and time to explore them provided by retirement from my job as university professor.

My admitted lack of musical credentials is reasonable cause for skepticism that I have anything useful to say, so the opinions of PKP of a couple of experts are worth quoting before proceeding. Musical theorist Paul Steinbeck: “The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music.” Jazz pianist/composer/teacher Taylor Eigsti: “… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano.”

GUIDE TO READERS

The depth of the insights provided by PKP led me to think they might be of interest to others, such as newcomers to the piano who have begun to be daunted by the complexity of music notation, pop and jazz musicians who are not pianists but want to explore harmony on the piano, “wannabe” expert pianists who might appreciate a helpful notation that exposes fundamentals, and anyone with a stake in the piano and curiosity about these issues. Expert pianists are not in this list because they have already figured out ways of dealing with the complexity, to the extent that they probably don’t even see it anymore, but may be curious about a notation that makes these claims.

Chapter 2 develops the concepts and notation. In it, new terms are boldfaced and conventional terms from music theory are in quotation marks, for early appearances. Chapter 3 develops a mode hierarchy that provides the framework for understanding music in its terms, both illustrated at every step by example pieces presented in the notation. Chapter 4 provides a smorgasbord of advanced pieces that I found difficult on first encounter (the first example is the piece of footnote 2 on the opening page). Chapter 5 provides observations and conclusions. References, acknowledgements and comments from some readers of earlier drafts follow. Appendices are about the relations between conventional representations and PKP, for terminology (A), scales (B), chords (C), enriched parallel modes (D), and hidden symmetries (E).

There’s very little music notation in the form of notes on a staff in this document because the purpose is to see the emperor without the clothes, not to describe the clothes. PKP is intended to be used in conjunction with written music in the form of annotations that help in interpreting it, but this is is helpful only if the annotations are understood in their own terms, independently of music notation. This is possible because the concepts are simple and are directly related to how the ears hear music. Interpretations in terms of music notation are results not a starting points.

The main elements of PKP are **melody lines** in chromatic scale notation (annotated on the staff or written separately), **harmony lines** in alphabet notation (annotated above the staff or written separately), and simple **mode tables** that provide a cross reference between the two. These are supplemented by Lego-like pictures of shapes formed from building blocks that can be directly understood in terms of music notation because the building blocks can be so understood.

CHAPTER 2: CONCEPTS & NOTATION

PKP concepts and notation enable probing deep and complex waters in music notation without becoming overwhelmed by details. The first ten pages contain all that's needed to understand PKP. Following that, understanding is developed via example pieces. *Happy Birthday to You* illustrates one classical mode for harmony and melody. *Over the Rainbow* illustrates chromatic harmony with no mode implications. *I Got Rhythm* illustrates cascaded mode/tonic changes known in jazz as "rhythm changes" because of their use in this piece. *Giant Steps* provides an example of simplicity underlying music-notation complexity: it "pushes the envelope" of the basic concepts so far that a new kind of simplicity emerges. These examples build towards a simple, coherent view of classical modes and the non-classical modes described in the next chapter.

STARTING FROM COLD

The central concept of PKP is a position-independent, **universal home octave** for any piece of music. This conceptual octave has two parts: a 12-half-tone chromatic scale, and a DNA-like **alphabet** for representing **building blocks** from the scale. Building blocks — tritones and fifos (fifths or fourths) — are fundamental intervals that both define scales and provide chords.

Developing a simple notation requires starting cold from a simple but accurate conceptual representation of a home octave on the piano as a line divided into 12 equal parts identifying 12 half tones. A half tone is the musical interval played by adjacent piano keys, so the 12 half tones identify 13 piano keys. The bottom **@** is the home tonic and the top **@** is the 13th piano key, which is at once the top note of the same octave and the tonic of the next octave up, which looks the same on the keyboard and is harmonically equivalent. Thus one conceptual home octave represents a stack of actual home octaves in a position-independent way. The piano keyboard provides overlapping home octaves offset by half tones, in which the half tones in the overlapped parts are shared.

An important feature of tonal music is the existence, for most tonic scales, of a scale frame **@-$-@**, in which the pitch center **$**, a half tone above the keyboard center, splits an octave into equal pitch halves with a fifth (7 half tones, a half tone larger than a tritone) on the bottom and fourth (5 half tones, a half tone smaller than a tritone) on top. A tritone splits an octave into equal keyboard halves. Scales without pitch centers are special cases, derived from scales with them. This warrants calling scales with pitch centers **primary scales**. The terms fifth and fourth can be confusing in this context because the terms refer to the number of scale steps of these intervals in the scales of key signatures: a fifth is not 5 half tones but 7, and a fourth in not 4 half tones but 5.

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                          home tonic  1/2 tone      tonic of next octave
                              |        →|  |←              |
overlapping home octaves      @ . . . . . . $ . . . . @
                                @ . . . . . . $ . . . . @
                                  @ . . . . . . $ . . . . @
                                  |    fifth    | fourth  |
                                           pitch center
                                                            . . . and so on
```

The piano keyboard aligns the pitch sizes of half tones between overlapped octaves by the simple expedient of providing only twelve, fixed-pitch piano keys for any octave that are shared with overlapping octaves. The central feature of PKP is its exploitation of this alignment to provide position-independent notations for keyboard shapes. The pitch sizes of half tones must increase within an octave to provide the doubling of pitch required for mutual consonance of notes an octave apart. If the increases are uniform within an octave, the pitches of overlapping octaves will be misaligned. The piano forces alignment. The small errors that result in the intervals of half tones are compensated by equal temperament tuning the makes larger intervals sound uniformly the same across the board. This works because half tones are dissonant intervals, and human ears are insensitive to small errors in dissonant intervals. Not everyone agrees this is good (see the books *How Equal Temperament Tuning Ruined Music* and *Lies My Music Teacher Told Me* for an understanding of the difference) but the piano has stood the test of time in many cultures worldwide, so it must be good enough. This establishes half tones as a universal measure of interval size across music, understanding that their pitch sizes may vary slightly.

Building up from Solfege Scales

The singer's "solfege" scale **do-re-mi-fa-so-la-ti-do** is a natural starting point for developing PKP because it's a position-independent, primary scale, based on intervals measured in half tones. The scale has two half tones **mi-fa** and **ti-do** and otherwise is all whole tones (two half tones). It's tonality is major because **do-mi** is a major third. The scale is position independent because the pitches of the notes follow from knowing the intervals, which is opposite to music notation. Singers know the intervals by ear and pianists can know them by eye on the keyboard.

This scale is the master mode of a family of modes determined by rotating it to start on different notes. For example, the rotation **la-ti-do-re-mi-fa-so-la** is a primary minor scale. It tonality is minor because **la-do** is a minor third. It has the same intervals in a different order.

These modes may be aligned on a shared tonic tonic **do** as shown next (whole tones are spaces with dashes, half tones are spaces without dashes). The names on the left are of the classical modes that provide the interval sequences of the default major and minor modes of key signatures, which are the interval sequences of these modes. The alignment makes the modes **parallel**, which means that in music notation they would have different key signatures, which is a complex way of representing the simple change from major to minor of the same tonic (see footnote 1 of Chapter 1 and Appendix B). PKP is based on parallel modes understood independently of key signatures.

The note symbols of these modes will be replaced later by the the symbols of a shared, 12-half-tone chromatic scale, but the concepts are best understood at this stage without the distraction of unfamiliar chromatic scale symbols.

⬇ tritone anchor

Ionian	**do – re – mi fa – so – la – ti do**
Aeolian (aka "natural minor")	**la – ti do – re – mi fa – so – la**

⬆ tritone anchor

Three takeaways from this picture are independent of solfege notation and lead directly to PKP notation. The first is the concept of **symmetric constructor shapes** (highlighted in grey) that determine the scales. The constructors consist of the mode tritone with two adjacent half tones, outside it for Ionian (**xx-xx**, where the **x**s are tritone notes), and inside it for Aeolian (**xx-xx**). As shown, these symmetric shapes can be understood purely in terms of intervals, without any note symbols. The symmetric shapes are of opposite kind because the same shape for Aeolian would add additional half tones. The second is that **symmetry breaking** by whole tones creates the scale (only whole tones because the scale's two half tones are already in the symmetric shape). The third is that **tritone anchors** — the nearest note of a tritone above the home tonic identified by an anchor symbol that is not a note symbol — are sufficient identifiers of tritones. Because tritones are the same size in either inversion, knowing a tritone anchor means knowing the tritone as a **building block** of fixed size that goes up from the anchor in the home octave but could go down from it in a piece of music.

The Alphabet

The alphabet **PORMIL** identifies six tritone anchors a half tone apart in the lower fifth of the home octave. The anchors of the tritones identified earlier for the Ionian and Aeolian modes represented in solfege notation are **I** and **O** from this alphabet. The combination of the alphabet and the scale frame identified earlier forms an **alphabet octave @PORMIL$xxxx@** that determines keyboard shapes. The upper fourth has no anchors because the tritones in the lower fifth supply the notes. Anchored tritones may invert into tritones going up from these notes as bass notes, but the bass notes are no more than the treble notes of the anchored tritones. Every tritone has only one anchor within the home octave. Inversions are identified by underlining the anchor letter. The letters are in a distinctive font (Arial black) to distinguish them in text (circling them in handwritten annotations accomplishes the same purpose).

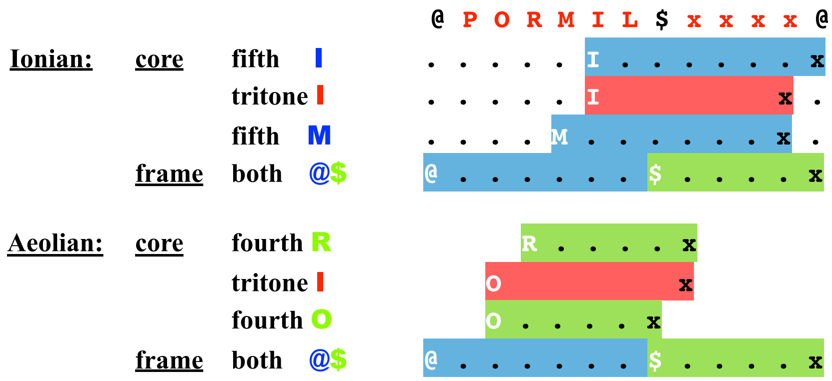
The full alphabet is provided by the names of classical modes listed in the unconventional order of their single tritones going up the keyboard by half tones: **P**hrygian, Ae**O**lian, Do**R**ian, **M**ixolydian, **I**onian and **L**ydian/**L**ocrian. Locrian is a derived mode of Lydian with the same tritone and all non-tritone notes different (one consequence of which is the absence of the pitch center). The first letters of Aeolian and Dorian names are not used because they're too easily confused with note symbols of music notation. The anchor letters refer to the modes but the anchored tritones exist independently of any mode.

The other modes will be presented later. The parallel Ionian and Aeolian modes identified earlier in solfege terms may now be summarized without reference to note symbols of any kind, as follows.

		@	P	O	R	M	I	L	$	x	x	x	x	@
Ionian:	**tritone I**	.	.	.	.	.	x	.	.	.	.	.	x	.
	constructor	.	.	.	.	x	x	.	.	.	.	.	x	x
	mode	o	.	o	.	x	x	.	o	.	x	x	.	o
Aeolian:	**tritone O**	.	.	x	.	.	.	.	.	x	.	.	.	.
	constructor	.	.	x	x	.	.	.	x	x	.	.	.	.
	mode	o	.	x	x	.	o	.	x	x	.	o	.	o

The most important building blocks of the Ionian and Aeolian modes are shown next in a Lego-like notation that's useful to have in the mind's eye. The core provides basic harmonic sequences. The frame provides a useful starting point for forming primary scales (ones with pitch centers). Building blocks go up from anchors and their inversions go down. As said earlier, inversions are identified by underlining anchor letters. For example tritone **O** going up from its anchor inverts into tritone **O** going down. Fifos are different because the color coding changes width inversions: the fourth **O** going up from its anchor inverts into the fifth **O** going down, and the fifth **R** going up from its anchor inverts into the fourth **R** going down (the underlining is not of the original, but of the result).

The scale frame shows that the symbols **@** and **$** anchor fifos. Preserving the uniqueness of tritone anchors requires that **@** and **$** never anchor tritones. However, inverted tritones may have them as bass notes (e.g., inverted tritones **L** and **P** go up from the tonic and pitch center as bass notes).

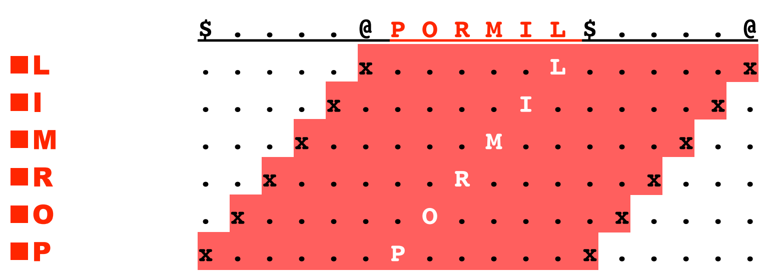


It's worth pausing here to reflect on the importance of tritones and fifos (fifths or fourths) as fundamental building blocks of music. They split the most fundamental interval of music, the octave symmetrically (tritones) and asymmetrically (fifos) — it's difficult to imagine that anything so structurally fundamental would not also be musically fundamental. The morphings of these building blocks into each other form a closed set that provides transitions between dissonance (tritones) and consonance (fifos) that are fundamental to the sound and emotional impact of music. The workhorse chords of harmony, namely triads, sixth chords and seventh chords are either split building blocks or overlapped combinations of pairs of building blocks. The "guide intervals" of standard chord voicing practice are these building blocks, except notated several levels removed from the keyboard (they're identified by pairs of degree numbers of chord scales relative to constantly changing chord roots).

Primitive Elements of PKP

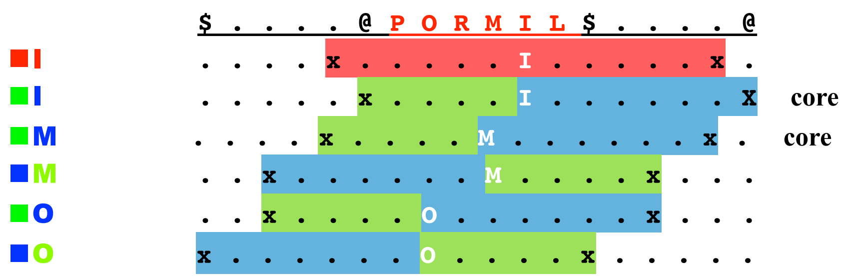
The primitive elements of PKP are not the building blocks themselves, but anchor-centered octave stacks formed of a building block and its opposite inversion. Adding the keyboard center to an octave yields a **symmetric octave stack** formed of a tritone and its opposite inversion. With one exception (tritone F-B), tritone piano keys are always of opposite kinds (black vs. white), which means that, with this exception, the keyboard center is always of the opposite kind to the octave notes. There are no all-black-key tritones. Raising the keyboard center a half tone to the pitch center yields an asymmetric octave stack formed of a fifth with a fourth on top. Thus octave stacks provide the opposite inversions of building blocks for free.

The six tritone stacks identified by the alphabet are shown next.

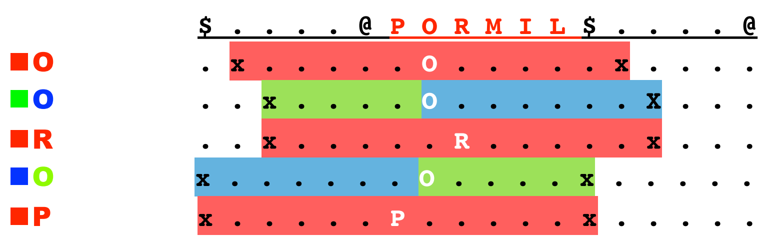


The alphabet octave is extended to include inversions going down into the next octave. The notations on the left identify octave completion intervals by colored box prefixes, which avoids having to repeat the anchor symbols, underlined for the inversions. Tritone stacks morphing into fifo stacks identify fifos as morphed tritones. Given that tritones determine modes, this opens the possibility of leaving fifo details to context. Octave stacks in harmony are almost chords and so provide a simple starting point for forming chords. Picturing music in terms of octave stacks is simple for single-tritone classical modes but is even simpler for multi-tritone modes used by much music. This is so because tritones are the simplest building blocks and more tritones provide more scale notes. This increasing simplicity is in stark contrast to the increasing complexity of music notation for the same thing.

As shown next, an Ionian tritone stack morphs directly into two core fifo stacks that provide four core fifos of the mode. As above, the notation on the left represents octave intervals by box prefixes that identify the color of the inversion (red completes red, green completes blue, blue completes green). The completed mode provides 6 more fifos morphed from these fifos.



This is only for the Ionian tritone. Every tritone has its own set of morphed fifos. The sets for different tritones overlap. For example, the **O** and **O** fifo stacks above are also morphed from the **P**, **O** and **R** tritone stacks not in the Ionian mode, as follows.

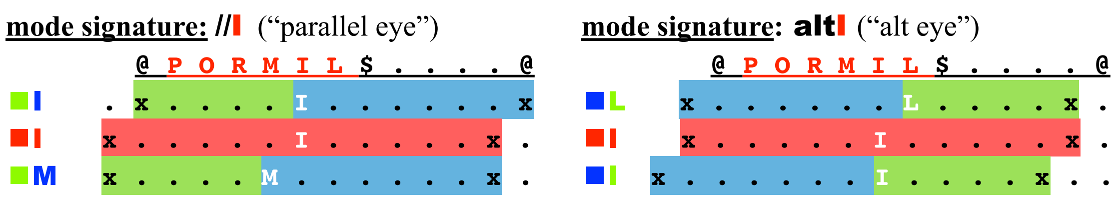


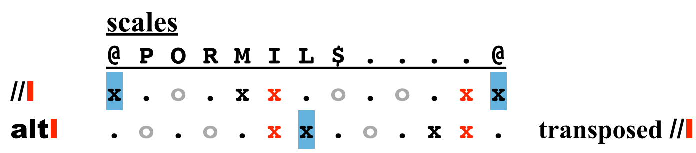
The simple, transparent nature of this way of understanding available building blocks is in sharp contrast to the complexity of the same thing in music notation. It makes possible the transformation of core building blocks into chords by adding core building blocks from context. It makes possible the substitution by eye of consonant fifos of chords for one another to fit context — think of this as chord substitution from the bottom up. It aids in the understanding of mode/tonic changes.

<u>"Alt" Modes</u>

The inversions of tritones provided by octave stacks are harmonically equivalent in established modes but become signifiers of change during changes between modes. At such times, opposite inversions identify possible modes of the same kind with tonics offset by tritone and all non-tritone notes different. This is so because inverting a tritone is equivalent to transposing it by a tritone. A tritone stack covers both inversions so the only difference in its terms is different morphings into fifo stacks that provide different non-tritone notes.

The modes identified by the different morphings are illustrated next for the ■I tritone stack. The mode signature **//I** identifies the the primary mode (**//** is proncounced "parallel"). The mode signature **altI** identifies a **tritone substitute mode** (borrowing a term from the chord domain) with the same tritone and all non-tritones notes different. The **altI** mode is actually a relative mode of an Ionian mode **transposed** by a tritone but the notes are the same either way and thinking in terms of the **alt** mode within the home octave is simpler. The different morphings yield different symmetric shapes **xx—xx** and **xx—xx** that determine different scales. This isn't math nor is it complex, it's only a compact symbolic notation.





These morphings answer a question in opening chapter: How can altering one particular note by a half tone not only tell the ear that the scale has changed but also identify the new scale? Given the establishment of a mode by one of these core sequences, morphing the tritone in the opposite direction from the one that established the mode will sound wrong to the ear, suggesting the opposite core. This means changing one core note of a core fifo by a half tone changes the mode. Knowing the simple relationship between the cores enables rejection of fifo possibilities that don't fit a particular mode. For example, the establishment of an Ionian mode by the core sequence **I-I-M** excludes the fourths of its tritone substitute mode determined by the core sequence **L-I-I** on the right.

Other Intervals in Alphabet Terms

Building blocks are not, of course, the only intervals of music. Intervals smaller than building blocks are major thirds, minor thirds, whole tones and half tones; intervals larger than building blocks are augmented fifths, major sixths, minor sevenths and major sevenths. The larger and smaller intervals spreading out in opposite directions from the building blocks are inversions of each other, so knowing one means knowing the other. These intervals appear in music mostly as inherent parts of shapes formed from building blocks determined by alphabet letters or words. The infrequent special cases when one of them acts as a separate building-block-like object are easily handled by annotations on letters. Details are explained as needed for example pieces, and summarized in Appendix A.

A UNIVERSAL CHROMATIC SCALE

The chromatic scale is not needed to understand the concepts, and introducing it earlier would only have been distracting. However, it's needed now to complete the picture. It consists of seven numbers identifying the fixed positions of the piano keys of the master Ionian mode within any home octave, and five prefixed numbers identifying the fixed positions in its 5 whole-tone gaps. The only reason for the choice of the Ionian mode as the basis for the scale is to provide a strong connection to music notation, in which this mode is conventionally understood as the master mode of a key signature.

conceptual home octave	@ P O R M I L $ x x x x @
master major scale (Ionian mode)	1 . 2 . 3 4 . 5 . 6 . 7 1
piano keys in its whole-tone gaps	p2 p3 p5 p6 p7
universal chromatic scale	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1

The prefix "**p**" stands for "phlat" and means "next piano key down." The prefix is not a conventional flat because it applies only to five specific notes that have whole tones below them in the Ionian mode. The numbers are not degree numbers that count scale notes, but fixed note positions within the home octave. There are no flats or sharps that can be attached to any piano key to raise or lower its pitch. The chromatic scale itself does the raising or lowering (e.g., **4-p5** and **p5-5** raise the pitch of of **4** and **5** by a half tone). The key-signature rule that the same note symbol (e.g., **p5**) cannot appear in successive scale symbols does not apply.

The simplicity of having a universal chromatic scale for any home octave contrasts strongly with music notation's different chromatic scales identified by sharps, flats and naturals relative to different key signatures.

The chromatic scale notation is not novel. It's adapted from a well known Roman-Numeral (RN) based scheme for identifying **chord roots** relative to a tonic, described, for example, in Mehegan's jazz piano instruction book. The possible chord roots in the chromatic scale are all the notes of the chromatic scale with plain numbers replaced by RN symbols. The only novel feature is the use of the "phlat" prefix **p** that avoids Mehegan's confusing use of conventional flat symbols that are not actual flats in music notation. His notation accurately represents the black keys of the C octave but is confusing for other other octaves with black keys in different scale positions.

chord roots in PKP **I-pII-II-pIII-III-IV-pV-V-pVI-VI-pVII-VII**.

In PKP, chords are identified as shapes defined by the alphabet, and this notation enables assigning a chord root to a shape. The shape determines the nature of the chord. These roots replace the letter-note roots of conventional chord symbols in explanations. The combination is somewhat awkward because the prefix **p** in the root notation is different in kind from sharps and flats in chord symbols, but the awkwardness is not a problem because chords represented this way are interpreted results, not starting points.

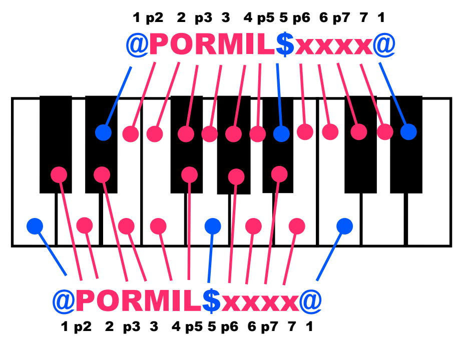
The home tonic is the reference for understanding changes to secondary tonics identified by **tonic pointers** of the form **@t**, where **t** is a chromatic scale symbol identifying a tonic (the home tonic pointer is implicitly **@1**). Tonic pointers enable parallel-mode changes to be interpreted as tonic changes.

A UNIVERSAL HOME OCTAVE

The universal home octave is a combination of the alphabet and the chromatic scale, typically used as a pair of header lines on mode tables in which entries identify note positions, as shown next.

	@ P O R M I L $ x x x x @
	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
Ionian mode	○ . ○ . x x . ○ . x x . ○

The universal home octave maps to the keyboard as follows for two possible home tonics (the color blue for the scale frame here is only for contrast with black piano keys, not an indicator of building-block size). The mix of black and white piano keys is visibly very different for different home octaves but the difference is manageable because of the intuitively simple nature of the notations. The chromatic scale mirrors the look of the C octave on the keyboard, enabling other octaves with different mixes of black and white piano keys to be understood in the same terms.



The only caveat is the necessity of keeping the home tonic fixed in the mind, to avoid confusion with recently played pieces with different home tonics that may still be in mind. This can be helped by putting a removable stick-on label on the tonic key, and by running through scales, arpeggios and chord sequences for a new tonic to get the new representations into the mind and fingers before approaching an actual piece. The difficulty of doing this is much less than the difficulty of dealing with the many and various different chromatic scales of music notation for different home tonics. The advantage of doing it is thinking of pieces of music with different key signatures in common terms. Music notation still has to be dealt with, but my experience has been that it's relatively easy to think in these terms when the PKP notation is annotated on the written music. With a bit of experience, the PKP notation can be used by itself to provide a shorthand description of the melody and harmony of an entire piece

PARALLEL CLASSICAL MODES

Here follows a simple table of all possible parallel classical modes. The changes are determined by the tritone moving in half-tone steps and the symmetric core shapes **xx–xx** and **xx–xx** alternating at each step. The alternation means the transposed master tonic changes by a fifth for every tritone change by a half tone (blue highlighting). For example, the **I-L** change up a half tone moves the Ionian tonic **@1** up a fifth to **@5**, and **I-M** change down a half tone moves it down a fifth to **@4** (up a fourth within the octave). This is true for any original mode but only the changes for the master Ionian mode are shown.

The letters in the **mode signatures** on the left are understood to be tritone anchors, so the color-coding shown is optional.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	tonality	transpositions	name
		@	P	O	R	M	I	L	$	.	.	.	.	@			
	//L	x	.	o	.	o	.	x	x	.	o	.	x	x	major	tr[//I]@5	Lydian
master	//I	x	.	o	.	x	x	.	o	.	o	.	x	x	major	//I@1	Ionian
	//M	o	.	o	.	x	x	.	o	.	x	x	.	o	major	tr[//I]@4	Mixolydian
	//R	o	.	x	x	.	o	.	o	.	x	x	.	o	major	tr[//I]@p7	doRian
	//O	o	.	x	x	.	o	.	x	x	.	o	.	o	minor	tr[//I]@p3	aeOlian
	//P	x	x	.	o	.	o	.	x	x	.	o	.	o	minor	tr[//I]@p6	Phrygian
	altL	x	x	.	o	.	x	x	.	o	.	o	.	x	minor	tr[//I]@p2	Locrian
	altI	.	o	.	o	.	x	x	.	o	.	x	x	.	minor	tr[//I]@p5	—
	altM	.	o	.	x	x	.	o	.	o	.	x	x	.	minor-major	tr[//I]@7	—
	altR	.	o	.	x	x	.	o	.	x	x	.	o	.	major-minor	tr[//I]@3	—
	altO	.	x	x	.	o	.	o	.	x	x	.	o	.	major	tr[//I]@6	—
	altP	.	x	x	.	o	.	x	x	.	o	.	o	.	major	tr[//I]@2	—

The modes are "classical" because they provide the interval sequences of the scales of key signatures, are "modes" because the interval sequence of one rotates into the interval sequence of any other, and are "parallel" because the **//** or the **alt** mode or both include the home tonic (among the **alt** modes, only **altL** includes it because the tritone includes it).

Two Sides of the Same Coin

The table shows that parallel modes and tonic changes are "two sides of the same coin." Put simply, the table covers all possible mode/tonic changes for classical modes. Corresponding tonic changes (highlighted in blue) by fifths down (alternating fifth down and fourth up in the table) determine the same modes transposed by a tritone. Rotating them into relative modes within the home octave yields the table. Either way alters the same notes. Tonic changes smaller than a tritone (fourths, major third, minor thirds, whole tones, half tones) are covered by other relative modes (not shown but easy to figure out — move the blue-highlighted tonic pointer to the desired different tonic).

Enriched Classical Modes

Looking ahead to the next chapter, double-tritone enrichments of these modes preserve the main properties of classical modes (7 notes, 2 half tones, no adjacent half tones). The sets of enriched modes are parallel "melodic" and "harmonic" modes determined by well known master modes called "melodic minor" and "harmonic minor" that are 1-note alterations of the Ionian and Aeolian classical modes. It's helpful to look at these master modes now, to get a sense of how tritone mode signatures identify multi-tritone modes in a very simple way. The dots in the mode signatures indicate skipped alphabet letters). The altered note forms a tritone with a scale note. The resulting double tritones provide constructors for the modes based on 4 whole tones and 3 minor thirds within the home octave that require only one note from context to complete.

		@	P	O	R	M	I	L	$	x	x	x	x	@	
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
melodic minor	//R.I	x	.	x	x	—*—	x	.	x	.	x	.	x	x	//I with 1 note altered (*)
harmonic minor	//O..I	x	.	x	x	.	x	.	x	x	——	*—	x	x	//O with 1 note altered (*)

The simple mode signatures developed in the next chapter for the sets of parallel modes identify one mode each, *by construction*. In contrast, the sets of parallel modes are known conventionally by an alphabet soup of many more names than there are modes, that identify the different members in different contexts in a bewilderingly complex way (Appendix D).

MELODY LINES

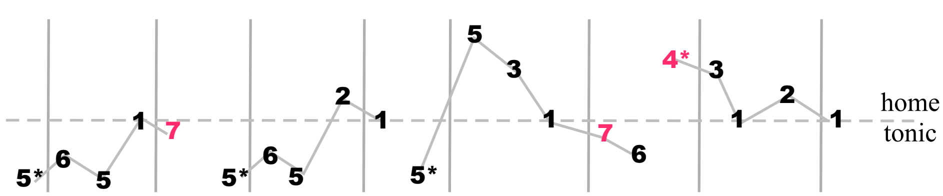
"Melody" and "harmony" are interchangeable concepts. Any single line of a piano piece music (sometimes a pair of them) may be treated as melody, automatically relegating other lines to harmony. A melody in tonal music is normally a single line that determines a "song" that can be sung, whistled or hummed, independently of the details of harmony, rhythm and timing (the only specified timing is one note after the other). The distinction between melody and harmony becomes blurred for melody lines accompanied by, for example, walking bass lines that can themselves be song-like.

As illustrated next for the piece *Happy Birthday to You* in F-major, the first step in representing a melody line in PKP terms is annotating chromatic scale symbols on the staff next to melody notes, with commas marking the ends of melody phrases. The basic purpose of the notation is to show melody plus harmony annotated on the written music. The melody is visibly Ionian. The harmony is left to an example coming up.

Independent Representations

The PKP representations of melody plus harmony can be written down separately from the written music, to provide a compact summary that's independent of it. This is done for most of the example pieces in this document, to avoid getting bogged down in details, and to highlight this independence. It's also useful in general to get a handle on pieces that are intricate, or lengthy (e.g., spread out over many pages of fully written out arrangement).

Sketching the line on graph paper and labeling the points with chromatic scale symbols is one way of getting an independent view. Asterisks indicate repeated notes. While this is useful for getting an independent view, it takes up too much vertical space on the page for a compact summary of pieces with longer melody lines that stretch over multiple page lines.



A compact summary follows from collapsing the line of chromatic scale symbols into a single, text line. The line of symbols by itself is ambiguous — is the next note up or down? — but the ambiguity is easily removed without adding any new symbols by highlighting pivot points: grey for low points, yellow for high points (think of yellow as mountain peaks illuminated by sunlight and grey as valleys in shadow). Notes within an up or down arc between pivot points are understood to go to the nearest note with that symbol. Zig-zags within arcs don't need highlighting because the trend remains in the direction of the arc. In rare cases, arrow prefixes (↑↓ not illustrated here) indicate moving the line up or down an octave.

```
5* | 6 5 1 | 7,     5*| 6 5 2 | 1,     5* | 5 3 1 | 7 6,     4* | 3 1 2 | 1
```

Timing may be represented by a line of downbeat markers (^) under the melody line. Downbeat markers identify timing independently of anything else, which means using a different timing requires only ignoring or changing the downbeat markers (or avoiding using them in the first place, often sufficient for familiar pieces). Timing choices for notes between identified downbeats are left open — the only constraint is squeezing them in. The choice between a rest and an extended note for a downbeat marker between melody notes is left open.

```
5* | 6 5 1 | 7,     5*| 6 5 2 | 1,     5* | 5 3 1 | 7 6,     4* | 3 1 2 | 1
^    ^ ^ ^  ^ ^     ^  ^ ^ ^ ^  ^      ^   ^ ^ ^ ^  ^       ^    ^ ^ ^  ^
```

Improving Readability

Readability is improved for longer or more intricate melody lines, without sacrificing simplicity, by offsetting the highlighted pivot notes up or down from the text line so that they stand out to the eye. Everything on the same line within pivot points is understood not to go outside the arc.

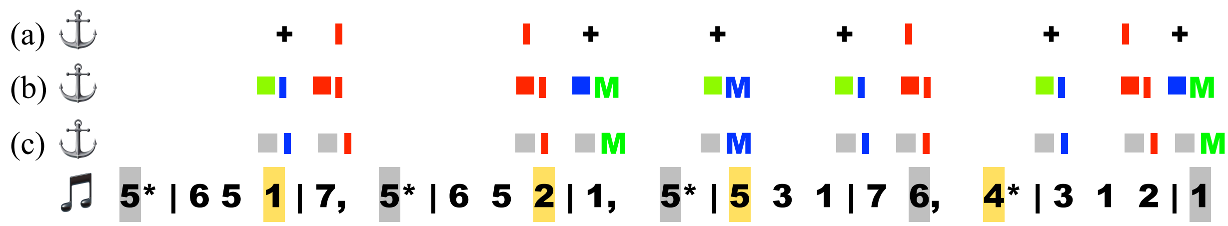
```
5* | 6 5 1 | 7,     5* | 6 5 2 | 1,     5* | 5 3 1 | 7 6,     4* | 3 1 2 | 1
```

EXAMPLE: *HAPPY BIRTHDAY*

This an example of one classical mode (Ionian) for everything follows from the earlier melody line in F-major. The relevance of this example goes beyond this particular piece or classical modes in general because the same concepts and notations apply across the board. The skeleton melody line seen earlier is identified by ♫ and the harmony line by ⚓ (these identifications are helpful for lengthier, more complicated pieces than this, but it's best to begin as one intends to continue). This harmony could follow from annotating anchors of building blocks next to chord symbols above the staff, but can also be developed independently of the written music, as is done here. This is short and simple, so pivot notes are not offset.

Core Harmony

The core harmony line developed in steps (a)-(c) shows building blocks within the home octave sliding or morphing into each other within the home octave, with attachments providing voicings.



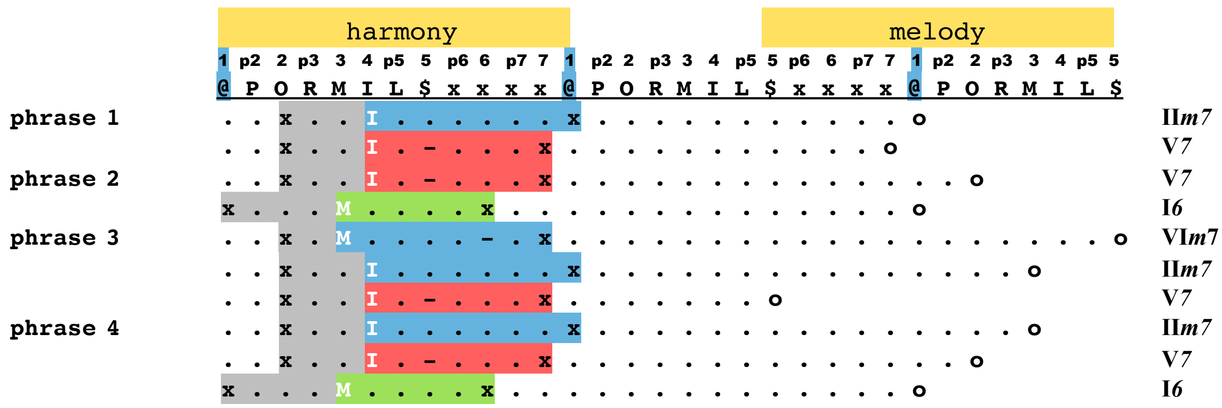
Here follows an explanation of the steps.

(a) The repetition of the single tritone I identifies the same mode (Ionian) as the melody. This example doesn't start from chords, but the logic of the tritone placement can be understood without reference to them. The tritones identify resolution in the mode and so are placed just before ends of melody phrases that resolve to the tonic, or at the ends of melody phrases that don't resolve, anticipating future resolution. The tritones establish a flow pattern — very simple here — that, with the known mode, constrains the choice of fifos. Harmony fifos come later and so need only be identified by placeholder symbols (**+**).

(b) Identify a voicing strategy by putting the tritone stacks in a form indicating completion on the bottom (here) or top. Add fifos and their stacks with corresponding completion notation. The anchor-line fifos are morphed from the tritone in a way that determines either **//I** or **altI**, and the former is the only choice that fits this melody line. Transitions in anchor-line harmony within the home octave are generally **slides** (e.g., **M-I**), **morphs** (e.g., **I-I**), or **wobbly slides** that combine the two (e.g., **I-M**). A slide moves a building block to a different keyboard position while holding its size. A morph changes its size while holding one end fixed. A wobbly slide changes both position and size in one step. This unusual term represents the hand movements exactly — slide the hand while moving the fingers ("wobbling" them) for the size change. The **I-M** wobbly slide may be understood as **I-M** transformed into **I-M** by fifo substitution. This decides **M-M-I** for the placeholder sequence in the middle. The **I-M** wobbly slide may also be understood as a contraction of **I-M-M** (**I-M** slides down a half tone, and **M-M** morphs the tritone into a fourth in the mode), which can be useful for mode changes.

(c) This is **core harmony** in which shape variety is provided by morphed octave shapes. The grey-box prefixes represent major or minor thirds determined by context (grey-box suffixes would produce a different, but harmonically equivalent result).

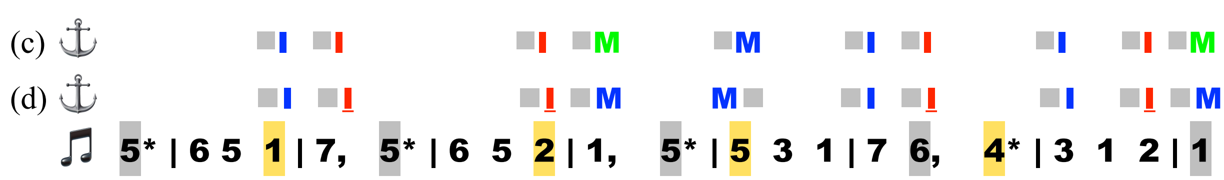
Here follows a Lego-like view of the (c) result, for the harmonized melody notes only (the others are understood to be interspersed). These voicings are compatible with the chords are shown on the right (omitted roots implied by context identified by dashes).



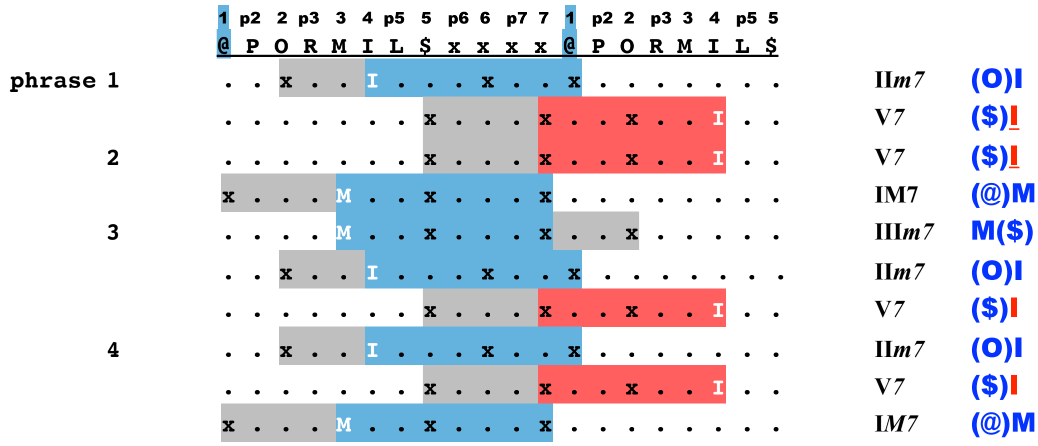
Core Harmony as a Basis for Any Harmony

The most obvious way of playing a written chord progression, namely as keyboard shapes going up from a chord root line, is technically difficult starting from cold because it requires lifting the entire hand and moving it accurately by often largish intervals between chord roots, while at the same time adjusting the inter-note intervals to fit the particular combination of white and black piano keys of the determining home octave. However, the step from **core harmony** to this way of playing harmony is simple and straightforward for classical modes, as illustrated next for our example.

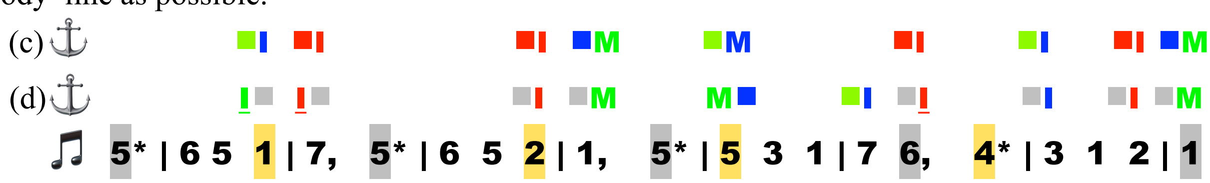
Constant-scale-shape harmony: This is harmony in which the set of inter-note scale intervals is the same for every shape. In this example, the intervals are all the same (2 scale steps), yielding basic seventh chords of classical modes. An interval of two scale steps in a classical mode is either a major third or a minor third. The voicing intervals identified by the grey box attachments are one or the other, determined by the scale position of the shape.



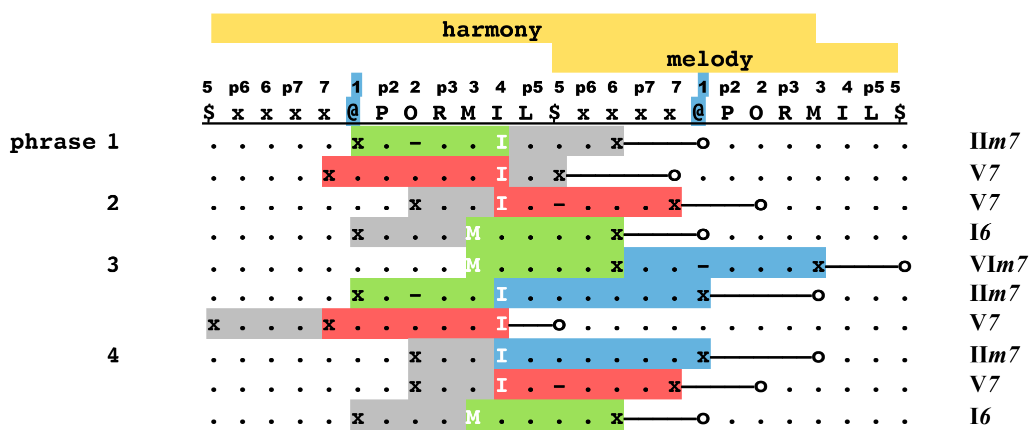
Here's how this looks on the keyboard. The result is spread over the octave-and-a-fifth interval below the melody line. The result is all basic seventh chords determined by the chord symbols on the right. Specifying the exact sizes of the voicing intervals identified by the grey boxes is redundant when their sizes are as obvious from context as they are here. For completeness, the pairs of building blocks that form the chords are shown at right, with the core-completion building blocks in parentheses. However, this is often both overly precise (completion tends to be obvious from context) and overly committed (too many symbols get in the way of understanding). That said, the notation is useful for understanding tricky chord sequences.



Voice-leading harmony: This is different in kind because it puts the harmony as close to the melody line as possible.



The original anchor line is transposed up an octave, with building blocks going down or up from it determined by the melody line. Here's how this looks on the keyboard. The (a)-(c) steps are the same as the original, except the (c) step is tweaked by inverting selected building blocks. The adjusted result in is (d) is overlapped melody and harmony. The octave shapes in phrase 3 fit the chords shown but the chords are completed in the flow rather than in place.



More About Chords

As illustrated by these examples, combinations of building blocks yield smaller and larger intervals as inner or outer intervals, without any need to notate them specifically.

Different inversions of the same combination of building blocks may have different chord symbols, increasing notational complexity without any increase in substance: for example, the inversion **I*6*/VI** is **VI*m7*** (the slash suffix indicates an inversion going up from a note identified by a root symbol); the inversion **II*m7*/IV** is **IV*M6***; the inversion **V*7*/II** is not normally given a separate chord symbol because it doesn't have the shape of a basic seventh or sixth chord it may be given an altered chord symbol).

Seventh and sixth chords can be understood as combinations of triad chords that share two notes: **I*M7*** is a combination of a **major triad** with root **I** and a minor triad with root **III**; **II*m7*** is a combination of a **minor triad** with root **II** and a major triad with root **IV**; **V*7*** is a combination of a major triad with root **V** and a **diminished triad** with root **IV**. Turning this around, a seventh or sixth chord may be implied by a sequence of triad chords.

A mix of triad chords and sixth and seventh chords tends to be complex because it increases the number of chord types. Triads are unlike chords formed of combinations of building blocks, in that inversions are not of the same form as the original (an inversion of a split building block is not a split building block). The different forms of inversions add to the complexity.

Triads are represented as split building blocks in PKP, identified by a superscript notation that preserves the visibility of the building blocks. Major or minor triads are asymmetric splits of fifths into a minor third and a major third in either order, symbolized by superscript "▲" indicating the larger interval is on the bottom, or superscript "▼" indicating the larger interval is on top. Diminished triads are symmetric splits of tritones, symbolized by superscript "●".   Fourths enter the picture via inversions of fifths.

Here follow some examples for the Ionian mode.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
	@	P	O	R	M	I	L	$	x	x	x	x	@	P	O	R	M	I	L	$	
@▲	@	.	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.	.	.	.	**major triad**
O▼	.	.	O	.	.	x	.	.	.	x	.	.	.	.	.	.	.	.	.	.	**minor triad**
M▼	.	.	.	.	M	.	.	x	.	.	.	x	.	.	.	.	.	.	.	.	**minor triad**
no symbol	.	.	.	.	.	I	.	x	.	.	.	x	.	.	.	.	.	.	.	.	**follows from a combination**
I●	.	.	.	.	.	I	.	.	x	.	.	x	.	.	.	.	.	.	.	.	**diminished triad not in the mode**
I▲	.	.	.	.	.	I	.	.	.	x	.	.	x	.	.	.	.	.	.	.	**major triad**
I●	.	.	.	.	.	.	.	.	.	.	.	x	.	.	x	.	.	I	.	.	**diminished triad in the mode**

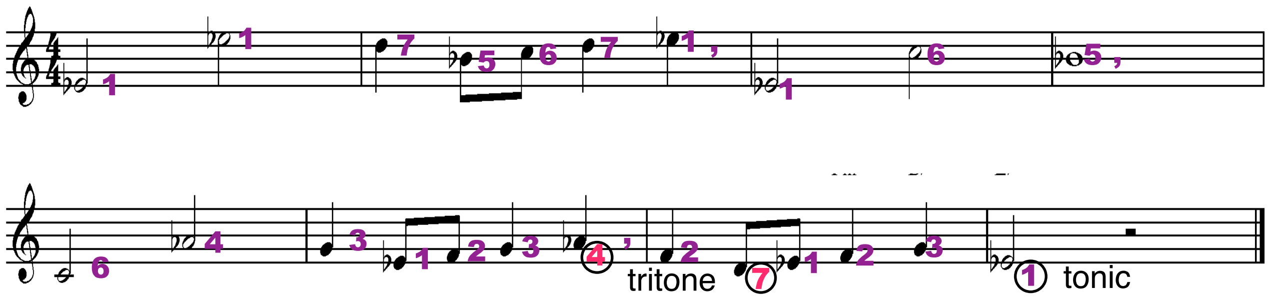
If the superscripts seem complex at first glance, keep in mind that the basis is always simple building blocks on the keyboard, not complex chord symbols several levels of abstraction distant from the keyboard.

It's sometimes useful to represent inversions of minor or major triads by outer intervals (not diminished triads because this would obscure the tritone). For example, the outer interval of the first inversion of minor triad @▼ is the major sixth R▷▷ (a fifth expanded by two half tones) with an inner note that may be left to context.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	I	L	$	x	x	x	x	@
@▼	@	.	.	x	.	.	.	x	.	.	.	.	.
R▷▷▼	.	.	.	x	.	.	.	x	.	.	.	.	@
R▷▷	.	.	.	R	.	.	.	.	.	.	.	.	x

EXAMPLE: *OVER THE RAINBOW*

This example develops strongly chromatic ornamental harmony from scratch for the opening melody phrase of the well known piece *Over the Rainbow*. This illustrates that written harmony may be just harmony, without any other purpose than to sound good. It need not originate in the melody scale and may not have any intended meaning as a harmony scale. The first eight bars of this piece are sufficient for the purpose. The melody line is straight Ionian, trending downwards in zig-zags over an octave range to home tonic Eb (the implied Ionian key signature is 3 flats).



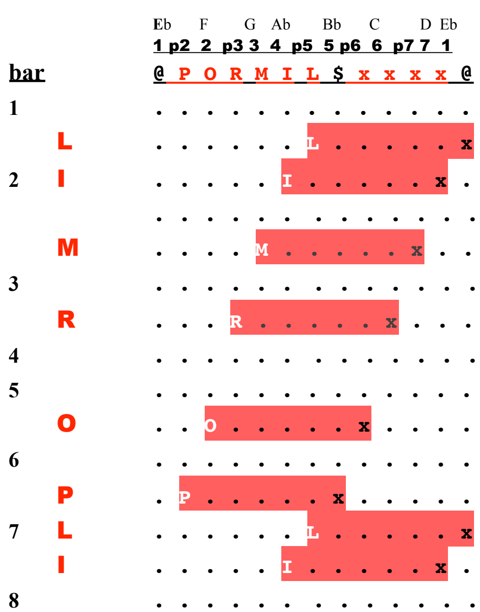
Here follows the skeleton melody line plus a skeleton harmony line consisting of all the tritone anchors in reverse order starting on **L** and wrapping around: **L-I-M-R-O-P-L-I**. This line is an example of constant-keyboard-shape harmony: the same keyboard shape (a tritone) moves to different positions in the chromatic scale.

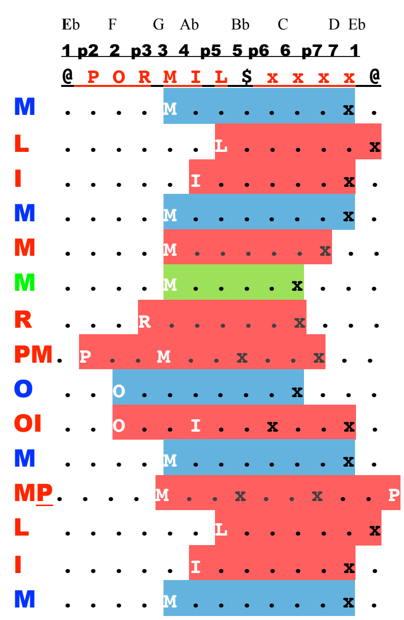
Over the Rainbow (orginal home tonic = Eb)

⚓ L I M R O P L I

♫ | 1 1 | 7 5 6 7 1, | 1 6 | 5, | 6 4 | 3 1 2 3 4, | 2 7 1 2 3 | 1, |

As shown next, the tritones provide a framework for adding core fifos and some other variations.





The double tritones provide elegant symmetric relationships between successive shapes (e.g., **PM-O** shrinks the outer notes of the double tritone inwards a half tone; and **O-OI-M** moves a fifth aligned with the bottom note of **OI** to a fifth aligned with the top note).

Here follows the melody plus new harmony. Play the single building blocks as octave shapes that go with the flow. The harmony could but isn't intended to imply parallel mode changes, or the tonic changes that are the other side of the same coin. It's only ornamentation.

Over the Rainbow (orginal home tonic = Eb)

⚓ **M L I M M M R PM O OI M MP L I M**

♫ **| 1 1 | 7 5 6 7 1, | 1 6 | 5, | 6 4 | 3 1 2 3 4, | 2 7 1 2 3 | 1, |**

TOWARDS AN ARCHITECTURAL VIEW

In a famous session of NPR's *Piano Jazz*, Bill Evans, in conversation with Marian Macpartland, said words to the effect that he advocated taking a piece apart to understand its "architecture" before putting it back together in an improvisation. I thought this was a great concept but wondered how "architecture" could be conceptualized.

My take on architecture starts by identifying different **domains of chromaticism**. "Chromaticism" means departures from a written key signature, which means, for classical modes, departures from a single classical mode that go to one or more other classical modes in succession. The domains are **basic classical** (chromaticism is absent or ornamental), **chromatic classical** (chromaticism is introduced via parallel classical modes or associated tonic changes that are "two sides of the same coin") and **non-classical** (chromaticism uses hierarchy of a non-classical parallel modes introduced in the next chapter). Our immediate concern is with the chromatic classical domain but seeing it up front as one of three domains is helpful.

Two Sides of the Same Coin

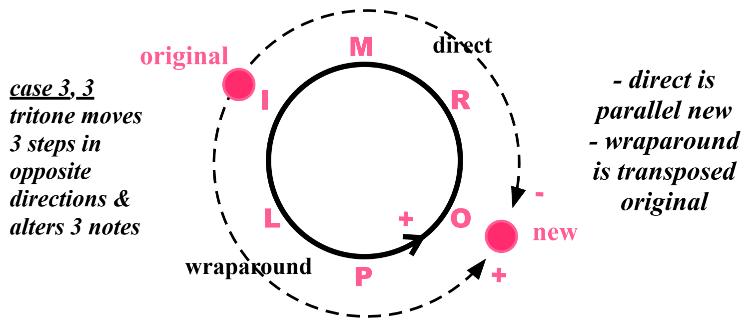
Parallel modes changes and tonic changes are architectural features that are "two sides of the same coin." For example, the possible mode changes determined by tritone changes **I-O** down a minor third (half a tritone) and **I-<u>O</u>** up a minor third (also half a tritone) are, respectively, the *same-tonic mode change* **//I-//O** and the *original-mode tonic change* **//I-altO**. The former is one of the most fundamental changes in music, namely major to natural minor of the home tonic. The latter is different in kind — a transposition of the original mode up a major sixth (equivalent to down a minor third), notated as **tr[//I]@6**, not **//I@6** because the latter indicates a relative mode (same notes), not a transposed mode (altered notes). The transposed original is a relative mode of the **alt** mode of the new tritone.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	I	L	$	.	.	.	.	@	
new (-)	//O	x	.	x	x	.	x	.	x	x	.	x	.	x	parallel mode change
original	//I	x	.	o	.	x	x	.	o	.	o	.	x	x	
new (+)	altO	.	x	x	.	x	.	x	.	x	x	o	x	.	transposed original

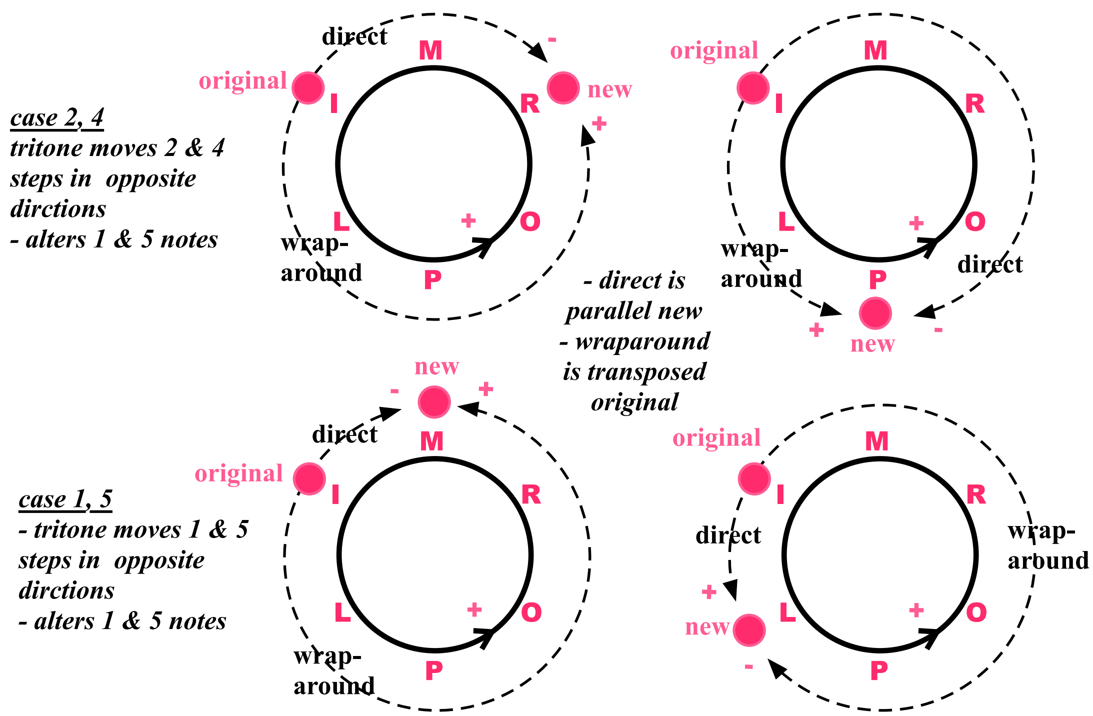
A Helpful Supplementary View: The Alphabet as a Circular Loop

This section may be skipped without loss of continuity, but it's worth having in mind when trying to understand a complex piece of written music. Parallel mode changes and corresponding tonic changes are complex in music notation and seeing them this way is extraordinarily simple once you "get it." The pictures are particularly helpful in dealing with pieces that have different kinds of key signatures for the same tonic (sharp vs. flat, as illustrated by footnote 1 of the opening chapter), or that change between distant or rapidly changing parallel modes, or all of these at once (as illustrated by Coltrane's jazz standard *Giant Steps* presented at the end of this chapter).

The first pictures captures the essence of the example just described. The picture is for a particular original mode but is of the same form for any original mode. The **alt** mode is determined by the path that wraps around between opposite ends of the alphabet (**LP**). The tranposed original is not the only interpretation, but is the simplest one, relative to which other tonic changes may be identified by going to relative modes that alter no notes. The common element is "3" — 3 steps up or down for the tritone, 3 steps for the tonic change, and 3 notes altered.



This is one of only three possible cases of mode/tonic change. The other two are determined by other tritone changes that add up to a tritone, namely 2 and 4 steps in opposite directions, or 1 and 5 steps in opposite directions (which follows directly from the earlier table of classical modes listed in tritone order). The biggest change in terms of difficulty is 4 steps altering 4 notes (these are the "giant steps" of Coltrane's jazz classic). The case of 5 steps altering 5 notes is conceptually simpler because it can be understood as all notes altered by a half tone, including the tritone notes.



EXAMPLE: *RHYTHM CHANGES*

The Gershwin piece *I Got Rhythm* (*The Standards Real Book*, Sher Music (2000), p191) is the origin of widely copied chord changes called “Rhythm Changes” by jazz musicians. These changes define the nature of the chromatic classical domain. The changes move through modes of the home tonic. Dealing with these changes can be challenging in music notation, depending on the key signatures involved. In the following skeleton summary, parallel mode changes occur at points marked “ ┌ .”

I Got Rhythm (original has home tonic = Bb)

┌ **//I**

⚓ **I I I I I M D O I I M I**

♫ **| — 5 6 | 1 2, | 2 1 | 6 5 , | 5 6 | 1 2 4 2, | 3* 2 3 2 | 1, — |**

┌ **altA** ┌ **altP** ┌ **//L** ┌ **//I**

⚓ **O O O P $ L I I**

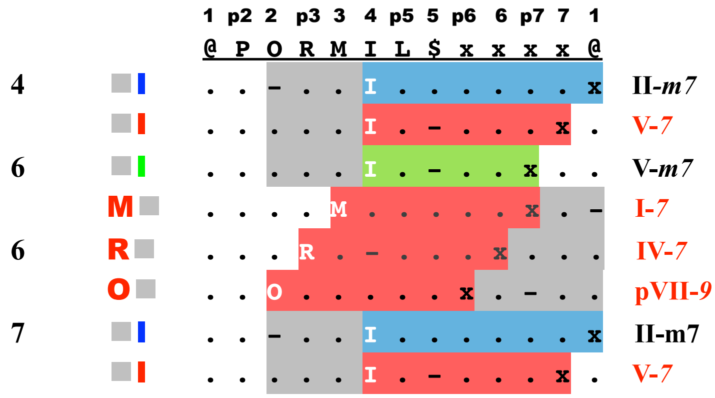
♫ **| 3 3 | 3 p5 , | 3 3 | 3 6 , | 2 2 | 2 3 , | 2 2 | 5 , |**

The tritone changes in bars 1-8 are purely ornamental because the melody line stays in the Ionian mode of the home tonic. They suggest mode changes that can be interpreted as tonic changes, but don’t follow through. The melody line in bar 10 goes to chromatic note **p5** that’s visibly and audibly not a passing note, identifying a mode change determined by the **O** tritone. The mode is not **//O** because this is a minor mode and the melody line in bars 10-11 is major: the mode can only be **altO** (same tritone, all non-tritone notes different). The same goes for tritone **P** in bars 12-13: the mode can only be **altP**. The mode in bars 14-15 is **//L** and bar 16 returns to **//I**. These are very simple mode changes: in bars 10-16, one note changes from one mode to the next, supplied by the tritone. The only big change (4 notes) is **//I-altO** in bars 8-10.

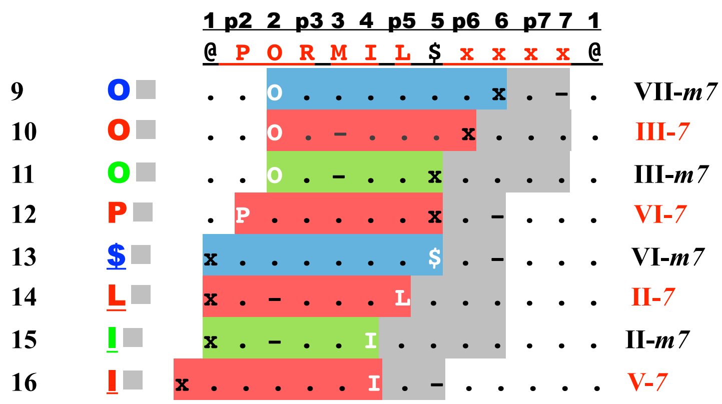
The parallel mode changes are shown next (altered notes highlighted). The other side of the coin of parallel mode changes is Ionian tonic changes shown on the right (up a major sixth and then down by fifths). The parallel modes provide the notes.

		Bb		C		D	Eb		F		G		A	Bb	<—written key
		1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**	
		@	**P**	**O**	**R**	**M**	**I**	**L**	**$**	**x**	**x**	**x**	**x**	**@**	
bar 1-8	**//I**	x	.	x	.	x	x	.	x	.	x	.	x	x	**//I@1**
bars 9-10	**altO**	.	x	x	.	x	.	x	.	x	x	.	x	.	**tr[//I]@6**
bars 11-12	**altP**	.	x	x	.	x	.	x	x	.	x	.	x	.	**tr[//I]@2**
bars 13-14	**//L**	x	.	x	.	x	.	x	x	.	x	.	x	x	**tr[//I]@5**
bar 15-16	**//I**	x	.	x	.	x	x	.	x	.	x	.	x	x	**//I@1**

The harmony of bars 4-7 is shown next. The grey box prefixes and suffixes on the left indicate voicing intervals of unspecified size less than a fourth that are left to context. This notation could be shown in the anchor line. The boxes do two useful things, namely identify the need for a voicing interval, and indicate its side. Dashes identify the roots of the chords shown on the right.



The harmonic sequence for bars 9-16 where the mode changes occur is shown next.



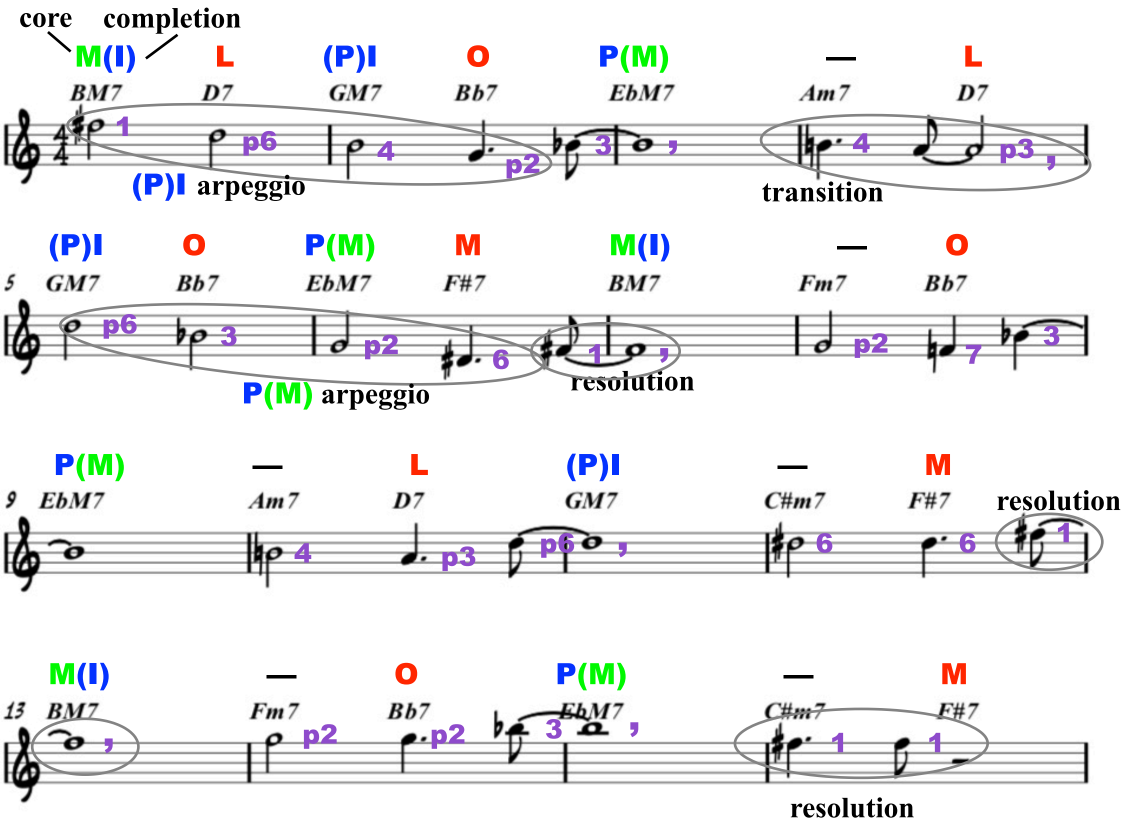
The VIIm7 chord in bar 9 includes note **p5**, of the **alt-O** mode. After that, the only notes that go outside the Ionian home tonic mode are one note from each of the three non-Ionian tritones, which is no different from the first eight bars. The parallel modes provide more differences than this, which could be reflected in the harmony or in improvised melody lines.

EXAMPLE: *GIANT STEPS*

Coltrane's famously difficult jazz classic *Giant Steps* "pushes the envelope" of the chromatic classical domain. In music notation, it presents as an example of successive classical modes. It's famously difficult because the modes (all Ionian) have home tonics from distant scales (the scales differ by 4 notes, the "giant steps"), and the changes are rapid. It's simpler than it looks for several reasons. The core harmony is composed of 3 tritones in rotation, with the same fifo always morphed from each. The melody line is a symmetric mashup of the 3 fifos, with a couple of passing notes added that break the symmetry to give an 8-note melody scale. The home tonic of the melody scale is different from any of the Ionian tonics and it's the only tonic of the piece.

The chart shown below is an annotated Sibelius copy of a fake-book chart (*The Real Book*, 6th Edition, Hal-Leonard). Notice first that the chord sequence suggests a succession of Ionian modes of tonics G (one sharp), B (4 sharps) and Eb (3 flats) that differ from each other by four notes, and the melody line has a home tonic a half tone above one of the these tonics (F#).

Relative to the home tonic, the three Ionian tritones are represented by anchors **O**, **L** and **M** for tonics Eb, G and B. The core tritone-fifo sequences of the classical modes are **O-P**, **L-I** and **M-M** in these terms. These core sequences normally imply parallel classical modes but the modes are actually not part of this picture because notes of the melody line are cherry picked from them to provide a rather simple melody scale formed only of the notes of the 3 fifos plus a couple of passing notes.



The melody scale is determined by the outlined arpeggios of the **G*M7*** and **Eb*M7*** chords in the opening bars. The chords are **(P)I** and **P(M)** in building-block terms, where the parentheses identify completion fifos for the core **I** and **P** fifos. Normally, core-completion fifos are left implicit in core anchor lines, but making them explicit here is important because the notes of the completion fifos determine the **PMI** melody scale, which is a 6-note, atonal, augmented scale morphed from the tritones of the 6-note, atonal, whole-tone scale **OML**. The **PMI** scale is called "augmented" because its 5th note is a half tone above the pitch center. The tritones determine the harmony and melody but are absent from the **PMI** scale (only one note from each is in the scale). Actually, the scale is a mashup of all three major seventh chords but the BM7 arpeggio is not in the melody line and would add no new notes if it was. Melody notes **p3** and **7** are ornamental passing notes relative to this scale. Identifying them as such is useful because, as actual scale notes they would form extra tritones **R** and **I** with scale notes **6** and **4**. Tritone **R** is equivalent to **O** and tritone **I** to **M** in the sense that they morph into the same fifos, but explicitly adding them would add symbolic clutter that would require interpretation, without adding compensating insight).

		F#	G				B			Eb				F#	
		1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**	
		@	P	O	R	M	I	L	$	.	.	.	.	@	
O		.	.	x	.	.	.	.	.	x	.	.	.	.	
M		.	.	.	.	x	.	.	.	.	.	x	.	.	
L		x	.	.	.	.	.	x	.	.	.	.	.	x	
OML		x	.	x	.	x	.	x	.	x	.	x	.	x	
M(I)	B*M7*	.	.	.	.	x	x	.	.	.	x	.	.	x	core **M**
(P)I	G*M7*	x	x	.	.	.	x	.	.	x	.	.	.	x	core **I**
P(M)	Eb*M7*	.	x	.	.	x	.	.	.	x	x	.	.	.	core **P**
PMI		x	x	.	.	x	x	.	.	x	x	.	.	x	
add passing notes		.	.	.	+	.	.	.	.	.	.	.	+	.	

Here is a skeleton summary of the piece, showing only the core building blocks above the staff (the plus signs are placeholders for fifos to be added from context). It's worthwhile for this piece showing the pivot points offset from the main line.

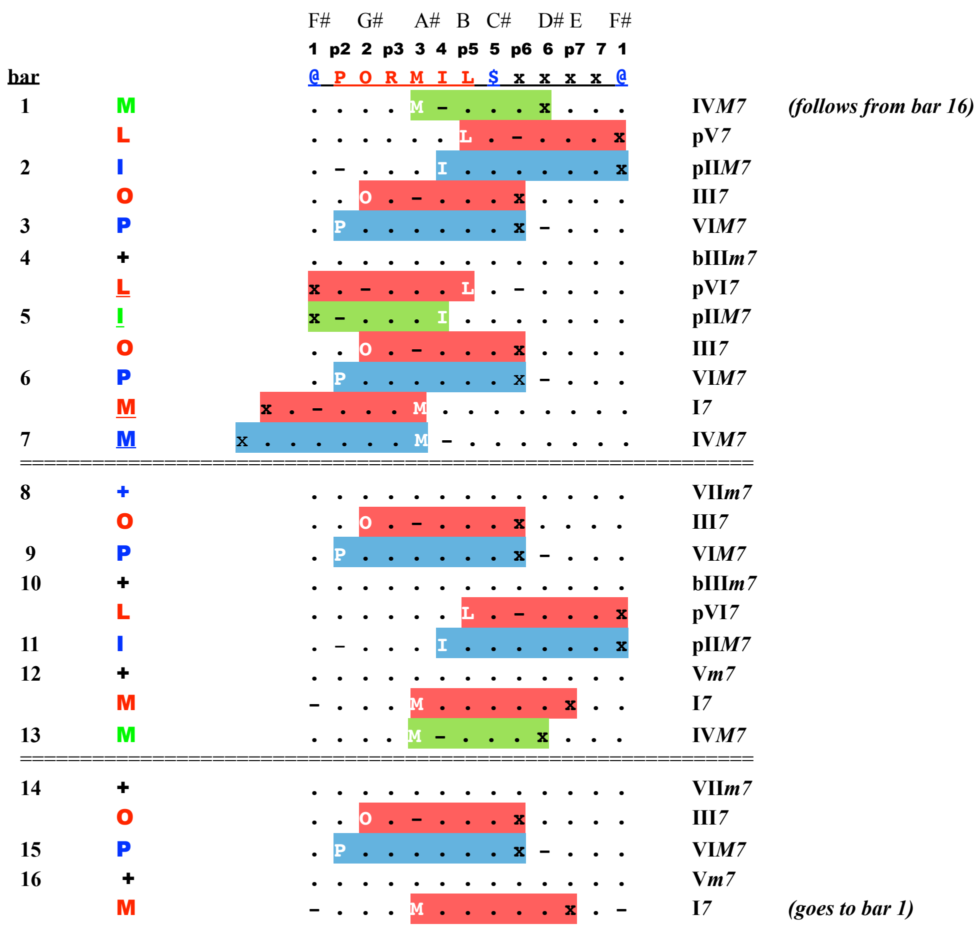
Giant Steps (original home tonic = F#)

```
⚓   M    L    I    O      P      +   L     I  O    P   M      M
♫ | 1   p6 | 4   p2  3 |— ,   | 4   p3 ,| p6  3 | p2   6   1 |— , |
    ^^  ^^   ^^   ^^    ^*      ^^   ^^    ^^ ^^   ^^   ^^     ^*

⚓   +  O     P     +  L      I     +  M    M    +  O     P   +   M
♫ | p2  7  3 |— , | 4   p3 p6 |— , | 6  6  1 |— , | p2 p2  3 |— ,| 1  1 — |
    ^^  ^^    ^*    ^^  ^^      ^*   ^^ ^^    ^*    ^^  ^^     ^*  ^^ ^^
```

The only trace left of three rapidly changing Ionian modes from distant key signatures is the **OML** harmony tritones morphing into the **PMI** fifos — each tritone always provides the same next fifo in the core harmonic sequence. The only thing to remember is the tritone sequence, which is determined here by working backwards from the fifos instead of the normal way of forward from the tritones — backwards because the this piece doesn't actually exercise the modes identified by the tritones. This skeleton gives a good sense of the sound of the piece and is easy to play. Once this is "in the fingers" adding the missing details is easy.

Here follows a Lego-like view of the core harmony that includes some inversions to fit the flow. This simple harmony fits the written chords shown on the right. Dashes in the building-block view indicate omitted roots. The blank spaces are easily filled in from the flow.



Observations

I spent countless hours trying to figure out this piece over the course of development of these ideas, and annoyed experts I consulted with half-baked ideas about its organization. An example of a half-baked idea was imagining that the essence of the piece was scales defined by double tritones from the whole tone **OML** scale (next chapter). No matter how much I massaged this view, it became complicated and sounded wrong. Experts concluded my "engineer's view" of piano music is inherently complex. It isn't — I just didn't fully understand it at the time. It took me a long time to understand the underlying symmetries that make it simple.

This interpretation is easy to understand and play and provides a dramatically simpler basis for variations than the rapidly changing classical modes of the original.

CHAPTER 3: A HIERARCHY OF MODES

This is the "other half of architecture" beyond the chromatic classical domain. The many different approaches to chromaticism beyond this domain may be organized into the following hierarchy of modes determined by words from the alphabet. These modes are chromatic to begin with and so reduce the need for mode/tonic changes to introduce chromaticism.

The reason for presenting classical modes before presenting this hierarchy is they provide the alphabet, which provides a link to music notation, thus grounding this unconventional view in something familiar to anyone who studies the piano. This hierarchy is comprehensive enough to cover all the modes and beyond in scale dictionaries such as *The Source* What's more it goes beyond scale dictionaries by covering the modes for all possible keyboard positions of the home tonic. The hierarchy is comprehensive but not exhaustive; there's room for identifying more modes, if anyone sees the need.

```
                    atonal scales
                   /             \
minor & major                     blues family
families        |  \  enriched  /  |
                |     classical    |
                |      modes       |
                |                  |
  classical modes                  |
                |            simple blues
                |                  |
                 pentatonic modes
```

The hierarchy provides the framework for developing and understanding chromaticism in a systematic way, bottom up, starting from simple pentatonic modes. Classical modes (e.g., minor Aeolian and major Ionian) are defined independently of this hierarchy because they're the scales of key signatures, but in this view of them, they're enrichments of pentatonic modes. Simple blues is an enrichment of a different kind. Classical modes and simple blues are at different levels because they're different in kind. Modes higher up in the two sides of the hierarchy are **mashups** of modes lower down, identified by words with more letters. Steps from pentatonic modes to basic blues and from pentatonic modes to classical modes yield scales that are different in kind, and the steps from there to family modes widens the difference. The family modes are similar in kind. Enriched classical modes (e.g., melodic minor, harmonic minor, harmonic major) are sub-modes of these that provide a bridge between the two sides of the hierarchy. These modes share with classical modes the properties of seven notes and no adjacent half tones; they differ from them by one note altered by a half tone that introduces a second tritone.

At the top of the hierarchy are atonal modes compatible with multiple tonics (e.g., diminished, whole tone) that can often be understood as parents of modes lower down (or as departures that go outside these modes).

There are no 5-letter mode signatures coming up because the implied scales would be too close to the chromatic scale to be usefully distinguished from it. The zone within the mode hierarchy ranging from 4-letter words down to 2-letter words that identify parallel modes is complex in music notation. Scales must be understood as enumerated notes relative to one of more than twelve possible different key signatures. The role of classical modes as the defining scales of key signatures makes music from scales that differ markedly from them misleadingly complex. The hierarchy makes this zone simple by pushing note symbols and interval inversions down to a lower conceptual level.

The chromaticism of such changes is identified in music notation by accidentals in melody lines and chromatic chords in harmony, without any explicit indication of its origin, which may be classical modes, non-classical modes, a mix of the two, or ornamentation that has no scale implications. Explicit key-signature changes indicate the origin is in classical modes, provided there are no accidentals in the melody and no chords that go outside the key signatures, but much strongly chromatic music is written with a single key signature.

The examples that illustrate the concepts in this chapter are as follows.

Backwater Blues (blues)
Summertime (minor family)
Traumerai (major family)
Laura (sequential families)

PENTATONIC MODES

Pentatonic modes with five notes and no half tones or tritones are the foundation of "folk" music in cultures worldwide, so called because anyone with a musical ear can sing or harmonize tunes from them. They're sub-modes of classical modes that omit their tritones and half tones, but the idea here is see classical modes as enrichments of pentatonic modes. Simple blues modes are enrichments of a different kind.

The scale frame provides three notes, one more note results from specifying major or minor tonality, and a final note follows from splitting a remaining major third into whole tones — the only way of splitting it that doesn't introduce half tones. This yields scales with inter-note intervals of whole tones and minor thirds. The minor thirds are made to stand out to the eye by joining their notes by horizontal lines that say "no notes here." The signature for these modes is, exceptionally, a single fifo anchor. It establishes the tonality in the lower fifth of the scale frame and adds a characteristic note in the upper fourth.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	.	.	.	.	@	
pentatonic major	x	.	x	.	x	—	—	x	.	x	—	—	x	//M
pentatonic minor	x	—	—	x	.	x	.	x	—	—	x	.	x	//R

The most visible instances of pentatonic modes on the piano are the clusters of 5 black piano keys: the major mode starts on the bottom black key of the 3-tritone cluster, and the relative minor mode on the next black key down. A lot of fun can be had and insight gained by experimenting with these black-key modes. They lead directly to simple, 6-note blues modes that are taught to beginners as "the" blues modes.

BLUES FAMILY

This development comes before moving up from classical modes on the left side of the hierarchy because it's simpler. I once heard a jazz musician say in an interview that learning the blues before learning music notation paved the way for understanding everything that music notation later threw at him. I thought this sounded right. Starting with pentatonic modes gives deep insight into the difference between classical modes and blues modes, which otherwise can seem somewhat arbitrary. This view of blues seems to be unconventional because no one I talked to and nothing I read explained things this way. It's a useful view because it gives the blues a logical coherence that it doesn't have in music notation.

Blues modes based on pentatonic modes are shown in (a) and (b) below. In (a), a mashup of the parallel pentatonic modes yields an **8-note blues mode //RM** that's very different from classical modes (8 notes, 2 tritones, 3 adjacent half tones). In (b), 6-note blues modes taught to beginners as "the" blues modes are 1-note extensions of the the parallel pentatonic modes. These are parallel modes with a shared interval sequence offset by a minor third. A mashup of these modes yields a **9-note blues family mode //RM.L** that's even more different from classical modes (9 notes, 3 tritones, 5 adjacent half tones). This is a family mode because all the other modes are sub-modes. This family mode is not my invention, only this representation of it is. I first learned of it in conventional terms in a blues piano workshop at the then Jazz School in Berkeley.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	I	L	$	.	.	.	.	@	
(a)	**//R**	x	—	—	x	.	x	.	x	—	—	x	.	x	**pentatonic minor**
	//M	x	.	x	.	x	—	—	x	.	x	—	—	x	**pentatonic major**
	//RM	x	.	x	x	x	x	.	x	.	x	x	.	x	**8-note mashup**
(b)	**//RL**	x	—	—	x	.	x	x	x	—	—	x	.	x	**6-note minor blues (//R + p5)**
	/RM	x	.	x	x	x	—	—	x	.	x	—	—	x	**6-note minor-major blues (p3 + //M)**
	//RM.L	x	.	x	x	x	x	x	x	.	x	x	.	x	**9-note family mashup**

These views of blues have an elegant simplicity. The family mode captures characteristic features of blues, namely mixed minor-major tonality, adjacent half tones, more scale notes, and the addition of three "bent" notes of the master major classical mode (Ionian), namely flatted 3rd, 5th and 7th. It's possible to extend this into a 10-note mode by including the **I** tritone, but this not only gets too close for comfort to the chromatic scale, it also loses something distinctive about the 9-note blues, namely a whole tone gap below the upper tonic and the top scale note. It seems more helpful to view the **I** tritone as ornamental in the blues. Incidentally, the key signatures of the **//M** and **//R** classical modes are good choices for blues pieces because only two accidentals are required to represent the family mode.

The distinctive *sound* of the blues relative to the parallel classical modes is conveyed, for example, by the **M•-R•** transition between diminished triads anchored by major and minor scale notes within the blues mode. A signature of the blues is "crushing" **3** into **p3** or vice versa, expressing the fact that they're in the same mode. They're in different modes in classical major to minor.

blues mixed major-minor

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	I	L	$	.	.	.	.	@
M•	.	.	.	.	x	.	.	o	.	.	x	.	.
R•	.	.	.	x	.	.	o	.	.	x	.	.	.

classical major-to-minor

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	I	L	$	.	.	.	.	@
M▼	.	.	.	.	x	.	.	o	.	.	.	x	.
R▲	.	.	.	x	.	.	.	o	.	.	x	.	.

Suites of enriched parallel classical modes called "melodic" and "harmonic" that are sub-modes of the master family mode will be covered later.

EXAMPLE: *BACKWATER BLUES*

This is a simple, 3-chord, 12-bar blues, which I learned in the blues piano workshop mentioned earlier, as representative of "probably half the blues pieces played by pop and jazz musicians." The home tonic of the source is F. It's a simple piece but exploring it plumbs musical depths. The simple melody line establishes the home tonic by resolving to it in several places. Commas at the ends of phrases only identify the last note of the phrase, leaving open whether it's sustained or not. Timing is left open. Swing-feel 4/4 time would be appropriate and could be notated by an extra downbeat line. The harmony is all tritone-based, which is not uncommon in blues. Offsetting the ups and downs isn't worth it here because everything is mostly downs.

Backwater Blues (original home tonic = F)

⚓ **M** **R** **M** **M**
♫ **| 5 4 p3 1, | 5 4 p3 1, |—— |—— |**

⚓ **R** **R** **M** **M**
♫ **| 5 4 p3 1, | 5 4 p3 1, |——| — 5 p7 5 |**

⚓ **I** **R** **M** **M**
♫ **| p7 p7, 5 | 1 p7 5 p5 4 p3 1, |—— |—— |**

The first bar illustrates a feature of many blues pieces that helps in establishing both the fact of a blues and identifying the home tonic, namely opposite tonality of melody (minor) and harmony (major). As shown next, this bar implies, all by itself, the shared blues scale **//RM**. The melody in bar 10 adds tritone **L**, rounding out the shared mode to **//RM.L**. The actual melody scale in all but bar 10 is pentatonic minor, and in bar 10 is the 6-note blues extension of it identified by **//RL**, but all of the melody and harmony originates in the family scale.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	x	x	x	x	@	
major harmony in bar 1	.	.	.	.	x	.	.	.	.	.	x	.	.	
minor melody in bar 1	x	.	.	x	.	x	.	x	.	.	.	.	x	
implied shared blues scale	x	.	+	x	x	x	.	x	.	x	x	.	x	**//RM**

All the tritones appearing in the piece are summarized next.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
@	P	O	R	M	I	L	$	x	x	x	x	@	
.	.	.	.	.	.	L	.	.	.	.	.	x	
.	.	.	.	.	I	.	.	.	.	.	x	.	**substitute for L**
.	.	.	.	M	.	.	.	.	.	x	.	.	
.	.	.	R	.	.	.	.	.	x	.	.	.	

The lone tritone **I** in the harmony in bar 10 is not in the family mode. The tritone is a "turnaround" marker that identifies the beginning of the last four bars of a 12-bar blues. Blues tritone **L** would seem to be a natural turnaround marker but is substituted by **I** because the seventh chord on root **V** that contains it is a dominant seventh chord, making all the chords the same kind. Harmony based on un-inverted and inverted tritone cores is shown next. It's easy to see how these simple voicing lines follow from altering tritone-based octave shapes. Grey box prefixes and suffixes could be used to suggest these shapes in anchor lines. The simple voicing lines are sufficient because the tritones provide sufficient harmonic variety. The omitted roots of the identified chords are are marked by dashes.

F		G		A	Bb		C		D	Eb		F	
1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**	
@	.	O	R	M	I	L	$	.	x	x	.	@	
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	.	R	.	–	.	.	.	x	.	.	.	*IV7*
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	.	R	.	–	.	.	.	x	.	.	.	*IV7*
.	.	.	R	.	–	.	.	.	x	.	.	.	*IV7*
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	.	.	.	I	.	–	.	.	.	x	.	*V7*
.	.	.	R	.	–	.	.	.	x	.	.	.	*IV7*
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	.	.	M	.	.	.	.	.	x	.	.	*I7*

F		G		A	Bb		C		D	Eb		F	
5	**p6**	**6**	**p7**	**7**	**1**	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	
$	.	x	x	.	@	.	O	R	M	I	L	$	
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*
.	.	x	.	.	.	.	.	R	.	–	.	.	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*
.	.	x	.	.	.	.	.	R	.	–	.	.	*IV9*
.	.	x	.	.	.	.	.	R	.	–	.	.	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*
.	.	.	.	x	.	.	.	.	.	I	.	.	*V7*
.	.	x	.	.	.	.	.	R	.	–	.	.	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	.	*I7*

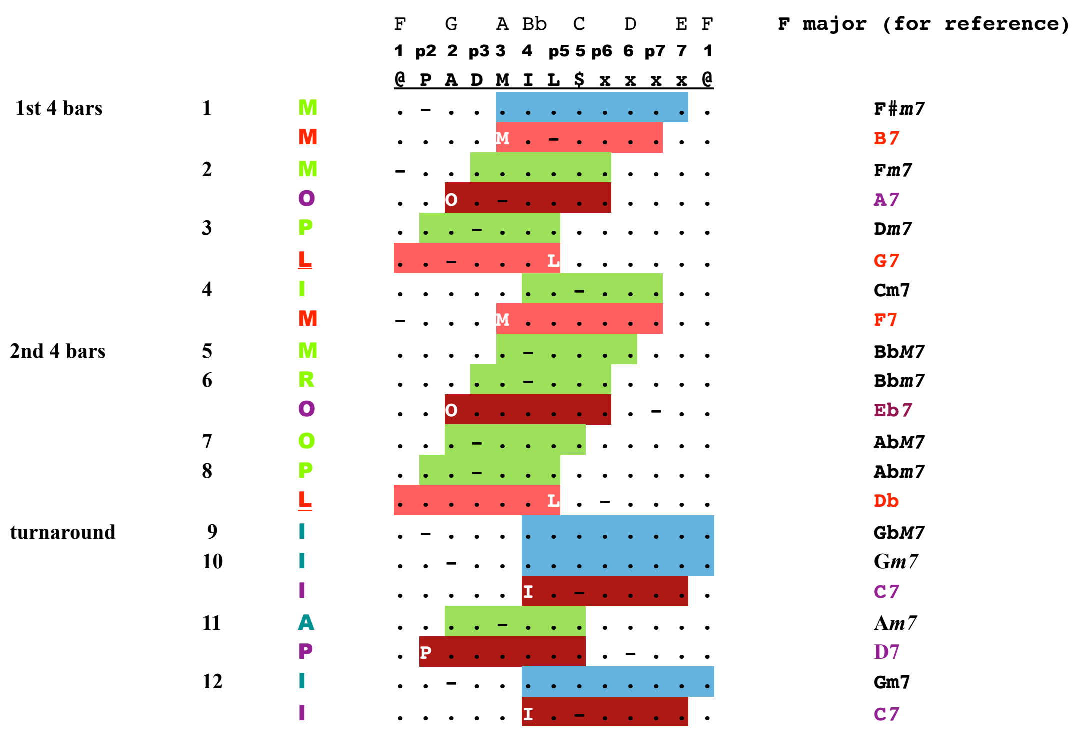
Blues Chord Progressions

Simple 3-chord blues pieces such as this often use dominant-7 chords **I***7*, **IV***7* and **V***7* containing tritones **M**, **R** and **I**, the last of which is not actually in the family scale represented by the mode signature **//RM.L**. The **V***7* chord containing tritone **I** is such a familiar feature of music that it tends to be borrowed for blues to round out a trio of chords of the same kind (the **V** chord containing tritone **L** is an altered major seventh chord). This is so common that many musicians understand basic blues to be defined by this chord trio. This is mistake because blues pieces in general use chords of too many different kinds for chord content to be a useful characterizing feature. Tritone content is more helpful and the **//RM.L** signature is particularly helpful because it captures fundamental features of the blues in a compact and intuitive way.

The following table digs deep, to reveal underlying simplicity of chromatic blues chord progressions that are forbiddingly complex even though they use only basic seventh chords. This is a representation of a table of chord progressions for tonic F handed out in the blues piano workshop mentioned earlier. The table vividly illustrates the simplicity of developing voicings of notationally complex chord progressions from very simple information. Tritone chords are represented by their anchors and fifo anchors are represented by placeholder plus signs, leaving fifos to be faired in from context. The highlighted anchor lines are representative of the two parts of the table: the anchor line of *Backwater Blues* in the top part (using segments from different lines); and a representative anchor line of a bebop blues progression in the bottom part.

									turnaround			
1	M	M	M	M	R	R	M	M	I	I	M	M
2	M	M	M	M	R	R	M	M	I	R	M	I
3	M	R	M	M	R	R	M	M	L	I	M	I
4	M	R	M	M	R	R	M	P	L	I	M	I
5	M	R	M	M	R	R	M	P	+	I	M	+ I
6	M	R	M	M	R	O	M	P	L	I	M	+ I
7	M	R	M	+ M	R	O	M	+ P	+	I	+ P	+ I
8	M	R	M	+ M	R	O	+	P	+	I	+ P	+ I
9	M	R	M	+ M	R	+ R	M R	O P	+	I	+ P	+ I
10	+	+ O	+ L	+ M	R	R	+	+ L	+ I	+ I	+ P	+ P
11	+	+ +	+ +	+ M	+	+	+	+	+	I	+ +	+ P
12	+	+	+ +	+ M	+ +	+	+	+	+	I	+ +	+
13	+	+	+ +	+ M	+	+ O	+	+ L	+	+ I	+ P	+ P
14	+	+ O	+ L	+ M	+	+ O	+	+ L	+	I	+ P	+ I
15	+	+ O	+ L	+ M	+	+ R	+	+ L	+	I R	+ P	+ I
16	+ M	+ O	+ L	+ M	+	+ O	+	+ L	+	+ I	+ P	+ I
17	+	+ M	+ +	+ +	+ +	+ R	+	+ P	+	+ +	+ +	+

Here follows the development of the highlighted bebop blues line:



Fifos are faired in between tritones in a visibly obvious way. Dashes indicate positions of assigned roots that yield conventional root-3rd-7th voicings of the seventh chords on the right. Notes and chords for tonic F are shown for concreteness.

The complexity of this chord progression in music notation follows from the use of tritone substitute chords (same tritone, all non-tritone notes different) that determine different parallel modes for the same tritone in different places in the progression. Different parallel modes mean different implied key signatures. The tritones of all possible dominant seventh chords and their tritone substitutes for any home tonic are easily determined from a table of the following form. The chord roots are the **X** entries in the table. All possible chords are listed on the right for home tonic F, with strikethroughs identifying unused chords (for this piece). This chord progression is doubly chromatic because each pair of tritone substitute chords determines two different implied key signatures, for a total here of eight different key signatures, which is complex by any measure. This table is good for any progression of dominant seventh chords from any home tonic by replacing the top line by the scale frame for that tonic.

	F							C					F		
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		
	@	P	O	R	M	I	L	$	x	x	x	x	@		
L	.	.	X	.	.	.	L	.	X	.	.	.	.	G7	Db7
I	.	X	.	.	.	I	.	X	.	.	.	.	.	~~Gb7~~	C7
M	X	.	.	.	M	.	X	.	.	.	.	.	X	F7	B7
R	.	.	.	R	.	X	.	.	.	.	.	X	.	~~E7~~	~~Bb7~~
O	.	.	O	.	X	.	.	.	.	.	X	.	.	A7	Eb7
P	.	P	.	X	.	.	.	.	.	X	.	.	.	~~Ab7~~	D7

Complex as such bebop blues chord progressions are, the chords themselves are plain seventh chords, which is not representative of blues in general. In fact, the bebop progressions don't sound like blues by themselves: they rely on a melody line to provide a blues sound. To give a blues sound to the harmony, requires non-tritone harmony notes to be from the blues family mode of the home tonic or a sub-mode of it, which isn't the case here. I have always wondered why some jazz pieces described as "blues" don't sound like blues to my ears; this offers an explanation.

Blues Chords

Blues modes are different in kind from the classical modes that determine the symbols for seventh chords (different numbers of notes, adjacent half tones, multiple tritones). Blue chords that are basic chords altered to fit a blues mode are often notationally complex ("banging square pegs into round holes"). A sampling of such chords is provided next. The sharp and flat suffixes are degree numbers in the 7-note scales that define the basic chords, and are quite confusing relative to blues scales with more or fewer notes. The chord symbols are intended only to illustrate the complexity: knowing them is not required to read on. The takeaway here is that combinations of building blocks *are* the chords. An amazing amount of chord complexity can be introduced by holding the tritone and changing the completion fifo (or vice-versa). Finding a chord symbol to identify such combinations can sometimes be difficult.

In this summary of chords types, dashes indicate omitted notes of voicings of 4-note chords.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
	@	.	O	R	M	I	L	$	.	x	x	.	@	.	O	R	M	I	L	$	<u>**examples**</u>
M•	.	.	.	.	M	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.	**III*dim***
(@)M	x	.	.	.	M	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.	**I*7*, III*m7b5#5*, III*m7b5b13***
M(<u>R</u>)	.	.	.	.	M	.	.	.	.	.	x	.	–	.	.	x	.	.	.	.	**rootless I*7#9***
R•	.	.	.	R	.	.	x	.	.	x	.	.	.	.	.	.	.	.	.	.	**pIII*dim***
R(I)	.	.	.	R	.	x	.	.	.	x	.	.	x	.	.	.	.	.	.	.	**IV*7***
R(<u>O</u>)	.	.	.	R	.	–	.	.	.	x	.	.	.	.	x	.	.	.	.	.	**rootless IV*13***
(O)M	.	.	x	.	.	.	M	.	.	x	.	.	x	.	.	.	.	.	.	.	**II*7***
(O)I	.	.	x	.	.	I	.	x	.	.	.	x	.	.	.	.	.	.	.	.	**V*7***
I(<u>O</u>)	.	.	.	.	.	I	.	x	.	.	.	x	.	.	x	.	.	.	.	.	**V*7***
L•	.	.	.	.	.	.	L	.	.	x	.	.	x	.	.	.	.	.	.	.	**pV*dim***
L($)	.	.	.	.	.	.	L	x	.	.	.	.	x	.	x	.	.	.	.	.	**V*M7#3***
(P)L	.	x	.	.	.	.	L	.	–	.	.	.	x	.	.	.	.	.	.	.	**tritone sub for V*M7#3***
ML	.	.	.	.	M	.	L	.	.	.	x	.	x	.	.	.	.	.	.	.	**I*7b5*, pV*7b5***
RM	.	.	.	R	M	.	.	.	.	x	x	.	.	.	.	.	.	.	.	.	**I*7#9(13)*, III*7b9b5***
RL	.	.	.	R	.	.	L	.	.	x	.	.	x	.	.	.	.	.	.	.	**I*dim7*, IV*7b9***

MAJOR AND MINOR FAMILIES

Classical modes, already explained, follow from enriching the pentatonic modes by splitting their minor third intervals in different ways. The minor and major families follow from a mashup of modes **//O** and **//I** that's analogous to the mashup of minor and major pentatonic modes on the blues side of the hierarchy. As shown below, the mashup yields a 10-note minor-major mode identified by **//ORMI** that sometimes appears as a melody mode in strongly chromatic pieces (e.g., *Lush Life* covered in Chapter 4), but is mainly a parent of 9-note minor and major family modes that differ by one note in the lower fifth of the scale frame (grey shading). The 9-note modes follow from omitting tritone anchor **R** or **M** while leaving its upper note untouched (the plus superscript on the signature identifies this). The modes shown all have visibly simple asymmetric forms. These modes are close to the chromatic scale but are unambiguously tonal because of asymmetry provided by the whole-tone gaps.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	.	.	.	.	@	
//ORMI	x	.	x	x	x	x	.	x	x	x	x	x	x	**10-note mashup of //O & //I**
//OR.I+	x	.	x	x	.	x	.	x	x	x	+	x	x	**9-note minor family (+ in upper fourth)**
//O.MI+	x	.	x	.	x	x	.	x	x	+	x	x	x	**9-note major family (+ in upper fourth)**

Here follows a summary of important sub-modes and variations of them. This makes logical sense of modes that can seem very ad hoc in conventional terms. As with the blues, the melodic and harmonic modes (names underlined below) are master modes of suites of parallel modes (described later) that are analogous to the suites of parallel classical modes in having 7 notes and no adjacent half tones; the difference is two tritones instead of only one. The 8-note harmonic minor-major mode is different in kind, namely a mashup of harmonic minor and harmonic major sub-modes. It's particularly useful as a composite master mode because of its simple, regular form, which reduces in a simple way to the irregular forms of the sub-modes.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	.	.	.	.	@	
//OR.I	x	.	x	x	.	x	.	x	x	x	.	x	x	**8-note minor (bebop melodic minor)**
//R.I	x	.	x	x	.	x	.	x	.	x	.	x	x	**7-note <u>melodic minor</u>**
//O.MI	x	.	x	.	x	x	.	x	x	.	x	x	x	**8-note major (bebop melodic major?)**
//O.MI	x	.	x	.	x	x	.	x	x	x	.	x	x	**8-note variation (bebop major)**
//O.M	x	.	x	x	.	x	.	x	.	x	.	x	x	**7-note <u>melodic major</u>**
//OxxI	x	.	x	x	x	x	.	x	x	—	—	x	x	**8-note harmonic minor-major**
//Ox.I	x	.	x	x	.	x	.	x	x	—	—	x	x	**7-note <u>harmonic minor</u>**
//O.xI	x	.	x	.	x	x	.	x	x	—	—	x	x	**7-note <u>harmonic major</u>**

MINOR FAMILY: *SUMMERTIME*

I learned this version of this well known minor piece by Gershwin some years ago in a piano comping course given by Susan Muscarella at the then Jazz School in Berkeley (now the Jazz Institute). The home tonic of the source is D and the key signature is one flat, identifying Aeolian mode **//O**, but the piece departs significantly from this mode. In the following summary, the anchor line is from a chord progression that will be presented later as a result. The only reference I have for this version of the piece is my course notes. Like *Backwater Blues*, it's a simple piece, the exploring of which plumbs musical depths.

The 6-note melody line is pentatonic minor with one added note (**2** in bar 8) that's in most minor scales. The harmony is strongly chromatic but is more than ornamental because, with the exception of bars 5-6, the tritone harmony self identifies a succession of sub-modes of the minor family mode with signature **//OR.I⁺** (recall that the plus superscript means fill in the top fourth).

Summertime (original home tonic = D)

┌//RI ┌//RM ┌//OI

⚓ R I R M R O OI

♫ | — **5** p3 | 5 | — , 4 p3 4 5 p3 | 1 **5** | — , **5** p3 | 4* — | — , p3 1 p3 1 p3 | **2** , |

┌//O ┌//OI

⚓ O O OI $

♫ | ——————— same ——————— | **5** p7 5 p7 1 p3, | **5** 4 p3 — | **5** , |

The modes are summarized next. The arrows at the right indicate some possibilities for inter-mode segues that are exploited in the above anchor line.

	D		E	F		G		A	Bb		C		D	
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	x	x	x	x	@	
6 melody notes	x	.	x	x	.	x	.	x—	—	—	x	.	x	
are from Dorian //R	x	.	x	x	.	x	.	x	.	x	x	.	x	
family //OR.I⁺	x	.	x	x	.	x	.	x	x	x	+	x	x	
yields melodic minor //R.I	x	.	x	x	.	x	.	x	.	x	.	x	x	↓ **Dorian-*#7***
and harmonic minor //O..I	x	.	x	x	.	x	.	x	x—	—	—	x	x	↓ **Aeolian-*#7***
and Dorian //R	x	.	x	x	.	x	.	x	.	x	x	.	x	↕
//RM is ornamental	x	.	x	x	x	x	.	x	.	x	x	.	x	↑ **8-note blues**

The skeleton melody line and the anchor line shown above are sufficient to play the melody with 3-note octave shapes for the harmony. The result has depth because of the three notes, and variety because of the tritones.

Adjusting the harmony to provide more shape variety is easy, as illustrated next for bars 1-8 plus the resolution bar 16. The numeric suffixes or prefixes on anchor symbols indicate offsets of voicing notes above or below the core, measured in half tones. These could be added to the above anchor symbols to indicate desired voicings. These shapes voice the chords on the right, if not always in place then in the flow (omitted chord roots are identified by dashes). All the voicings except for the **II** chord are rootless, demonstrating once again that chord roots are not inherently fundamental to the sound of a chord progression.

The suffixes for on anchors show that fit the scales. For example, the opening tritone with a fourth on top (often called "all fourths" because a tritone is an augmented fourth) has a size of 6+5=11 half tones, a half tone less than an octave. This makes finding the shape on the keyboard easy — find the treble note a half tone below the bass note an octave up, and complete the shape by adding an internal note a tritone above the bass note. All-fourths shapes generally imply complex chord symbols because this is not a basic chord shape from a classical mode.

bar	core	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	a voicing of	
		@	.	O	R	.	I	.	$	x	x	x	x	@	.	O	R		
2	R	.	.	.	R	.	–	.	.	.	x	.	.	.	.	x	.	IV-*13*	
3	I	.	.	.	.	.	I	.	–	.	.	.	x	.	.	.	x	V-*7#5*	
4	R	.	.	.	R	.	–	.	.	.	x	.	.	.	.	x	.	IV-*13*	
5	M	–	.	.	.	M	.	.	.	.	.	x	.	–	.	.	x	I-*7#9*	ornamental
6	R	–	.	.	R	.	.	.	.	.	x	.	.	–	.	x	.	I-*m6(9)*	
7	O	.	.	O	.	.	.	.	.	x	.	.	.	.	.	.	.	II-*m7b5*	
8	OI	.	.	O	.	.	I	.	–	x	.	.	x	.	.	.	.	V-*7b9*	
16	$	–	.	.	x	.	.	.	$	.	.	.	.	x	.	.	–	I-*m*	triad resolution

As summarized next, there's a strong contrast between the simplicity of these shapes and the complexity of the corresponding chord symbols. The adjustment of basic chord symbols to fit scales, or places in scales, they don't quite fit is like "banging square pegs into round holes" — the results tend to be messy.

chord scale notes (bars 1-8)	chromatic scale notes
root of ***I-m6(9)***, "5" of ***IV-13***, "7" of ***II-m7***	**1**
root of ***II-m7b5***, "9" of ***I-m6(9)***, "13" of ***IV-13***	**2**
"7" of ***IV-7***, "#5" of ***V***, "#9" of ***I-7#9***	**p3**
"b5" of ***II***, "b9" of ***V-7b9***	**p6**
"b5" of ***IV-7b5*** (not in the above but could be)	**7**

MAJOR FAMILY: *TRAUMERAI*

The example is Schumann's beautiful *Traumerai* (*Classical Fake Book*, 2nd Edition, Hal Leonard (2013). The home tonic of the source is F. As shown below, the melody is straight **//I** with a mashup of it and **//M** in [b]-[c] that adds one note in a few places (**p7**). The 8-note mashup could be identified as **//MI** (no other mode in this document has this signature) but it seems simpler to think of the melody plus harmony of all sections as governed by the major family mode **//O.MI⁺**, with tritones **P** and **L** providing ornamental harmonic transitions. The melody line resolves to relative tonics **@2-@6-@2-@5-@1** of this mode that are also relative tonics of the **//I** mode, but thinking in terms of the more general mode helps in remembering melody and harmony resolutions.

Rhythm is 4/4 with most chords on downbeats. The exceptions are diminished seventh chords (double tritones) on upbeats that provide ornamental transitions to following chords on downbeats.

The written harmony looks misleadingly simple but is actually complex in aggregate due to a mix of different chord types in different inversions, on a jumpy bass line formed of chord roots and bass notes of inversions. This interpretation replaces the jumpy bass line by a smoothly flowing anchor line that represents both building blocks and outer intervals of compound shapes (augmented fifths or major sixths) going up or down from it. It makes its own decisions about chord inversions.

The thin core harmony captures the essential sound of the piece effectively, and implies thicker harmony that's easily filled in from context (most of the missing notes are the pitch center or the home tonic). The ups and downs of the harmony roughly follow those of the melody line, but by smaller intervals. The main challenge is remembering how the melody and harmony go together in the latter half of each section, where the resolution patterns to different relative tonics are different.

The notation offsets the pivot points of the melody line up or down for greater readability.

Traumerai (original home tonic = F)

[a] ⚓ ↓M▹ I M I M▹ M M▹ O▹▹

♫ 5 ||: 1 , 7 1 3 | 5 1 1 , 7 6 | 5 1 2 3 4 6 1 2 | 3 5 2 , 5 |

[b] ⚓ ↓M▹ O■ M▹ R▹▹ O▹▹ PM L I

♫ | 1 , 7 1 3 | 5 3 3 , 2 1 7 | 1 3 6 1 7 — p7 | 6 7 5 , 5 :||

[c] ⚓ ↓M▹ I M O PM O M O P @

♫ | 1 , 7 1 3 | 5 p7 p7 , 6 5 | 4 6 2 — 3 4 3 | 2 6 — , 1 |

[d] ⚓ ⊲I I▹▹ O■ M▹ OI M▹ I M▹ O I I

♫ | 4 , 3 4 6 | 1 4 4 , 3 2 | 1 3 6 — 7 1 7 | 6 3 — 2 , 5 |

repeat [a]

[e] ⚓ ↓M▹ ■L ■$ ■I O P▹ O I M▹

♫ | 1, 7 1 3 | 5 3 3 , 2 1 6 | 5 1 2 3 4 6 2 3 | 4 6 6 7 1 |

Here follow explanations of the notated harmony (omitted chord notes identified by dashes).

```
[a] ⚓     ↓M▹                  I          M    I        M▹  M  M▹ O▹▹
    ♫  5 ||: 1 ,  7  1  3 | 5  1  1 ,  7  6  | 5  1  2  3  4  6  1  2 | 3  5  2 ,  5 |
```

Explanation

```
              F   G   A Bb  C   D   E F
              1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
              @ . O . M I . $ x x x x @
F             | . <— M . . – . . . . x      ↓M▹   inverted major tonic triad an octave down
Bb            | . . . . I . . . – . . x     I     as written
F/C           | . . . M . . – . . . . x     M▹    inversion ignored
C7            | . . . . I . – . . . x |     I     inverted chord
F/A           | . . . M . . – . . . . x     M▹    as written
C             | . – . M . . – . . . x |     M▹    bass note held
F             | . . . M . . – . . . . x     M▹    aug. fifth
C             | . O . . . . – . . . x |     O▹▹   maj. sixth (inverted major triad)
C/G           | . O . . . . – . . . x |     –     held
```

```
[b] ⚓     ↓M▹               O■          M▹   R▹▹    O▹▹ PM  L   I
    ♫  | 1,  7  1  3 | 5  3  3,  2  1  7 | 1  3  6  1   7  p7 | 6  7  5 ,  5 :||
```

Explanation

```
              @ . O . M I . $ x x x x @
F             | . <— M . . – . . . . x          ↓M▹   as in [a]
A7/C#         | . O . – . . . X . . . . . O     O■    inversion ignored
Dm            | . . . M . . . . – . . x         M▹    inverted chord
Fm            | . . R . . . – . . . . x         R▹    inverted chord
C/G           | . O . . . . – . . . x |         O▹▹   as written
F#dim7        | P . . M . . x . . x . |         PM    as written
G7          x . – . . . L | . – . . |           L     inverted chord
C7        x . . – . . I . – . . . . |           I     inverted chord
```

```
[c] ⚓     ↓M▹               I  M        O      PM  O  M  O  P  @
    ♫  | 1,  7  1  3 | 5  p7 p7,  6  5 | 4  6  2   3   4  3 | 2  6  — , 1 |
```

Explanation

```
              @ . O . M I . $ x x x x @
F             | . <— M . . – . . . . x      ↓M▹   as in [a]
Cm7           | . . – . I . – . . x . |     I     inverted chord
D             | . . . M . . . . . x . –     M     tritone implied by  chord sequence
Gm            | . O . . – . . . X . . |     O     as written
F#dim7        | P . . M . . x . . x . |     PM    as written
Gm/D          | . O . . – . . . x . . |     O     inversion ignored
Adim/Eb       | . . . M . . x . . x . |     M     as written
Gm/D          | . O . . – . . . x . . |     O     inversion ignored
D7            | P . . – . . x . – . . |     P     inverted chord
Gm            | . O . . – . . . x . . |     skip
F             @ . . . – . . x . . . . |     @     as written
```

[d] ⚓ ◁I   I▷▷ O■   M▷   OI M▷ I M▷ O I I

♫ | 4, 3 4 6 | 1 4 4, 3 2 | 1 3 6 7 1 7 | 6 3 — 2 ,5 |

Explanation

```
                @ . O . M I . $ x x x x @
Bb       x . . - . . . . . I . | . . . . |          ◁I     inverted chord
Gm          | . . . . I . . . - . . . . O            I▷▷    inverted chord
A           | . O . - . . . . x . . . - . . O        O■     tritone implied by chord sequence
Dm          | . . . M . . . . - . . . x              M▷     inverted chord
C#dim7      | . O . . I . . . x . . . x |            OI     inverted chord
Dm/A        | . . . M . . . . - . . . x              M▷     as written
Edim        | . - . . I . . . . . . x |              I      inverted chord
Dm/A        | . . . M . . . . - . . . x              M▷     inverted chord
A7          | . O . - . . . . x . . . - |            O      inverted chord
Bb          x . . . . . I . | . - . . . |            I      inverted chord
C7        x . . . . . . I . - . . . . . . |          I      chord an octave down
```

[e] ⚓ ↓M▷   ■L   ■$ ■I   O P▷ O   I M▷

♫ | 1, 7 1 3 | 5 3 3 , 2 1 6 | 5 1 2 3 4 6 2 3 | 4 6 6 7 1 |

Explanation

```
                @ . O . M I . $ x x x x @
F           | . <— M . . - . . . . . x               ↓M▷    as in [a]
G7          x . - . . . . L . - . . . . x            ■L     inverted chord
F/C         x . . . . - . . $ . . . . . x            ■$     inversion ignored
C7        x . . - . . . I . - . . . . . x |          ■I     inverted chord
Gm          | . O . . - . . . . x . . . |            O      as written
D           | P . . . - . . . . x . . . |            P▷     inverted chord
Gm          | . O . . - . . . . x . . . |            O      as written
C7          | . . . . . I . - . . . . x |            I      inverted chord (optionally an octave down)
F           | . . . M . . - . . . . . x              M▷     ditto
```

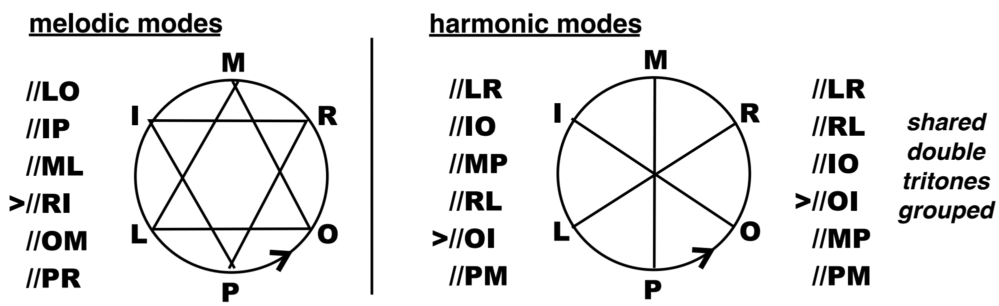
Observations

The meanings of additions or alterations identified by prefixes or suffixes on building blocks are much simpler than for additions or alterations to chord symbols because only two kinds of building blocks exist, namely tritones and fifos, and only six anchors place them directly on the keyboard.

MELODIC AND HARMONIC MODES

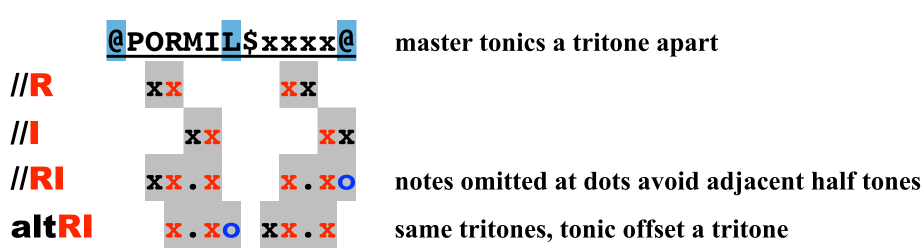
The major, minor and blues family modes provide suites of melodic and harmonic modes as sub-modes. These are enriched classical modes that add a second tritone, while preserving the properties of classical modes of seven notes and no adjacent half tones. These are the only multi-tritone modes that have been formally developed this way in music notation. They're parallel modes of master melodic minor and harmonic minor modes but, as with straight classical modes, have different tonalities and so are best referred to as "melodic" and "harmonic" modes, leaving tonality to be understood from mode signatures. This is a deep and complex area in music notation that tritone symmetries make simple. See the book *Modalogy* and Appendix D for more on the complexity. The scales are difficult to keep straight in the mind in conventional terms. The large number and wide variety of exotic names by which they're are conventionally known has to seen to be believed.

The following picture sets the stage. Think of the alphabet as a circular loop in which transitions off one end wrap around to the other end (harmonically equivalent to going up into the next octave). Classical modes step around the circle without going across it. The melodic and harmonic modes do the same, except they gather a second tritone at each step from across the circle. The order of the double tritones in the mode signatures implies inversions when the second tritone goes off the end of the alphabet.



> identifies the master mode

Melodic Modes. A simple way of understanding these modes is as 9-note mashups of parallel classical modes with tritones a whole tone apart, reduced to 7 notes by eliminating adjacent half tones. The defining symmetric shape of the master **//RI** mode follows from the mashup of the defining symmetric shapes of the **//R** and **//I** modes shown next. The **altRI** mode follows logically. This pair accounts for two of 12 possible modes. As with classical modes, the symmetric pairs include the tonics, so there's no need to think about them up front. The new thing is twice the number of modes containing **L**, which automatically includes the tonic, causing the distribution of modes among the two different kinds of mode signatures to be different.



Here follows a summary of the twelve modes, with ones containing the home tonic identified by check marks. The master tonics a tritone apart are identified by o. The symmetric shapes provide six notes leaving only one note to be added to complete the scale by whole tones. Some shapes extend into the next octave to bring forward the pattern to the eye, but the projecting parts wrap around to the beginning to put the scale within the home octave. The construction process shows that each of these modes is the classical mode of one of the tritones with the replacement of one scale note by a note of the second tritone (the other note is already in the scale).

			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1				
			@	P	O	R	M	I	L	$	.	.	.	.	@				home tonality
	✔	//LO	x					x	x		x	o		x	x		x		major
	✔	altLO	x					x	x		x		x		x		x	o	minor
		//IP					x	x		x		x		x		x	o		major
		altIP				x		x		x	o		x	x		x			minor
	✔	//ML	x		x		x		x	o		x	x		x				major
	✔	altML	x			x	x		x		x		x		x	o			major
master	✔	//RI	x		x	x		x		x		x		x	o				minor
		altRI		x		x		x	o		x	x		x					minor
	✔	//OM	x		x		x	o		x	x		x		x				major
		altOM		x	x		x		x		x		x	o					major
	✔	//PR	x	x		x		x		x		x	o		x				minor
		altPR		x		x	o		x	x		x		x					minor-major

Here follows an example of the **//ML** mode as a sub-mode of the blues.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	.	.	.	.	@	
//RM.L	x	.	x	x	x	x	x	x	.	x	x	.	x	blues family
//M.L	x	.	x	.	x	.	x	x	.	x	x	.	x	Lydian dominant

Parallel Harmonic Modes

The harmonic modes are different in kind because only 3 of the double tritones have different notes. The alphabet-as-a-circle figure makes clear that the last 3 double tritones of the harmonic modes are inversions of the first 3. Instead of 2 possible tonics a tritone apart, there are 4 possible tonics a minor third apart for each of the harmonic minor and harmonic major modes. The upshot is 12 possible tonics for each master mode of which 7 include the home tonic, for a total of 24 modes, of which 14 are parallel modes. Untangling the possibilities in these terms gets complicated. There's a simpler approach suggested by the simplicity of the symmetric shapes of the classical and melodic modes. The 8-note mashup of the parallel harmonic modes covers all the possible modes. Each of these modes is easily reduced by context into one of the parallel modes on the left by omitting one of the notes in the full minor third.

harmonic modes (12x2 tonics)

```
        @PORMIL$xxxx@
//Ox.I  o xx.x xx..xo
//O.xI  o x.xx xx..xo
```

enhanced harmonic modes (3x4 tonics)

```
         @PORMIL$xxxx@POR
//OI+    x xxxx xx..xo
altOI+     x..xo xxxx
//IO+       x x..xx x..xo
altIO+        x..xo xxxx
```

Here follows a summary of the construction of the twelve modes from the symmetric shapes, with ones containing the home tonic identified by check marks. The shapes ending on the upper tonic or projecting into the next octave are only to help the eye see the repeated pattern — they wrap around to the beginning of the home octave. The check marks identify modes containing the home tonic. The master tonics shown as o are not in the symmetric shapes and so have to be added but this is easy because the additions are determined for the top mode of each set of 4 modes by the following master tonic sequence highlighted in yellow: **5-1-4**, from which everything else follows.

			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	
			@	P	O	R	M	I	L	$	.	.	.	.	.	@	P	O	R	M	**home tonality**
	✔	**//RL⁺**			x	x	—	—	x	o		x	x	x	x						**min**
	✔	**altRL⁺**		o		x	x	x	x		x	x	—	—	x	o					**min-maj**
	✔	**//LR⁺**					o		x	x	x	x		x	x	—	—	x	o		**maj**
	✔	**altLR⁺**						x	x	—	—	x	o		x	x	x	x			**min**
master	✔	**//OI⁺**	o		x	x	x	x		x	x	—	—	x	o						**min-maj**
		altOI⁺		x	x	—	—	x	o		x	x	x	x							**ambiguos**
	✔	**//IO⁺**					o	x	—	—	x	o		x	x	x	x				**maj**
		altIO⁺				o		x	x	x	x		x	x	—	—	x	o			**min**
	✔	**//PM⁺**		x	—	—	x	o		x	x	x	x		x			`			**maj**
		altPM⁺		x	x	x	x		x	x	—	—	x	o							**min-maj**
	✔	**//PM⁺**				x	x	—	—	x	o		x	x	x	x					**min-maj**
		altPM⁺			o		x	x	x	x		x	x	—	—	x	o				**maj**

Here follows an example of the **//RL⁺** mode as a sub-mode of the blues, with the removal of one out-of-context note from the full minor third.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	.	.	.	.	@	
//RM.L	x	.	x	x	x	x	x	x	.	x	x	.	x	**blues family**
//R..L⁺	x	.	x	x	—	—	x	x	.	x	x	.	x	**Romanian**

OTHER MASHUPS

Mashups have proven to be useful way of generating new scales. Are there more of them that might be useful? Let's see. Here follows an example of a mashup of classical modes a half tone apart that produces an 8-note blues scale **//RM** seen before. This scale has not, to my knowledge, been formally developed into a suite of parallel modes in music notation but it could be. This is a minor-major mode but an all major mode such as **//MI** (not shown) could be useful for pieces that switch back and fourth between **//M** and **//I** (e.g., *Traumerai*).

```
       @PORMIL$xxxx@
//R    x xx x x xx x
//M    x x xx x xx x
//RM   x xxxx x xx x   seen before as a blues mashup of parallel pentatonic modes
```

SUCCESSIVE, MIXED, MINOR & MAJOR FAMILIES: *LAURA*

My source for the summary shown next is *The Jazz Book*, John Brimell, CPP/Belwin, 1989, p24. The home tonic of the source is C. The source book is sub-titled *Today's Easy Adult Piano* but this piece is "easy" only in the sense that the key signature is empty, the density of notes on the page is low, and playing it without thinking about the changes is easy. The not-so-easy part is getting a handle on the changes, due to the presence of adjacent half tones in often-sparse melody lines, and to seemingly irregular relationships between the melody lines and the richly chromatic harmony. In the following summary, fifos are omitted at points marked ✚ because understanding the changes requires only the tritones, and the fifos are easily added from context determined by the tritones.

Laura (original home tonic = C)

(a) ⚓ ✚ RL ✚
♫ | 7* | — , p7 7 p7 p5 5 | 6* 3 | — , |

(b) ⚓ ✚ PM ✚ R
♫ | 6* | — , p6 6 p6 3 4 | 5 — | — , |

(c) ⚓ OI ✚ I ✚ L RL
♫ | — p3 4 | — , p3 4 | p3 4 | 5* 2 | — , p3 5 p7 | p3* p6 |

(d) ⚓ LO L ✚ ✚ I OI
♫ | — , 6 | 2 — | — , 4 1 — 2 |

repeat (a)-(b) and then go to (e)-(f)

(e) ⚓ ✚ I● ✚ L
♫ | 4 5 | — , 3 4 p6 7 | 2 1 2 | — , 7 1 |

(f) ⚓ LO OI I OI ✚ ✚
♫ | p3 p3 | — , 3 p3 2 6 7 | 1 — |

An interpretation in terms of mixed, parallel, classical modes: The melody lines are from successive classical modes that go from major in (a)-(b) to minor in (c) to major in (d)-(f). Ornamental notes (highlighted) "bleed" between the classical modes. The pentatonic major mode of (d) is a sub-mode of the resolution Ionian mode of (e)-(f). The successive classical modes are determined by single tritones going down by whole tones through (a)-(b)-(c) and then up a minor third to (e)-(f). This may be interpreted as Ionian tonics going down by whole tones through (a)-(b)-(c) and then down a minor third to (e)-(f), except this is not very helpful because the modes are daisy-chained together instead of resolving. It's much simpler to think of this in terms of parallel mode changes.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	@	P	O	R	M	I	L	$	x	x	x	x	@	
(a) melody notes	.	.	.	.	.	.	x	x	.	x	x	x	.	
//L	x	.	x	.	x	.	x	x	.	x	.	x		**major**
(b) melody notes	.	.	.	.	x	x	.	x	x	x	.	.		
//M	x	.	x	.	x	x	.	x	.	x	x	.	x	**major**
(c) melody notes	.	.	x	x	.	x	.	x	x	.	x	.	.	
//O	x	.	x	x	.	x	.	x	x	.	x	.	x	**minor**
(d) melody notes	x	.	x	.	x	.	.	.	.	x	.	.	x	
//M	x	.	x	.	x	.	.	x	.	x	.	.	x	**major**
(e) melody notes	x	.	x	.	x	x	.	x	x	x	.	x	x	
//I	x	.	x	.	x	x	.	x	.	x	.	x	x	**major**
(f) melody notes	x	.	x	x	x	.	.	.	.	x	.	x	x	
//I	x	.	x	x	x	x	.	x	.	x	.	x	x	**ornamented major**

<u>An interpretation in terms of parallel harmonic-minor-major modes:</u> I find the local irregularity of (a)-(b)-(c) unsatisfying. The piece sounds like each melody phrase is from a fully defined parallel mode of its own. Out of curiosity, I looked for an interpretation in terms of non-classical modes and found the following one in terms of parallel modes of the harmonic-minor-major mode determined by double tritones in the harmony. There's an elegant regularity about this that fits the elegant regularity of the melody phrases to the ear.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	I	L	$	x	x	x	x	@	**<u>home tonalities</u>**
(a)	**melody notes**	.	.	.	.	.	.	x	x	.	x	x	x	.	
	//(RL)⁺	x	.	x	x	—	—	x	o	.	x	x	x	x	**minor**
(b)	**melody notes**	.	.	.	.	x	x	.	x	x	x	.	.	.	
	//(PM)⁺	x	x	—	—	x	o	.	x	x	x	x	.	x	**major**
(c)	**melody notes**	.	.	x	x	.	x	.	x	x	.	x	.	.	
	alt(IO)⁺	—	—	x	o		x	x	x	x		x	x	—	**same notes as (c) above**
(d)–(e)-(f) as before															

ATONAL MODES

The focus of PKP is tonal music but atonal scales summarized next are included because shared letters of their defining words make them conceptual parents of scales lower down in the hierarchy; and also because they may be used ornamentally in tonal music. Atonal scales have no minor, major or minor-major tonality by themselves but representing them by words that place them in the context of the home tonic gives them tonality relative to it. The last mode is familiar from the earlier *Giant Steps* example. It has no no tritones but is determined by tritones because it's morphed from the tritones of **O.M.L**. There are no all-atonal example pieces in this document.

Atonal modes have a symmetry that makes them compatible with multiple tonics (the shape going up from different tonics is the same). Whole tone scales have circular symmetry and diminished scales have mirror symmetry. Representing them by alphabet words selects the home tonic as a reference. Breaking their symmetry creates related home-tonic scales.

		1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
word	type name	@ P O R M I L $ . . . . @	related to
ORMI	—	———x x x x———x x x x——	**//ORMI, //OR.I, //O.MI**
P.R.I	whole tone	. x . x . x . x . x . x .	**//RI**
O.M.L	"	x . x . x . x . x . x . x	**//OM**
P.RM.L	diminished	x x . x x . x x . x x . x	**//RM.L**
PO.MI	"	. x x . x x . x x . x x .	**//O.MI**
OR.IL	"	x . x x . x x . x x . x x	**//OR.I**
PMI	augmented	x x———x x———x x———x	**O.M.L**

CHAPTER 4: A SMORGASBORD OF ADVANCED EXAMPLES

This chapter explores a smorgasbord of example pieces that I found difficult to understand when I first encountered them in music notation because of strong departures from classical modes, often combined with difficult or multiple written key signatures. The idea of a smorgasbord is to offer something for everyone. Any one of these pieces rewards deep study and, when the study is in these terms, what's learned is portable between different pieces with different tonics and key signatures.

A takeaway from this chapter is confirmation that tritones are structurally fundamental to music understood in interval terms, and that a DNA-like alphabet is sufficient to cover any tonic scale or chord progression that may be played on the piano, independently of the key signature(s) in which a piece may be written. This is different in kind from "the score is sacred" view of many classical musicians.

I have heard it said, and believe it to be true, that composition is frozen improvisation. Therefore it, like improvisation, requires a sense of architecture. I suggest that exactly such a sense of architecture is revealed by the examples in this chapter. The fact such a sense can be extracted from such a wide variety of pieces without any information other than that provided by the piece itself suggests that it was put there by a composer or arranger, based on an intuitive understanding that's no doubt different in kind from PKP, but that can be represented simply by PKP.

The examples are intended to be understood by playing them, not just looking at pages, and so should be approached in front of a piano on which they can be tried. Start with the melody line and the tritone core to establish basic flow. Play the tritone core as octave shapes first. Then play the actual shapes shown, including interpolated fifo shapes. Then try variations.

The following pieces explore a wide variety of home tonics, modes and key signatures. All are in the chromatic non-classical domain. There are distinct differences in style between the different pieces that range from loose and somewhat ambiguous, to systematic and unambiguous. The examples speak for themselves.

Blues

Goodbye Pork Pie Hat
When Sunny Gets Blue
Prelude to an Afternoon of a Faun (interpreted as a blues)
Straight No Chaser
Blue Monk

Minor or Major Family

You Must Believe in Spring
Round Midnight
All of Me
All the Things You Are
No Greater Love
Body and Soul
Every Time We Say Goodbye
All the Things You Are
Chelsea Bridge
Lush Life
The Peacocks

GOODBYE PORKPIE HAT

This blues in Eb, the chord progression of which is shown in footnote 2 of the opening chapter, is a poster child for chromatic music that's difficult in music notation for all but experts. My source for it is the *Mingus Fakebook*, Hal Leonard (1991). Trying to learn this piece from this source was one of the stimuli that sent me down the path to PKP. The key signature of 3 flats and the home tonic of Eb jointly identify the Ionian mode **//I** as the reference mode for the accidentals that determine blues notes. This is an example of the often misleading nature of key signatures. A 5-flats or 6-flats key signature indicating Dorian or Aeolian modes of Eb would be closer to the **//RM.L** blues that governs the piece for this tonic.

The simple melody line is mostly in the 6-note minor blues sub-scale. Bars 6-7 are a variation that goes "outside" the blues. In bar 7, the **1.p2** notation means the first note is "crushed" into the second note on the same beat. The melody is played in swing-feel 4/4 time. No downbeat line is required because commas indicate phrases terminated by a downbeat, with the next note on upbeat.

Keep in mind here and later that this notation is only to illustrate the concepts independently of the obscuring clothes of music notation, and that, most of the time, all that's required is chromatic scale symbols annotated on the staff and augmented anchor lines annotated above the staff next to chord symbols. That said, this is a useful way of capturing the essence of tricky passages, to say nothing of it being valid for any home tonic, not just the written one.

Goodbye Porkpie Hat (original home tonic = Eb)

⚓ ■M L■ ■I ML ■I L■

♫ 5 | 1 p3 , 1 p3 4 p3 , 1 p7 | 1 p3 , 1 p7 1 , 5 | 1 p3 , 1 p3 p5 p3 , 1 p7 |

⚓ ■I ■M ■I L■

♫ | 1 p3 , 1 p7 1 , 1 | 5 p7 , 1 p3 4 p3 , p7 p3 |

⚓ O■ I■ P■ L■) *variation*

♫ | 5 1 , 1 p3 p7 5 p3 p7 p6 , | p3 p5 , 1.p2 7 p5 , 2 |)

⚓ L■ ■I M■ R■ I■ L■ ML) *resolution*

♫ | p3 1 , p5 4 | p3 1 , | 4 p3 1 p7 , p5 4 p3 1 | p3 1 , |)

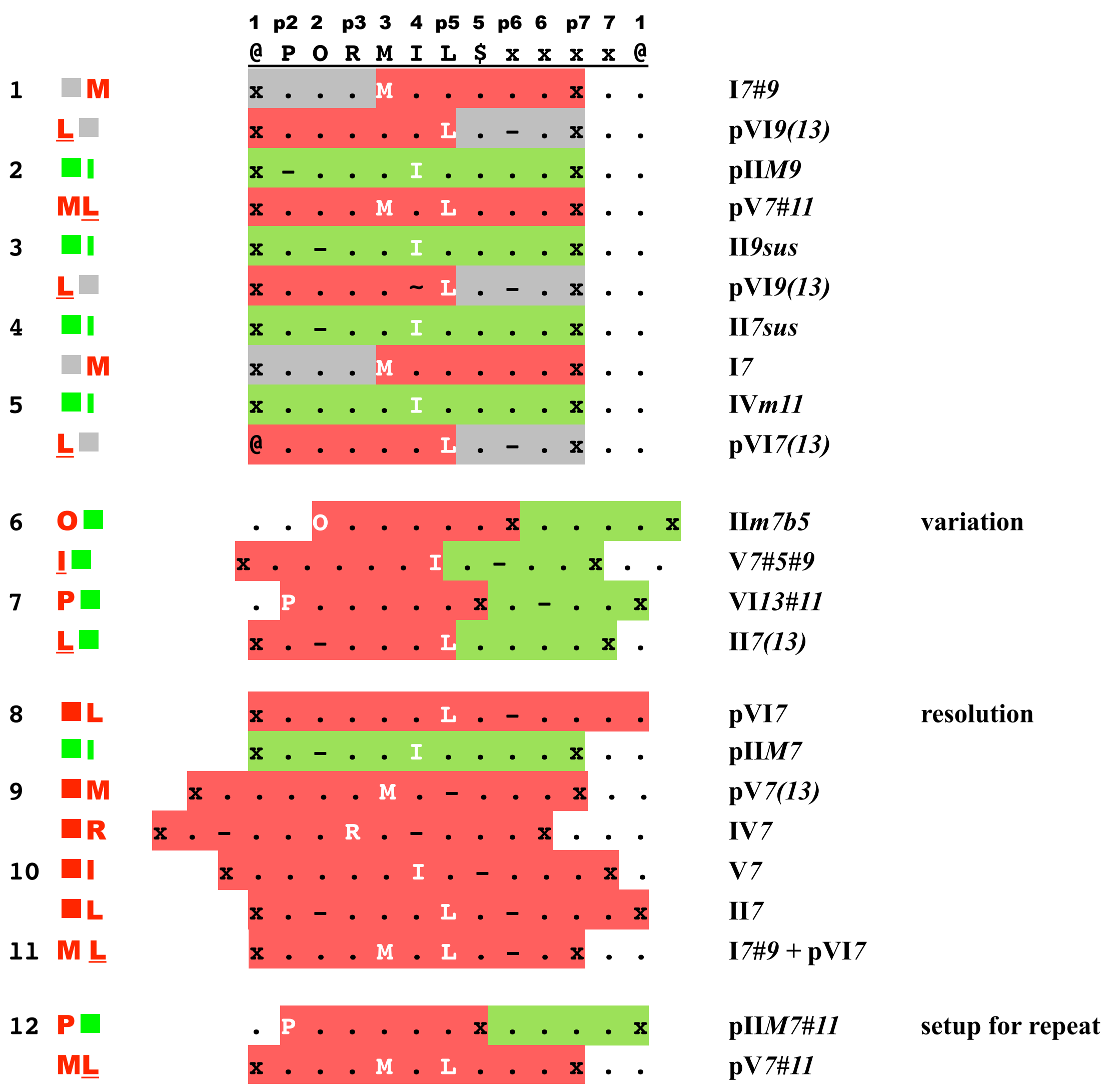
⚓ P■ ML) *turnaround*

♫ | 5 4 p3 , 1 p7 5 |)

Three styles of harmony are illustrated: in bars 1-5, the outer notes stay fixed (determined by the double tritone **ML**) while one inner note moves; in the variation in bars 6-7, the tritone shapes are "all fourths," providing an easy-to-play sequence that gives a bluesy sound to this departure from the blues; in bars 8-12, the single-tritone chords are all octave shapes, on the principle that there's so much harmonic variety in the tritone sequence that the tritone-based shapes don't have to provide it.

The Lego-like structure of this harmony is shown next, along with the written chords. Many of the alterations in the chord symbols on the right are provided by context. For example, *#9* of the **I***7#9* chord in bar 1 is melody note **p3**. The free use of tritone substitute chords makes for a complex written chord progression. These chords have all non-tritone notes different (including roots a tritone apart) but the same harmonic function; in other words, they combine different fifos with the same tritone. The **pII** roots of the major seventh chords are byproducts of tritone substitutions, not indicators of visits to a mode containing this note. Several repetitions of the non-tritone shape ■I voice a variety of chords, the nuances of which can be left to context. A more accurate voicing of the *sus* chords would be provided by the shape ■$, a whole tone up from ■I, but the two shapes are mutually consonant and the the difference is relatively unimportant to the ear in this context; the ■I shape fits the flow better.

The bass line provided by the chord roots is missing but could easily be added back underneath if desired. That said, this harmony evokes the haunting, rich sound of the piece in a very satisfying way: small note changes in a smoothly flowing keyboard pattern make big sound changes.



WHEN SUNNY GETS BLUE

My source for this piece is sheet music by Jack Segal and Marvin Fisher from Hal-Leonard (1956), publication number HL00351105. The home tonic is G and the key signature is one flat, identifying Dorian of G as the reference classical mode. The melody line is a **//RM.L** blues, with a 4-bar section in the bridge [c] that goes to the nearby **//L** mode. The harmony shown here is a modification of the written harmony, which I found to have a confusingly irregular flow. This harmony flows smoothly to my eye and ear. Ornamental tritones in [a] (not in the blues scale) are shown in a darker shade of red.

When Sunny Gets Blue (original home tonic = G)

[a] ⚓ M■ R■ L■ R■ M■ R■ L■ I■
♫ 4 ||: 2 1 2 4 , 4 | 4 p3 p7 p5 4 p3 | — 2 1 p7 6 4 1 | 6 , |
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^*

⚓ M■ R■ O■ P■ L■ I■
♫| 5 p7 1 p7 p2 p7 1 p7 , | 4 p7 1 p7 p2 p7 1 p7 , |
^ ^ ^ ^ ^ ^ ^ ^ ^

⚓ ■M ■R ■L ■M ■R ■M ■L
♫| 5 p7 1 p7 4 — p3 p7 | 2 , 5 :|| p2 , |
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

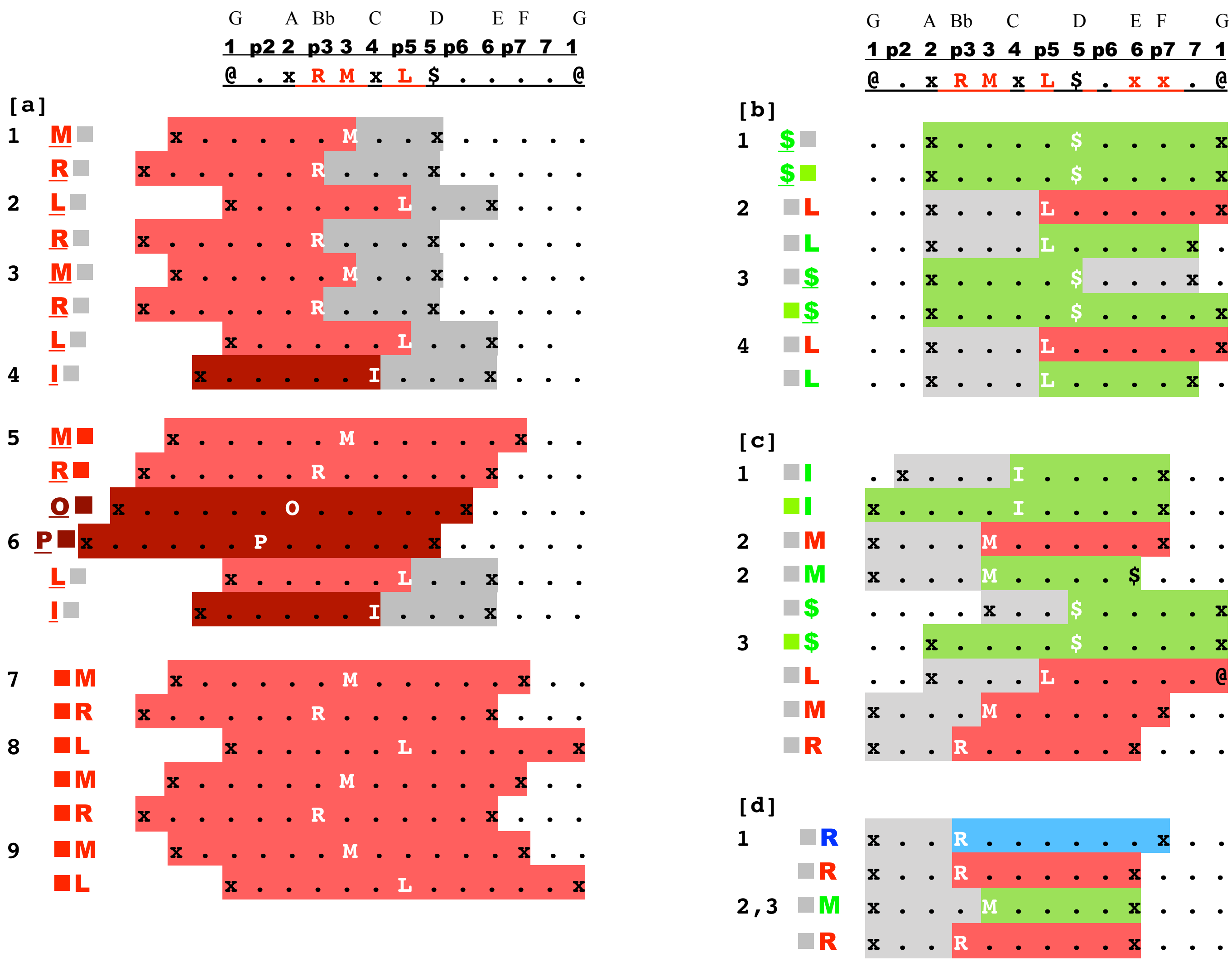
[b] ⚓ $■ $■ ■L ■L $■ $■ ■L L
♫| 5 1 7 1 2 3 | p5 2 2 2 7 7, | 7 5 6 7 1 | 2, |
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^^

[c] ⚓ ■I ■I ■M ■M ■$ ■$ ■L ■M ■R
♫| p7 1 p7 1 2 p3 | 3 1 1 1 6 6 , | 6 4 5 6 p7 | 2 , 5 |
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

repeat [a] followed by this ending, repeated twice

[d] ⚓ ■R ■R ■M ■R ■M ■R
♫| 2 4 5 4 p6 4 p2 | 1 , | |
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

Here’s what this harmony looks like on the keyboard.

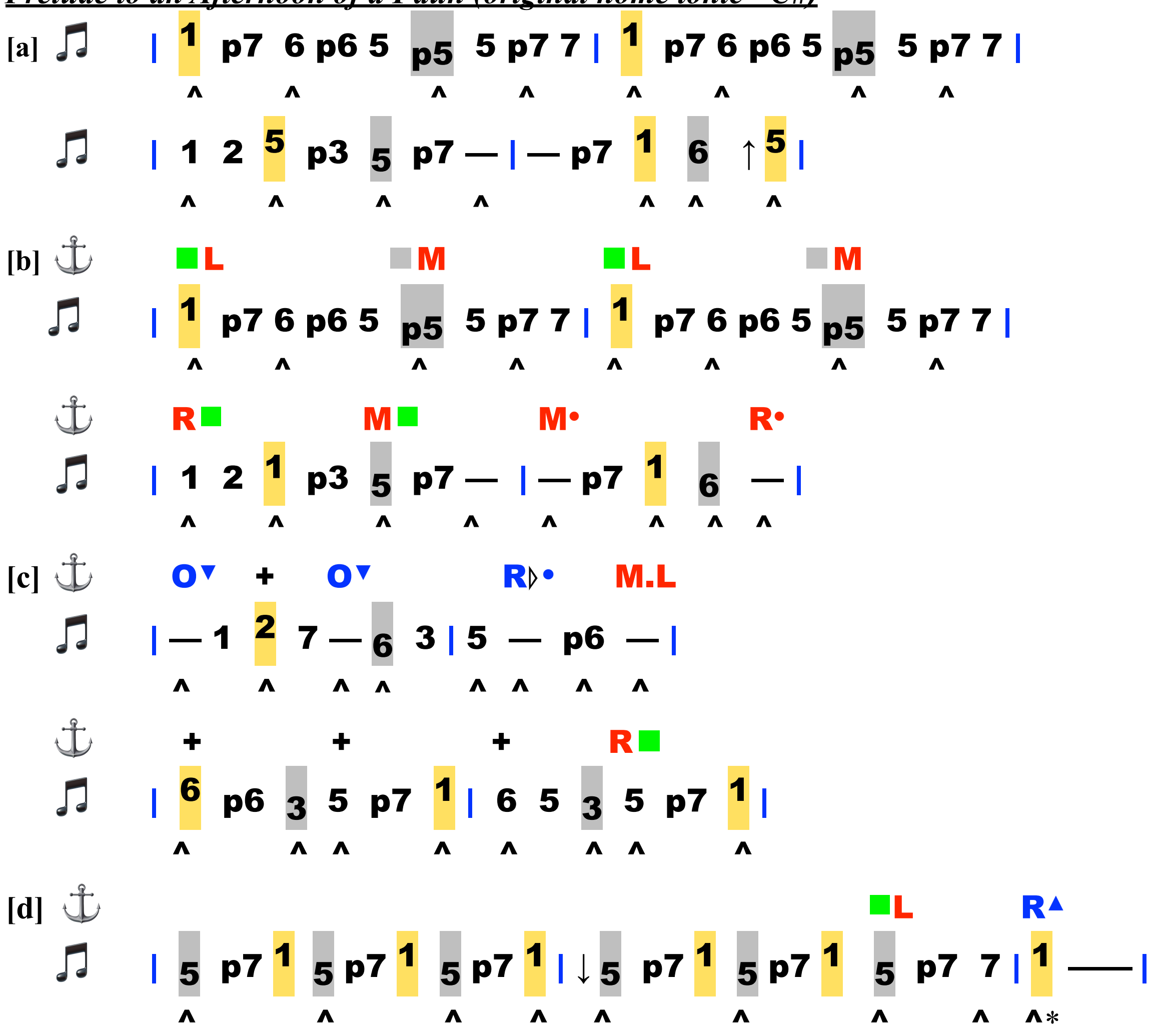


PRELUDE TO AN AFTERNOON OF A FAUN

I was motivated to investigate this hauntingly beautiful Debussy piece by things said about its creative violation of music convention in the article *Beauty in the Void,* Alex Ross, The New Yorker, Oct. 29, 2018. The violations were explained in the article in terms of unusual sequences of different keys. I wondered if there might be a way of understanding it in terms of non-classical modes. My source for the written music is the *Classical Fake Book*, 2nd Edition, Hal Leonard (2013), page 222. The piece looks innocently (and misleadingly) simple there — a melody line with a few accidentals and some mostly simple chord symbols above it for mostly triad chords. The problem is, it's written in C# minor with accidentals that bring in Db major, a combination that's inherently complex in music notation (recall footnote 1 in the opening chapter).

Here follows a summary of a variation that's faithful to the written melody line but takes some liberties with the harmony (described following this). The written time signature of 9/8 divides each bar into 3 groups of 3 eighth notes, but playing it in 4/4 time sounds very close to the original timing and leads naturally to a swing-feel blues. The "**+**" entries in [c] are placeholders, to be filled in, if desired, from the ensuing description.

Prelude to an Afternoon of a Faun (original home tonic =C#)



Getting a handle on the piece is helped by the following identification of the melody scales. The piece uses all the notes of the chromatic scale for both melody and harmony, but organized in a way that suggests **//RM.L** blues with ornamental passing notes **p6** and **7** (they're passing notes because omitting them, while maintaining the same timing, has no substantive effect on the sound).

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
	C#		D#		E	F#		G#		A		B	C#	<—written key sig.
	@	P	O	R	M	I	L	$	x	x	x	x	@	
[a]-[c]	x	.	x	x	x	x	x	x	+	x	x	+	x	//RM.L blues melody scale with passing notes
[d]	x	.	.	x	.	x	.	x	.	.	x	+	x	//R pentatonic minor sub-mode

The original harmony is mostly from the blues scale, with the striking exception of note **p2** as the root of a major triad in the opening bars of [b]. This is striking not only because this note is not in the melody scale, but also because it's dissonant with the melody note it harmonizes (the tonic). Adding the tonic to the triad yields a major seventh chord, a simple alteration of which yields the "all fourths" shape ■**L** from the blues scale (a tritone is an augmented fourth). This basic blues shape sounds as appropriate as the original major triad and paves the way for more blues-centered harmony in the rest of the piece, in which the same shape appears for different tritones. No chord symbol is given because the convention of identifying chromatic notes by altered degree number suffixes is confusing for scales that have more notes than the classical modes that determine the basic chords.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	I	L	$	x	x	x	x	@	
pII	P▲	.	x	.	.	.	x	.	.	x	.	.	.	.	1st written chord of [b]
pII*M7*	—	.	x	.	.	.	x	.	.	x	.	.	.	x	forms this chord with the tonic
—	■L	.	x	.	.	.	.	x	.	.	.	.	.	x	altered chord fits the //RM.L scale

A complete Lego-like picture of the harmony is shown on the next page, including the original harmony and the new blues-based harmony. A comparison for [b] leading into [c] shown below. The new harmony is entirely from the blues scale, and is evocative of the blues. The scale and the flow prompt the shapes, and the result sounds fine. The proof is in the playing.

Here follows the complete harmony. ▷ or ◁

		original		here	C# D# E F# G# A B C#
					1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
					@ P O R M I L $ x x x x @
[a]	1-4	—		—	. . . .
[b]	5	pII	P▴	■L	. x . . . . . L x
		pVII*dim*/pII	■M	■M	. x . . M . . . . . x . .
	6	pII	P▴	■L	. x . . . . . L x
		pVII*dim*/pII	■M	■M	. x . . M . . . . . x . .
	7	II*dim*	O•	R■	. . . R . . . . . x x
		pVII*7*	O■	M■	. . . . M . . . . . x x
	8	IV*dim*	I•	M•	. . . . M . . x . . x . .
		—	—	R•	. . . R . . x . . x . . .
[c]	9	IV/II	O▾	O▾	. . O . . x . . . x . . .
		V	O▷▷▴	+	. . O . . . . x . . . x .
		IV/II	O▾	O▾	. . O . . x . . . x . . .
	10	VII+	R▷•	R▷•	. . . R . . . x . . . x .
		pVII*7b5*	M.L	M.L	. . . . M . L . . . x . x
	11	VI	M▷▷▴	+	. . . . M . . . . x x
		pVII*6*	I▷▷▴	+	. . . . . I . . . . x . . . x
	12	VI	M▷▷▴	+	. . . . M . . . . x x
		IV*dim*	I•	R■	. . . R . – . . . x x
[d]	13	—	—		. . . .
	14	—	—	■L	. x . . . . . L x
	15	pIII*6*	R▴	R▴	. . . R . . . $ . . x . .

<——written key sig.

Row comments: 9 (V): split major seventh; 10 (VII+): split aug. fifth; 11 (VI): split major seventh; pVII6: ditto; 12 (VI): ditto.

STRAIGHT NO CHASER

Walking bass lines provide an alternative to chordal harmony in many kinds of music, for interest and variety. They are easy to represent in the melody-line notation, easily portable in this form between different pieces with the same or different home tonics, easy to create knowing mode scales of the tonics, and easy to play. These things are illustrated by the following use of a walking bass line from Monk's *Straight No Chaser* (coming up) for *Backwater Blues* in Chapter 2. The interleaving of this bass line and the melody line of *Straight No Chaser* is a bit tricky, so this introduces the bass line in a simpler way, besides making *Backwater Blues* more fun to play. The bass line is identified as harmony by an anchor symbol on the left, but is shown under the melody line where it's actually played, because showing it above would be confusing (it's not confusing for anchor lines shown above the staff because they're not represented in note symbols). There are some transient dissonances between this melody and harmony that could be avoided by some syncopation of the melody, but such dissonances are often a feature of the blues.

Backwater Blues with the Walking Bass Line of Straight No Chaser

♫ | **5** 4 p3 — **1** | **5** 4 p3 — **1** | ———— | ———— |
⚓ 1 2 p3 3 4 p3 2 p2 1 3 5 6 p7 6 5 3

♫ | **5** 4 p3 — **1** | **5** 4 p3 — **1** | ———— | ——— 5 **p7** **3** |
⚓ 4 p3 2 p2 4 p3 2 p2 1 3 5 6 3 p7 6 2

♫ | **p7** p7 ———— **5** | **1** p7 5 p5 4 p3 — **1** | ———— | ———— |
⚓ 1 2 p3 3 4 5 p6 6 1 2 p3 3 1 2 p3 3

Walking bass lines are easily created (or understood) using a mode table, as illustrated next. Bass-line notes **p2** and **p6** and **7** ornamental passing notes. To be concrete, the corresponding notes of music notation are shown on top for tonic F blues.

	F		G	Ab	A	Bb	B	C		D	Eb		F	<==enriched F Dorian			
	1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**				
	@	P	O	R	M	I	L	$	x	x	x	x	@				
//DML	x	.	x	x	x	x	x	x	.	x	x	.	x				
bar 1	x	.	x	x	x	.	.	.	.	.	.	.	.				up
bars 2, 5, 6	.	x	x	x	.	x	.	.	.	.	.	.	.				down
bar 3, 7	x	.	.	.	x	.	.	x	.	x	.	.	.				up
bar 4	.	.	.	.	x	.	.	x	.	x	x	.	.				down
bar 8	.	.	.	.	x	.	.	.	.	.	x	.					up
	.	.	.	x	.	.	.	.	.	x	.	.	.				down
bars 9-11	x	.	x	x	x	x	.	x	x	x	.	x	.	x	x	x	up
bar 12	x	.	x	x	x	.	.	.	.	.	.	.	.				up

Here follows the melody line of *Straight No Chaser* with home tonic F using this walking bass line (the original in the Monk fake-book has home tonic Bb but I learned it in F). The transition to Bb is easy, simply replace the F Dorian scale in the table below by the corresponding Bb scale. In either case, the key signature identifies the Dorian melody mode **//R** of the home tonic which is altered to **//RM** by an accidental. This is not the full blues scale because **p5** is missing.

Straight No Chaser with the Same Walking Bass Line

♫ 5 | 1 2 p3 3 — 5 1 2 | p3 3 4 p3 —— 5 | 1 2 p3 3 — 5 1 2 | p3 3 — 5 1 2 p3 |
⚓ 1 2 p3 3 4 p3 2 p2 1 3 5 6 p7 6 5 3

♫ | — 5 1 2 p3 4 p3 | ———————— | — 5 1 2 p3 3 — 5 | 1 2 p3 3 — 5 1 2 |
⚓ 4 p3 2 p2 4 p3 2 p2 1 3 5 6 3 p7 6 3

♫ | p3 3 4 5 p6 6 p7 7 | — 1 p2 2 p3 3 4 5 | 1 2 p3 3 —— | ——— 5 |
⚓ 1 2 p3 3 4 5 p6 6 1 2 p3 3 1 2 p3 3

The melody line is in the **//RM** sub-mode of the blues family mode with two obvious passing notes (each appears only once). Recall that the mode is a mashup of parallel pentatonic modes).

F		G	Ab		Bb		C		D	Eb		F	<== Dorian of tonic F
1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
@	P	O	R	M	I	L	$	x	x	x	x	@	
x	.	x	x	x	x	.	x	.	x	x	.	x	//RM
	+							+			+		passing notes

The intricate interleaving of the two lines makes the whole sound like more than the sum of the parts, and requires considerable practice to get right, and also considerable effort to write down correctly. The benefit of this effort is portability of concepts and of keyboard patterns to other pieces.

BLUE MONK

This piece is so chromatic that it seems as if several different home tonics might fit. Only one fits, namely Bb of the written 2-flats key signature, established by the first notes of a double melody line consisting of two lines offset by thirds (major or minor). This choice of tonic makes clear that the appearance of all notes of the chromatic scale is a byproduct of three passing notes.

```
Bb    C     D  Eb    F     G     A  Bb   <== key signature notes
1  p2 2  p3 3  4  p5 5  p6 6  p7 7  1
@  P  O  R  M  I  L  $  x  x  x  x  @
x  .  x  x  x  x  x  x  .  x  x  .  x    //RM.L
   +                    +        +       passing notes
```

As shown next, the double melody line formed of main line with a secondary line a third (major or minor) down from it. The thirds are determined by a simple rule: switch between a major and a minor third when one note in either line moves by a whole tone. This is to avoid the "outside" notes **p2** or **p6** or **7** wherever possible; the rule is broken only when "outside" notes are unavoidable (thus they become passing notes). The positions of the resulting major thirds are marked below by over lines. For concreteness, the secondary line is shown as a sequence of lighter note symbols, but it's simpler to think of it as determined by construction from the rule. The opening sequence of a major third and three minor thirds establishes the melody scale.

Blue Monk (original home tonic = Bb)

```
♫ | 3  4  p5 5 — —  | 6  p7  7  1  ——  |  5  6  5  p5  4  p3 — 3 | — p3 —  2   —— |
    1  2  p3 3        4  5  p6  6           3  4  3  p3  2  7     1      7     p7
⚓   1     2    p3  3   4     5   p6  6    1    1    5    p5    1    2    p3  3

♫ | 6  p7 7  1  — — | 1  p2  2  p3 — — |  5  6  5  p5  4 p3 — 3 | — — —  5   5   5 |
    p5 5 p6  6          6  p7  7  1        3  4  3  p3  2  7    1
⚓   4    5    p6  6   p5    5    p6 6    1    1    5    p5    1  p7  6    p6

♫ | 5  5  — — —  | ↑ 5  6  5  p5  4 p3 — 3 | —  5  6  5  p5  4 | p3  3 — — — |
                    3  4  3  p3  2  7     1     3  4  3  p3  2    7   1
⚓   5    6  p7  7    1   p7    6    p6     5   6   p7    7   1    2  p3  3
```

Construction of melody thirds from the rule is illustrated next for bars 1-4 (**x** marks blues-scale notes, **o** marks "outside" passing notes, grey shading highlights major thirds)

```
                  Bb      C       D   Eb      F       G       A   Bb
                  1   p2  2   p3  3   4   p5  5   p6  6   p7  7   1
                  @   P   O   R   M   I   L   $   x   x   x   x   @
//DML             x   .   x   x   x   x   x   x   .   x   x   .   x
bar 1             x---------------x   .   .   .   .   .   .   .   .   1-2-p3-3
                  .   .   x-----------x   .   .   .   .   .   .   .
                  .   .   .   x-----------x   .   .   .   .   .   .
                  .   .   .   .   x-----------x   .   .   .   .   .
bar 2             .   .   .   .   .   x---------------x   .   .   .   4-5-p6-6
                  .   .   .   .   .   .   .   x-----------x   .   .
                  .   .   .   .   .   .   .   .   o-----------o   .
                  .   .   .   .   .   .   .   .   .   x-----------x
bar 3             .   .   .   .   x-----------x   .   .   .   .   .   3-4-3-p3-2-7-1
                  .   .   .   .   .   x---------------x   .   .   .
                  .   .   .   .   x-----------x   .   .   .   .   .
                  .   .   .   x-----------x   .   .   .   .   .   .
                  .   .   x-----------x   .   .   .   .   .
              o---------------x   .   .   .   .   .   .   .   .   .
                  x---------------x   .   .   .   .   .   .   .   .
bar 4         o---------------x   .   .   .   .   .   .   .   .   .   7-p7
          x---------------x   .   .   .   .   .   .   .   .   .   .
```

The sequence starts with a major third to avoid "outside" note **p2**. It continues with a minor third for the same reason. And so on. The few appearances of an outside note (**o**) are unavoidable in order not to disrupt the flow. The result is a smoothly flowing sequence of thirds that's easy to play and sounds good. This simplicity is obscured in the written music by the clutter of sharps, flats and naturals on different staff lines and spaces required to represent the notes relative to the written key signature.

YOU MUST BELIEVE IN SPRING

My source for this piece is *The Michel Legrand Songbook*, 1997, Warner Bros. Publications. The strongly systematic organization of this piece is difficult to see in the written music, which is a full arrangement spread over 3 pages with 2 key signatures (2 sharps, 3 flats) and 32 tritone chords of many varieties (next page) that establish two parallel modes with all notes a half tone apart (bottom of this page). This notation brings the systematic organization forward to the eye.

home tonic @1 = D

[a] ┌ //O.MI (bebop major)

⚓ I OI OI I I OI

♫ | — 2 3 4 6 4 p6 3 | 2 1 2 7 1 , | — 4 5 6 1 6 7 5 | 4 3 4 2 3, |

^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

[b] ┌transition //O.MI (bebop major)

⚓ L RL M PM I I O

♫ | — 6 7 1 3 p5 p3 | — 5 6 p7 2 3 p2, | — 4 5 6 1 6 7 5 | 4 3 2 1 3, |

^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

[c] repeat [a]

[d] ┌transition ┌ //R.IL (bebop major 1/2 tone up)

⚓ L RL I OI L L R

♫ | — p7 1 2 3 p5 p3 | — 6 p7 7 p3 4 2, | — p5 6 p7 2 p7 1 6 | p5 4 p3 2 p3 , |

^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

[e] ┌ //R.IL continued

⚓ L RL RL L L

♫ | — p3 4 p5 p7 p5 6 4 | p3 p2 p3 1 p3, | — p5 p6 p7 p2 p7 1 p6 | p5 4 p5 p3 4,|

^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

[f] ┌ //R.IL with some variations

⚓ P PM I OI L L R

♫ | — 6 p7 7 p3 4 2 | — p6 6 p7 p7 p3, | — p5 p6 p7 p3 p7 1 p6 | p5 4 p3 p2 1 p7 , |

^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

[g] ┌ //R.IL@p7 (relative mode)

⚓ L L R

♫ | 4 p3 p2 1 p7 6 | p7, |

^ ^ ^ ^ ^*

Modes (tonics highlighted)

	D		E		F#	G		A		B		C#	D
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	I	L	$	.	.	.	.	@
//O.MI	x	.	x	.	x	x	.	x	x	x	.	x	x	bebop major of D
//R.IL	x	x	.	x	.	x	x	.	x	x	x	.	x	same of Eb (all notes 1/2 tone up)
//R.IL@p7	x	x	.	x	.	x	x	.	x	x	x	.	x	relative mode resolves to C

Tritone chords (roots highlighted)

The 32 tritone chords provide core harmony consisting of 6 tritones and 2 triple-tritone mode signatures offset by a half tone. The notes of the full chords are provided by context. The fifo chords are not shown here because the core fifos are context-determined morphs of the tritones.

			D	.	E	.	F#	G	.	A	.	B	.	C#	D	
			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
			@	P	O	R	M	I	L	$	.	.	.	.	@	
[a]	1	C#m7b5	.	.	x	.	.	x	.	.	.	x	.	x	.	I
		F#7b9	.	.	x	.	x	x	.	.	x	.	.	x	.	OI
	2	F#7b9/B	.	.	x	.	x	x	.	.	x	x	.	x	.	OI
	3	A7	.	.	x	.	.	x	.	x	.	.	.	x	.	I
	4	A7b9/D	x	.	x	.	.	x	.	x	x	.	.	x	.	OI
[b]	1	G#m7b5	x	.	.	.	x	.	x	.	.	x	.	.	x	L
		C#7b9	.	.	.	x	.	.	x	.	.	x	.	x	x	RL
	2	F#m7b5	.	.	.	.	x	.	x	.	.	x	.	.	.	M
		B7b9	.	x	.	.	x	.	.	x	.	x	x	.	.	PM
	3	A7	.	.	x	.	.	x	.	x	.	.	.	x	.	I
	4	C#m7b5	.	.	x	.	.	x	.	.	.	x	.	x	.	I
		F#7	.	.	x	.	x	.	.	.	x	.	.	x	.	O
[c]=[a]																
[d]	1	G#m7b5	x	.	.	.	x	.	x	.	.	x	.	.	x	L
		C#7b9	.	.	.	x	.	.	x	.	.	x	.	x	x	RL
	2	Gm7b5	.	.	.	x	.	x	.	.	x	.	.	x	.	I
		C7b9	.	.	x	.	x	x	.	.	x	.	x	x	.	OI
	3	Bb7	x	.	.	x	.	.	x	.	x	.	.	.	x	L
	4	Dm7b5	x	.	.	x	.	.	x	.	.	x	.	.	.	L
		G7	x	.	.	x	.	x	.	.	.	x	.	.	x	R
[e]	1	Dm7b5	x	.	.	x	.	.	x	.	.	.	x	.	x	L
		G7b9	x	.	.	x	.	x	x	.	.	x	.	.	x	RL
	2	G7b9/C	x	.	.	x	.	x	x	.	.	x	x	.	x	RL
	3	Bb7	x	.	.	x	.	.	x	.	x	.	.	.	x	L
	4	Bb7b9/Eb	x	x	.	x	.	.	x	.	x	x	.	.	x	RL
[f]	1	Am7b5	.	x	.	.	.	x	.	x	.	.	x	.	.	P
		D7b9	x	x	.	.	x	.	.	x	.	.	x	.	.	PM
	2	Gm7b5	.	.	.	x	.	x	.	.	x	.	.	x	.	I
		C7b9	.	.	x	.	x	x	.	.	x	.	x	x	.	OI
	3	Bb7	x	.	.	x	.	.	x	.	x	.	.	.	x	L
[g]	1	Dm7b5	x	.	.	x	.	.	x	.	.	.	x	.	x	L
		Dm7b5/G	x	.	.	x	.	x	x	.	.	.	x	.	x	L
		G7	x	.	.	x	.	x	.	.	.	x	.	.	x	R

ROUND MIDNIGHT

The source for this evocative piece in Eb minor by Monk are *The Ultimate Jazz Fakebook*, Hal Leonard, 1988, p. 322 for the main body of the piece (shown first) and *Standards Real Book*, Sher Music, 2000, p. 369 for an optional introduction (next page). The following summary is easy to play without a downbeat line if you've ever heard the tune.

Round Midnight (original home tonic = Eb)

[a] ⚓ @▾ R R O I @▾ R M I R M
♫ | , 5 1 2 5 p3 5 | 1 7 1 5 4 , | , 1 p3 5 p7 6 1 | p5 4 4 3 , |

⚓ O▾ O @▾ R ⊕ L I @▾
♫ | , 4 p6 1 p3 2 p6 | 5 , 1 7 p7 6 5 , 1 2 | p3 2 1 2 1 7 | 1, 1 2 |

[b] ⚓ R L I R L I
♫ | p3 2 1 2 1 | 7 5 , 1 2 | p3 2 1 2 1 | 7 5 , ↑5 |

⚓ R O O L I M I L L I I
♫ | p6 p6 5 5 | 4 p3 2 , 5 | 1* 1 p7* p7 | p6 1 p7 5 4 p2 7 |

⊕ ⚓ L I R
♫ | p3 2 1 p3 1 p7 | 1 , |

The melody scale is the minor family scale with passing notes, as follows. Much of the harmony is from this scale, with a few ornamental elements that fit the flow (e.g., **L-I**, **I-M**).

	Eb		F	Gb		Ab		Bb	B	C	Db	D	Eb
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
	@	P	O	R	M	I	L	$	x	x	x	x	@
//OR.I+	@	.	x	x	.	x	.	$	x	x	+	x	@
ornamental		+			+		+						

Here follows an optional, 8-bar introduction (or conclusion) that's visibly based on whole-tone intervals and tritones in a highly structured way. Each 2-bar segment repeats the previous one a whole tone down. The harmony tritones are in the reverse alphabet order **LIMROP**. The final 2-bar segment is a rhythmic pattern with no specific melody notes. The up arrows say go up an octave.

⚓ **L** **I**

♫ **| — 7 1 2 3 p5 7 p5 3 7 p5 3 | 2 — 1 5 p3 ↑4 |**

^ ^ ^ ^ ^ ^ ^ ^

⚓ **M** **R**

♫ **| — 6 p7 1 2 3 6 3 2 6 3 2 | 1 — p7 4 p2 ↑ p3 |**

^ ^ ^ ^ ^ ^ ^ ^

⚓ **O** **P**

♫ **| — 5 p6 p7 1 2 5 2 1 5 2 1 | p7 — p6 p3 7 ↑ p2 |**

^ ^ ^ ^ ^ ^ ^ ^

⚓ **M.L** **I.P**

♫ **| ——— | ——— |**

^* **^***

ALL OF ME

I learned this version of *All of Me* (along with *No Greater Love* coming up) in Susan Muscarella's piano comping course some years ago at the then Jazz School in Berkeley (now the Jazz Institute). The objective was learning about 4-note "open" voicings of extended and altered seventh chords often used to accompany soloists. The home tonic is C and the key signature empty, nominally identifying the Ionian mode, but the melody line uses all but two notes of the chromatic scale and the harmony uses all of them. Open voicings of the harmony chords rearrange the notes and sometimes omit the roots. The standard way of explaining such voicings is by rearranged degree numbers of chord scales, in which the same notes from adjacent chords have different degree numbers relative to different roots, which are sometimes omitted. Such representations are indirect relative to the keyboard, difficult to comprehend as a whole or remember, and too cumbersome to annotate routinely on the written music. This complexity goes away when the shapes are understood in terms of building blocks.

The following summary shows the melody line with 3-note shapes to voice all chords (except double tritones). The melody line is from the mode **//O.MI⁺**, with two appearances of minor note **p3** as ornamental passing notes. Harmony tritones **P** and **L** are ornamental substitutes for **O** and **I**. The 3-note shapes are incomplete versions (top notes omitted size) the 4-note open voicings shown on the next two pages. Grey boxes for voicing intervals of of uncommitted size (■) are replaced by circled numbers ④**,** ③ and ② that commit to sizes of 4, 3 or 2 half tones (more compact than **xxxx**, **xxx** and **xx**). Many of the voicings sound interesting; some are easy-to-play "all fourths" shapes (counting tritones as augmented fourths). Play octave shapes based on the anchors to begin with (except for double tritones) and then shrink them into the shapes shown. The latter are easy to expand into the 4-note open voicings by adding one obvious note. Rhythm is implied swing-feel 4/4 time.

All of Me (original home tonic = C)

⚓ **M■ I■ M■ O■ MO P■ P④ @④ P④**

♫ **| 1 5 3, — | — 1 2 1 | 7 p6 3, | — | 6 5 3, | — p3 3 p7 6 | 5 4, — | —— |**

⚓ **O② OI @■ OI @■ $④ L④ I④ I■ I④**

♫ **| 3 p3 2 , | — 3 p6 7 | 2 1, — | — | 7 p7 6, | — 6 2 7 | 6 7, | —— |**

repeat first 8 bars

⚓ **L■ L④ O② P■ P④ I④ I■ I④ M■ P④ $④ I④**

♫ **| 2 1 7 2 | — , 1 | 7 3 5 | 7 , 6 | 1 6 , 1 3 | — 3 — 1 | —, — | ——— |**

Open Voicings

Four-note "open" voicings of the written chords are shown next. The **X\Y** notation identifies non-overlapping building blocks **X** on the bottom and **Y** on top. The shapes are voicings of the chords shown on the right. The chord symbols are all satisfied serially in the flow. The four notes are difficult to play with the left hand, but easy-to-play 3-note voicings of the same chords follow from playing the core shown in the foregoing skeleton summary with a note added on top (in other words by knocking off the top note below); the top notes are easily added by the right hand under the melody line, if desired.

Chords on roots I, II and V are from the Ionian mode. The other chords (highlighted in yellow) provide visibly simple segues between the Ionian voicings, when seen in building-block terms.

Bars 1-8

```
                1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
       A.MI+    @ - O - M I - $ x + x x @ - O - M I - $ x + x x @

1    M \ O      . . . . M . . . . x . . . – . A . . . . $ . . . . . .      I-6(9)
     I \ M      . . . . . I . – . . . x . . . . . M . . . . x . . . .      V-9(13)
2    M \ O      . . . . M . . . . x . . . – . A . . . . $ . . . . . .      I-6(9)
3    O \ L      . . O . – . . . x . . . . . x . . . . L . . . . . . .      III-9
4    M \ O      . . . . M . . . . . x . . . . O . . . . . . . . . . .      III-9b5
---------------------------------------------------------------------------
5    P \ L      . . . . . . . x . – . . . P . . – . L . . . . x .          VI-9(13)
6    P \ ▷I     . . . . . . . x . – . . . P . . . . I . . . x . . . .      VI-7b13
7    @ \ M       . . . . . .  x . . . . . @ . – . M . . . . . x . . . .    II-m9
     P \ ▷I     . . . . . . . x . – . . . P . . . . I . . . x . . . .      VI-7b13
8    @ \ M      . . . . . . . x . . . . . @ . – . M . . . . . x . . . .    II-m9
```

Bars 9-16

```
                1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
                @ - O - M I - $ x + x x @ - O - M I - $ x + x x @
9    O \ M      . . . . . . . . x . . . . . O . M . . . . . x . . .        III-7
10   O \ I      . . . . . . . . x . . . . . O . . I . . . . . . x .        III-7b9
11   @ \ M      . . . . . . . x . – . . @ . . . M . . . . . x . . .        VI-m7
     O \ I      . . . . . . . . x . . . . . O . . I . . . . . . x .        III-7b9
12   @ \ M      . . . . . . . x . – . . @ . . . M . . . . . x . . .        VI-m7
---------------------------------------------------------------------------
13   $ \ M      . . . . . . . $ . . . . @ . – . M . . . . . x . . .        II-9sus
14   L \ M      . . . . . . L . . . . . @ . – . M . . . . . x . . .        II-9
15   I \ M      . . . . . I . . . . . . @ . – . M . . . . . x . . .        II-m9
16   I \ M      . . . . . I . – . . . x . . . . . M . . . . . x . . .      V-9
     I \ R      . . . . . I . – . . . x . . . . R . . . . . x . . . . .    V-7b9b13
```

Bar 17-24 - repeat bars 1-8

Bars 25-32

```
                 1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
                 @ -  O -  M I -  $ x  + x  x @ -  O -  M I -  $ x  + x  x @
25  L \ M        . .  . .  . . L  . .  - .  x . .  . .  M .  . . .  x .  . .     VI-6(9)
26  L \ M        . .  . .  . . L  . .  - .  . @ .  . .  M .  . . .  x .  . .     VI-m6(9)
27  O \ M        . .  . .  . . .  $ .  . .  . . .  O .  M .  . . .  x .  . .     III-m9
28  P \ L        . .  . .  . . .  $ .  - .  . . P  . .  . .  L . .  . .  x .     VI-9(13)
    P \ I        . .  . .  . . .  $ .  - .  . . P  . .  . I  . . .  . x  . .     VI-7b9b13
-----------------------------------------------------------------------------
29  I \ M        . .  . .  . I .  . .  . .  . x .  - .  M .  . . .  x .  . .     II-m9
30  I \ M        . .  . .  . I .  - .  . .  x . .  . .  M .  . . .  x .  . .     V-13
    I \ R        . .  . .  . I .  - .  . .  x . .  . R  . .  . . x  . .  . .     V-b9(13)
31  M \ O        . .  . .  M . .  . .  x .  . - .  O .  . .  . x .  . .  . .     I-6(9)
    P \ I        . .  . .  . . .  x .  - .  . . P  . .  . I  . . .  . x  . .     VI-7b9b13
32  $ \ M        . .  . .  . . .  $ .  . .  . @ .  - .  M .  . . .  x .  . .     II-m9(11)
    I \ R        . .  . .  . I .  - .  . .  x . .  . R  . .  . . x  . .  . .     V-7b9b13
```

Understanding how the shapes satisfy the chord symbols requires knowing the relationship between the numeric suffixes of the chord symbols and the chromatic scale. I explain this only to make clear that the shapes satisfy the written chords, not to suggest figuring the shapes out this way. In bars 3-4, chord suffix "9" is chromatic scale note **p5** and chord suffix "b5" is chromatic scale note **p7**. The 3-note voicings in bars 3-4 don't include **p5** but it's included in bar 5, thus satisfying the bar-4 chord in the flow. The bar-4 shape jumps up a minor third in bar 5, with a wiggle that puts a fourth on top instead of a major third. The wiggle provides degree "13" of the chord. It also provides degree "9" of the bar-4 chord, thus satisfying two chord symbols at one stroke.

NO GREATER LOVE

I learned this piece in the same piano comping course as *All of Me*. It uses the same major family mode but is interestingly different.

No Greater Love (original home tonic = Bb)

♫ 7 1 2

⚓ M■ R■ O④ P I■ M■ I■ I■

♫ | 1 7 6 3 | 5 p5 4 1 | 3 | — , 3 7 p7 | 7 | — , 6 3 p3 | 2 | — , 7 1 2 |

⚓ L④ I④ I■ M■ R■

♫ | ———————repeat bars 1-4 ——————— | 6* 3 2 | 1 2 | 1, | —— |

⚓ OI $③ IO $③ IO $④ L④ I③

♫ | 3 p6 7 2 | 1, | p6 7 2 4 | 3, | 3 p6 7 2 | 1 7 6 3 | 2 — | — , 7 1 2 |

⚓ | I■

♫ | ———————————repeat bars 9-11——————————| —— |

Here's the same thing with a walking bass line provided in class (blue text).

♫ 7 1 2

♫ | 1 7 6 3, | 5 p5 4 1 | 3, | — 3 7 p7 | 7, | — 6 3 p3 | 2, | — 7 1 2 |

⚓ 1 5 1 3 4 1 4 7 p7 4 p7 4 6 3 6 p3 2 6 2 p3 2 6 2 p2 2 6 2 p6 5 2 5 p2

♫ | ———————repeat bars 1-4 ——————— | 6* 3 2 | 1 2 | 1, | —— |

2 6 2 p2 2 p6 5 7 1 3 4 p2 5 7 1 p6

♫ | 3 p6 7 2 | 1, | p6 7 2 4 | 3, | 3 p6 7 2 | 1 7 6 3 | 2 , — | — 7 1 2 |

7 4 3 p6 6 p6 6 p7 7 4 3 p7 6 3 6 p7 7 4 3 p6 6 p6 6 p3 2 2 6 p7 5 7 2 7

♫ | ———————————repeat bars 9-11——————————| —— |

1 5 1 1

Here follows a development of the bass line for bars 1-4. The line outlines a building block and then transitions to the next one via an internal or external note. The only passing notes outside the major family scale are **p2** and **p3** at the ends of bars.

```
         1 p2 2 p3 3  4 p5 5 p6 6 p7 7  1
         @  P O  R M  I  L $  x x  x x  @
//O.MI+  @  . x  . x  x  . @  x x  x x  @
         x  . .  . x  .  . x  . .  . .  .
         .  . .  . .  x  . .  . .  . x  x

         .  . .  . .  x  . .  . .  x .  .
         .  . .  x x  .  . .  . x  . .  .

         .  . x  x .  .  . .  . x  . .  .
         .  x x  . .  .  . .  . x  . .  .

         .  . x  . .  .  . .  x x  . .  .
         .  x x  . .  .  . x  . .  . .  .
```

BODY AND SOUL

This piece is strongly chromatic in a way that's particularly complex in music notation, namely multiple key signatures of different kinds (sharp vs. flat). My source for the written music is *The Ultimate Jazz Fakebook*, Hal-Leonard (1988), p67. I worked out the details before I heard the most astounding jazz performance of this piece I have ever heard, by Esperanza Spaulding on the CD *Ezperanza* in the cd HUCD 3140, 2008 from wwi.headsup.com. Following the changes in this performance is amazing and inspiring. Three successive sections have three different key signatures: A 5 flats (Db-Ionian); B 2 sharps (D-Ionian); C 1 flat (D-Aeolian, but actually Dorian due to a natural in the written melody line). Ionian and Dorian of tonic D are nearby parallel modes (only 2 notes different). Ionian of Db is a distant from these (5 notes different from B). The obvious home tonic is Db because A starts and ends the piece. The large distance between A and B is more than compensated by the simplicity of every note of B being a half tone up from every note of A.

Body and Soul (original home tonic = Ab)

A ┌ //I (Ionian@1)

⚓ O P M M R LR

♫ | — 2 3 2 3 2 | 6* — , | — 5 6 5 6 5 | 2 1 7 5 , |

1______

⚓ O I O M IO I

♫ | — 1 6 4 2 | 3 2 , | — 1 2 3 5, 5 6 p3 | 1 — | 1 — |

B ┌ altL@p2 (Ionian@p2)

2______

⚓ L M L

♫ | — | p2 p3 4 p6* | p2* 4 p6* — p5 | 4* p2 p3 1 p6 , | — |

C ┌ altM@p2 (Dorian@p2)

⚓ M O RL M M R O P PR

♫ | — p2 p3 3 p6* | p3* p7 p2* p6 | 7* p6 p7* p5 | p3 2 p2 6 |

The melody scales are classical modes. The strongly chromatic harmony is not, but is consistent with the melody. When I first encountered this piece, I struggled with trying to understand how this harmony implied the melody scales and finally realized it doesn't.

		Db		Eb		F	Gb		Ab		Bb		C	Db	
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	P	O	R	M	I	L	$	x	x	x	x	@	
A	//I	@	.	x	.	x	x	.	$	.	x	.	x	@	Ionian of tonic 1
B	altL@p2	x	x	.	x	.	x	x	.	x	.	x	.	x	Ionian of tonic p2
C	altM@p2	.	x	.	x	x	.	x	.	x	.	x	x	.	Dorian of tonic p2

EVERY TIME WE SAY GOODBYE

This Cole Porter piece is hauntingly lovely, with major-minor changes that go well with the words about love and loss. The home tonic is Eb and the key signature is 3 flats, identifying the Ionian mode. The melody line sequence **5-4-3-2-1-7-1** at the end confirms this as the resolution mode. Otherwise, much of the of the melody line is in the major family mode **//O.MI**$^+$. This and much of the harmony are strongly chromatic. The changes are often subtle and understated.

Although the main tonality is major, minor or minor-ish segments appear in both melody and harmony. Section (d) is solidly minor. The words in section (f) include the famous phrase “how strange the change from major to minor.” The change may be interpreted as between family modes **//O.MI**$^+$ (grey highlighting) and **//OR.I**$^+$. The melody makes no explicit change of tonality here but could. The sound of a fourth going down a half tone in the harmony conveys this change to the ear in the context.

Every Time We Say Goodbye (original home tonic = Eb)

(a) ⚓ **M I I M PM I I**

♫ **| 3 3 3 — | — 3 3 3 | 3 — — 3 | 4 — 2 1 7 |**

(b) ⚓ **M PM I I M M R A**

♫ **| — 5 5 5 | — 5 5 5 | 5 — — 3 4 5 | p6 — 4 3 2 |**

(c) ⚓ **R R O I R M M**

♫ **| — 1 1 1 | 1 7 — 5 | p7 — 3 4 5 | 6 —— |**

(d) ⚓ **R O O O L L O I**

♫ **| — p6 p6 p6 | p6 5 — 1 2 | p3 p3 — 1 p3 2 | ——— |**

(e) **repeat (a)-(b)**

⌐ //O.MI$^+$ **⌐//OR.I**$^+$

(f) ⚓ **$ RL O I M M R O**

♫ **| — 1 1 2 1 | 1 7 — 5 6 | p7 3 6 2 | 1 7 1 5 4 |**

(g) ⚓ **M M L I M RL OI**

♫ **| 3 3 3 — | — 2 1 7 | 1 —— | ——— |**

The major to minor bars of (f) are shown next. The harmony lead-in to these bars is **RL-OI** down a half tone. The family modes follow from filling in **OI** differently. The chords shown are from these modes.

			1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		
			@	.	O	R	M	I	.	$	x	+	x	x	@		
(f)	**M**	*I-7*	–	.	.	.	M	.	.	.	.	.	x	.	.	|	**//O.MI$^+$**
	M	*IV-M7*	.	.	.	.	M	–	.	.	.	x	.	.	.	|	
	D	*IV-m7*	.	.	.	D	.	–	.	.	x	.	.	.	.	|	**//OR.I$^+$**
	A	*pVII-7*	.	.	A	.	.	.	.	.	x	.	–	.	.	|	

ALL THE THINGS YOU ARE

I learned this beautiful piece early in my musical adventure, and found it easy to learn because the keyboard sequences are simple, but difficult to understand in music notation. The source is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). The home tonic is Ab and the key signature is four flats, identifying the starting and ending melody modes as Ionian. In the following interpretation, the piece visits successive parallel and alt classical modes. The determining tritones of these modes in (a)-(b)-(c)-(d) are **I-R-L-M**. The implied Ionian tonics are **1-3-5-7** but this isn't helpful because the passages don't all resolve to them, but transition to the next passage via shared notes. Better to think in terms of the home-tonic modes because they provide everything needed to play the changes.

The skeleton melody line and associated core harmony of this interpretation are shown next for bars 1-26, where all the changes occur.

All The Things You Are (original home tonic = Ab)

(a)//**I** ⚓ **$ I I M M**
♫ **1 | 4 1 | 7* | 7 3 7, | 6*|**

(b)**altD** ⚓ **R R**
♫ **| 6 p3 6 | p6, | — |**

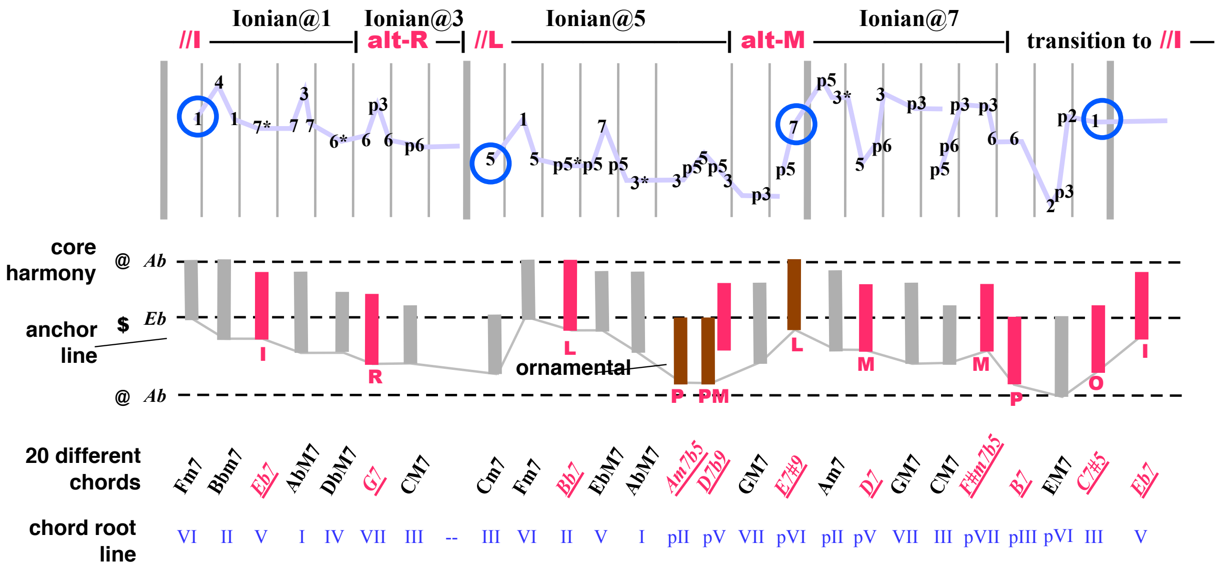
(c) //**L** ⚓ **O $ L L M P PM**
♫ **| 5 | 1 5, | p5* | p5 7 p5, | 3* | 3 p5 5 p5 3**

(d)**altM** ⚓ **L M M R R**
♫ **| p3, p5 7 p5 | p5 3*, | — 5 p6 3 | p3, | — |**

⚓ **M**
♫ **| — p5 7 p3 | p3 p2*, |**

(e)trans. ⚓ **P @ O … I**
♫ **| — 2 p3 p2 | 1 | — |**

The following overview picture is helpful in getting a handle on the piece as a whole.



Here follows a summary of the modes of this interpretation ("notes" in the left column means actual melody notes). The sometimes-sparse melody lines are open to different interpretations but this interpretation provides a useful handle on the piece.

		Ab		Bb		C		D	Eb		F		G	Ab	
		1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**	
		@	P	O	R	M	I	L	$	x	x	x	x	@	
(a)	notes	x	.	.	.	x	x	.	.	.	x	.	x	x	
	//I	x	.	x	.	x	x	.	x	.	x	.	x	x	Ionian@**1** (Ab)
(b)	notes	.	.	.	x	.	.	.	.	x	x	.	.	.	
	altR	.	x	.	x	x	.	x	.	x	x	.	x	.	Ionian@**3** (C)
(c)	notes	x	.	.	.	x	.	x	x	.	.	.	x	x	
	//L	x	.	x	.	x	.	x	x	.	x	.	x	x	Ionian@**5** (Eb)
(d)	notes	.	x	.	x	x	.	x	.	x	.	x	x	.	
	altM	.	x	.	x	x	.	x	.	x	.	x	x	.	Ionian@**7** (G)
(e)	notes	x	x	x	x	.	.	.	.	.	.	.	.	x	sparse, ambiguous
(f)	**//I**	x	.	x	.	x	x	.	x	.	x	.	x	x	Ionian@**1** (Ab)

The (d)-(e)-(f) harmony transition shown next is "interesting." My piano teacher at the time suggested I just memorize it. And so I did, but I kept worrying away at understanding it in more fundamental terms. This shows that it shares notes between modes, resulting in an unusual sequence of chord symbols.

		Ab		Bb		C		D	Eb		F	Gb		Ab	
		1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**	**1**	
		@	P	O	R	M	I	L	$	x	x	x	x	@	
(d)	③M	.	x	.	.	M	.	.	.	.	.	x	.	.	**pVII*m7b5***
(e)	P ■	.	P	.	–	.	.	.	x	.	.	.	.	x	**pIII*7***
	@ ■	@	.	.	.	.	.	.	x	–	.	.	.	x	**pVI*M7***
	O④	.	.	O	.	–	.	.	.	x	.	.	.	x	**III*7#5***
(f)	③I	.	.	x	.	.	I	.	–	.	.	.	x	.	**V*7***

CHELSEA BRIDGE

My source for this hauntingly beautiful Strayhorn piece is *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988). There are two written key signatures but only one home tonic on the piano, the black key immediately above middle C. The piece moves from Ionian major of this tonic (Db major, 5 flats) to Aeolian minor of the same tonic (C# minor, 4 sharps), and then back to Ionian major. This is complex to begin with and the complexity is multiplied by staying with the 4 sharps key signature to return to Ionian major. Between these parallel classical modes of the same tonic are chromatic passages that multiply the notational complexity once more. The result is a confusing muddle of many more than twelve note symbols relative to the two written key signatures, with tricky timing in some places. The piece is musically sophisticated by any measure, but the notational complexity is out of proportion to the sophistication.

I first learned this piece by rote with great difficulty from the written music, but thought that anything that sounds this good must have simple musical logic behind it. Tritones identify scales; fifos implied by them are mostly omitted. Digging this kind of thing out of written music is a struggle, but doing it sharpens the ability to think in terms of keyboard intervals instead of the notes of key signatures. It may look complex at first glance, but it's valid for any home tonic, and shines a light on commonly used patterns, which are obscured in music notation by showing the same piano key in different places by sharps or flats or naturals in spaces or lines of a musical staff.

Chelsea Bridge (original home tonic = Db/C#)

(a) ♫ | — 6 7 1 2 3 4 5 | ... //I
^ ^ ^ ^

(b) @ L @ L ... play aug. fifths
⚓ L.O M.L L.O M.L P ... major modes
♫ ||: p6 ——, 6 p5 | ——, 5 p6 | — , 6 p5 — 5 p2 | —, | ... major with ⚓
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

(c) 1________ 2________
⚓ I I M M R O
♫ | 3 1 —— | 3 5 —— | 3 1 —— , :|| ——, 5 p6 | ... 1: maj. triad, 2: segue
^ ^^^ ^ ^^^ ^ ^^^ ^^^ ^

(d) ⚓ O A
♫ | p7* p7 p7* | p7 1 p3 5 p7 2, — p3 | 7 p6 p3 1 p7 5 —, 2 | ... //A
^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^

(e) ⚓ P I
♫ | p2 — p3 4 | 5 — p6 p3 p2 7 | p7 p2 ——, 7 p7 | ... minor mode **alt-IP**
^ ^ ^^ ^ ^ ^ ^ ^ ^ ^^^ ^

(f) ♫ 6 p5 6 p2 p5 p2 ... segue
^ ^ ^ ^

(g) ⚓ M R O P ... sets up for a repeat
♫ | p5, 6 7 1 2 3 4 5 | ... //I
^ ^ ^ ^

Here are the melody modes and transitions between them. The main sequence is **//I-//O-//I**. The in-between modes and segues are related to them in simple ways. The changes are musically interesting but easy to understand and play as shown here, once you “get it.” I can imagine that virtuoso pianists are able to read the written music and understand these changes instinctively in whatever terms they use to conceptualize music, but this leaves everyone else stuck with learning the piece laboriously by rote, without any real understanding. Knowing the changes below enables even inexpert pianists to approach the piece with understanding.

```
          1 p2 2 p3 3  4 p5 5 p6 6 p7 7 1
          @ P  O R  M  I L  $ x  x x  x @
(a)       x .  x .  x  x .  x .  x .  x x   //I    Ionian
(b)       x .  x .  x  . x  . x  x .  x x   //LO   major mode
          @ .  . .  .  . .  . x  . .  . .          imply by aug. fifth @▷
          x .  x .  x  . x  x .  x x  . x   //ML   major  mode
   x . .  . .  . .  .  L .  . .  . .  . .          imply by aug. fifth ◁L
(c)       x .  . .  x  . .  x .  . .  . x   //I    major triad
——————————————— minor of home tonic ———————————————
          @ .  . .  .  . .  . x  . .  . x          segue to minor via aug. fifth @▷
(d)       x .  x x  .  x .  x x  . x  . x   //O    Aeolian
(e)       . x  . x  .  x .  x x  . x  x .   alt-IP Phrygian flat one (minor)
(f)       . x  . .  .  . x  . x  . .  . .          ambiguous but suggests Locrian next
          x x  . x  .  x x  . x  . x  . x   alt-L  Locrian goes down 1/2 tone to Ionian
——————————————— major of home tonic ———————————————
(g)       x .  x .  x  x .  x .  x .  x x   //I    Ionian
```

LUSH LIFE

This beautiful Strayhorn piece is melodically and harmonically rich, and challenging to play from the written music because there are often two or more chords per bar, many of them chromatic relative to the written key signature of five flats (major tonic Db). The piece is so chromatic that there is more than one way of understanding it. It can be seen as daisy-chained, short segments of successive parallel classical modes, but this quickly bogs down in complexity. The view of it here is in terms of two non-classical parallel modes of the home tonic for both melody and harmony: one is the 10-note **//ORMI** minor-major mode in [A] and [C]; the other is the tonic-less **//IP** mode (Ionian#*1*) in [B].

Lush Life (original home tonic = Db)

[A] ⚓ @ O M O M O M I O R

♫ 5 ||: 1* 2* | 3* 2* | 3* 2* | 3* 4* 5* p6*|

1 2

⚓ I I O I M M R

♫ | p7*— p6 p7 p6 | 5* p7 p3*| 2*, — 5 :|| 2*, — 7 |

[B] ⚓ I P O P RL R P R P RL

♫ | 7* p2* | 3* 5* | 7 p2 6 7*| 7 — 7 | 7* p2* | 3* 5* | 7 1 6 7*

⚓ I M MP RL OI

♫ | 6 7 5 6 7 3 | — | —, 5 4 3 | 2 | —, 2* 3 1 | 5 —, | — |

[C](a) ⚓ ▹M I ▹M I R O M L I

♫ | 5 p7 p3 p6 | 5 7 p3 p7 | 6 5 1 3 4 5 | p7 — p6 |

(b) ⚓ M R P O

♫ | repeat bars 1-2 of (a) | 6 5 1 3 5 p6 | 7 — 1 |

(c) ⚓ O L RL M O P L I

♫ | 2 3 4* — p3 | 2 3 4* p3 | p6 p7 7* — p7 | p6 p7 7 6|

(d) ⚓ RI OI P

♫ | repeat bars again | 6 5 1 3 4 5 | 6 — 3 |

(e) ⚓ O L I RL P

♫ | 5 4 5* 4 | 3 2 3 7 | 2 1 2 — 2 1 | 1 p7 6 — 3 |

(f) ⚓ O L I O R M M O R M

♫ | 5 4 5* 4 | 3* 2 3 | p7 7 1 p2 | — 2 p3 3 |

These modes lead naturally to short, easy-to-remember segments in classical and other modes that follow from the flow. For example, **alt-IP** morphs into **//I** halfway through [B] by altering one note.

From time to time, the harmony of the different sections substitutes **P** and **L** for **O** and **I**, but thinking of these as ornamental relative to the **//ORMI** keeps things conceptually simple.

Harmony fifos are mostly omitted for simplicity, except for characteristic sequences of them in [A] and the ending bars of [C].

The double tritones **PM**, **OI** and **RL** that appear in many places in the harmony are diminished seventh chords, commonly used ornamentally in any context in which they sound good. The double tritones have circular symmetry (all intervals are minor thirds), so only three of them with different notes exist. They can be played as four notes but they can also be voiced with three notes (a tritone with a minor third on top, or the opposite) or even with two notes (outer notes a major sixth apart), when the context implies the other note(s). A sequence of two or three of them, often used as an ornamental transition between modes, can be played as as sequence of major sixths anchored by any three of six tritone anchors (e.g., **P-O-R** going up and **L-I-M** going down, to name just two possibilities — others require inversions of individual tritones).

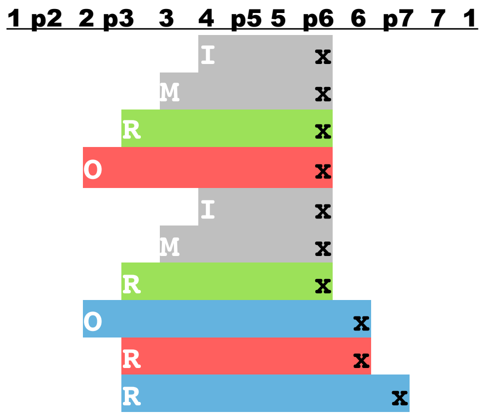
The chromatic melody-line run in the final two bars is a signature of the piece.

THE PEACOCKS

This hauntingly beautiful piece by Jimmy Rowles wraps up the chapter with an example in annotated music notation (in handwritten annotations, circling tritone anchors, or their chords of origin, or both is helpful). This is a Sibelius score created from a borrowed fake book to which I no longer have a reference. The annotations use the original **PADMIL** alphabet, requiring **AD** to be understood as **OR**. The piece sounds so “right” as written that jazz improvisations rarely stray far from it. The home tonic is determined to be F by the final notes of bars 9 and 17. In bars 1-9, the combination of the 5 flats key signature, the home tonic and the lack of accidentals in the melody line identifies the melody scale as Phrygian of the home tonic (mode signature **//P**). The melody line in bar 8 runs up through all the notes of this scale starting a fourth up (fifth down) from the tonic. The contrasting harmony is chromatic. The over-lines in bars 2 and 4 indicate held treble notes.

The ups and downs of the melody line are directly visible in the notes on a staff but the drawback is this commits to a particular tonic and key signature, and gives little direct sense of context.

The core harmony in opening bars is visibly from the **//ORMI** scale (which is minor-major relative to the home tonic).



The melody of the second section consists mainly of individually simple sequences from the full chromatic scale that are prompted by the harmony. These are better remembered in terms of simple patterns shown by **x** entries in a mode table than from note sequences of any kind.

The two groups of four sixteenth notes in bars 11-15 follow a repeated pattern of down a minor third, down a half tone, and up a minor third (net down a half tone).

```
               1 p2 2 p3 3  4 p5 5 p6 6 p7 7  1
               @  P O  R M  I  L $  x x  x x  @
bar 11         .  . .  x .  .  x .  . .  . .  .    down
               .  . x  . .  x  . .  . .  . .  .    up
               .  x .  . x  .  . .  . .  . .  .    down
               x  . .  x .  .  . .  . .  . .  .    up
bar 13         same, up a whole tone
bar 15         same, up a minor third
```

The eighth note triplets of bar 16-17 follow a repeated pattern of down a minor seventh and up a half tone, (net down a major sixth).

CHAPTER 5: OBSERVATIONS & CONCLUSIONS

Music notation is here to stay and must be lived with, but this this doesn't preclude alternate views of piano music from a different perspective. I looked for an alternate view of harmony and found one based on intervals.

The central role of tritones in PKP tends to seem inappropriate to people savvy in music notation, for several reasons. In general, it goes against received musical wisdom. More specifically, tritones cannot even be written down in music notation until a key signature is established, so imagining them as identifiers of scales is difficult. Anything involving multiple tritones is complex in music notation because it implicitly brings in other key signatures — the more tritones, the more complexity. The very concept of fifos is unconventional and even more so is the concept that they can be treated as morphed tritones, understood from context. The payoff is insight into deep structure that is difficult to see in music notation. Appreciating these things requires stepping outside the box of music notation, at least momentarily.

Thinking in notes and thinking in intervals *is* different in kind. Many experts who have learned the hard way to think in notes apparently find thinking in intervals too alien to contemplate. Comments from them have run the gamut from "obviously wrong" to "wrong headed" to "naively simplistic" to "overwhelmingly complex." One expert said, dismissively, I had "found a tritone hammer and saw everything as a nail." The book *Modalogy*, devotes 50 or so complex pages to parallel modes without ever mentioning the possibility that tritones might simplify things.

Because I started out seeing intervals and notes as dual views of the same thing, I came to find it natural to switch back and forth. I have made my best effort in this document to convey the simplicity of this. I believe, based on my personal experience as an adult beginner, that any beginner could easily learn to think this way.

I know from personal experience that PKP helps with learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. It turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

The jumpy chord root lines in many of the examples demonstrate that referring chords to constantly moving roots creates misleading complexity. This complexity is both conceptual (the underlying, smoothly flowing core harmony is obscured) and actual (playing the jumpy harmony is technically difficult because all the fingers must be lifted and moved by large intervals). It's easier to learn the core harmony and introduce the jumps later, if desired, via inversions, which are always easy on the keyboard, given an anchor line.

GENERAL BENEFITS

PKP annotations above the staff tell you many things. Combined with context provided by the melody line, they tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not right at any point by identifying tonic scales in play in a very immediate way. They tell you about transitions between major and minor tonality and between different tonics. The provide a basis for enrichment and improvisation.

Scale, modes and chords are seen as part of a continuum expressed in the same terms. This seems to me to be a very powerful benefit, compared to seeing everything beyond classical modes as special cases identified by accidentals (which includes chromatic chords with suffixes that imply accidentals). The representations of scales and chords in terms of tritone clusters are unfamiliar to expert pianists but the interval stacks that they imply on the keyboard are completely familiar.

With PKP, playing music is guided by note-based symbology on the music page (one dual view) plus PKP annotations above the staff (the other dual view) that suggest core harmony and make changing tonic scales and tonics directly visible to the eye in terms of changing alphabet words.

A simple mode hierarchy covers much ground. The mode signatures provide a novel, high-level, conceptual view of tonic scales that's easier to hold in the mind's eye than the very much larger number of spelled out tonic scales in music notation. The modes cover 12x39=468 scales of 12 different tonics in music notation (more than is covered by scale dictionaries such as *The Source*). Words of 1-4 letters from the 6-letter alphabet identify, by tritone content, modes with 7 or more notes, 1-4 tritones, and no intervals larger than a minor third. The count of 39 modes includes 11 single modes covered by single words and 4x7=28 parallel modes covered by transpositions of 1-2 letter master words. A selected home tonic anchors all of this to the keyboard.

The words are analogous to the words of biological DNA in the sense that they identify deep structure without spelling out the details. Words of biological DNA are "expressed" as proteins, the building blocks of life. **PORMIL** words are "expressed" as building blocks of scales and harmony. Biological DNA is "extracted" from biological samples and interpreted by sophisticated machines called "sequencers." **PORMIL** scales and building blocks are "extracted" and "sequenced" from musical lines by sophisticated machines called human eyes. Biological DNA is "inherited" from parents. **PORMIL** words are "inherited" from general knowledge about them encoded in the hierarchy.

Symmetry breaking in the scale hierarchy and symmetry breaking in core harmony are two sides of the same coin. In the hierarchy, it yields scales with fewer tritones. In harmony, it releases the tension of dissonance to produce consonance. Seeing the simplicity of this requires forming mental images of it relative to the schematic tonic octave that are independent of specific assignments of the octave to the keyboard.

Understanding harmony is possible independently of practicing. Keyboard symmetries centering around tritones enable high-level representations of harmony that are simple in their own terms and deep in harmonic meaning in any terms.

Music is freed from its key-signature straightjacket. Harmony scales are understood at a glance from alphabet words aggregated by eye, instead of having to be figured out or remembered from key signatures, accidentals and chromatic chord symbols. Tonic changes are no more difficult than any other changes.

Anchor lines that define harmonic cores replace chord root lines as the fundamental musical lines of harmony. Chord roots are visibly secondary elements in the flow of the music because inversions move them into the body of the chord, where they become no more than another chord note that may be omitted if implied by context.

Sight reading anchor lines provides deep context. Anchor lines tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not by identifying tonic scales in play. They tell you about transitions between major and minor tonality and between different tonics.

Multiple explicit or implicit key signatures become a non-issue. This is true whether the key signatures imply tonic changes or only scale changes for the same tonic. Tonic changes amount to no more than moving (conceptually) a tonic pointer within the home octave and reinterpreting the alphabet letters relative to it, without changing any notation.

Chord symbols become a non-issue. Chord complexities that result from banging square pegs (chord symbols) into round holes (places in scales where they don't quite fit) are misleading. Chords described by chord symbols are, on the keyboard, almost universally composed of combinations of PKP's two kinds of building blocks, of which only tritones are always core. A melody line plus an anchor line above the staff imply a voicing of an unidentified chord progression. Specific chords are identified by assigning roots.

Relating the notation to the keyboard is simple. Different home octaves look very different on the keyboard because of different mixes of black and white piano keys, but the difference is manageable for two reasons: mapping the notation to the keyboard boils down to assigning a 6-letter alphabet to 6 adjacent piano keys; and the C octave provides a mental reminder of the meaning.

Learning blues prepares you for anything music notation can throw at you. Blues is an example of "folk music" that evolved independently of music notation and therefore tends to be regarded as naive when measured against its depth and complexity. This condescending view is unwarranted. Blues is harmonically sophisticated in a highly organized way that makes it, to my mind, a better starting point than classical modes of key signatures for understanding many forms of music.

Sharps and flats are not needed. Building blocks are the same number of half tones no matter how they are notated. This enables PKP to dispense with sharps and flats without any loss of musical accuracy. Expert pianists tend to think they are still needed to identify context because they have learned to use them for that, but note positions relative to tonics are sufficient.

The simple flow of harmony in harmonic cores, in which building blocks slide or morph into other building blocks, is universal. Nothing constrains harmony to be played this way but the flow is so natural and easy to play that much is to be gained by learning pieces this way and then transitioning to other playing styles if desired.

Tritone-intensive harmonic cores of much music provide so much harmonic variety that voicing them requires only adding depth. Depth is easily added by doubling the treble line of the harmonic core an octave down, or the bass line an octave up, and then making adjustments to move points on the line closer to the core, if desired.

No more piano wizardry is required to have fun playing music than the ability to sight-read melody lines in music notation. Anyone with eyes to see intervals on the keyboard can play harmonic cores from anchor lines annotated above the staff, and remember them.

The simple way harmony is represented provides great freedom in choosing how to play it. Playing styles may range between the extremes of smoothly flowing harmony within the octave of the home tonic of piece to jumpy harmony in which all harmony notes go up from a chord root line. Improvisation follows naturally and is fun, even if not expertly done.

The simplification provided by the alphabet is different in kind from simplifying chord symbols. Any sophistication in the harmony is in the tonic scales of origin of the chords.

Mode signatures provide an above-the-staff markup notation for the concept of "modal jazz" which is otherwise a concept without a notation.

PKP annotations above the staff are not always necessary. With experience, straightforward chord progressions may be sight-read in PKP terms.

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2. Mehegan, *Jazz Improvisation 1: Tonal and Rhythmic Principles*, Watson-Guptil (1984), for Roman-numeral chord notation, and for teaching me (unintentionally) that it is not a solution for complex chromatic chord progressions, but a problem if pushed beyond its basic function of specifying chord root lines.
3. Eskelin, *Lies My Music Teacher Told Me*, Stage Three Publishing (1994) for insight into the nature of scales and musical "perfection," and for encouraging me to think outside the box.
4. Dmitri Tymoczko, *A Geometry of Music* (2011) for stimulating discussions of how to think about music from different angles.
5. Mark Levine, *The Jazz Theory Book*, Sher Music Co. (1995) for providing examples of well known jazz scales and harmonic forms in conventional notation, against which to verify PKP coverage.
6. George Russell, *The Lydian Chromatic Concept of Tonal Organization*, http://www.georgerussell.com/lc.html, for making me aware that PKP covers the concept, because nothing is changed by replacing the Ionian mode by the Lydian mode as the default reference major mode for any piece of music.
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9. Ross W. Duffin, *How Equal Temperament Tuning Ruined Harmony (and Why You Should Care)*, W.W. Norton (2007), for an understanding of the piano's deviation from musical perfection.
10. Stephon Alexandar, *The Jazz of Physics: The Secret Link Between Music and The Structure of the Universe*, Basic Books (2016). This amazing book resonates strongly with my own ideas. The author is a jazz musician and physicist who speaks of doing physics as informing the playing of jazz and playing jazz as informing the doing of physics. Both require rising above formalisms to form conceptual representations, with symmetries highlighted as particularly important.

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My piano teacher in San Francisco for several years, Ken Fishler, provided inspiration and how-to information on chord voicings, while patiently tolerating and responding to my question-everything approach. My first piano teacher in Ottawa, Canada, Sally Robinson, started me off right as an adult beginner by helping me to learn favorite harmonically sophisticated pieces by following her fingers on the keyboard without understanding anything about what I was doing except that it sounded right — being able to play these pieces, even if clumsily and by rote, enabled understanding to seep in gradually. I think that without this particular way of starting out — fingers on the keyboard first, written music later — I would not have started thinking about keyboard harmony in the way I did. It forced to my attention the large gap between simplicity on the keyboard and complexity of the full notation that represents it.

I wore out the patience of many music professionals by bombarding them with unsolicited email requests for comments on my ideas. I would like to thank four in particular, who politely responded to numerous emails in spite of being uncomfortable with my ideas: Harry Likas, Dmitri Tymoczko, Jeff Brent and Daniel Glover. Their criticisms helped me change my explanations without changing my mind.

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My wife Sheila's accurate musical ear helps me know what does and does not sound good on the piano, in addition to her making my life generally interesting. My grandsons Joshua and Ethan Feiber provided encouragement and comments; Joshua set up the website www.pianotheoryman.com as birthday gift; Ethan, who learned very young to play the piano impressively well by ear, has experimented with the notation and found it helpful for approaching written music.

SOME COMMENTS FROM READERS

The following comments provide a kind of history of the development of PKP: the dates on the left identify when the commenters read different drafts of this material; the material has evolved considerably since then. The unconventionality of putting tritones on center stage has tended to get in the way of expert musicians accepting the ideas at all, let alone seeing them as simple, which has not been helped by the fact that it took me a long time to find the simple way of explaining the ideas that appears in this document. I continue to think the ideas are important for learning the piano in a less restrictive manner than is conventional but have not yet been able to find an audience. At the very least, the more recent comments show that the PKP method is more than just a fantasy of an over-enthusiastic amateur.

(2016) Taylor Eigsti more recently than below. "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

(2011) Paul Steinbeck. Assistant Professor of Music Theory; Washington University, St. Louis

"The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music."

(2009) Robert Rawlins. University Music Department Chair (Rowan University); jazz musician; teacher; author of several books on jazz

"I became aware of Raymond Buhr's novel method for analyzing and voicing chromatic chord progressions in 2008 through a draft of a paper he wrote on the subject. I have kept up to date on developments of the method and we have had many email exchanges discussing issues of interpretation and application. I am a member of his intended target audience—a jazz musician who is not a pianist who needs to work out harmonic patterns on the piano from time to time. I am also a music teacher who has actually tried out aspects of his method on students. I can vouch from personal experience for his method's helpfulness in dealing with complex chromatic chord progressions."

(2009) Susan Brand. Board Chair, The Jazz School in Berkeley; amateur pianist

"When Raymond Buhr consulted me about his theory of chromatic chord progressions, I was immediately struck by his ability to analyze and attempt to simplify this complex subject. Mr. Buhr brings a unique perspective and a great deal of enthusiasm, depth of understanding and originality of viewpoint to the subject. Over the years I have watched the continuous work that he has put into editing and rethinking his work. He has had ongoing consultations with many knowledgeable musicians/ teachers/editors and all have contributed to the development of the method described in this book. His ideas offer a way of understanding musical theory that will add greatly to the field and will be extremely helpful to musicians and music educators."

(2008) Taylor Eigsti. Jazz pianist, composer, teacher, former Artist in Residence at The Jazz School in Berkeley

"Through the brilliant lens of an engineer, Raymond Buhr has laid out an analysis of harmony that is a unique and complex look at the right-brain from the left-brain's perspective."

APPENDIX A : UNCONVENTIONAL ELEMENTS

TERMINOLOGY & NOTATION

- **anchor:** identifies a building block by the position of its bottom end relative to the home tonic
- **anchor set:** set of tritone and fifo anchors that define a scale by construction
- **anchor line:** anchor sequence written above the staff
- **alphabet: PORMIL** identifies anchors by the first letters of the names of classical modes (internal letters in the case of **O** and **R**)
- **building block:** tritones or fifos (fifths or fourths) anchored at alphabet positions (size distinctions determined by color coding: red for tritones, blue for fifths, green for fourths).
    - words determine combinations, with secondary letters parenthesized (the others are **core**)
    - superscripts ▼(minor), ▲(major), ● (dim) identify building-block splits that provide triads
    - attachments ■, ■, and ■ identify octave completion building blocks that form octave **stacks**
    - attachment ■ identifies an interval smaller than a building block to be determined from context
    - attachment ▷ or ◁ identifies an expanded or shrunk building block
- **chromatic scale of the home octave: 1-p2-2-p3-3-4-p5-5-p6-6-p7-7-1**
    - **chord roots use RN symbols**
- **context:** provided by mode signatures for melody and harmony plus flow of both
- **core:** set of octave shapes morphed asymmetrically by a half tone from a tritone shape
- **family:** a set of sub-scales of a family scale defined by a single master mode
- **fifo:** fifth or fourth that are opposite inversions (add up to an octave)
- **flow:** formed by morphs and slides of building blocks
- **frame:** defined by the tonic **@** and pitch center **$** of a tonic octave
- **morph:** change in the size of a building block by a half tone at one or both ends
- **mode signature:** word prefixed by **//** or **alt**
    - plus superscript identifies a family mode with an implied extra note
    - a transposed mode signature transposes a mode, e.g, **tr[//I]@6**
- **outside:** not in a given tonic scale, as distinct from "chromatic" meaning not in a key-signature scale
- **pattern:** organized arrangement of intervals on the keyboard or over time
- **pitch center:** note identified by a fifth/fourth octave split, symbolized by **$**
- **shape:** an object on the keyboard determined by split or combined **building blocks**
- **slide:** size-preserving movement of a building block
    - **wobbly slide:** combined morph and slide
- **phlat:** prefix **p** identifying chromatic-scale notes in the whole tone gaps of the major scale of a tonic
- **tonic pointer:** symbol of form **@t** indicating a secondary tonic
- **word:** set of alphabet letters
    - optional dots indicate skipped letters
    - underlining indicates inversions of building blocks
    - parentheses indicate non-core building blocks
    - backslashes indicate non-overlapped building blocks

APPENDIX B: ABOUT SCALES

KEY-SIGNATURE SCALES

The following summary of the standard interpretation of the major and natural minor tonic scales of key signatures is a helpful reference. It shows the sharped notes of sharp scales in red and the flatted notes of flat scales in blue. Although there are only five black piano keys, scales with six flats or sharps exist because of the scale-spelling rule that the same letter note cannot appear on both sides of a half-tone interval. For example, note B is C*b* in a 6-flats scale.

Major Ionian and relative minor Aeolian) scales are shown on the same line because they have the same key signature. Parallel minor scales (diagonal lines) have the same tonic, which does not necessarily mean the same note symbol because the same piano key may be represented by different note symbols (e.g., the parallel D*b* major and C# sharp minor scales, which have the same black-key tonic on the piano, have different note symbols for it).

Major and minor scales have a fundamental note in the lower fifth of the scale that's a major or minor third above the tonic (PKP makes this explicit by symbols **3** and **p3** for these note positions).

Ionian tonics going down by fifths

Ionian	rel. Aeolian	key sig.	Ionian scale
C	A	empty	C-D-E-F-G-A-B-C
F	D	1 *b*	F-G-A-B*b*-C-D-E-F
B*b*	G	2 *b*	B*b*-C-D-E*b*-F-G-A-B*b*
E*b*	C	3 *b*	E*b*-F-G-A*b*-B*b*-C-D-E*b*
A*b*	F	4 *b*	A*b*-B*b*-C-D*b*-E*b*-F-G-A*b*
D*b*	B*b*	5 *b*	D*b*-E*b*-F-G*b*-A*b*-B*b*-C-D*b*
G*b*	E*b*	6 *b*	G*b*-A*b*-B*b*-C*b*-D*b*-E*b*-F-G*b*
F#	D#	6 #	F#-G#-A#-B-C#-D#-E#-F#
B	G#	5 #	B-C#-D#-E-F#-G#-A#-B
E	C#	4 #	E-F#-G#-A-B-C#-D#-E
A	F#	3 #	A-B-C#-D-E-F#-G#-A
D	B	2 #	D-E-F#-G-A-B-C#-D
G	E	1 #	G-A-B-C-D-E-F#-G
C	A	empty	C-D-E-F-G-A-B-C

switch from flat to sharp keys

parallel (same tonic) Ionian & Aeolian modes
- - - - same tonic symbols
—— different tonic symbols

Avoiding multiple written key signatures by using accidentals to identify implicit key changes replaces one kind of complexity by another because accidentals have many more uses than this (e.g., ornamental passing notes, or identifying scales that originate in no key signature). Sorting out the possibilities requires considerable head-scratching at a very low conceptual level.

Key-signature-based music notation has stood the test of time and is here to stay because of the huge legacy of music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation.

CROSS REFERENCES

The following table cross-referencing music notation and PKP notation. This is the simplest case. More generally, naturals may enter the note-symbol picture to cancel sharps or flats of key signatures, and white piano keys may be represented by sharps or flats (B=Cb, C=B#, F=E#, E=Fb) to satisfy the scale-spelling rule that the same letter note must not appear twice. Experts develop rules of thumb about sharps and flats as indicators of what's going on, and see PKP as discarding these clues — not so, the clues are present in PKP in a different way.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7
@	P	O	R	M	I	L	$	x	x	x	x
C	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**
C#/Db	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**
D	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**
D#/Eb	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**
E	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**
F	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**
F#/Gb	**G**	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**
G	**G#/Ab**	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**
G#/Ab	**A**	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**
A	**A#/Bb**	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**
A#/Bb	**B**	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**
B	**C**	**C#/Db**	**D**	**D#/Eb**	**E**	**F**	**F#/Gb**	**G**	**G#/Ab**	**A**	**A#/Bb**

APPENDIX C: ABOUT CHORDS

Chords identified by chord symbols with four or more notes boil down to combinations of the two kinds of building blocks. Here follow some examples. Two fifos (fifths or fourths) form major-7, minor-7 or major-6 chords. One fifo and one tritone form dominant-7, half-diminished-7 (a.k.a. minor-7-♭5) or minor-6 chords. Three fifos or two fifos and a tritone form 9th, 1lth and 13th extensions of these chords. When tonic scales depart from key signatures, the same kinds of building blocks are available from the scales but now more than one tritone is available. For example, two tritones form diminished-7 or dominant-7-♭5 chords. Two tritones and one fifo (that may share a note with a tritone) form variations of other chord types such as dominant-7♭9. Omitting inner notes of combinations may yield different intervals, or thinned voicings of chords. For example, an augmented fifth (8 half tones) is combination of two tritones a whole tone apart with their inner notes omitted; and a 3-note voicing of a 4-note seventh chord is composed of the outer notes plus the most important inner note.

ANCHOR LETTERS FROM TRITONE CHORDS

The following table of tritone chords summarizes the simplest examples of chords with tritones in different positions.

offset of tritone bass note above root R	suffixes on root symbol R										
	7	7♯9(13)	9♭13 (or ♯5)	7♭9	7♭5 (or ♯4 or ♯11)	dim7 (or o7)	m7♭5	m6	M7(11)	♭9	M7♯11 (or ♭5)
fourth									x		
major third	x	x	x	x	x						
minor third		x				x		x			
whole tone			x								
half tone				x						x	
0					x	x	x				x

Only chord variations that alter tritone content are included. For example, R7, R9, R7#9, R13 and R9(13) are all variations of R7 with the same tritone content and so are all represented in the table by R7 (variations are left to context). Diminished seventh chords (Rdim7) have no counterpart in classical modes. Sus chords are not shown because they have no tritones. They are typically V-7♯3 chords. A Vsus-V-I progression is a substitute for a II-V-I progression in which one note is altered between the first two steps.

Inverting the tritone of a chord into the home octave (if it isn't already there) determines its PKP anchor. For example, The tritone of a V-7 chord is partially outside the home octave but its inversion is within it. The bass note of its inversion is a fourth above the home tonic which makes its anchor **I**.

Double tritones are the normal upper limit for tritone content of chords encountered above the staff in fake books, lead sheets and sheet music.=

FIGURED BASS NOTATION; EXTENDED CHORDS

Figured bass notation provides a simple representation of chords from scales with 7 notes and no adjacent half tones. It represents chords by stacks of numbers going up from a bass note, in which each number is a count of the scale steps to the next note up. The stacks are annotated on a staff as literal stacks of numbers (vertical lists). I write them here as horizontal lists separated by plus signs. For classical modes, a major or minor third is always two scale steps and whole tone is always one scale step (e.g., a seventh chord is **2+2+2** and a sixth chord is **2+2+1**).

Figured bass notation can be helpful for understanding extended chords of classical modes that follow from splitting chord steps of size **2** into **1+1** sequences and then moving the added note up an octave. The **+4** on top of two of the extended chords on the right is a consequence of avoiding **1+1+1** sequences in the in-place forms. The corresponding chord symbols have degree-number suffixes 6, 9, 11 or 13 added to basic symbols to identify the extensions.

chord type	(2) = split	in place	extended
seventh	**2+2+2**	**2+2+2**	**–**
ninth	**(2)+2+2**	**1+1+2+2**	**2+2+2+2**
eleventh (seventh+11th)	**2+(2)+2**	**2+1+1+2**	**2+2+2+2+4**
thirteenth (ninth+13th)	**(2)+2+(2)**	**1+1+2+1+1**	**2+2+2+2+2+4**

DOUBLE TRITONES

The number and variety of chords formed of or voiced by double tritones is startling. The chord symbols are all over the map but the double tritones have only three basic keyboard shapes exemplified by **RM**, **M.L** and **R..L** from the blues family mode, namely two tritones offset by a half tone, a whole tone or a minor third. Let's symbolize these shapes by **XX**, **X.X** and **X..X** , where the **X**s represent alphabet letters and the dots represent skipped letters. The same shapes appear for different mixes of tritones on both sides of the mode hierarchy. Double tritones with whole tone and half tone separations are progressively more dissonant than single tritones when their notes are sounded together. Double tritones with minor third separations have a "sweet" sound that's somewhere between dissonance and consonance.

The basic shapes are simple but finding any particular inversion of one on the keyboard can be difficult if you try to do it by finding the individual inversions and combining them." Two things make it difficult: one is different mixes of black and white keys in different home octaves; the other is different outer and inner intervals for different inversions. Better to work inwards from the outer intervals, as shown next.

←major sixth (9h)→

all inversions of X..X x . . x . . x . . x

←augmented fifth (8h)→

X.X & X.X x . x . . . x . x

←minor seventh (10h)→

X.X & X.X x . . . x . x . . . x

←fifth (7h)→

XX & XX x x . . . . x x

←maj. seventh (11h)→

XX & XX x . . . . x x x

Find the bass note, establish the treble note, then move inwards from both ends by the offset interval. Either all inversions have same shape or all inversions have only two shapes.

MISLEADINGLY COMPLEX CHORD SYMBOLS FROM CLASSICAL MODES

This illustrates, for the Ionian mode, that a startling variety of chords are voiced by simple shapes formed of a tritone combined with different fifos (some are rootless, all are in the mode).

```
1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
@ P  O R  M I L  $ x  x x  x @
. .  . .  . x .  . .  . .  x .     V7, IIm7(13), IM9(11)
. .  x .  . . .  x .  . .  . .

. .  . .  . x .  . .  . .  x .     V9(13), IVM7b5
. .  . .  x . .  . .  x .  . .

. .  . .  . x .  . .  . .  x .     V7(11), IIm7(13)
x .  . .  . x .  . .  . .  . .

. .  . .  . x .  . .  . .  x .     V13(11), IM7b5
x .  . .  . . .  x .  . .  . .

. .  . .  . x .  . .  . .  x .     VIIm7b5, IVM6b5, IIm6
. .  x .  . . .  . .  x .  . .
```

CHORDS FROM NON-CLASSICAL MODES.

This illustrates that non-classical modes provide many new chords.

```
                1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1
                @ P  O R  M I L  $ x  x x  x @
minor family    @ .  x x  . x .  $ x  x x  x @
OR              . .  O R  . . .  . x  x .  . .    IV7#9(13), VII7#9(13)
R.I             . .  . R  . I .  . .  x .  x .    IV7b5, VII7b5
O..I            . .  O .  . I .  . x  . .  x .    IIdim7, IVdim7, pVIdim7, VIIdim7
                                                  rootless 7b9 rel. to roots a 1/2 tone down
③R+             x .  . R  . . .  . .  . .  x .    ImM7, pIIIM+
④$              . .  . x  . . .  $ .  . .  . @    Im (minor triad @▼, spread out)

major family    @ .  x .  x x .  $ x  x x  x @
MI              . .  . .  M I .  . .  . x  x .    pII7#9(13), V7#9(13)
O.M             . .  O .  M . .  . x  . x  . .    III7b5, pVII7b5
O..I            . .  O .  . I .  . x  . .  x .    as above
③$              . .  . .  x . .  $ .  . .  . @    I (major triad)

blues family    @ .  x x  x x x  $ .  x x  . @
RM              . .  . R  M . .  . .  x x  . .    I7#9(13), pV#9(13)
M.L             . .  . .  M . L  . .  . x  . x    I7b5, pV7b5
R..L            . .  . R  . . L  . .  x .  . x    analogous to O..I
```

The strongly dissonant chords formed of two tritones offset by half tone would tend to be voiced in context as a stacked tritone and fourth with no internal half tones. This shape has a rich sound with a dissonant edge. The weakly dissonant chords formed of two tritones offset by a whole tone can be played with all notes in any inversion. The chords formed of two tritones offset by a minor third have a circular symmetry (same shape in all inversions) that produces a unique sound I have come think of as "sweet."

CHORDS DIRECTLY FROM TRITONES ON THE KEYBOARD

Tritone chords may be identified directly on the keyboard as indicated by the following selection of examples. Context would determine which one fits.

```
3-note chords          X . . X . . X             Xdim
                 x . . x . . . . . X             Xdim (inverted)
                       x . . . . . x . . X       Xdim (inverted)
4-note chords  X . . . x . . x . . x             X7 (3 note voicing omits inner note)
                 X . . x . . . x . x             Xm6 (3 note voicing omits inner note)
                       x . X . . . x             X7 (3 note voicing of inversion)
                       x . . x . . x . X         X7 (inverted)
                       X . . x . . x . . x       Xm7b5
                       x . . . x . X             Xm7b5 (3-note voicing of inversion)
```

TRITONE SUBSTITUTE CHORDS

The tritones of all possible dominant seventh chords and their tritone substitutes for any home tonic are easily determined from a table of the following form, the top line of which is the actual scale frame of the home tonic on the keyboard (**Eb-Bb-Eb** provides an example). The chord roots are the **X** entries in the table. All possible chords are listed on the right for the home tonic.

	Eb							Bb					Eb		
	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1		
	@	P	O	R	M	I	L	$	x	x	x	x	@		
L	.	.	X	.	.	.	L	.	X	.	.	.	.	F7	B7
I	.	X	.	.	.	I	.	X	.	.	.	.	.	E7	Bb7
M	X	.	.	.	M	.	X	.	.	.	.	.	X	Eb7	A7
R	.	.	.	R	.	X	.	.	.	.	.	X	.	D7	Ab7
O	.	.	O	.	X	.	.	.	.	.	X	.	.	Db7	G7
P	.	P	.	X	.	.	.	.	.	X	.	.	.	C7	Gb7

APPENDIX D: ABOUT ENRICHED CLASSICAL MODES

The enriched classical modes identified as “melodic” and “harmonic” presented in the book *Modalogy* are summarized next, except with notes in numeric-chromatic-scale notation, instead of in the RN (Roman Numeral) notation with sharps, flats and naturals used in the book. Corresponding PKP mode signatures are shown on the left.

The purpose is twofold. One is to verify the correctness of the PKP view of these modes. The other is to highlight the complexity that results from using sharps, flats and naturals. For example, the tritone anchored by **R** that is a component of many of the minor modes is understood as **p3-6** in the symbolic chromatic scale, and that’s it. Inversions in different places in harmony are left to context. In *Modalogy*, tritone anchor **p3** is ♭III or ♯II and tritone anchor **6** is VI or 𝄫VII, and that’s only the anchors.

The alphabet soup of names and the intricate details of the scales makes them very difficult to distinguish functionally, or to remember as enumerated scales. Seeing the possibility that tritones might be fundamental scale-defining objects for these modes is effectively impossible in the terms used in *Modalogy*. This impossibility is demonstrated by a discussion of defining and non-defining notes of the many and various modes that never mentions tritones.

PKP’s way of knowing the modes by 2-letter mode signatures from a 6-letter alphabet is simple and unambiguous. It enables seeing the modes as emerging from family context instead of as independent entities that give no hint of where they might be used. It simplifies the harmonic modes by making the master mode harmonic minor-major, thus reducing 2x7=14 parallel modes to 8 parallel modes. An 8-note minor-major mode is easily reduced to a 7-note harmonic minor or major mode to fit context.

Parallel Modes of the Melodic Minor |WhWW|WWh|

//LO	1-2-3-p5-p6-6-7-1	Lydian Augmented
alt-LO	1-2-p3-4-p5-p6-p7-1	Aeolian Diminished
//ML	1-2-3-p5-5-6-p7-1	Lydian Dominant
alt-ML	1-p2-p3-3-p5-p6-p7-1	Jazz Altered
//RI	1-2-p3 -4-5-6-7-1	Melodic, or Jazz, Minor
//OM	1-2-3-4-5-p6-p7-1	Jazz Mixolydian
//PD	1-p2-p3-4-5-6-p7-1	Jazz Phrygian

Parallel Modes of the Harmonic Minor |WhWW|hW+h|

//RL	1-2-p3-p5-5-6-p7-1	Romanian, Dorian ♯4, Mishebarakh
alt-RL	1-p2-p3-3-p5-p6-6-1	Leading Tone Minor Diminished, Super Locrian 𝄫7
//LR	1-p3-3-p5-5-6-7-1	Lydian Blues Major, Lydian ♯2
alt-LR	1-p2-p3-4-p5-6-p7-1	Jazz Phrygian Diminished
//OI	1-2-p3-4-5-p6-7-1	Harmonic Minor, Aeolian ♮7, Jazz Minor ♭6, Mohammedan
//IO	1-2-3-4-p6-6-7-1	Ionian Augmented
//PM	1-p2-3-4-5-p6-p7-1	Phrygian Dominant

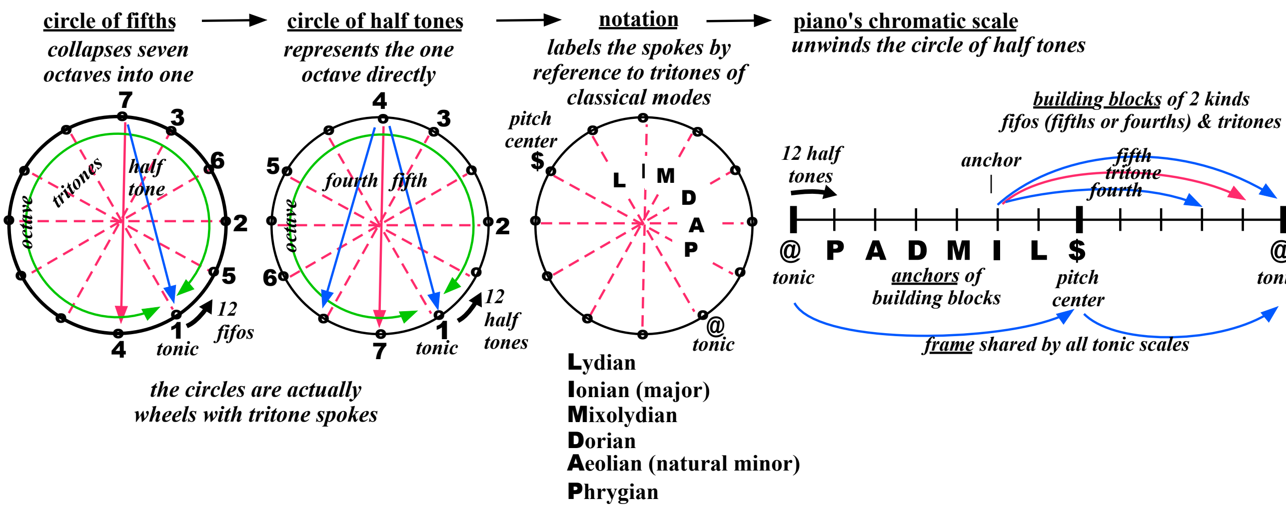
Parallel Modes of the Harmonic Major |WWhW|hW+h|

//RL	1-p3-3-p5-p6-6-7-1	Lydian Blues Augmented, Lydian Augmented ♯2
alt-RL	1-2-p3-4-p5-6-p7-1	Jazz Minor ♯4, Lydian Diminished
//LR	1-2-p3-p5-5-6-7-1	Lydian Melodic Minor, Lydian ♭3
alt-LR	1-p2-p3-4-p5-p6-6-1	Leading Tone Major Diminished, Locrian 𝄫7, Locrian Diminished-7
//OI	1-2-3-4-5-p6-7-1	Harmonic Major
//PM	1-p2-3-4-5-6-p7-1	Jazz Phrygian Dominant, Mixolydian ♭2
alt-MP	1-p2-p3-3-5-p6-p7-1	Altered Phrygian Dominant, Phrygian ♭4, Superlocrian ♮5, Superphrygian

APPENDIX E: ABOUT SYMMETRY & SYMMETRY-BREAKING

The elements of PKP are summarized here in a way that provides a link between the concept of symmetry breaking in these pages and in the book *The Jazz of Physics*. This kind of thing is fascinating to anyone with a mathematical bent but possibly not to anyone else. (This uses the original version of the alphabet, namely **PADMIL**, before I replaced **AD** with **OR** to avoid confusion with letter notes.)

The circle of fifths shown below left presents the basic elements of tonic scales and chord progressions from them in one simple picture. The points going counterclockwise around the circle are 12 notes a fifth apart (7 half tones) across 7 keyboard octaves (12 fifths x 7 half tones = 7 octaves x 12 half tones). The circle is manifestly a wheel with tritone spokes. The notes of the major scale are the numbered points around the half circle delimited by the highlighted tritone spoke (6 half tones).



The circle visibly establishes two kinds of Lego™-like **building blocks** as fundamental elements of music, namely fifths or fourths referred to jointly as **fifos** for simplicity, and tritones. The circle as labeled identifies fifths going up the keyboard through 7 octaves or fourths going down the keyboard through 5 octaves. For the major scale, it identifies a basic chord root line going down by five fifths (3½ octaves) or five fourths (2½ octaves) to the tonic (equivalent to progressively zig-zagging down a fifth and up a fourth within one octave).

The pictorial geometry of the circle of fifths and the musical geometry of its interpretation are different in a way that is counterintuitive (different numbers of octaves going around the circle for fifths and fourths, very different musical sizes of lines of almost the same pictorial length that cut across the circle). This becomes confusing for more general chord progressions of this and other scales.

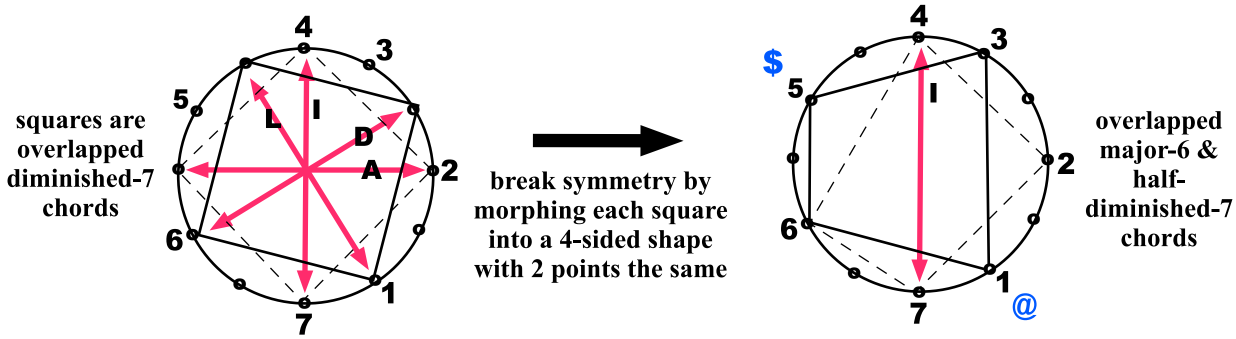
The circle of half tones rearranges the spokes of the circle to put the labeled points around it in scale order. Around the circle remains an octave and across any spoke remains a tritone. The half tones across the circle now go around it and the fifos around the circle now go across it.

The end points of all the spokes of either circle visibly identify all the points around it, suggesting a notation that labels spokes (building blocks) instead of points (notes). This simple observation is the twist that leads to PKP's unique combination of simplicity and depth. The spoke labels, called **anchors**, directly identify tritones but also identify fifos. In the chromatic scale that unwinds from the circle, each anchor letter identifies a tritone and two fifos morphed from it. The letters identify relative positions of the anchors in the chromatic scale. Tritone anchors are unambiguous independently of context and fifo anchors are ambiguous but the ambiguity is generally resolved by context. No anchors are needed in the top pitch half of the octave because inversions cover them.

The letters of the **PADMIL** alphabet mark the relative positions of the anchors of the single tritones of classical modes, but this is only to provide a correspondence between PKP notation and music notation. It does not bind the letters to the classical modes, or to tritones.

<u>SYMMETRY BREAKING IN THE CIRCLE OF FIFTHS</u>

According to *The Jazz of Physics*, symmetry-breaking is a deep feature of how both music and the universe work. Here's a view of breaking the symmetry of a diminished seventh scale to yield a major scale that corresponds to an example on page 66 of the book (in the book, the scale is diminished, not diminished seventh, so the dotted square and its transformation are missing). No mention is made of tritones).



The symmetry-breaking takes place in progressive steps that morph tritones into fifos.