THE EMPEROR HAS NO CLOTHES:
MUSIC AS IT ACTUALLY IS ON THE PIANO KEYBOARD

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INTRODUCTION

I approached the piano as an adult beginner interested in jazz, which eventually led me to wonder how jazz pianists can improvise in terms of a music notation that seemed to me to be misleadingly complex for the piano.[1] I thought, jazz improvisation must tap into deeper musical structures that are obscured by this notation. Music notation has stood the test of time and is here to stay for piano music, even if for no more reason than the huge legacy of piano music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation. In effect, "the emperor has no clothes" — piano music is much simpler than the "clothes" of this notation make it seem.

Trying to understand the deep structure of piano music without the obscuring clothes of music notation became an absorbing hobby that eventually led me to a simple system I call **PKP** (standing for "Picturing Keyboard Patterns"). The scope of PKP is tonal music in which a melody line is harmonized at selected points by chords formed of clusters of piano keys. Before you stop reading because of my admitted lack of musical credentials, consider the opinions of PKP of a couple of music professionals. Musical theorist Paul Steinbeck: "The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music." Jazz pianist/composer/teacher Taylor Eigsti: "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

The basic notation of PKP is an alphabet of six letters that's analogous to the alphabet used to describe biological DNA in the sense that words from the alphabet identify deep structure. The identification is in terms of **building blocks** identified by the letters. The building blocks are Lego-like in concept: "snapped together" combinations of them form the keyboard shapes of tonic scales and chords within the **home octave** of a piece of music.

The home octave of a piece goes up from the home tonic, which is the piano key on which its melody line comes to rest at the end, or "wants to" to the ear (it may also be the one on which the melody line starts on a repeat, which amounts to transposing the end to the beginning). A piece may visit secondary tonics along the way but the home tonic provides the reference for understanding all such visits. The piano provides a stack of harmonically equivalent home octaves. The stack can be represented by a single conceptual home octave that provides the twelve half tones into which any octave is divided. Harmonically equivalent chords of different shapes that extend outside the home octave may be formed by playing selected piano keys of the building blocks in the same positions in higher or lower actual octaves on the keyboard.

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[1] [1] The following chord progression written by Mingus for the hauntingly beautiful E♭ blues *Goodbye Porkpie Hat* is a "poster child" for music notation that's misleadingly complex: E♭7♯9—B9(13)—**[**EM9**]**—A7♯11—**[**D♭9sus**]**—B9(13)—**[**D♭7sus**]**—E♭7—**[**A♭m11**]**—B7(13)—Fm7♭5—B♭7♯5♯9—C13♯11—F7(13)—B7—**[**EM7**]**—A7(13)—A♭7—B♭7—D♭7—E♭7♯9—B7—EM7♯11—A7♯11. There are 24 chords, an average of 2 per bar for 12 bars, only 5 of which (highlighted in blue) don't contain tritones. The tritone content is the basis of a deep structure of startling simplicity.

There's an unusual combination of simplicity and depth in this way of understanding scales and chords. Words of one to four letters from the alphabet provide a highly condensed representation of them that enables the details to be known in terms of intervals on the keyboard measured in half tones, without spelling them out in note symbols of any kind. I call the words **mode signatures** because they identify **parallel modes** of the same tonic. The simple deep structure of music revealed by mode signatures is profoundly obscured by music notation. To discover it, I had to enter uncharted territory. Nothing I read said anything about it and no expert I approached knew anything about it. I suspect I speak for legions of amateurs when I say the piano is not just for experts but the conventional approach to teaching and learning it tends to make it so: understanding is expected to emerge by osmosis from the same extensive practicing that develops "chops." The idea that understanding can be had independent of chops is unconventional.

PKP represents shapes of scales and chords in the same terms, enabling simple annotations above the staff of a piece of music to provide contextual cues that help in learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. This turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

Is this combination of simplicity and depth only a serendipitous side effect of the organization of the piano keyboard? Or is it a fundamental property of music that's obscured by music notation? Either way, it works in practice.

Why would a beginner with no formal musical training discover something as profoundly unconventional and useful as this? The answer has many parts: 1) Curiosity. 2) A skeptical cast of mind. 3) No received reverence for music notation. 4) Personal experience as a university professor developing notations to deal with complexity in another field, computer software, that led me to see an analogy between computer software and piano music. Both use abstract notations to describe things that are performed on "hardware." I thought ways of thinking I had developed for the former might apply to the latter. 5) Time to indulge my curiosity following retirement from my day job as university professor just as I was beginning to "get into" the piano.

GUIDE TO READERS

This document began as notes to myself to explain the ideas I was developing. I tried to write it so anyone with no knowledge of music theory and very little experience with the piano could read it — in other words with myself when I started out in mind. The potential audience includes novices at this level, pop and jazz musicians who are not pianists but want to explore harmony on the piano, music teachers interested in a different approach to understanding piano music, and "wannabe" expert pianists interested in understanding music at a higher conceptual level than notes. Chapter 2 develops the concepts sufficiently to understand many of the examples in Chapters 4 and 5. Chapter 3 develops details independently of examples. Chapter 4 and 5 develop the details via examples, independently of music notation in Chapter 4, in terms of annotated music notation in Chapter 5 (the final example in this chapter is the piece with the footnoted chord progression on the previous page). Chapter 6 provides observations and conclusions. References, acknowledgements, comments from some readers, and appendices follow. Appendix A summarizes unconventional terminology. Appendices B-D are about scales, chords and parallel modes in music notation vs. PKP. Appendix E is about the role of symmetries in keyboard shapes.

CHAPTER 2: CONCEPTS & NOTATION

TONIC SCALES FROM SPLIT HOME OCTAVES

The emperor-has-no-clothes view of the piano sees every octave as twelve half tones between thirteen piano keys — thirteen because the top of the octave is the bottom of the next octave up, which has the same keyboard shape, and is harmonically equivalent. As shown next, a useful conceptual model of a home octave is a horizontal line split into twelve equal parts representing twelve musically equal half tones — musically equal because their pitch sizes increase within the octave to make the pitch of the top note double that of the bottom note. The top and bottom notes are home tonics an octave apart symbolized by **@**. The big difference between the piano and music notation for it is the sharing of intervals between overlapping octaves, symbolized by the vertically aligned dots. This simple picture is the jumping off point for PKP.

Everything in PKP is referred to the single home tonic of a piece, so keeping it fixed in the mind is essential. It's useful while learning or revisiting a piece to put a temporary label (e.g., a small stick-on circle) on the piano key that is the home tonic, to avoid interpreting things relative to the different home tonic of some recently played piece that's still in the mind.

overlapping octaves

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@ . . . . . . . . . . . @
  @ . . . . . . . . . . . @
    @ . . . . . . . . . . . @
      and so on
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Profoundly simple relationships are revealed by two different kinds of splits of the conceptual home octave into equal halves: a geometric split that yields dissonant tritones (6 half tones); and a musical split that yields a consonant fifth and fourth (7 and 5 half tones, respectively) that are of equal pitch sizes because of the increasing pitch sizes of half tones going up within an octave. The musical split defines a **scale frame** formed of the octave notes with the pitch center of the octave in between (**$**). This scale frame is the starting point for forming tonic scales (tonic scales without pitch centers exist but the references for understanding them are scales with pitch centers).

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octave           @ . . . . . . . . . . . @    12 half tones
geometric split  x . . . . . x . . . . . x    tritone-tritone
musical split    @ . . . . . . $ . . . . @    fifth-fourth scale frame
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Tritones, fifths and fourths are the Lego-like building blocks of PKP. Fifths and fourths are, visibly, the same kind of building block, different in kind from tritones. This warrants a special term, **fifo**, standing for "fifth or fourth." This term is more than just a textual shorthand, it identifies a fundamental property that contributes to PKP's combination of simplicity and depth: the identification of fifos as morphed tritones (hold one note and move the other by a half tone). This is directly visible for the special cases shown above — the lower tritone morphs into the lower fifth, the upper tritone morphs into the upper fourth — but it pervades harmony viewed in terms of the home octave. This greatly simplifies situations that are misleadingly complex in music notation, without sacrificing substance.

This small set of basic building blocks is sufficient for most purposes because smaller and larger intervals emerge as inner or outer intervals of shapes formed of them. The smaller or larger intervals are occasionally needed as independent building-block-like quantities but PKP covers this possibility in a

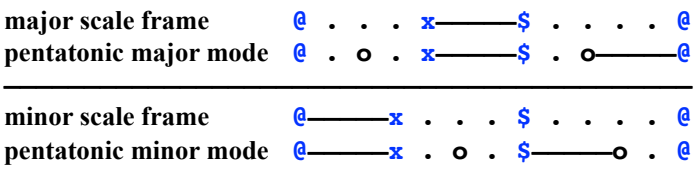
simple way that doesn't require increasing the size of the building-block menu.

The terms fifth and fourth refer to the numbers of notes these intervals contain in the classical modes that define the scales of key signatures (5 and 4, respectively). It would be better for our purpose if these terms were "augmented tritone" and "diminished tritone" because this would correspond to their actual keyboard sizes in half tones. As it is, the mismatch between their names and their keyboard sizes must be kept constantly in mind to avoid confusion — a fifth has 7 half tones, not 5; a fourth has 5 half tones, not 4; a major third has 4 half tones.

Pentatonic Modes are the Foundation

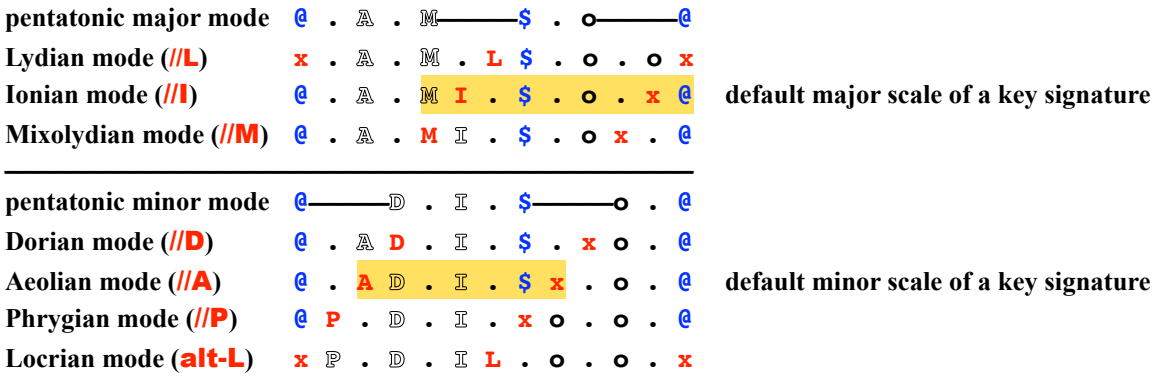
Tritones and associated half tones bring dissonance to music. Simple scales called "pentatonic" don't contain them, but provide a foundation for understanding the contribution this makes to music, as shown next. Pentatonic major and minor modes follow from splitting the bottom fifth of the scale frame into unequal parts that determine major or minor tonality, and then splitting the results again to yield scales with intervals of whole tones (2 half tones) and minor thirds (3 half tones), in a way that avoids adjacent minor thirds which form tritones. Minor thirds are shown as solid lines to make them stand out. These are the foundation scales of music in many cultures worldwide because anyone with a musical ear can sing tunes from them, harmony is simple, and they generalize simply and directly to more general scales.

"Modes" is the term conventionally used for scales with interval sequences that are rotations of each other (same interval sequence starting from different points), which is visibly the case for these scales. These are, visibly, parallel modes that share the same tonic (relative modes share the same notes).



Classical Modes Provide the Alphabet

The development of more general musical scales from pentatonic scales goes in different directions. One direction, shown next, leads to the scales of key signatures called "classical modes." The single letters are **anchors** of building blocks. The anchor concept is explained in the next section.

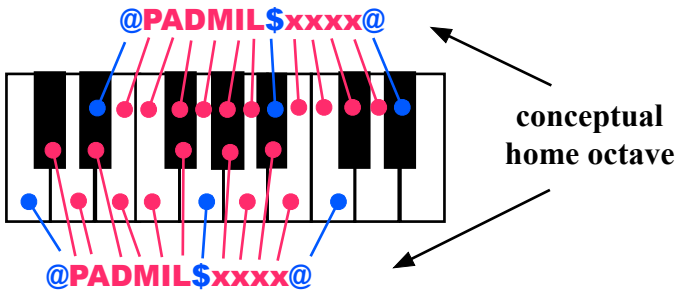


On the left, the same letters prefixed by "*II*" (standing for "parallel") are mode signatures that identify the modes by tritone content. The prefixes distinguish the modes identified by tritones from the tritones themselves, which exist independently of the modes. The term "default" on the right means that these two modes are implied by default, when, for example, a piece is said to be "in" C major or "in" C minor.

These modes split the minor thirds of pentatonic scales into a whole tone and a half tone in different ways. The splits yield tritones (red text) and associated half tones as side effects. Symmetric shapes formed by these side effects — same interval sequence going up and down, highlighted in yellow for the Ionian and Aeolian modes — determine the notes. The scale is completed by whole tones. Symmetry is well known in math and physics to be a cornerstone for understanding complexity, so it's logical that it might do so for music, which is complex by any measure. It's surprising only in relation to music notation, which obscures these symmetries and others to come.

The Anchor Concept

The anchor concept is illustrated next for two of the twelve possible home octaves on the keyboard, which is sufficient to give the sense of it for any home octave. The concept dramatically simplifies the representation of building blocks compared to using note symbols, without sacrificing substance.



Tritones and fifos are identified by labels called **anchors**, which are the first letters of the names of the classical modes containing them (the mode names are mouthfuls but the letters are only letters, pronounced as such). The origin of the letters is classical modes but the anchored tritones and fifos are independent of the the modes. Tritones determine the alphabet and fifos emerge from it. The solid letters (shown) provide the alphabet **PADMIL** of tritone anchors. The outline form of the same letters (not shown, but implied) provides the alphabet PADMIL of fifo anchors. The same letter in two different versions of the same font anchors a tritone and two fifos — a fifth or fourth morphed from the tritone by raising or lowering its upper note a half tone. Two more fifos on different anchors are formed by raising or lowering the anchor a half tone. The alphabet is in a more distinctive font than in the above table (the table font is a fixed width, yielding nicely aligned columns, but is too easily confused with other ways some of these letters are used in music notation and theory). Red text for tritones highlights them for explanation purposes where this seems appropriate, but the meaning is in the letters, not in the color. Fonts cannot be used in handwritten annotations on the music page but the same objective can be achieved with colored pencils or different outlines for letters (circles for tritones, boxes for fifos). Fonts are simpler in text.

The default meaning of an anchor is a building block going up from it. Building blocks going down from an anchor are identified by underlining. Put another way, the bass note of a building block is its

anchor unless the anchor is underlined, making its opposite end the bass note. No anchors are needed in the top fourth of the octave because this covers all possible bass notes of building blocks.

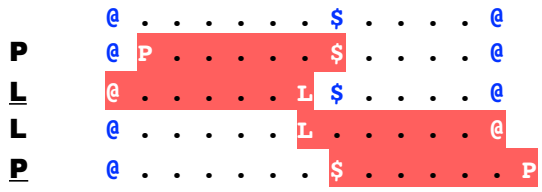
Tritone anchors are unambiguous within the tonic octave (they always go up), except for upper anchor of the alphabet, which may go up (**L**) or down (**L**). This constraint makes for very simple representations of the tritone content of keyboard shapes within the home octave.

The underlining convention for inversions applies also to fifo anchors, except the inversions of fifos are different sizes (a fifth inverts into a harmonically equivalent fourth and vice versa). Fifos of the same size going up or down from an anchor are not inversions of each other but they are mutually consonant because one inverts into a size a whole tone different from the other. Fifos anchored more than a half tone apart are also mutually consonant because there notes are at least a whole tone apart. This harmonic equivalence and mutual consonance of fifos makes them relatively interchangeable. Often, the only notation needed for a fifo is its anchor, leaving its size to emerge from context. The notation for such an uncommitted fifo anchor is an anchor symbol in angle brackets (e.g., ⟨𝕄⟩). Uncommitted anchors open the possibility of admitting sizes smaller than fourths or larger than fifths. Such sizes emerge in harmony as inner or outer intervals of shapes formed of combinations of building blocks of the basic three sizes, but sometimes these inner or outer intervals need separate representation. Expanding the alphabet to include them would add unneeded complexity because the need is infrequent and the sizes are often obvious from context (e.g., a treble or bass line held from one step to the next).

Specifying fifo sizes is simple, when needed. The default size is a fifth (e.g., 𝕄 anchors a fifth). A fourth is identified by a letter with a strikethrough, symbolizing cutting down the size (e.g., 𝕄̶ anchors a fourth). The fifth or fourth goes up from the anchor unless the anchor is underlined. Opposite sizes up and down are harmonically equivalent opposite inversions (e.g., 𝕄̶ and 𝕄). The same sizes up and down are not opposite inversions, but are mutually consonant because the inversion of one is smaller or larger than the other by a whole tone (e.g., 𝕄 and 𝕄).

The symbols **@** for the home tonic and **$** for the pitch center of the home octave can be tritone bass notes but never tritone anchors. This rule ensures the uniqueness of tritone anchors and therefore of mode signatures. The convention that a fifo anchor symbol identifies a fifth unless the symbol has a strikethrough requires the fourth **$-@** to be identified as $̶ ($̶ implicitly anchors a fifth going outside the home octave).

Considering the alphabet as a circular loop (traversing it in one direction goes off one end to proceed in the same direction from the other end), it may be said that adjacent anchors identify tritones offset by a half tone. This means for tritones **P** and **L** at opposite ends of the alphabet that the bass notes of opposite inversions are a half tone apart, as illustrated next. It also illustrates the rule that the tonic and pitch center can be bass notes of tritones but not anchors of them.

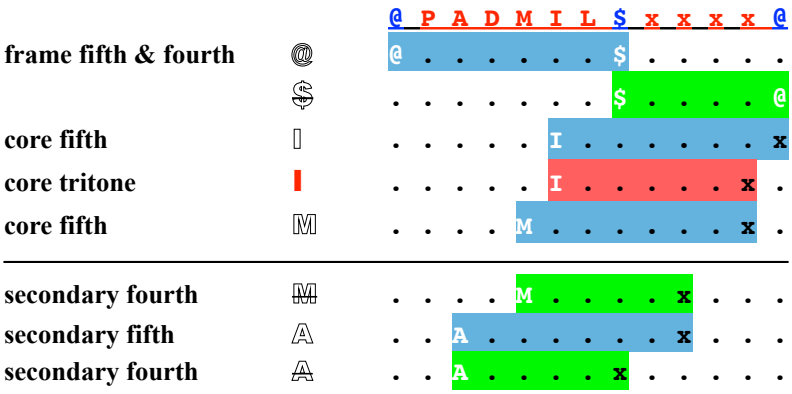


Returning to Classical Modes

The earlier table of classical mode begins with the Lydian mode containing the **L** tritone that splits the octave into geometric halves, and continues with modes containing tritones anchored successive half tones down. The Locrian mode is an obvious outlier because it's missing the pitch center of the home octave. It's not an independent parallel mode but an **alt** mode of Lydian in which the tritone is inverted and all non-tritone notes are different.

The five modes other than Lydian also have **alt** modes, but these are not parallel modes of the home tonic because they don't include it, and so are not shown (which doesn't mean they aren't useful). They are parallel modes of a secondary tonic a tritone away (e.g., the **alt** mode of Ionian of C is Ionian of F# or Gb, same piano key).

The building blocks identified by the anchors are shown next in Lego-like terms for the Ionian mode. Accumulating the anchors and their opposite ends from all the building blocks yields the scale. This is not to suggest actually constructing a scale this way, only to show that the building blocks are implied by this way of representing scales. **A scale is constructed without any reference to note symbols by combining the anchored tritone with the scale frame, adding symmetric half tones to the tritone, either outside or inside it (the position alternates between successive modes), and completing the 7-note scale by whole tones**.



Properties of Classical Modes

One piano key changes from one mode to the next in alphabet order, provided by the next tritone (one of its piano keys is in both modes).

Every primary classical mode has a 3-part **anchor set** within the lower fifth of the scale frame that distinguishes it from the other primary classical modes (e.g., 𝔸.𝕄𝕀 for the Ionian mode, where the optional dot indicates a skipped alphabet letter). The tritone anchor also provides a fifo anchor that's not listed in the anchor set because it's understood (e.g., 𝕀 for the Ionian mode). Classical modes are constrained by a rule of no adjacent half tones: the tritone anchor can have only one fifo anchor adjacent to it (e.g., 𝕄 for the Ionian mode), which leaves only one other alphabet letter available as a fifo anchor (e.g., 𝔸 for the Ionian mode) because any other choice would yield adjacent scale half tones (e.g., 𝕃, 𝔻 and ℙ are excluded from the Ionian mode because they would yield adjacent half tones 𝕄𝕀𝕃, 𝔻𝕄𝕀 and **x**@ℙ, where **x** is the upper note of the 𝕀 tritone).

The fifo that shares its note with the mode tritone (e.g., 𝕃 of the Ionian mode) alternates between a fourth and a fifth for successive modes in dictionary order. The other scale fifos may be either fifths or fourths. The core fifo anchors identify opposite harmonic roles (e.g., for the Ionian mode, 𝕄 establishes tonality as major and so may characterized as a **resolution anchor**; 𝕃 is offset a half tone away from it and so is automatically a **non-resolution anchor** because of the relative dissonance.

Most of the fifos of classical modes are substitutable for each other in harmony because of either harmonic equivalence (e.g., the frame fifth and fourth) or mutual consonance (e.g., all but the mutually dissonant core fifos). This opens the possibility of different combinations of building blocks and different permutations of their notes (following from inversions) being used for essentially the same harmonic function in different places in the same piece. This substitutability makes for harmonic richness but also for notational complexity in music notation, because of the large number and wide variety of different chord symbols that may result. PKP keeps the possibility of harmonic richness but does an end run around the notational complexity.

A small amount of notational complexity is unavoidable with fifo-intensive classical modes (7:1 ratio of fifos to tritones) because decisions must be made about sizes of anchored fifos but this is small potatoes compared to music notation. The PKP anchor concept eliminates most of the complexity at one stroke. It permits dividing harmony into **core** and **enrichment** parts, in which the core parts are determined by anchor sequences and the enrichment parts by context provided by mode signatures determined by the melody line and the core harmony. Beyond classical modes, the more tritones in the tonic scales, the simpler the core harmony.

Tritones are inherently simpler than fifos because of their fixed size and their lack of mutual substitutability (different tritones are neither harmonically equivalent nor mutually consonant). Ornamental tritones sometimes act as ornamental surrogates for mode tritones in the flow of harmony (e.g., tritone **L** sometimes acts as a surrogate for **I**, or tritone **P** for **A**), but that's different in kind from fifo substitution.

Parallel classical modes are rotated transpositions of a master mode, conventionally assumed to be the Ionian mode ("rotated" means starting from a note of the mode other than the tonic). This simple property means that parallel modes of the same tonic and tonic changes for the same mode are two sides of the same coin: one implies the other. This hugely simplifies understanding tonic changes. Details are presented at the end of this chapter under the heading *Two Sides of the Same Coin*.

The alphabet order is different from the order in which classical modes are conventionally understood. The conventional order identifies them as relative modes with the same key signature but different starting positions. The list of the first letters of the relative modes in the conventional order is **IDPLMAL**, in which **L** appears twice because relative Lydian and Locrian modes have different starting notes. The **PADMIL** order identifies parallel modes that share the same tonic. The letter **L** appears only once because the same tritone appears in opposite inversions in the Lydian and Locrian modes.

A BIG CONCEPTUAL LEAP

Accepting that tritone content is in any way fundamental to music is a big conceptual leap but only a small notational step from here. Tritones don't seem fundamental in music notation because the presence of any tritone beyond the one in the tonic scale of the written key signature introduces notational complexity — other tritones come from a different key signatures. The emperor-has-no-

clothes view of the piano brings tritones forward to the eye as structurally fundamental. Skeptics should glance through the examples of Chapter 4.

The small notational step is illustrated by family scales, details of which are deferred until later, but concepts of which are easy to understand now. Parallel classical modes have already been seen as as a family of scales developed from parallel pentatonic modes by splitting their minor thirds in a systematic way. A very different family scale follows from the union of the pentatonic modes. Their union with an added twist yields a 3-tritone scale determined by tritone anchor set **DM.L** (the dot indicates a skipped alphabet letter). The **DM** pair of anchors provides the mixed minor-major tonality that gives blues its characteristic sad-happy sound. The **L** anchor adds the characteristic blues "flatted 5th" (which is the flatted 5th note of a 7-note classical mode, not of the 9-note blues scale). This scale is easy to complete from the mode signature: seven scale notes are provided by the scale frame and the tritones, and the remaining notes follow from the foundation pentatonic scales. It's very different from classical modes (9 notes, 3 tritones, 5 adjacent half tones), so different that inherently simple blues is a misleadingly complex genre in music notation. The blues family scale provides many sub-scales that follow from omitting selected notes: the two main ones used for melody are shown here ("minor" and "major" refer to the tonalities of the pentatonic scales that provide the five basic notes).

(a) pentatonic major	@ . A . M———$. x———@
(b) pentatonic minor	@———D . I . $———x . @
//DM.L = (a) + (b) + L	x . A D M I L $. x x . x
6-note major blues sub-scale	@ . A D M———$. x———@
6-note minor blues sub-scale	x———D . I L $———x . x

In an analogous manner, the union of classical modes **//A** and **//I** (that are themselves developments of the foundation pentatonic scales) yields a 4-tritone scale determined by **ADMI** that reduces, by morphing the **M** or **D** tritone into fifos, to 3-tritone family scales of minor or major tonality identified by simple augmentations of the words **AD.I** or **A.MI** (without going into the details, the augmentations are identified by **AD.I⁺** or **A.MI⁺**). As with blues, the family scales provide many sub-scales ranging all the way down to minor classical modes **//A** or **//D**, or major classical modes **//I** or **//M**.

Scale families provide a simple, unified view of a very large number of scales of music notation of different kinds in different key signatures that are, in aggregate, overwhelmingly complex for all but experts. That said, key signatures must be lived with.

Blues pieces illustrate the confusing mishmash of choices of key signatures displayed by pieces that go outside classical modes. Shown next are the differences (highlighted in yellow) between the blues family scale and the different parallel classical modes that are determined by different key signatures.

The Mixolydian and Dorian modes are the closest of the parallel classical modes to the blues family scale, suggesting a key signature for one of them might be a good choice. However, given a melody line in one of the 6-note blues scales, any key signature that provides the corresponding pentatonic scale as a sub-scale can also be a good choice, which broadens the scope to include the Ionian and Aeolian modes. All of these possibilities, and others not shown, are displayed by different blues pieces. The effect is to make blues seem, misleadingly, an entirely ad hoc musical genre.

//DM.L	x	.	A	D	M	I	L	$	.	x	x	.	x	differences
Lydian mode (//L)	x	.	A	.	M	.	L	$	.	o	.	o	x	3 omissions, 1 addition
Ionian mode (//I)	@	.	A	.	M	I	.	$	.	o	.	x	@	3 omissions, 1 addition
Mixolydian mode (//M)	@	.	A	.	M	I	.	$	.	o	x	.	@	2 omissions
Dorian mode (//D)	@	.	A	D	.	I	.	$	.	x	o	.	@	2 omissions
Aeolian mode (//A)	@	.	A	D	.	I	.	$	x	.	o	.	@	3 omissions, 1 addition
Phrygian mode (//P)	@	P	.	D	.	I	.	x	o	.	o	.	@	3 omissions, 2 additions
Locrian mode (alt-L)	x	P	.	D	.	I	L	.	o	.	o	.	x	4 omissions, 2 additions

Accidentals (sharps, flats, naturals) identify departures from key signatures. Accidentals don't distinguish between notes that determine a scale and ornamental passing notes that have no scale implications. Many low-level details must be sorted out. The complexity over all possible tonics and key signatures can be mind boggling. Mode signatures determined by tritone content sort things out at a high conceptual level that's actually simple.

SYMBOLIC CHROMATIC SCALE

A basic principle of PKP is harmony and melody of tonal music form a coherent whole that provides context for understanding a piece of music. As has just been illustrated, one can go a long way with tonic scales and harmony from them without needing note symbols. Melody scales can be understood in terms of building blocks within the home octave but melody lines need all possible intervals. Melody lines could be written down as sequences of these intervals but this requires mentally adding and subtracting interval sizes to keep track of where a melody line is at any time. Needed are twelve symbols for the twelve piano keys of an octave (twelve half tones require thirteen piano keys, but only twelve symbols are required because the bottom piano symbol is repeated on top to represent both the top of the octave and the bottom of the next octave). Music notation provides no such scale because overlapping octaves require different key signatures (explicit or implicit). In principle, any set of twelve different symbols could represent the chromatic scale (**o-p-q-r-s-t-u-v-w-x-y-z-o**, for example) but symbols that are more easily related to music notation are helpful.

A suitable notation begins with the convention of music notation that the Ionian mode is the default reference scale of a key signature, relative to which all other scales are understood. Labeling the piano keys of the Ionian mode of the home octave **1-2-3-4-5-6-7-1** provides a set of symbols for piano keys going up from the home tonic by whole tones, except for the two half tones **3-4** and **7-1**. Completing the chromatic scale requires five symbols for the five piano keys in the five whole-tone intervals. A sufficient notation for them is provided by attaching prefix **p** to the five numbers above the whole tone intervals. The prefix stands for "phlat" and means next piano key down. The result is shown next, aligned with the conceptual home octave.

chromatic scale of the home octave	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1
conceptual home octave	@	P	A	D	M	I	L	$	x	x	x	x	@

These are the only symbols (**7** is never **p1**, **3** is never **p4**). There are no symbols corresponding to sharps. The beauty of this notation is it mirrors the look of the C-octave on the piano, giving visibility to the musical functions of the notes in any octave. The notation is simple enough to annotate on the

staff next to important note symbols of a melody line.

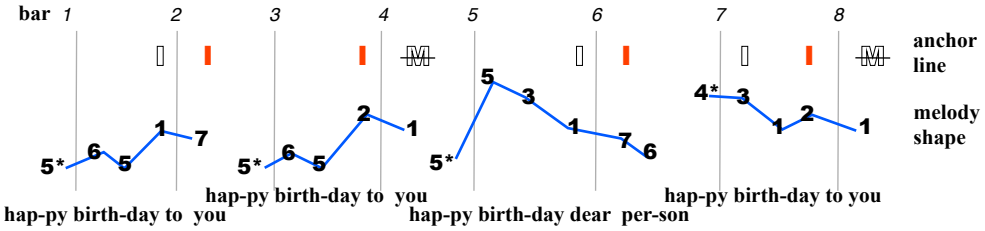
From now on, all explanations of harmony in the tabular form that shows Lego-like building blocks will include both representation along the top, to enable easy cross reference (for example a tritone anchored by **M** is understood to be **3-p7** in the chromatic scale without spelling it out as such.

This scale with the numbers replaced by RN symbols provides a set of twelve symbols for the twelve possible chord roots of the home octave: ***I, pII, II, pIII, III, IV, pV, V, pVI, VI, pVII, VII.***

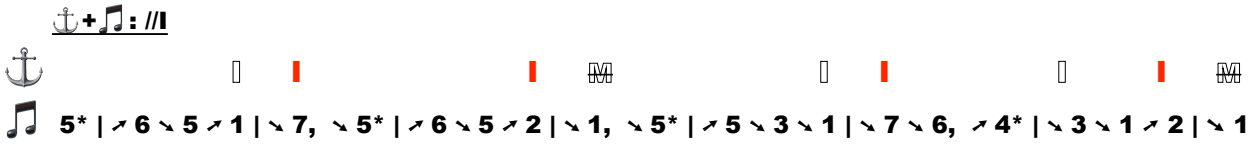
Mehegan's jazz piano instruction book uses exactly this notation for chord roots except with actual flat symbols of music notation as prefixes, which causes endless confusion when the written roots are sharped or flatted letter notes, or suffixes of the written chord symbols contain sharped or flatted letter notes that alter notes of the chord scale. The "phlat" prefixes avoid the confusion.

INTEGRATED MELODY PLUS HARMONY

A shorthand representation of integrated melody and core harmony by two simple lines is shown next for a familiar piece in which melody and harmony are both in the Ionian mode (having both melody and harmony in the same scale is not the general case, but serves to introduce the concepts). The anchor line that defines core harmony is shown above the melody line to symbolize annotating it above the staff in the written music. The ziz-zag melody line symbolizes chromatic scale symbols annotated on the staff next to selected note symbols (asterisks indicate repeated notes, leaving the number to be determined by the piece). The intervals of the zig-zags represent the essence of how people with musical ears remember melody shapes as sequences of pitch intervals going up or down, independently of rhythm, timing and duration of notes and rests. The building blocks in the harmony are easily correlated with note symbols of the melody (is the fifth **4-1**, ▮ is the tritone **4-7**, Ⓜ is the fourth **3-6**).



As shown next, this may be condensed into two textual lines by representing the melody shape by a single line interspersed with diagonal up and down arrows.

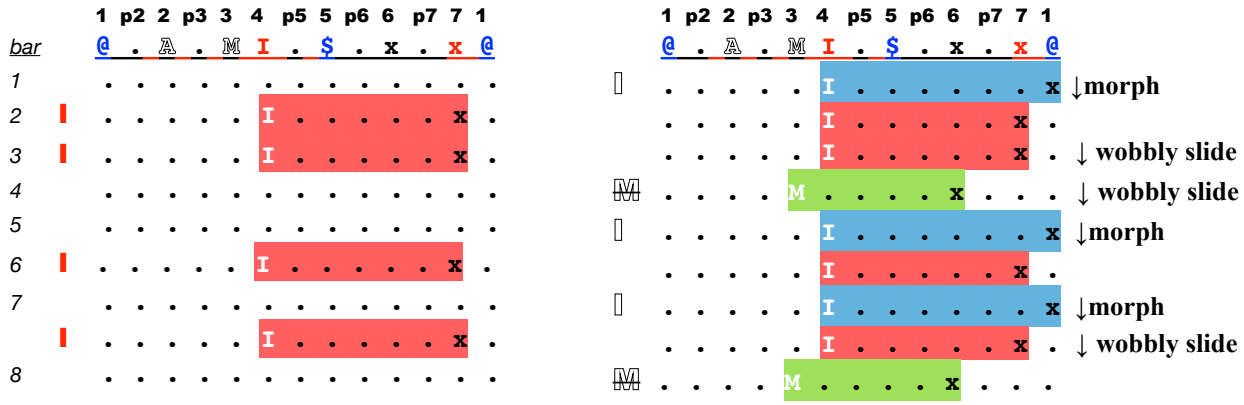


The anchor line is identified by ⚓ and the melody line by ♫ (the annotation ⚓+♫: //I indicates harmony and melody use the same Ionian mode). In the melody line, commas indicate ends of phrases. Dashes not shown may be used to mark melody gaps where harmony may appear. This is not as graphic as above, but provides a useful shorthand summary of the melodic and harmonic essence of piece that's independent of music notation and of the choice of home tonic.

The core harmony established by this anchor line is explained next in Lego-like terms, starting from tritone content as the "seed." This is useful because the tritones are both fixed size and unique to the mode, and the fifos are neither.

The tritone appearances are strategically positioned relative to the melody line to indicate immediate or anticipated resolution to the tonic in the mode of the tritone. Positioning the tritone before the end of melody phrases that resolve indicates immediate resolution. Positioning it at the end of ones that don't indicates anticipated resolution. The resulting all-tritone core (on the left) establishes a fixed framework for adding core fifos (on the right) morphed from the tritones in the forward or reverse direction (the reverse direction while figuring things out becomes the forward direction while playing).

The fifos fill in the gaps in visibly obvious ways. For a start, there's only one fifo anchored by in this mode, a fifth. An appearance of this fifth precedes each appearance of the mode tritone, forming a characteristic pre-resolution sequence of major harmony in which a fifth **morphs** downward into a tritone. For resolution in bars 4 and 8, the tritone could **morph** downward into a fifth anchored by 𝕄 but doesn't here, to avoid putting a dissonant half tone below the melody tonic (assuming the melody is played in the next octave up). Instead it morphs into a fourth anchored by 𝕄. This isn't a simple morph, but a combination of a **slide** (the tritone slides down a half tone) and a morph (the tritone morphs into a fourth) that I have come to call a **wobbly slide**. This odd term accurately represents how the fingers must move, namely lift them and move them as one (the slide), while spreading them apart or squeezing them together to make size changes (the wobble).



Chords in general will be dealt with later but the usual chords implied by this core harmony are easily understood now. They're determined by assigning roots to building blocks as follows (these are the usual roots but far from the only possible ones): root ***V*** for the tritones in bars 2, 3, 6 and 7 voices dominant seventh chord ***V-7***; root ***II*** for the fifths in bars 1, 5 and 7 voices minor seventh chord ***II-m7***; root ***VI*** for the fourth in bar 4 voices minor seventh chord ***VI-m7*** (in this context); and root ***I*** for the fourth in bar 8 voices major sixth chord ***I-6***, of which ***VI-m7*** is an inversion. As shown later, a large number and wide variety of chords with very different chord symbols can be implied by adding different roots to the same core. Chord symbols are not nearly as important for understanding harmony as they look.

Beyond This Example

Wobbly slides may be understood as literal sequences of a **slide** and a **morph**. For example, -𝕄

is a contraction of **I-M-𝕄** (a tritone slides down a half tone and then shrinks into a fourth); and **𝕄-𝕀** is contraction of **𝕄-𝕄-𝕀** (a fourth expands into a fifth which slides up a half tone). This is a useful way of thinking in any case because the contractions are of possible multi-step anchor lines that may identify mode changes (e.g., **I-𝕄** or **𝕄-𝕄-𝕀** stays in the Ionian mode, and **I-M-𝕄** goes to the parallel Mixolydian mode).

Slides that move off one end of the alphabet wrap around to the other end. For example **I-A** slides a tritone down a minor third, and **I-A** slides a tritone up a minor third, which amounts to wrapping around to **A** and sliding the result up a tritone (in other words, inverting it). Such simple movements may imply mode or tonic changes that can be complex in music notation (there's more on this at the end of this chapter).

Fifo-only anchors @, $ and 𝔸 don't appear in this example but may appear in general in core harmony. A good way of thinking of the appearance of these anchors in core harmony is as fifo substitutions to provide more variety. For example, the 𝔸 fifth might be substituted for the 𝕄 fifth in bar 1 of the example.

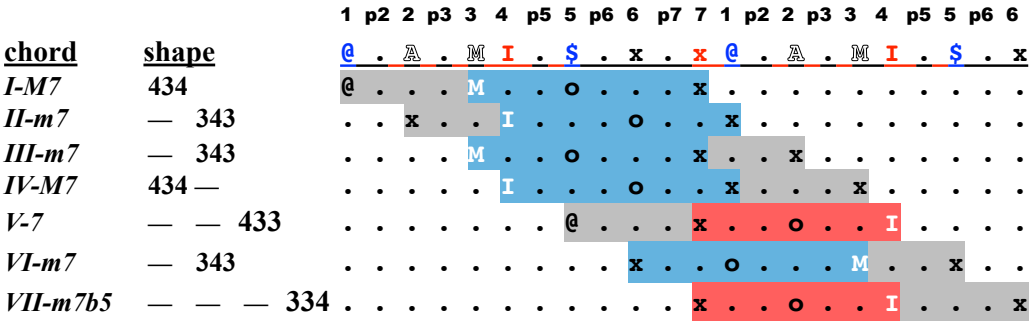
FROM CORE HARMONY TO CHORD PROGRESSIONS

When piano music is fully written out on the grand staff, the flow of the music is directly visible. The problem is the amount of complex detail is overwhelming for all but experts. When music is separated into melody on a staff and harmony represented by chord symbols above the staff, only the flow of melody is visible because chords may be voiced in different ways to give different harmonic flows that complement the melody line in different ways.

A piano teacher once told me to learn chords by picturing the chord symbols on the keyboard. I bought a chord symbol wall chart that did exactly this. As I looked at it while trying to learn pieces with these chords, I began to see it as hiding simple keyboard shapes that move in simple ways behind a facade of misleading complexity. This led me to see voicings of chord progressions as built up from harmonic cores without reference to chord symbols beyond identifying tritones. Knowing the scales of origin of the harmony from mode signatures provided by the tritones makes transforming cores into shapes to voice chords simple.

Chords of Classical Modes

The seventh chords from the Ionian mode shown next provide the basic chord symbols for chords in general.



Their core building blocks shown in color are analogous to "guide intervals" used in standard chord

voicing practice. In this practice, guide intervals are usually the 3rd-7th intervals of chord scales. These are the core building blocks of chords I, II and V here, but not of the others. The core building block of a chord in PKP is the most unique building block of the chord in the tonic scale. This is always a tritone for chords containing a tritone. Otherwise it's one of the two fifos of the scale anchored a half tone apart (e.g., fifths anchored by and 𝕄 for the Ionian mode). Think of the bottom or top segments shown in grey as the visible parts of enrichment fifos, of sizes to be determined, snapped onto the core building blocks underneath them at the anchor points (the fifos are all fifths for basic seventh chords going up from these roots, but these are not the only possibilities).

The numeric shape notation on the left is a variation of a standard notation called "figured bass notation" (Appendix B). The variation represents keyboard shapes as lists of numbers that identify the sizes of inter-note intervals in half tones (**1** identifies a half tone, **2** a whole tone, **3** a minor third, **4** a major third, and so on). The lists are called **interval stacks** because the intervals are stacked going up the keyboard. The numbers are the same as the numbers in the symbolic chromatic scale and also for degree numbers that simply count scale notes (1st, 2nd, 3rd, and so on) but the meaning is generally clear from the different textual formats and from context. Basic seventh chords from classical modes are stacks of thirds, either minor thirds (**3**) or major thirds (**4**). In classical modes, the intervals are two scale steps, but this is not so in general for other scales.

Omitting the inner notes of the core building blocks reduces the chords to a core building block with a major or minor third above or below it, with their sizes determined by scale position (for example, for the first chord, the "third" below the core fifth can only be major). The resulting 3-note shapes are thin voicings of the chords that are often sufficient in the context of a piece of music. The missing inner notes are likely to be implied by most contexts because they're the tonic or pitch center of the home octave, or notes a whole tone above them.

This yields at one stroke a way of voicing seventh chords of classical modes starting from core building blocks (add a major or minor third from the scale above or below a core building block), and a way of notating the voicings (add the size number as a suffix or prefix to the anchor). The suffix is different in kind from that defining smaller or larger interval than building blocks **********

Chords From Core Building Blocks

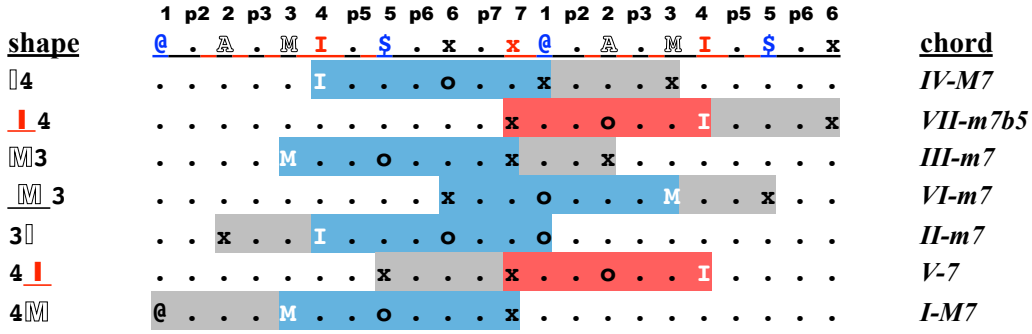
Putting all the core building blocks of the chords in the bottom octave, and rearranging the root order, provides the following revealing picture within the home octave.

	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
shape	@	.	A	.	M	I	.	$	.	x	.	x	@	.	A	.	M	I	.	$	**chord**
4	.	.	.	.	.	I	.	.	.	o	.	x	.	.	.	x	.	.	.	.	*IV-M7*
3	.	.	.	.	.	I	.	.	.	o	.	x	.	.	x	.	.	.	.	.	*VII-m7b5*
M3	.	.	.	.	M	.	.	o	.	.	.	x	.	.	x	.	.	.	.	.	*III-m7*
M3	.	.	.	.	x	.	.	o	.	x	.	.	x	.	.	.	.	.	.	.	*VI-m7*
3	.	.	x	.	.	I	.	.	.	o	.	x	.	.	.	.	.	.	.	.	*II-m7*
3	.	.	x	.	.	I	.	o	.	.	.	x	.	.	.	.	.	.	.	.	*V-7*
4M	x	.	.	.	M	.	.	o	.	.	.	x	.	.	.	.	.	.	.	.	*I-M7*

This is a voicing of the famous "diatonic cycle" in which chord roots go down by fifths (conceptually) starting from ***VII*** (***IV*** is at the end in the down-by-fifths sequence but is put at the

beginning here to put I at the end). The core is the same 1-5-3 for the first three steps and the last three steps. The building blocks and the 3-note shapes symbolized on the left morph from one to the next. The 3-note shapes identify an enriched core that's sufficient to imply the chord in context. Playing these shapes is very easy: two fingers are held and two lowered at each step.

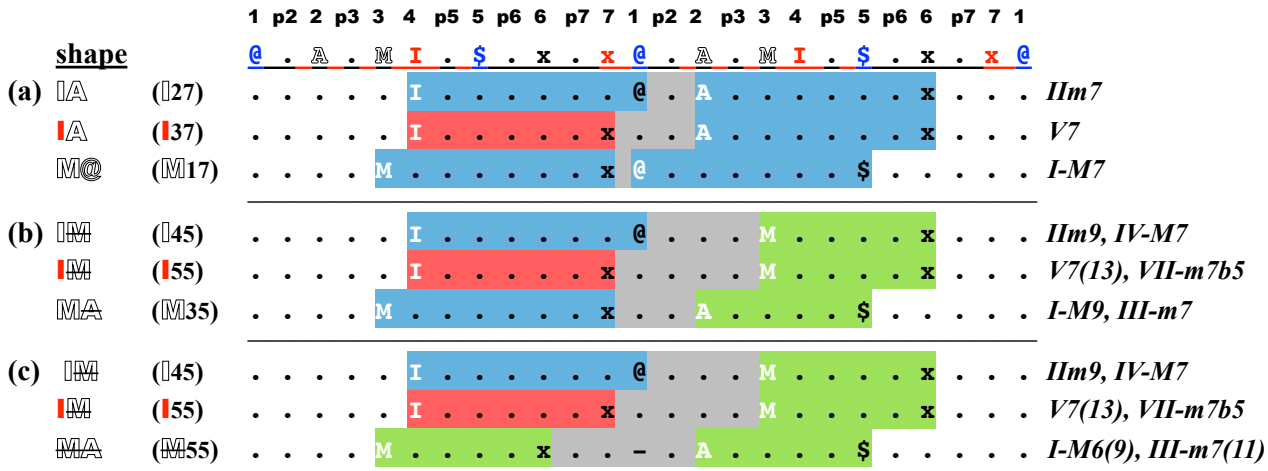
Transforming this so the chords go up from the roots yields the difficult-to-play result shown next. The zig-zags up a fourth and down a fifth are harmonically equivalent to down by fifths. This is difficult to play because all the fingers must be lifted and moved by large jumps, while also adjusting them for "wobbles." Such shape progressions may be understood as sliding a "scale shape" to different positions in the same scale. The scale shape of a seventh chord from a classical mode has two scale steps between successive notes, but many other scale shapes are possible. This is conceptually simple but the practical reality is wobbly slides with different inter-note intervals on the keyboard (two scale steps in a classical mode may be a major or a minor third).



The morphed version within the home octave and the zig-zag slide version that goes outside it are harmonically equivalent. There's obvious benefit in learning the easier-to-play morphed version first and then switching to the other version, if desired, by inverting selected building blocks (which is always easy when building blocks are known by their anchors).

Open Voicings

As shown next, open voicings separate the overlapped building blocks of chords within the home octave, putting them one above the other in adjacent home octaves.



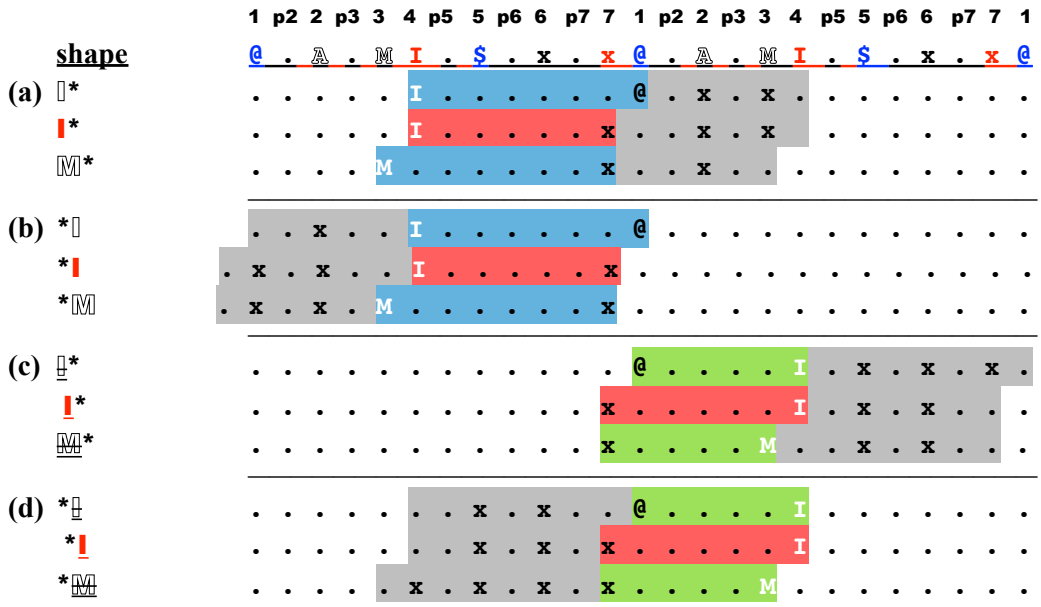
These shapes can be represented by pairs of anchors that are equivalent to the anchor-with-numeric-offsets notation. The figure vividly illustrates the banging-square-pegs-into-round-holes nature of chord symbols. Simple variations in the building blocks yield complex variations in the chord symbols because of having to adjust them to shapes they don't quite fit.

Example (a) is directly from the earlier diatonic cycle. Examples (b) and (c) are simple variations in building block terms that provide voicings of a variety of complex-looking chord symbols. Example (c) is particularly simple: first the middle note moves down a scale step and then all notes move down a scale step. The shapes in the last two steps are "all fourths" (counting the tritone as an augmented fourth). These are far from the only possible variations but they give a good sense of the often deceptively complex nature of chord progressions.

The double anchor notation covers the 3-note shapes described earlier: simply omit the top note of the upper building block. That said, it's simpler to work with core building blocks and offsets from them if 3-note shapes are satisfactory.

Octave Shapes

Octave shapes provide a simple way transforming thin core harmony into 3-note shapes with all notes different. There are four different ways of forming octave shapes from a core, illustrated next for the Ionian core -I-M. Start with the core un-inverted (a, b) or inverted (c, d) and add a voicing note to complete an octave, on the side indicated by the asterisk prefix or suffix. Then move the voicing note to a selected scale note closer to the core (the likeliest choices are marked by x in the grey-shaded areas) to form a 3-note shape that goes well with the melody line. The octave shapes of (a) and (d) or of (b) and (c) have the same notes an octave apart and so are harmonically equivalent. The difference is in the final, less-than-octave shapes. The asterisks could be replaced by numeric offsets as illustrated earlier (e.g., I* of (a) could become I3 or I5) but this is often overly precise because choices are few and tend to be obvious from context.



Playing shapes of this kind is a standard keyboard exercise for beginners but the ability to write

them down in this simple way as shapes that voice chords without using chord or note symbols is novel. Voicings of a wide variety of chord progressions with the same core are easily created this way.

This picture is for single-tritone classical modes but the concept is the same for multi-tritone modes. The details are different because multiple tritones provide a more definite core that requires fewer core fifos to complete (sometimes none). The tritones overlap, which suggests, going forward, voicing them on opposite sides for smooth flow.

DIFFERENT WAYS OF PLAYING HARMONY

Chords that seem very different from their chord symbols may be hardly different at all in the flow of harmony on the keyboard. A dominant seventh chord such as *V-7* and a half-diminished seventh chord such as *V-m7b5*(or a *II-m6* chord with the same notes) differ only in one note that amounts to using an overlapped fifth or fourth to complete the chord. The alteration of the chord symbol is like banging a square peg into a round hole — the result is messy. This example is from the same classical mode. Things only get messier with more general scales.

Early in my musical adventure, I learned about simple "open" voicings of complex chords in a "piano comping" course given by jazz pianist Susan Muscarella at the Jazz School in Berkeley (now the Jazz Institute). The voicings were based on chord progressions that were complex to begin with, making figuring out the voicings in terms of reorganized degree numbers a complex process with difficult-to-remember results.

At the time, jazz pianist Taylor Eigsti was an artist in residence at the Jazz school who, I heard, was teaching a chord-symbol-free way of creating voicings based on representing chords as "scale shapes" determined by counts of scale steps between adjacent chord notes. This resonated with my developing ideas about building blocks, so I contacted him to learn about his method. I learned that he recommends, to beginning jazz piano students, a practicing regime of moving scale shapes of chords to different positions in and between classical modes, without reference to chord symbols. The objective is developing the instinctive moves required of jazz pianists. Although the scale shapes are fixed, the keyboard shapes are wobbly, requiring adjustment of the fingers to play. This combined with lifting all the fingers off the keyboard and moving them by often largish intervals to other keyboard positions makes this style of playing difficult. The method has no notation and is learned only from exercises.

I thought there must be a simple way of notating these approaches in common terms. After much experimentation, I found the way just described.

The big difference from what I learned from Susan Muscarella is core-based representations of chords that are independent of chord symbols (apart from identifying tritones from the chord table in Appendix C); Chapter 4 presents examples from her comping course in full.

The big difference from Taylor Eigsti's method is a notation for wobbly slides on the keyboard that covers all kinds of shapes from all kinds of scales. Given that wobbly slides in keyboard terms are the general case, it seems useful to start out thinking in their terms, using a notation that represents them.

The examples so far have all been from the Ionian mode but the concepts are general. The general case is actually simpler because more tritones identify more scale notes.

HARMONY WORKSHEETS

The Lego-like views of harmony provided in many figures illustrate how *given* anchor lines may be interpreted on the keyboard. The figures are only for explanation of the notation, and are helpful for forming mental models.

That said, they suggest **worksheets** for *figuring out* anchor lines to provide desired harmonic

sequences. All that's needed for this is pages marked off with vertical and horizontal grid lines. The columns created by the vertical grid lines identify the half tones of the home octave. The rows identified by the horizontal grid lines identify successive bars and positions within bars. Entries penciled into the columns are sufficient for identifying building blocks. The colored shading in the illustrative views is not needed for this. Translating such worksheets into music notation can be useful in particular cases and is so straightforward that it could easily be automated. However, describing such translations is outside the scope of this document.

TWO SIDES OF THE SAME COIN

The table of parallel classical modes shown earlier demonstrates that parallel mode changes and tonic changes for the same mode are two sides of the same coin. The two sides are summarized in the following simple table. Everything follows from word changes in the alphabet (single letter changes in the case of classical modes covered by the table, but the concept is general)

word change	// mode change	altered notes	same-mode tonic change (same direction)	altered notes	same-mode tonic change (opposite direction)	altered notes
± 1 step	next mode	1	half tone	5	fourth	1
± 2 steps	skip one mode	2	whole tone	2	maj third	4
± 3 steps	skip two modes	3	minor third	3	minor third	3

In the table, changes of single letters words up or down a half tone (1 step), whole tone (2 steps) or minor third (3 steps) that don't settle on intermediate modes cover all possibilities. Changes in opposite directions always add up to a tritone. Cells outlined in red identify the cases that alter the fewest notes. That said, changes that alter more notes may occur.

Inspection of the earlier parallel-mode table shows that the tritone anchor change may be interpreted as an Ionian-Aeolian parallel mode change down a minor third (**//I-//A**) or an Ionian-Ionian tonic change up a minor third. The former is a change from major to minor of the same tonic that's one of the most fundamental changes in music. It's simplicity is directly visible here, but deeply obscured by music notation, which requires an explicit or implicit change of key signature that can be complex (e.g., 5 flats to 4 sharps, with different note symbols for the same home tonic). The tonic change is by interpretation: the **A** tritone is interpreted as an inversion of an "**I**" tritone relative to a secondary tonic a minor third up. No change in notation is required for the tonic change because the scale notes are the same, except in a different order. The secondary tonic up a minor third is determined by a melody line, which determines the scale order. This example assumes the secondary Ionian tonic is in the **//A** mode. If it isn't, **alt-A** supplies the scale of the secondary Ionian tonic.

Cascaded mode or tonic changes, known to jazz musicians as "Rhythm Changes" because of their origin in Gershwin's *I Got Rhythm*, are determined in PKP by alphabet sequences such as **A-P-L-I**, which moves down the alphabet as a circular loop (**A-P** down wraps around to **L-I** down).

CHAPTER 3: DETAILS DEVELOPED FROM BASIC PRINCIPLES

This chapter fills in notational details independently of specific pieces of music. The next chapter fills them in by exploring example pieces. The concepts needed to understand the examples are sufficiently well explained in the previous chapter that proceeding to the next chapter with only a quick skim of this one to see what's in it should be possible, returning here if and when needed.

DICTIONARY OF MODE SIGNATURES

The following short dictionary of tonic scales identified by mode signatures covers in half a page all the scales and more in scale dictionaries such as *The Source*. The scales above double line are atonal, with symmetric shapes (same interval sequence going up and down). The scales below it are tonal, with asymmetric shapes. These scales are completed by populating the anchor sets in the lower fifth of the scale frame with fifo-only anchors. The letters **ADMI** are fundamental to these anchor sets, with different omissions of letters or different identifications of them as fifo-only anchors determining different scales. Minor-third intervals of scales are shown as solid lines to make them stand out.

Parallel mode changes and tonic changes are two sides of the same coin; the difference is generally indicated by the melody line. These are by no means the only possible scales but the dictionary is easily extended if anyone sees a need.

signature	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	scale type
PADMIL	x P A D M I L x x x x x x	**chromatic (12 notes)**
P.DM.L	. P . D M . L x . x x . x	**diminished (8-notes, min-maj)**
AD.IL	. . A D . I L . x x . x x	**diminished (8 notes, min)**
PA.MI	. P A . M I . x x . x x .	**diminished (8 notes, maj)**
ADMI	. . A D M I———–x x x x .	**no name (8 notes, min-maj)**
A.M.L	. . A . M . L . x . x . x	**whole tone (6 notes, maj)**
P.D.I	. P . D . I . x . x . x .	**whole tone (6 notes, min)**
DM.L	@ . A D M I L $. x x . @	**min-maj blues family (9 notes)**
AD.I+	@ . A D . I . $ x x + x @	**minor family (9 notes)**
A.MI+	@ . A . M I . $ x + x x @	**major family (9 notes)**
DM	@ . A D M I . $. x x . @	**pentatonic union (8 notes), a basic blues scale**
A..I	@ . A D M I . $ x———–x @	**harmonic minor-major (8 notes)** **(master of //PM,AI,DL,MP,IA,LD)**
D.I	@ . A D . I . $. x . x @	**melodic minor (7 notes)** **(master of //PD,AM,DI,ML,IP,LA)**
I	@ . A . M I . $. x . x @	**Ionian (7 notes)** **(master of //P,A,D,M,I,L)**
pentatonic	@ . A . M———–$. x———–@ @———–D . I . $———–x . @	**foundation**

The focus of PKP is tonal music but atonal scales are included because shared letters of their

defining words make them visible structural parents of scales lower down in the dictionary; and also because they may be used ornamentally in tonal music. Atonal scales have no minor, major or minor-major tonality by themselves but representing them by words that place them in the context of the home tonic gives them the relative tonality shown on the right (the presence of **D**, **M** or **DM** identifies minor, major or minor-major tonality).

The prefix "**//**" that distinguishes parallel modes determined by words from the tritones determined by the same words is only really needed for words of one or two letters because two tritones is the practical limit for chords containing tritones. Beyond that, no confusion of meaning is possible. That said, it's useful to use the prefix for clarity.

There are no 5-letter words because the implied scales would be too close to the chromatic scale to be usefully distinguished from it. As scale identifiers, single-letter words are reserved for scales with seven or more notes, which includes classical modes but not 6-note blues scales containing single tritones that are understood as sub-scales of higher level blues scales.

The zone within the dictionary ranging from 4-letter words down to 2-letter words that identify parallel modes is complex in music notation. Scales must be understood as enumerated notes relative to one of more than twelve possible different key signatures. The role of classical modes as the defining scales of key signatures makes music from scales that differ markedly from them misleadingly complex. The dictionary makes this zone simple by pushing note symbols and interval inversions down to a lower conceptual level. The three scales in the middle formed around three asymmetric triple tritones provide important families of tonal scales.

SCALE FAMILIES

Scale families provide a simple, unified view of a very large number of scales of music notation of different kinds in different key signatures that are, in aggregate, overwhelmingly complex for all but experts. These scales are built up from a foundation of parallel pentatonic minor and major modes, the scales of simple "folk music" in many cultures worldwide. The classical modes that provide the alphabet were developed in the previous chapter from these modes by splitting their minor-third intervals. Scales beyond these follow from forming unions of parallel modes of opposite tonalities.

Blues Family

The union of the pentatonic major and minor modes with an added note that splits the octave into geometric halves yields a 9-note blues family scale.

	1 p2 2 p3 3 4 p5 5 p6 6 p7 7 1	
pentatonic major	@ . A . M———–$. x———–@	**foundation scales**
pentatonic minor	@———–D . I . $———–x . @	
8-note union (//DM)	@ . A D M I . $. x x . @	
add geometric octave split	x . . . . . x x	
blues family scale (//DM.L)	x . A D M I L $. x x . x	

The 8-note union is a natural consequence of "bending" the tonality-establishing notes of simple pentatonic music downwards to give a major piece a sad twist, or upwards to give minor piece a happy twist. The bent notes in the lower fifth join with pentatonic notes in the upper fourth to form two tritones. Also bending the pitch center adds the center note of the geometric octave split seen earlier, which forms a tritone with the tonic. The result is a 9-note scale with three tritones and five adjacent

half tones that differs markedly from the classical modes that are the scales of key signatures, which have seven notes, one tritone and no adjacent half tones. This explains why blues pieces are often complex in music notation (e.g., the footnoted chord progression in the opening chapter).

I learned the 9-note scale in conventional terms some years ago in a blues piano workshop at the then Jazz School in Berkeley, but this intuitively simple way of understanding and remembering it is my own invention and appears to be novel. Pianists and music teachers I have asked don't seem to know about it. Books I have consulted don't say anything about it.

The three tritones of **DM.L** determine six notes, the scale frame adds one note (the **L** tritone already includes the tonic), and the other two non-tritone notes are from the foundation pentatonic scales. This scale provides many sub-scales that are stalwarts of blues pieces, shown next listed in order of decreasing tritone content.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
sub-scales of //DM.L	**1.**	x	.	A	D	M	I	L	$	.	x	x	.	x	**| lines 5+6**
//DM (8 notes)	**2.**	@	.	A	D	M	I	.	$	.	x	x	.	@	**| lines 9+10**
//D..L (7 notes)	**3.**	@	.	A	D	.	I	L	$	.	x	x	.	@	**| // mode of harmonic minor**
//M.L (7 notes)	**4.**	x	.	A	.	M	I	L	$	.	x	x	.	x	**| // mode of melodic minor**
minor blues (6 notes)	**5.**	@	–	–	–	D	.	I	L	$	–	–	–	x	. x **| line 9+L (not //L)**
major blues (6 notes)	**6.**	@	.	A	D	M	–	–	–	$	.	x	–	–	– @ **| line 10+D (not //D)**
//D (Dorian) (7 notes)	**7.**	@	.	A	D	.	I	.	$	.	x	x	.	@	
//M (Mixolydian) (7 notes)	**8.**	@	.	A	.	M	I	.	$	.	x	x	.	@	
pentatonic minor (5 notes)	**9.**	@	–	–	–	D	.	I	.	$	–	–	–	x	. @
pentatonic major (5 notes)	**10.**	@	.	A	.	M	–	–	–	$	.	x	–	–	– @

A contrasting top-down view to this bottom-up view is provided by the following hierarchy of scales and sub-scales starting from parent scale **P.DM.L** (an atonal scale called "diminished"). This scale morphs into the **DM.L** scale (the **P** tritone morphs into a fifo while holding its upper note, the pitch center, fixed). The morphing breaks the symmetry of the diminished scale, yielding an asymmetric tonal scale that contains the other scales as sub-scales that follow from pruning notes.

P.DM.L

|| morph

DM.L

**/ | ** sub

DM M.L D..L

\ | / sub

D, M

The words **M.L** and **D..L** determine parallel "melodic" and "harmonic" modes, (described later) that share with classical modes the properties of seven notes and no adjacent half tones.

Conspicuously missing are tritones **A** and **I** from the classical modes conventionally regarded as

the default minor and major modes of key signatures. Fifo anchors 𝔸 and 𝕀 identify in-scale fifos but tritones anchored by **A** and **I** are ornamental extras.

Blues has high tritone content but it also has high fifo content (five possible fifo anchors from the **DM.L** scale). A strong blues sound follows from core harmony that has correspondingly high tritone content (for example, the footnoted chord progression of *Goodbye Porkpie Hat* in the opening chapter). Weakening the core tritone content in favor of in-scale fifos weakens the blues sound (see *When Sunny Gets Blue* in Chapter 4 for an example).

Minor and Major Families

The union of the parallel Aeolian and Ionian modes forms a 10-note scale of mixed minor-major tonality determined by the word **ADMI** that is a master scale of family scales of major and minor tonality. This scale reduces to 9-note minor or major family scales by morphing the **M** or **D** tritone into a fifo with the same top note. The morphed scales are identified by the augmented words **AD.I⁺** or **A.MI⁺** where the plus superscript indicates the top note of the missing tritone is retained. The result is both interesting and simple: an all-half-tone top end for both scales, with bottom ends of different tonalities.

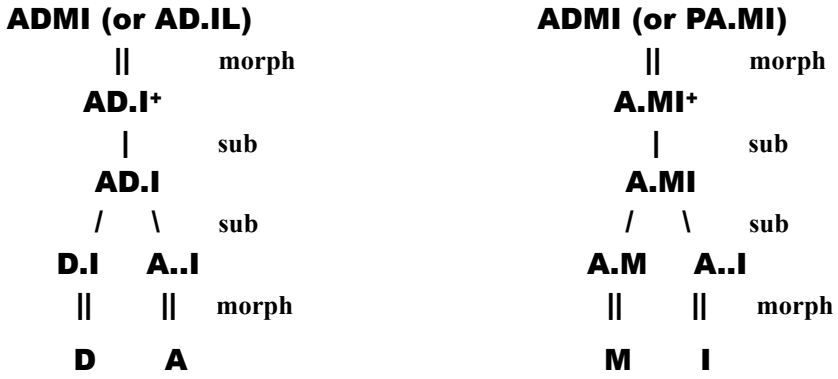
		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
Ionian mode	**//A**	@	.	𝔸	.	𝕄	I	.	$	.	o	.	x	@	**foundation scales**
Aeolian mode	**//I**	@	.	A	𝔻	.	𝕀	.	$	x	.	o	.	@	
10-note union	**//ADMI**	@	.	A	D	M	I	.	$	x	x	x	x	@	
minor family scale	**//AD.I⁺**	@	.	A	D	.	I	.	$	x	x	+	x	@	**morph major tritone**
major family scale	**//A.MI⁺**	@	.	A	.	M	I	.	$	x	+	x	x	@	**morph minor tritone**

As for the blues, the 9-note scales provide sub-scales shown next that are stalwarts of minor and major music.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	
sub-scales of AD.I⁺		@	.	A	D	.	I	.	$	x	x	+	x	@
AD.I	**bebop melodic minor**	@	.	A	D	.	I	.	$	x	x	.	x	@
//A..I	**harmonic minor**	@	.	A	𝔻	.	I	.	$	x	.	.	x	@
//D.I	**melodic minor**	@	.	𝔸	D	.	I	.	$	.	x	.	x	@
//D	**Dorian**	@	.	𝔸	D	.	𝕀	.	$	.	x	+	.	@
//A	**Aeolian**	@	.	A	𝔻	.	𝕀	.	$	x	.	+	.	@
sub-scales of A.MI⁺		@	.	A	.	M	I	.	$	x	+	x	x	@
A.MI	**no specific name**	@	.	A	.	M	I	.	$	x	.	x	x	@
//A..I⁺	**bebop major**	@	.	A	.	𝕄	I	.	$	x	+	.	x	@
//A..I	**harmonic major**	@	.	A	.	𝕄	I	.	$	x	.	.	x	@
//A.M	**// mode of melodic minor**	@	.	A	.	M	𝕀	.	$	x	.	x	.	@
//M	**Mixolydian**	@	.	𝔸	.	M	𝕀	.	$	.	+	x	.	@
//I	**Ionian**	@	.	𝔸	.	𝕄	I	.	$	.	+	.	x	@

The minor and major hierarchies are slightly more complex than the blues hierarchy because **ADMI** is not the only parent atonal scale, and the classical modes at the bottom are morphs of the

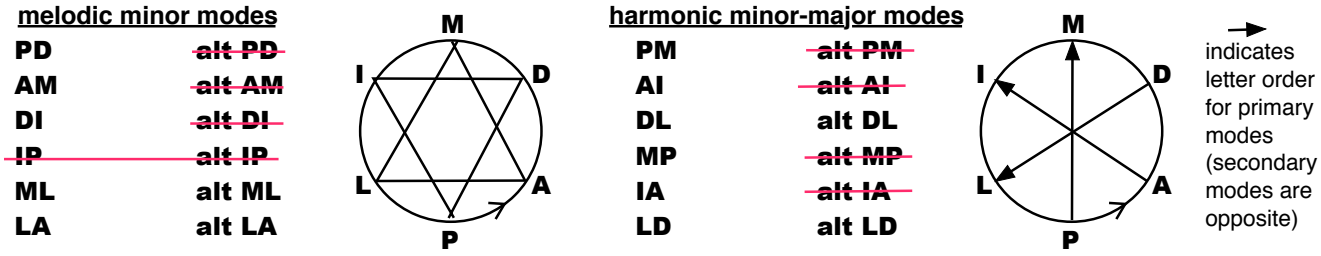
modes above them, not sub-scales (they're sub-scales of **AD.I⁺** and **A.MI⁺**).



The double tritones **D.I** and **A..I** on the left, and **A.M** and **A..I** on the right, determine parallel "melodic" and "harmonic" modes that share with classical modes the properties of seven notes and no adjacent half tones.

PARALLEL "MELODIC" AND "HARMONIC" MODES

The concept of parallel modes applies to any scale — they're rotated transpositions of a master scale — but its systematic development in music notation is restricted to single- and double-tritone modes identified by the terms "melodic" and "harmonic" that share with classical modes the properties of seven notes and no adjacent half tones. In PKP, the parallel modes are determined by transpositions of a master word within within the alphabet as a circular loop (meaning wrapping the letters around to the other end of the alphabet). The transpositions may be understood graphically as shown next (spacer dots are omitted for brevity).



crossed-out modes don't contain the home tonic but may be used ornamentally

The melodic and harmonic modes are different in kind because each of the words is unique for the former and half of the words are repetitions of the other half in the opposite order for the latter. This is because the double tritones of the latter are circularly symmetric (all inter-note intervals are minor thirds in any inversion). The harmonic modes can be dauntingly complex in conventional terms (Appendix D) because the irregularity of the master mode makes rotated transpositions doubly irregular. The irregularity can be finessed by making the master mode mixed minor-major, leaving the choice of a minor or major master to context. The choice is a minor detail, except for the master mode itself.

The three words **A.M**, **D.I** and **M.L** that fall out of the earlier scale hierarchies determine four

parallel modes of the melodic minor mode identified by **D.I**. The three words determine four modes because any word containing **L** determines a primary parallel mode and an **alt** mode with all non-tritone notes different.

The two words **A..I** and **D..L** that also fall out of the foregoing scale hierarchies determine six possible parallel modes of a harmonic minor-major master scale (two modes for the former and modes for the latter).

Details follow for information, but most of the modes fall directly out of blues, minor or major family scales.

Parallel "Melodic" Modes

The master mode is conventionally understood to be the melodic minor identified here by **D.I** but but **A.M** is logically just as valid. In any case, a simple way of knowing them is as morphed whole tone scales. Whole tone scales are determined by **A.M.L** and **P.D.I** are the simplest keyboard shapes possible (5 stacked whole tones). The **A.M**, **M.L** and **L.A** modes are morphed from the **A.M.L** scale; the **P.D**, **D.I** and **I.P** modes are morphed from to **P.D.I** scale. The morphs split one of the whole tones into two half tones. Here follows a summary of the parallel modes that highlights the splits. The bottom note of the split is the starting note of a transposed melodic minor scale.

mode	source	1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	tonality determiners highlighted
alt-LA	AML	x	.	A	D	.	I	L	.	x	.	x	.	x	
//LA	AML	x	.	A	.	M	.	L	.	x	x	.	x	x	
✖ alt-IP	PDI	.	P	.	D	.	I	.	x	x	.	x	x	.	pseudo-parallel mode (Phrygian-*b1*)
✖ //IP	PDI	.	P	A	.	M	I	.	x	.	x	.	x	.	pseudo-parallel mode (Ionian-*#1*)
alt-ML	AML	x	P	.	D	M	.	L	.	x	.	x	.	x	
//ML	AML	x	.	A	.	M	.	L	x	.	x	x	.	x	
//DI	PDI	@	.	A	D	.	I	.	$	.	x	.	x	@	
//AM	AML	@	.	A	.	M	I	.	$	x	.	x	.	@	
//PD	PDI	@	P	.	D	M	.	L	x	.	x	.	x	@	

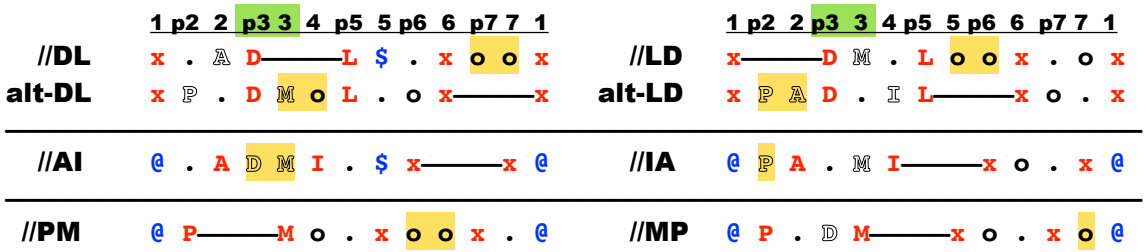
The **//IP** mode marked by ✖ isn't a proper parallel mode because it doesn't include the tonic but it's included as a pseudo-parallel mode because it's so close to being one (it's the Ionian mode with the tonic raised a half tone). The **//IP** pseudo mode has major tonality and is partnered by an **alt-IP** mode of minor tonality with all non-tritone notes different.

Parallel "Harmonic" Modes

Making the minor-major mode the master reduces fourteen parallel modes that are often intricate and difficult to comprehend or remember, into the eight simple parallel modes summarized next.

The circular symmetry of the double tritones of these modes makes them remarkably simple on the keyboard — a stack of three minor thirds. One of the minor thirds is empty in the scale and the one a minor third away from it is, for a minor-major master, filled with two notes a half tone apart. Choosing a minor or major or minor master boils down to omitting one of these notes (candidates for omission are highlighted in yellow — sometimes there's only one possible choice). Except for the master mode itself, the choices don't affect the tonality of the result. This simplifies thinking because the difference between a major or minor master mode is a detail that can be left to context. An example from the blues

follows.



Examples of **//DL** and **alt-DL** modes from the blues are shown next (the examples of conventional names for them are from Appendix D).

CHAPTER 4: DETAILS DEVELOPED VIA EXAMPLE PIECES

This chapter develops the details via example pieces, avoiding the use of music notation to keep things simple and independent of specific tonics. The next chapter provides a few example pieces in annotated music notation, culminating in the piece with the footnoted chord progression in the opening chapter.

My hope for the example chapters is that the visible simplicity and depth that PKP provides for each of many different kinds of pieces will accumulate a weight of evidence to convince skeptics that the approach is both correct and practically useful. Practicality is demonstrated by the simplicity and depth of the annotations needed on the written music. The annotations are independent of the actual home tonic of a piece, enabling lessons and patterns learned from one piece to be applied to other pieces with the same or different home tonics. The meaning of the annotations in keyboard terms is demonstrated by tabular representations of Lego-like harmony shapes on the keyboard. The resulting harmony isn't itself novel, the only novelty is in the unconventional representation.

This chapter presents a selection of example pieces in these terms. The examples are independent of each other and so may be approached in any order. That said, they get progressively more sophisticated as the chapter proceeds. I worked them out at different times, sometimes years apart, so they sometimes present things in slightly different ways, but the ways are always consistent.

The examples are intended to be understood by playing them, not just looking at them. Start with the melody line and the tritone core to establish basic flow. Play the tritone core as octave shapes, to add depth to the sound and mute tritone dissonance without introducing any additional notes. There are four ways of doing this: un-inverted or inverted tritone core, with an added tritone below or above the core to form an octave shape. One of these ways suffices to get of the ground for any piece, but experimenting with different ones helps to understand the different choices made for different pieces. Then play the actual choices, including interpolated fifo shapes. The simplicity of finding flow "bottom up" from core harmony is in stark contrast to the complexity of finding it "top down" from chord symbols.

Understanding the relationship between melody and harmony always benefits from starting with the melody first because it determines the home tonic and any secondary tonics, and often suggests harmony scales.

Tonic changes and parallel-mode changes are two sides of the same coin, so identifying scales of secondary tonics as parallel modes of the home tonic is sufficient; scales of secondary tonics identified by melody lines are understood to be relative modes of the corresponding parallel modes (same notes).

A large range of different types of melody and harmony scales are covered by examples in this chapter. Many pieces can be understood in terms of specific mode scales with ornamentation. PKP visually distinguishes the types in a direct and simple manner. For example:

1. Melody and harmony use the same scale.
2. Melody and harmony use different scales.
3. Melody and harmony share a scale family but use different elements of it in different ways.
4. Melody is in a definite scale and harmony is ambiguous.
5. Harmony is in a definite scale and melody is ambiguous, in which case harmony may imply a melody scale.
6. Neither melody nor harmony seem to be in definite scales.

A SIMPLE BLUES

The simple piece called *Backwater Blues* provides surprisingly deep insight into a number of things that are complex in music notation. It's a 3-chord, 12-bar blues in F, which I learned some years ago in a blues piano workshop at the then Jazz School in Berkeley, as representative of "probably half the blues pieces played by pop and jazz musicians." A summary of the melody and core harmony is shown next. The melody is in the pentatonic minor scale except for bar 9 that adds note **p5** (tritone anchor **L**) to form the 6-note, minor blues scale visited in reverse in bar 9. Opposite tonalities in melody and harmony illustrated by the opening **M** bar (and implied for improvisations in all **M** bars) are a common feature of the blues. The opposite tonalities may also be the other way round (improvised major melody in the **D** bars). Another common feature of the blues is all-tritone core harmony.

⚓+🎵: //DM.L

⚓	M	D	M	M
🎵	| 5↘4↘p3↘1,	| ↗5↘4↘p3↘1 | 1 ,	| —	|
⚓	D	D	M	M
🎵	| ↗5↘4↘p3↘1,	| ↗5↘4↘p3↘1 | 1 ,	| —	↗5↗p7↘5 |
⚓	I	D	M	M
🎵	| ↗p7*, —	↘5 | ↗1↘p7↘5↘p5↘4↘p3↘ 1 | 1 ,	| —	|

Worksheets for un-inverted and inverted cores are shown next.

1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
@	.	A	D	M	I	L	$	.	x	x	.	@	
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
.	.	x	.	.	I	.	–	.	.	.	+	.	*V7*
@	.	.	D	.	–	.	.	.	x	.	.	.	*IV7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*
@	.	.	.	M	.	.	.	.	.	x	.	.	*I7*

5	p6	6	p7	7	1	p2	2	p3	3	4	p5	5	
$	.	x	x	.	@	.	A	D	M	I	L	$	
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	.	+	.	.	.	.	.	I	.	$	*V7*
.	.	x	.	.	.	.	.	D	.	–	.	$	*IV9*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*
.	.	.	x	.	–	.	.	.	M	.	.	$	*I7*

The simple voicing lines shown satisfy the chords shown. This simplicity works because there's

so much harmonic variety in the tritones that the voicing lines only have to add depth. Omitted chord notes are identified by dashes. The anchor symbols in the shorthand summary could have prefixes or suffixes to identify the bass or treble lines, but the lines are so simple it's hardly worth it. Keep in mind that chord root ***I*** and anchor symbol **I** (shown as **I** in the Lego views) mean different things.

The previous chapter gives **DM.L** as the mode signature of general 9-note blues scale that's represented in chromatic scale notation as **1-2-p3-3-4-p5-5-6-p7-1** (tritone notes in red, pitch center in blue, completion notes in black): think of it as the chromatic scale without notes **p2**, **p6** and **7**. This is not a new scale introduced by PKP but only a novel way of representing a known scale by a mode signature. The same blues workshop that taught me this piece also taught me this general blues scale in conventional terms; mode signatures came later.

The melody scale of this piece is visibly a sub-scale of the **DM.L** scale. Harmony tritone **I** stands out like sore thumb as not in this scale (it's anchor is a fifo-only anchor). The single appearance of this tritone in bar 9 provides a harmonic "turnaround" marker indicating the start of the last four bars (useful for improvisors). The in-scale tritone **L** could provide this marker but tritone **I** forms a familiar dominant seventh chord on root ***V***, providing a simple chord progression of all dominant-seventh chords.

Chord substitution is a favorite trick of jazz musicians, often having the contrary effects of simplifying the keyboard flow of the harmony while complicating its symbolic representation in chord notation. In PKP terms, the basic form of chord substitution boils down to holding the core and changing other notes. For example, raising the bass note a whole tone for the last four bars on the left voices the chord sequence ***V7-IV7b13-I9-I9*** (among other possible interpretations). Altering the treble line for the sequence at bottom right to a minor third above the inverted core yields a voicing of "tritone substitute" chords on a root line a tritone away from the original, namely ***bII7-VII7-bV7-bV7*** (among other possible interpretations). A tritone substitute chord is a tritone chord with its root a tritone away, containing the same tritones in opposite inversions. Other possibilities have different chord root lines for different types of chords.

A more advanced kind of chord substitution alters the core. An example is substituting tritone **I** for blues tritone **L** in bar 9. The V-chord with tritone **L** is ***V-M7(11)*** which is different in kind from the other chords. The substitution turns it into ***V-7***. The substitution is different in kind from that achieved by substituting harmonically equivalent or mutually consonant fifos: tritone **I** is made **functionally equivalent** to tritone **L** by its position in the core sequence.

Make enough substitutions and symbolic chord progressions can quickly become difficult to comprehend for all but experts. Chord progressions become complex, but thinking and working in terms of Lego-like building blocks instead of chord symbols and chord scales is child's play in comparison.

The "swing feel" with which such pieces should be played is a performance property not represented by the notation (e.g., bounce downbeats off delayed upbeats). The general scale has many sub-scales that may be used for improvisation beyond the two used in this example.

Context

An all-tritone core containing the tritones of the **DM.L** mode signature establishes a solid blues context. That said, blues pieces often include fifos in core harmony. The scale provides an ample supply of fifos because its anchor set **ADMIL** includes all possible fifo anchors from the alphabet

except 𝙿. The problem is fifos don't establish a definite context because there are twice as many of them as tritones and they're widely shared between different kinds of tonic scales. Too many of them in core harmony weakens the definitive look and sound of a blues, and indeed of any music from a multi-tritone scale. Combine this kind of harmony with an ambiguous melody line and the piece as a whole becomes musically ambiguous. This may be a desired musical effect but it makes learning and remembering such pieces difficult because of the lack of definite guiding context.

More tritones in core harmony make it not only simpler on the keyboard but also more definite, musically. Ignoring repetitions and replacing **I** by **L**, for reasons just explained, the core tritone sequence is **M-D-M-D-M-L-D-M**. The order is not as important for the blues sound as is the presence of the tritones in some order. The actual order is determined by the melody line.

BLUES WALKING BASS LINES

Walking bass lines provide an alternative to chordal harmony. They're ubiquitous in blues. The following simple walking bass line (indicated by ▼) is from Monk's *Straight No Chaser* (coming up). The line is very easy to learn and remember and may be used for many blues pieces. The notes are all on downbeats. No up or down arrows to mark zig-zags are needed because the next note is always the nearest one up or down. The notes of the tritones from the **DM.L** mode signature are highlighted in red to bring forward their strong presence in this harmony in a distributed fashion over time.

▼ |1-2-p3-3	|4-p3-2-p2	|1-3-5-6	|p7-6-5-3
▼ |1-2-p3-3	|4-p3-2-p2	|1-3-5-6	|3-p7-6-3
▼ |2-3-4-p5	|5-6-p7-7	|1-2-p3-3	|1-2-p3-3

The use of this line with *Backwater Blues* is shown next. Interleaving the bass and melody lines in the way suggested by long and short dashes enables the lines to cue each other, and avoids a few direct dissonances. The bass line is on down beats, melody notes in blue text are on upbeats. This is very easy and satisfying to play.

♫ | 5-4—p3—1	| 5-4—p3—1_____| 1—	| —	
▼ 1—2—p3—3	4—p3—2—p2	1—3—5—6	p7—6—5—3
♫ | 5-4—p3—1	| 5-4—p3—1_____| 1 —	| ———— 5-p7-5	
▼ 1—2—p3—3	1—p3 —2—p2	1—3—5—6	3—p7—6—p3
♫ | p7-p7————5	| 1-p7-5-p5-4-p3—1_____| 1—	| —	
▼ 2——3—4—p5	5——6—-p7——7	1—2—p3—3	1—2—p3—3

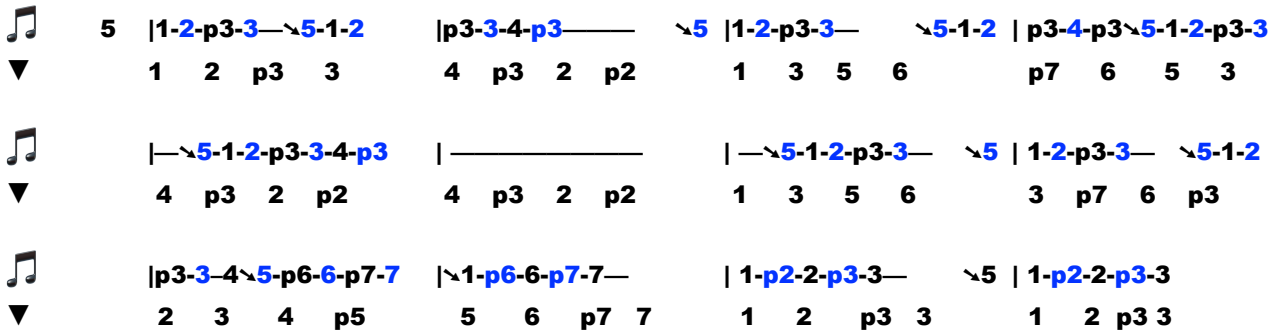
Walking bass lines are replacements for PKP anchor lines, not additions to them. Annotating both above the staff is not practical for most written music because there isn't enough space. Anchor lines are basic so where do walking bass lines go? One answer is to put them in a separate notebook of walking bass lines cross-referenced to different pieces.

A COUPLE OF MONK EXAMPLES

Presented here are two Monk blues pieces with walking-bass harmony. The lines have few direction change arrows because few are needed. The lines mostly move steadily in one direction or the other.

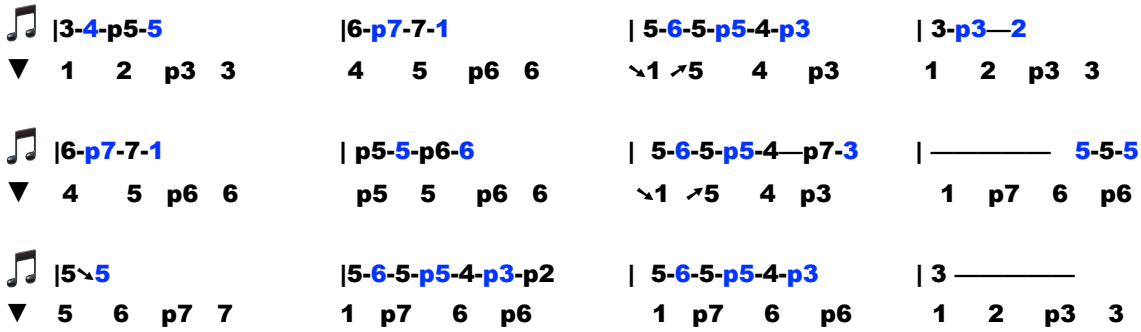
Straight No Chaser

A walking bass line was borrowed above from Monk's blues in F called *Straight No Chaser*. The original is shown below (melody notes on upbeats highlighted in blue to bring the simple pattern forward to the eye). A very simple melody line is combined with a very simple walking bass line, dovetailed such that the whole sounds more than the sum of the parts: the melody and harmony notes cue each other. The passing dissonances on a few downbeats are typical of the blues and sound right because they anticipate notes to come (and repeat earlier notes).



Blue Monk

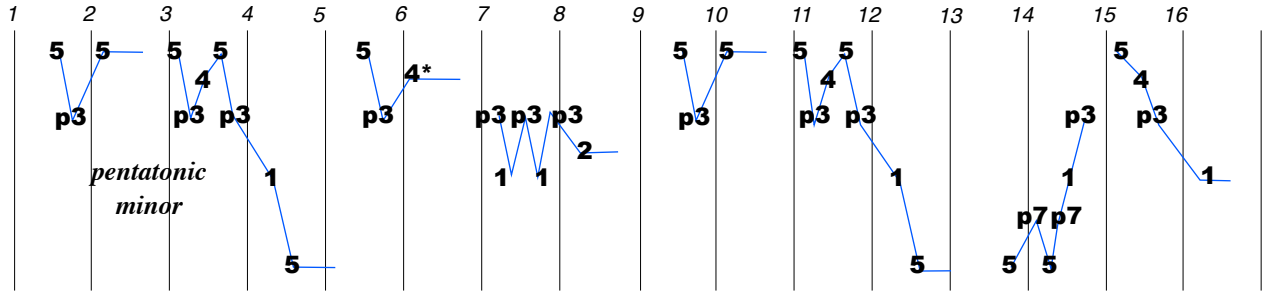
Other Monk blues pieces are useful sources of model walking bass lines, to say nothing of providing much playing pleasure. *Blue Monk* in Bb an example (same format as above).



In the written piece, another harmony line tracks the melody line just shown so closely that the effect is of a double melody line, as shown below for the first 4 bars. The lower line uses bass-line note sequences shifted to melody-line beats. The result is a mix of minor and major thirds between the lines that don't have to be held in mind because the easy-to-remember lines deliver them. Knowing this, the rest of the double melody line (not shown) is easy to figure out.

A FAMILIAR MINOR PIECE

I learned *Summertime* in D-minor (one flat) with associated interesting harmony some years ago in Susan Muscarella's Piano Comping course at the then Jazz School in Berkeley. The melody scale is plain pentatonic minor.



Shown next is a summary of the melody and harmony. The governing harmony scale is the minor family scale with mode signature **AD.I⁺**, which is a 9-note extension of the **AD.I** scale known to jazz musicians as "bebop melodic minor." The plus means retain the upper note of the omitted **M** tritone (think of the tritone as morphed into a fifth anchored by 𝔻 or a fourth anchored by 𝔸). This scale has many sub-scales identified by sub-words: from double-tritone intermediate modes identified by **//D.I** (melodic minor) and **//A..I** (harmonic minor) down to classical modes **//A** (Aeolian) and **//D** (Dorian). The melody scale is sub-scale of **//A**. The only departure from these scales is in bar 5, where an 8-note minor-major scale identified by **DM** comes into play briefly (recall that this scale is formed of the union of the pentatonic major and minor scales); this scale provides an ornamental transition from melodic minor to natural minor (Aeolian). Annotating **AD.I⁺** at the beginning as the governing mode is sufficient, but for illustrative purposes, the individual scales are annotated above the applicable segments. The chord root line is the one I learned in the course but many others are possible for the same core. Only an anchor is specified for the concluding ***I*** chord

⚓: //AD.I⁺, 🎵: pentatonic minor

IV V IV I I II V

//DI→ //DM→ //AI→

⚓ D I D M D A A..I

🎵 | - 5↘p3 | ↗5 | 5, ↘4↘p3↗4↗5↘p3 |↘1↘5 | 5, ↗5↘p3 |↗4 - 4 | 4,↘ p3↘1↗p3↘1↗p3 |↘2, |

II pVII V I

//A→ //AI→

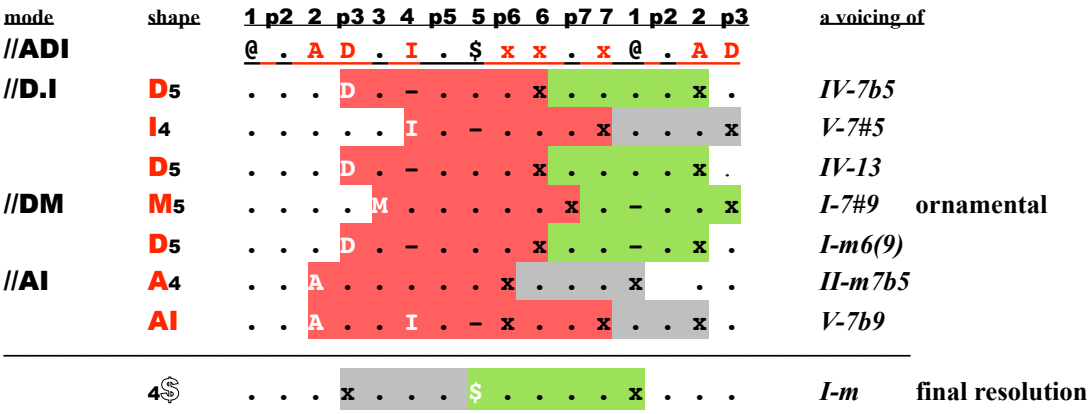
⚓ A A AI $

🎵 | ——————repeat bars 1-4—————— | 5↗p7↘5↗p7↗1↗p3 , | ↗5↘4↘p3 - |↘1 |

Shown next is a 3-part voicing of the tritone core of bars 1-8 that voices chords on the assigned roots shown in blue (double tritones are voiced here by three notes just to show how). All the voicings but one are rootless (the missing roots are identified by dashes). The simplicity of the building block

view stands out in contrast to the complexity of the implied chords. Creating these shapes is no more complex than what a child does when it snaps plastic building blocks together. The harmony is only 3-part but the chords are satisfied partly in place and partly in the flow. The voicings are all rootless except for the II chord just before the end but this is no problem because the roots are implied by the joint flow of melody and harmony. Counting tritones as augmented fourths, four of the seven shapes may be said to be "all-fourths." The other three tritones have either a major third on top (it and its inversion, an augmented fifth, count as a fifos) or a minor third on top (one case) that completes the double tritone of the **AI** mode.

The ornamental insertion of **M** provides a transition from melodic minor to natural minor via a wavelike movement up and down by a half tone. This amounts to using the 8-note **DM** mode as an ornamental scale.



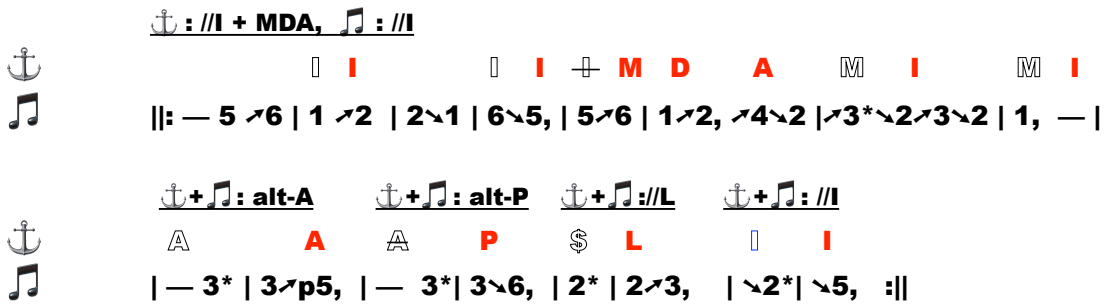
Nothing could better illustrate that taking chord symbols from classical modes and adjusting them for non-classical modes is like "banging square pegs into round holes." As shown below, six piano keys are identified in thirteen different ways in the chord symbols.

chord scale notes	**piano keys**
root of ***I-m6(9)***, "5" of ***IV-13***, "7" of ***II-m7***	**1**
root of ***II-m7b5***, "9" of ***I-m6(9)***, "13" of ***IV-13***	**2**
"7" of ***IV-7***, "#5" of ***V***, "#9" of ***I-7#9***	**p3**
"b5" of ***II***, "b9" of ***V-7b9***	**p6**
root of ***pVII-7*** (not in bars 1-8)	**p7**
"b5" of ***IV-7b5***	**7**

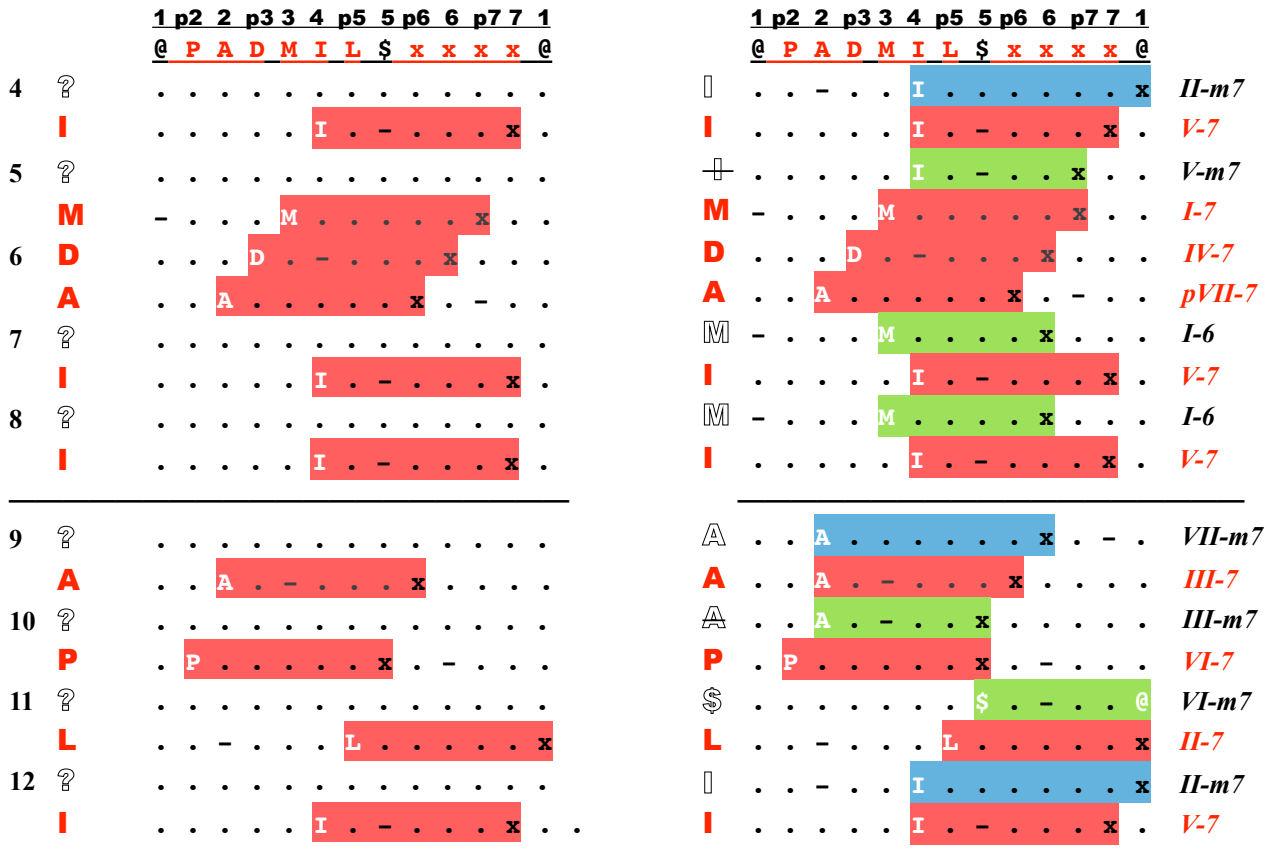
These chords and others like them may also appear in blues pieces on different roots. This is to be expected because the blues mode signature **DM.L** and the minor mode signature **AD.I** have the same half-tone-whole-tone form.

“RHYTHM CHANGES”

The piece *I Got Rhythm* by Gershwin is the source of widely used chord changes in jazz, popularly known as “Rhythm Changes.” The original is in Bb major with variations. The melody line of the first eight bars is straight Ionian mode. So is the harmony except for ornamental tritones anchored by **M**, **D** and **A**. The second eight bars are different in kind because the music in each of four 2-bar segments goes to classical modes that are identified by the tritones. These modes preserve the major tonality of the home tonic, which means that tritones **A** and **P** identify modes **alt-A** and **alt-P** that are of opposite tonality (major) to the primary minor Aeolian and Phrygian modes, and don’t include the home tonic. The rhythmic nuances are left to music notation or memory: the purpose here is only to correlate melody and harmony



As shown next starting from bar 4, completing the all-tritone core by fairing in fifos is simple.



Dashes indicate the positions of the roots of the chords shown, most of which may be omitted in 3-note shapes that are sufficient voicings of the chords in context.

In the first eight bars, the tritones anchored by **M**, **D** and **A** are purely ornamental. They contain notes **p7**, **p3** and **p6** that are outside the home Ionian mode. The outside note of the first chord of this trio is the root of the third chord. Otherwise, all notes of shapes to provide voicing notes for core elements are from the home Ionian mode.

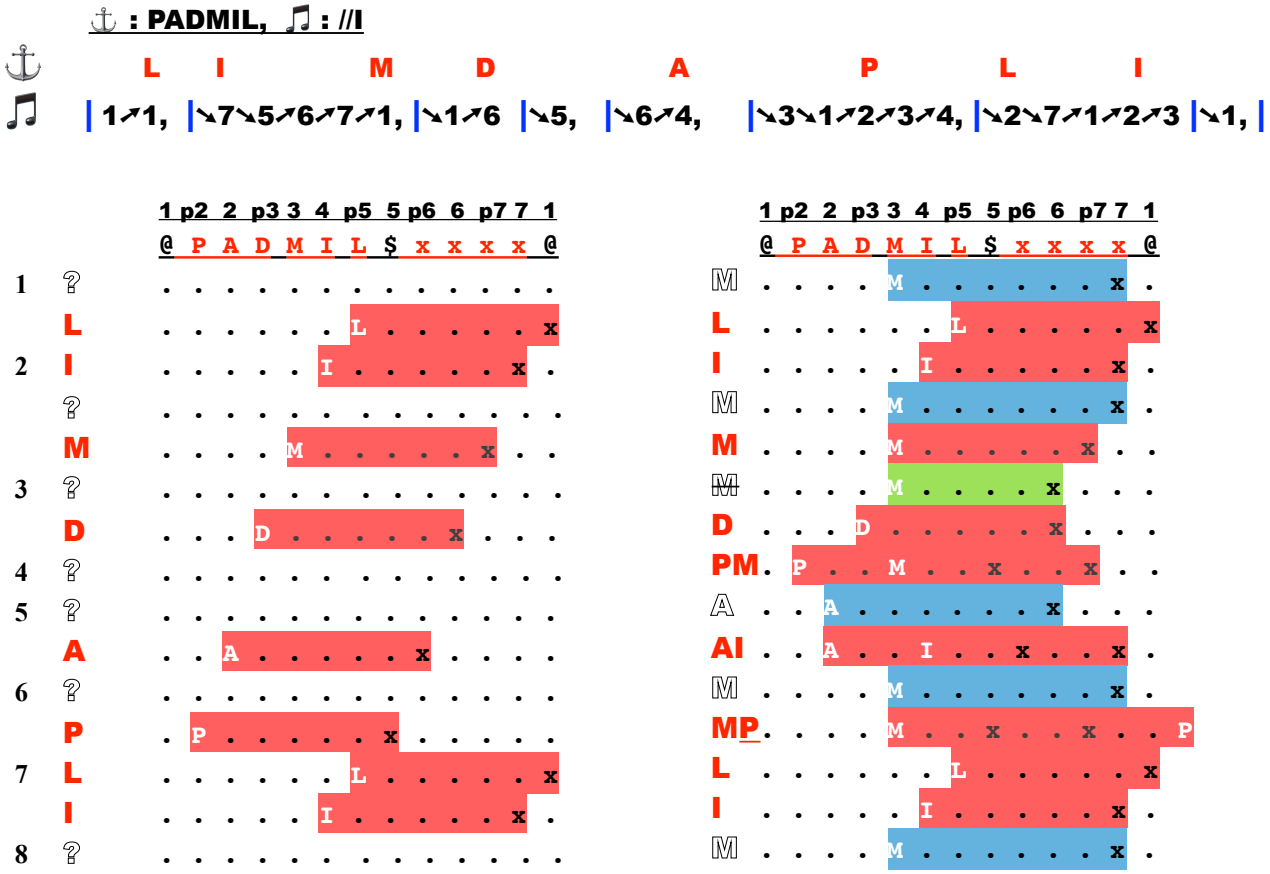
In the final eight bars, the piece visits four different parallel modes that bring in the following notes (working backwards from the end): **p5** replaces **4**, **p2** replaces **1** and **p6** replaces **5**. This leaves notes **5**, **1** and **4** unavailable as voicing notes for bar 9, **4** and **1** for bar 10 and **4** for bar 11.

This is very simple seen this way, but rather complex if you think in terms of degree numbers in chord scales relative to the different key signatures of seven different parallel modes. The piece includes all six tritones identified by the alphabet, each of which is compatible with parallel modes of two tonics a tritone apart. Tritone **A** appearing twice in different places, implies a different parallel mode with a different key signature for each appearance. The other five tritones imply one key signature each, for a total of seven. Thinking in terms of chord scales relative to seven different key signatures, all implied by accidentals relative to one of them, is complex by any measure.

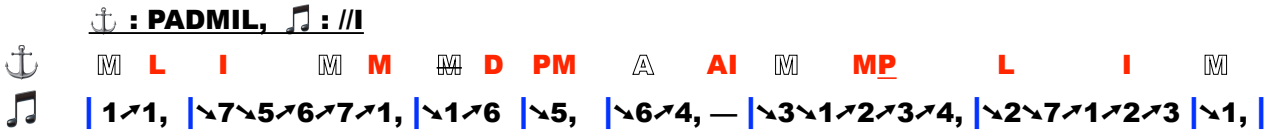
ORNAMENTAL HARMONY FROM SCRATCH

This is an exercise in creating ornamental harmony from scratch without even thinking about harmony scales or chords. The example is the melody line of *Over the Rainbow*. The original is in Eb major (3 flats). The Ionian melody line trending downwards is given core harmony defined by a reverse-alphabet tritone slide going downward by half tones. This harmony uses the full chromatic scale and yet is visibly simple. The only creative parts are positioning the tritones for consonance with the melody notes and adding enrichments.

The core harmony is visibly a variation of "Rhythm Changes." The variation uses the full reversed alphabet **L-I-M-D-A-P** with a tag **-L-I** at the end to bring it to resolution in the home scale. The **//I** annotation identifies all harmony except **I** as ornamental.



The core on the left is completed with interspersed fifos plus a few double tritones just because they sound good. Identifying implied chords is unnecessary but straightforward. Some of the simple symmetries between successive shapes are striking (e.g., **PM -** 𝔸 shrinks the outer notes of the double tritone inwards a half tone).



RE-HARMONIZING A BLUES BALLAD

This is an exercise in re-harmonizing the piece *When Sunny Gets Blue* that's complex as written because of an idiosyncratic bar structure (not the common 12 bars of many blues pieces) and written chords that are all over the map. It's a blues in G because the melody line is from the **//DM.L** blues scale of G, with very few departures. The key signature of one flat identifies neither of the usual modes Ionian of F or Aeolian of D, but the blues-compatible mode Dorian of G. The only different melody scale is **//L** (Lydian) in the first four bars of the bridge, which is not a blues sub-scale but fits here because it's the relative major mode of a minor Dorian mode with tonic a minor third down.

The re-harmonized piece is summarized next. The melody line is as written (within the limits of the summary notation) but the harmony shown is altered. Ornamental touches are highlighted in yellow — the ones in the melody line are from the written music and the ones in the harmony are part of the reharmonization.

//DM.L

⚓ M D L D M D L I

🎵 4 ||: ↗2↘1↗2↘4, — 4 | ↗4 ↘p3↘p7↘p5 ↗4↘p3 | p3, ↘2↘1↘p7↘6, ↘5 ↗1 | ↘6, — |

⚓ M D A P L I

🎵 | ↘5↗p7↗1↘p7, ↗p2↘p7↗1↘p7, | ↘4↗p7↗1↘p7, ↗p2↘p7↗1↘p7, |

⚓ M D L M D M L

🎵 | ↘5↗p7↗1↘p7, ↗4 — ↘p3↘p7 | ↗2 — — — , ↘4 :|| ↗2 — |

==========bridge==========

//L

⚓ $ $ L L $ $ $ L L

🎵 | ↘7↗1↘7↗1, ↗2↗3 | ↗p5 ↘2*, 2 ↘7*, | 7↘5↗6, ↗7↗1 | ↗2 — |

//DM.L

⚓ M M A A L M D

🎵 | p7↗1↘p7↗1↗2 — ↗p3 | ↗3↘1*, 1↘6*, | 6↘4↗5 ↗6 ↗p7 | ↗1 — — , ↘4 |

==========

repeat bars 1-6 (first two lines) here, followed by this ending, repeated twice

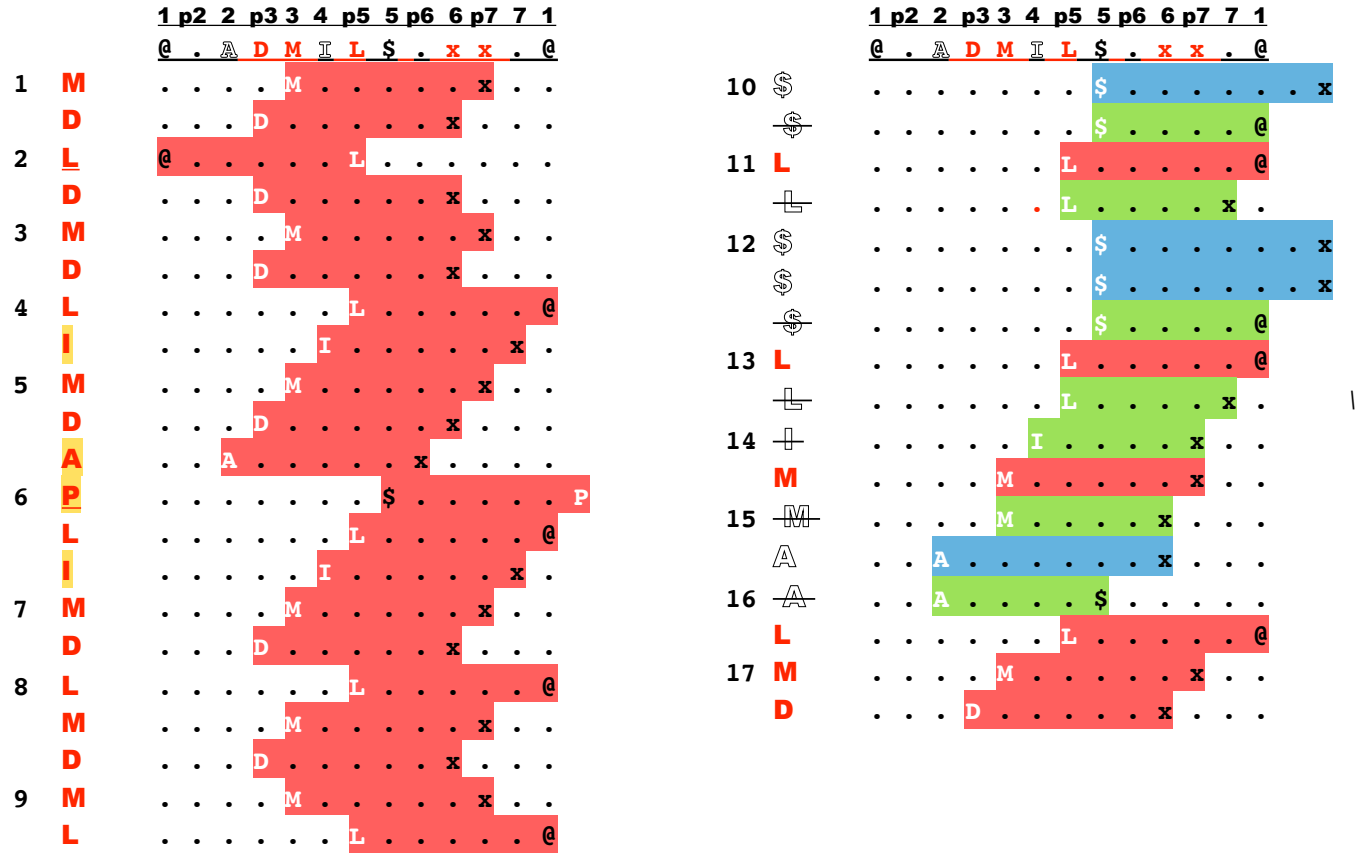
⚓ D D A D A D

🎵 | ↗2↗4↗5↘4, ↗p6 ↘4↘p2 | ↘1 — | 1 — |

The core harmony is based entirely on mode signatures **//DM.L** and **//L**, without reference to chord symbols. The big difference in the reharmonization is a coherence of the whole to the eye and ear. Play the harmony in the first instance as octave shapes by doubling the treble line of the core an octave down. The 5-tritone runs down by half tones in bars 4-5 and 6-7 are very easy to play and sound

good with the melody line. They include ornamental tritones **I**, **A** and **P** that are easily morphed into in-scale fifos, if desired, to provide a mixed core of the kind shown on the right, that remains more definite and easier to remember than the original. The harmony in these bars is so chromatic to begin with that the difference to the sound is almost too subtle to notice.

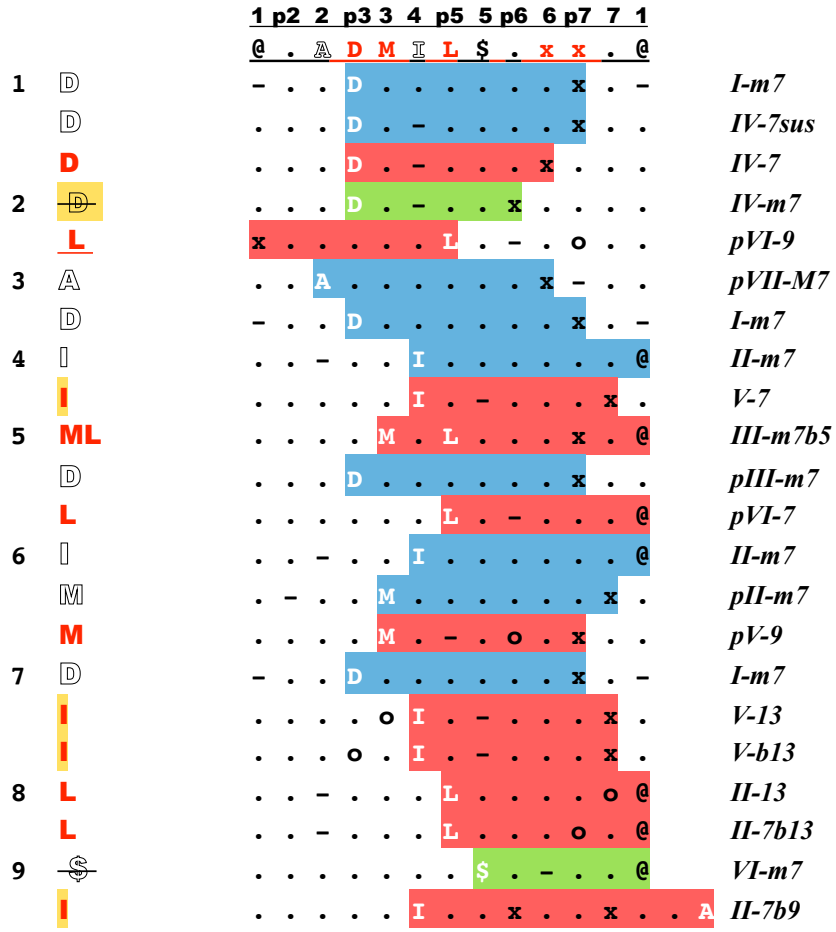
Melody and harmony of the first four bars of the bridge are in the **//L** mode (Lydian). The final four bars of the bridge return to the original blues scale. Unlike the original, all the harmony for these modes is from the two scales. As in the original, the transition between bars 13-14 is marked by the distinctive sound of a fourth sliding down a half tone (⏊ - ⊥).



This completes the re-harmonization. The original from which I learned this piece is shown next in the same terms.

Bars 1-8 from the Written Chords

The original written chords and associated core harmony are shown next for these bars and coming up for the remaining bars. Ornamental elements of the core are highlighted in yellow on the left. The notes identified by circles are ***9***, ***13*** and ***b13*** chord extensions/alterations that are not fundamental to the sound of the chords in this context.

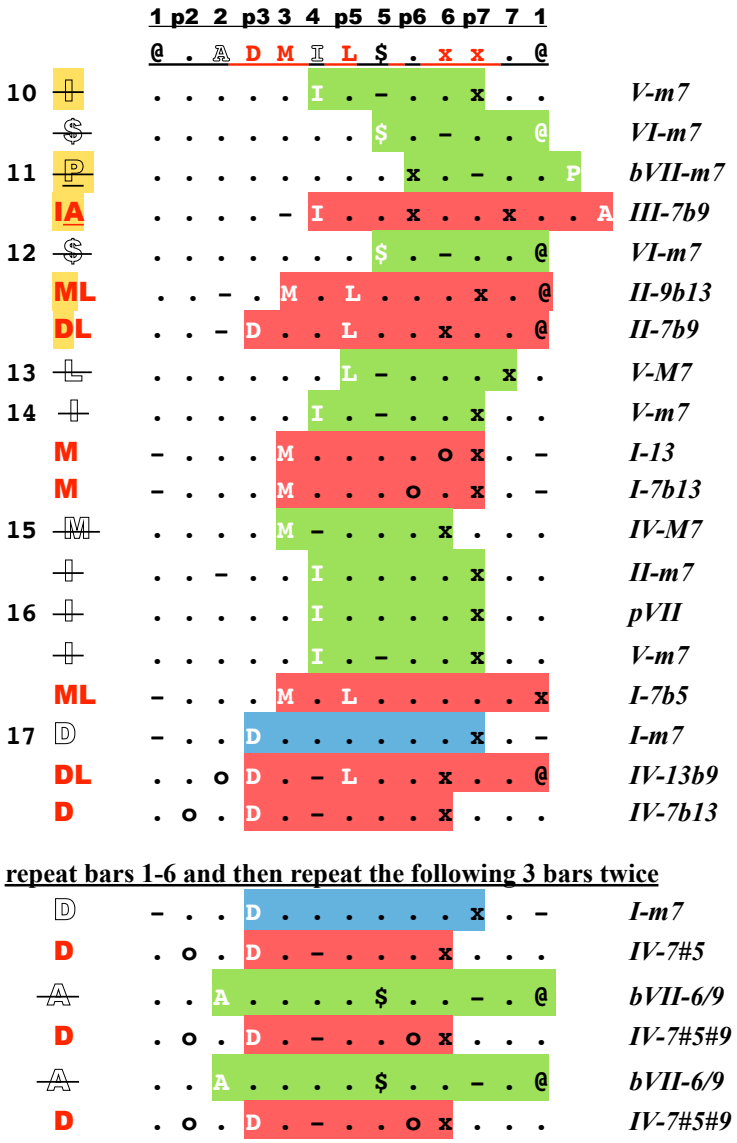


Remaining Bars from the Written Chords

The ornamental elements highlighted in yellow in bars 10-13 are at odds with the Lydian mode of the melody line in these bars, making for a confusing transition to and from these bars.

Everything in the blues bars 14-17 is from the family blues scale but the flow is irregular and difficult to remember, especially following the unclear nature of the first four bars.

The final ending also provides a harmonic introduction to the piece. The #5 and #9 notes of the ***IV-7*** chords are the out-of-scale melody notes **p2** and **p6** repeated in the chord symbols. The practice of sometimes including melody notes in chord symbols is not only redundant, it overcomplicates chord notation.

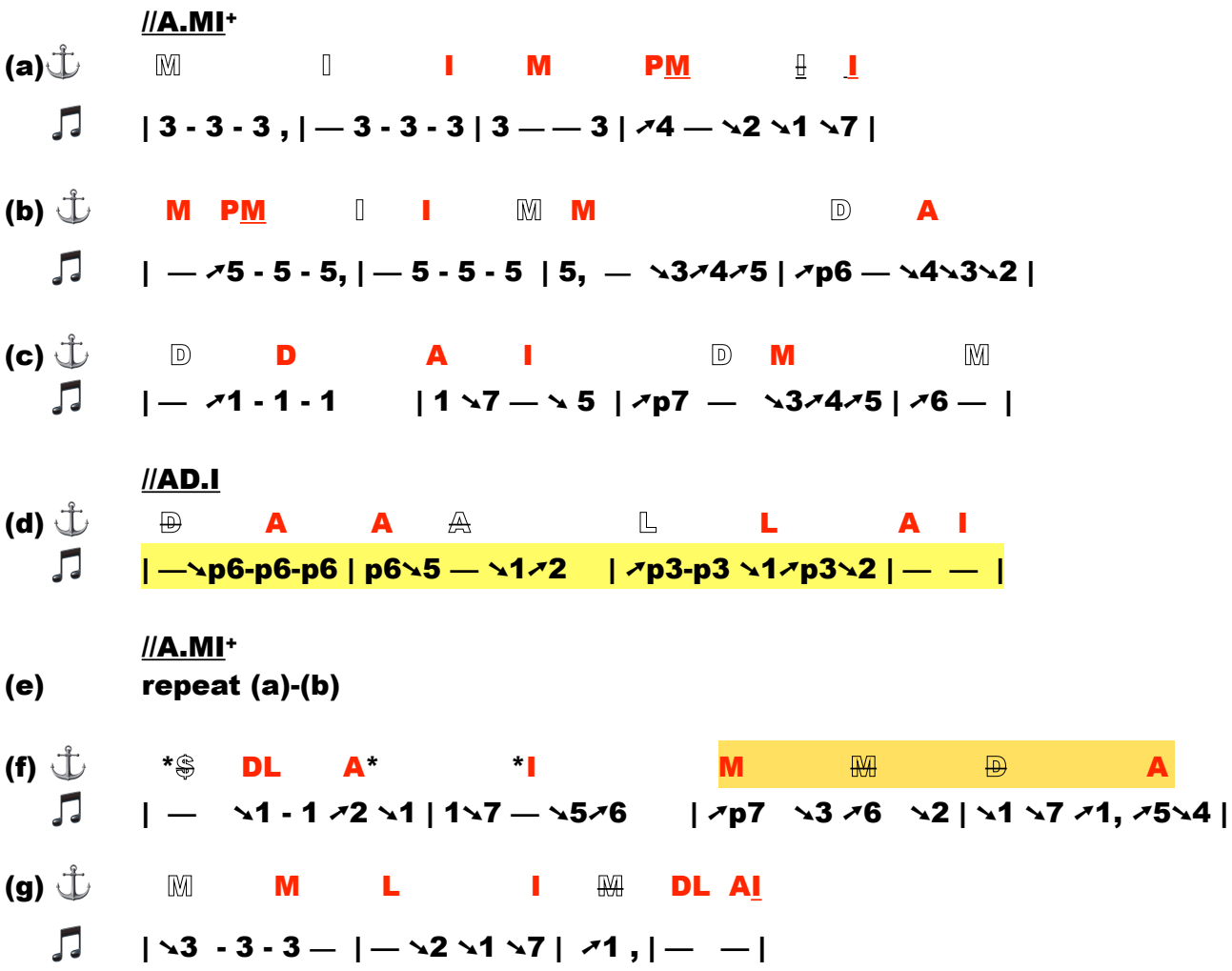


repeat bars 1-6 and then repeat the following 3 bars twice

A HARMONICALLY RICH BALLAD

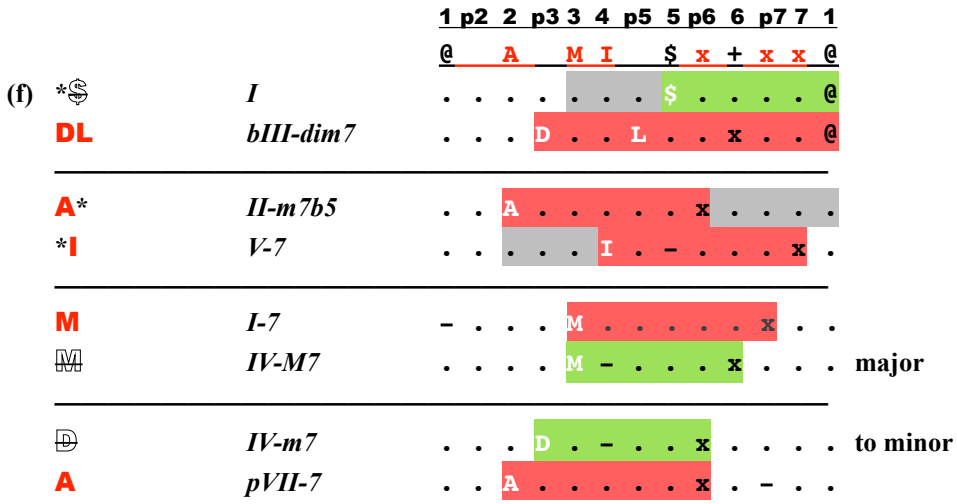
The piece is *Every Time We Say Goodbye* (original in Eb major). The mostly-simple melody and rich, mixed harmony are organized in an irregular way relative to each other that I have always found impossible to remember in chord terms. The chords sometimes harmonize melody notes and sometimes provide transitions between them, in patterns that are easy to get wrong. It's much easier to get the core harmony right because the flow cues what comes next.

The melody line is solidly in the major family scale identified by **A.MI+** except for highlighted 4-bar phrase (d), which is minor. The melody line in the major bars comes to rest on notes **6** and **2** of the family scale in a couple of places, but the simplest interpretation of these is resting points in the exploration of the mode. A couple of the double tritones in this harmony are from the flow rather than from the exact chord symbols.



In phrase (f) shown next, the two melody bars with the famous words "how strange the change from major to minor" stay resolutely in major for the melody, while the highlighted harmony, in these bars only, goes from major to minor. The harmony of the first two bars is intricate in chord terms but simple on the keyboard. The asterisk prefixes and suffixes help, by indicating the side of the core on

which to place obvious voicing notes. The final voicing for the second bar implies the double tritone **AI** that's a half tone down from **DL** in the first bar. In effect, the core sequence is **DL-AI** down a half tone. Octave voicings for the remaining bars are sufficient.



The sound of the piece is relatively insensitive to the exact placement of the harmony relative to the melody. When the melody is on a downbeat, playing the harmony on the following upbeat — and vice versa — often sounds fine. For this reason, the harmony placements shown don't necessarily align exactly with the chord placements in the written music. Many of the harmony shapes are rootless voicings of the written chords.

OPEN VOICINGS

I learned this version of *All of Me* in C (along with *No Greater Love* in Bb coming up) in Susan Muscarella's piano comping course some years ago at the then Jazz School in Berkeley (now the Jazz Institute) as an example of using 4-note "open" voicings of extended and altered seventh chords to accompany soloists. The chord progression is strongly chromatic and therefore complex in chord symbol terms. The voicings rearrange the chord notes. As presented in the course, the rearrangements are identified by rearranged degree numbers in which the same notes from adjacent chords have different degree numbers relative to different roots that are often omitted from the shapes. The representations are indirect relative to the keyboard and difficult to comprehend as a whole, or remember. The complexity goes away when the shapes are understood in terms of core building blocks directly on the keyboard.

The melody and core harmony are summarized next. The piece consists of four 8-bar sections in the form ABAC. The melody line overall is in the 9-note major mode identified by **A.MI⁺**. The core harmony goes outside this mode via ornamental tritones **P** and **L** and fifos morphed from them. This core harmony puts its own spin on "Rhythm Changes."

//A.MI⁺

⚓ M I M A M P P M P $

🎵 | 1↘5↘3 — | 3 , ↗1↗2↘1 | ↘7↘p6↘3 — | 3 , | ↗6↘5↘3 — | 3 , ↘p3↗3↗p7↘6 | ↘5 ↘4 | 4 , |

⚓ A A A A $ L I

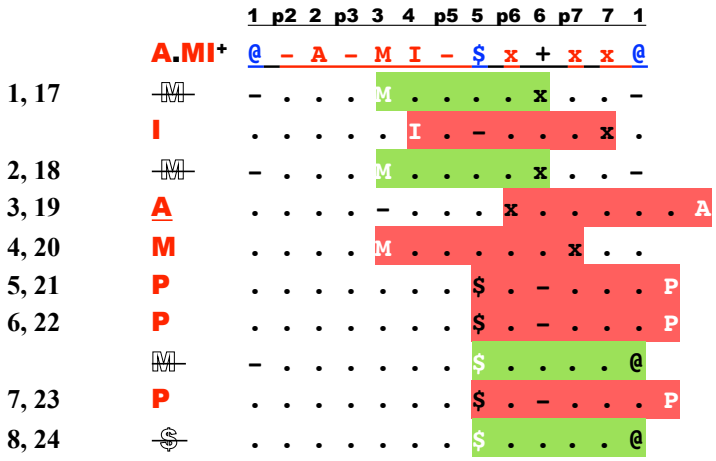
🎵 | ↘3↘p3↘2 — | 2 , ↗3↗p6↗7 | ↗2 — ↘1 — | 1 , | ↘7↘p7↘6 — | 6 , 6↗2↘7 | ↘6 — | ↗7, |

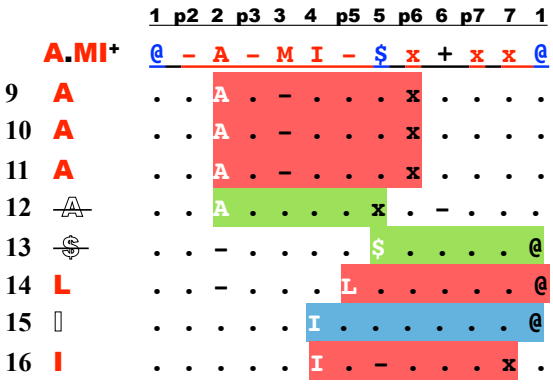
repeat first 8 bars and then finish with this

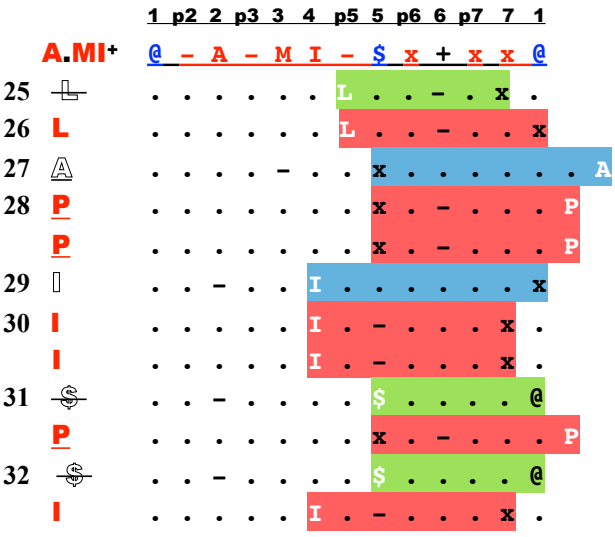
⚓ L L A P P I I I $ P $ I

🎵 | ↗2—↘1↘7 | ↗2 , ↘1 | ↘7 — ↘3↗5 |↗7 , ↘6 | ↗1 — ↘6↗1 | ↗3 3 | ↘1 — | — — |

As usual, the tritone core provides a framework for fairing in core fifos without reference to anything else. Play this harmony in the first instance as octave shapes formed by doubling the treble line of the core an octave down. The only exception is bars 3-4 in which the tritones establish outer notes a minor seventh apart that may be held for both, only moving the middle note.



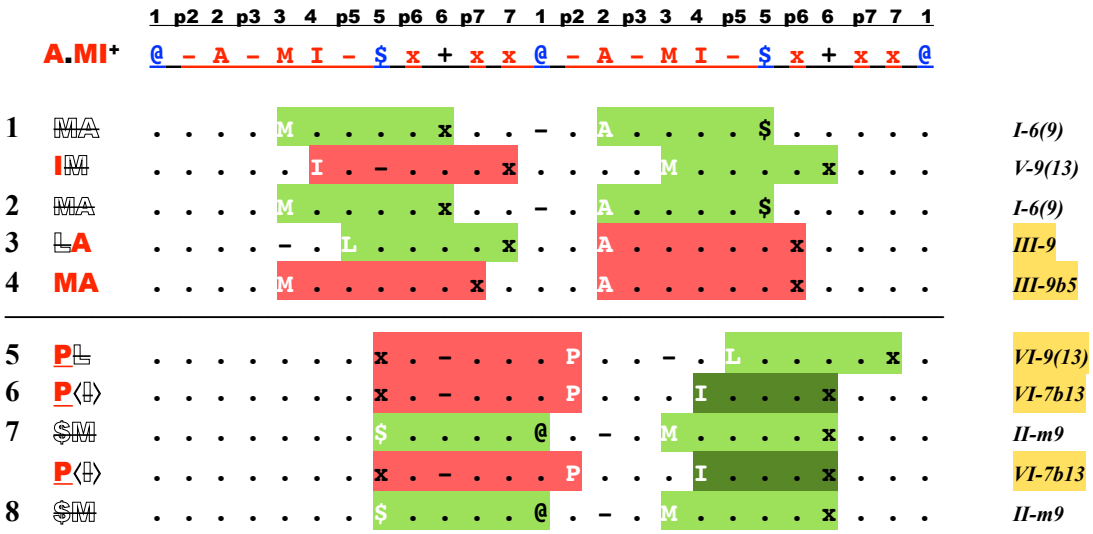




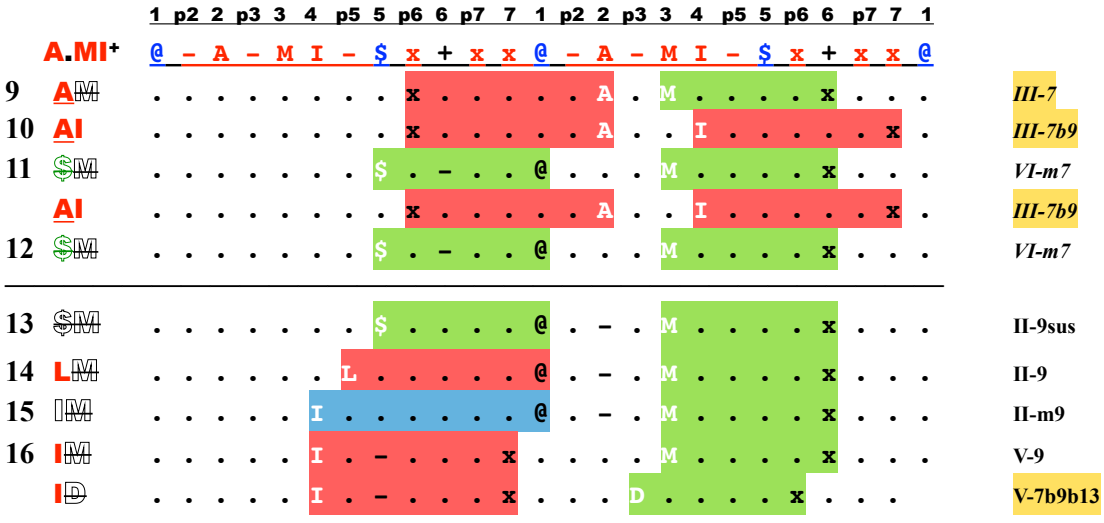
4-Note Open Voicings

The 4-note "open" voicings developed from this core are shown next. These are voicings of the chords on the right. Chords on roots I, II and V are from the Ionian mode. The other chords (higlighted in yellow) provide visibly simple segues between the Ionian voicings. The chord symbols are all satisfied serially in the flow.

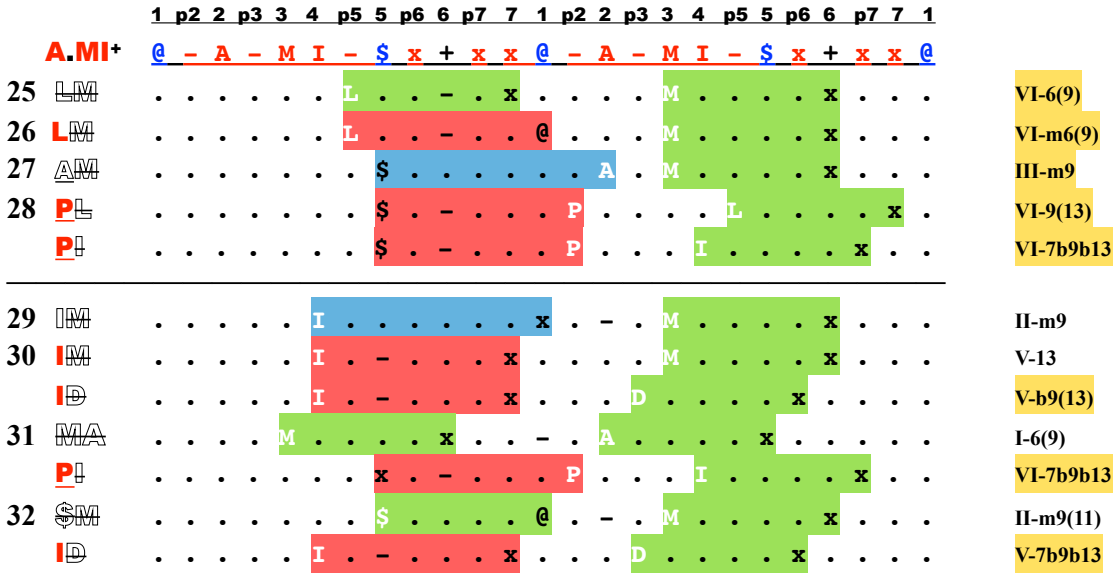
Bars 1-8



Bars 9-16



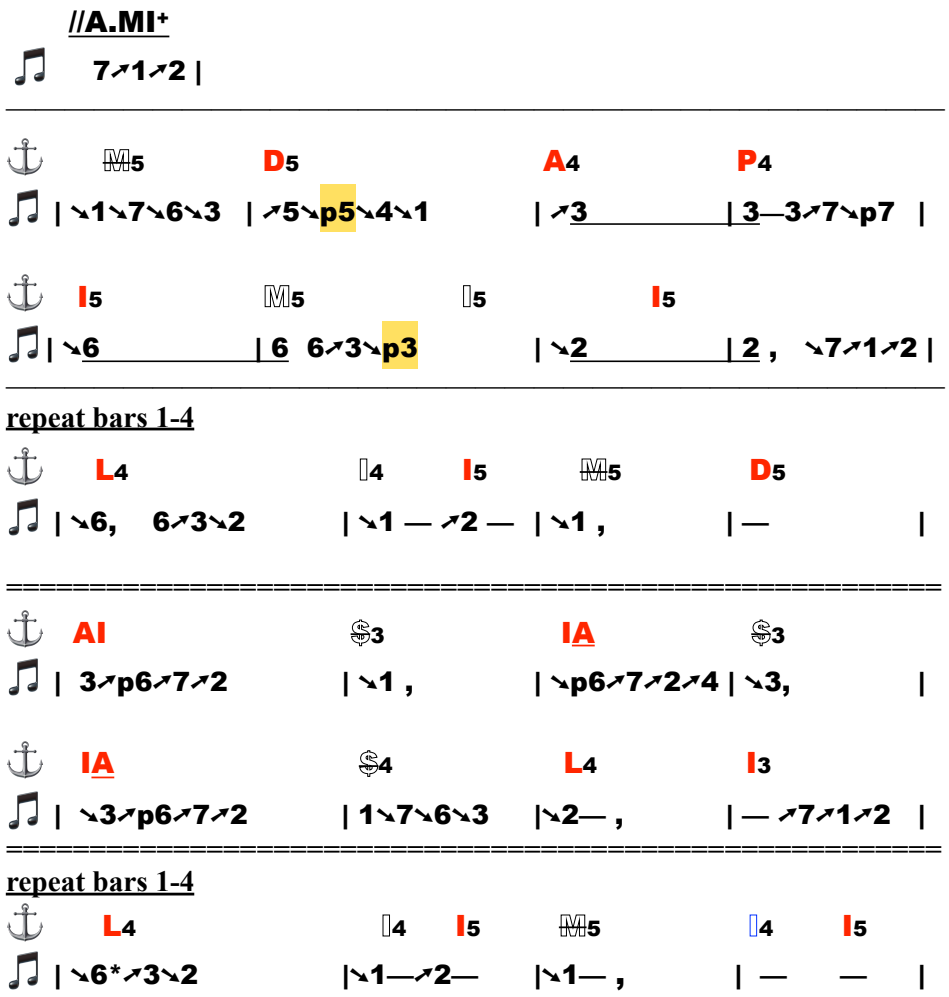
Bars 25-32



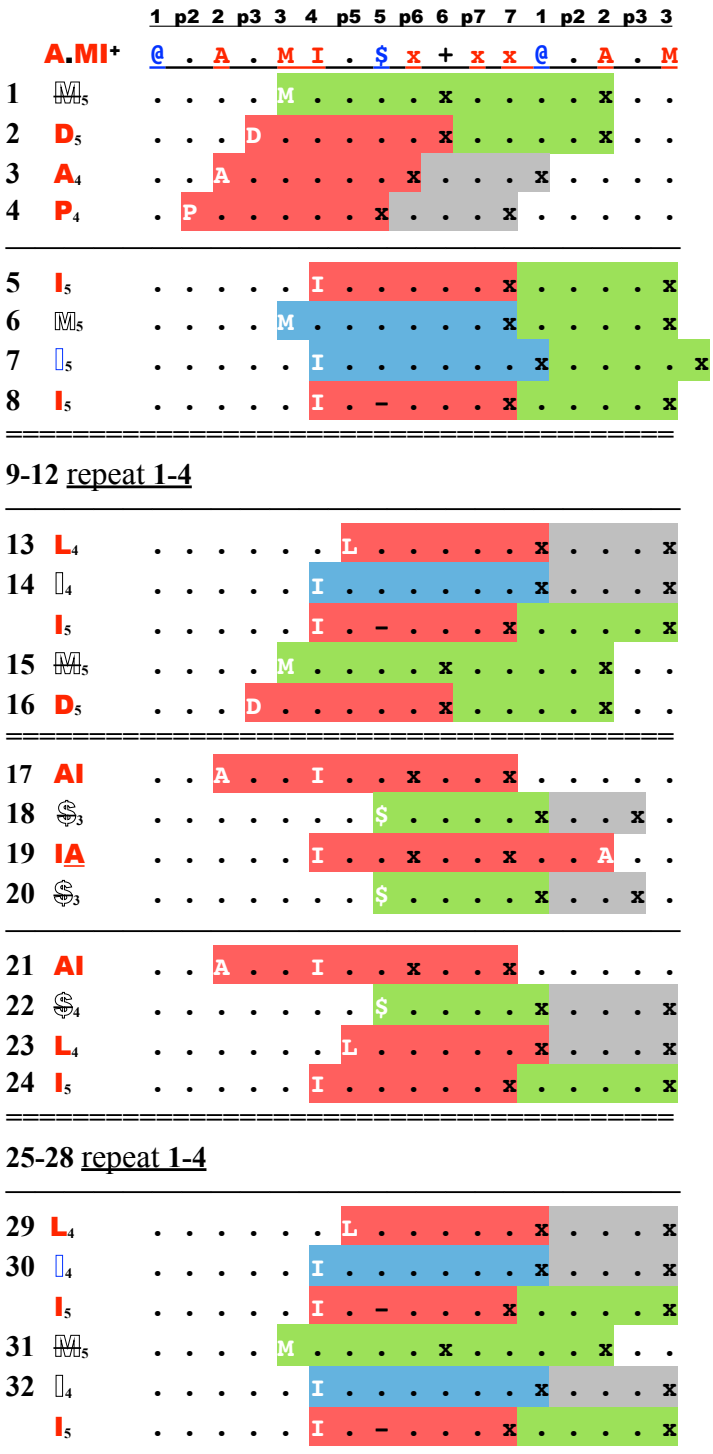
Understanding how the shapes satisfy the chord symbols requires knowing the relationship between the numeric suffixes of the chords and the chromatic scale along the top. I explain this only to make clear that the shapes satisfy the written chords, not to suggest figuring the shapes out this way. In bars 3-4, chord suffix "9" is chromatic scale note **p5** and chord suffix "b5" is chromatic scale note **p7**. The 3-note voicings in bars 3-4 don't include **p5** but it's included in bar 5, thus satisfying the bar-4 chord in the flow. The bar-4 shape jumps up a minor third in bar 5, with a wobble that puts a fourth on top instead of a major third. The wobble provides degree "13" of the chord. It also provides degree "9" of the bar-4 chord, thus satisfying two chord symbols at one stroke.

MORE CHROMATIC HARMONY

The version of *No Greater Love* (original in Bb major) has strongly chromatic harmony that runs through the entire **PADMIL** alphabet, which is always complex in music notation. The chords (not shown) are many and various. The melody mode is **//A.MI+** throughout (**1-2-3-4-5-p6-6-p7-7-1**). The harmony departs from this mode to bring in tritones **P**, **M** and **L** for transitions that are ornamental because they don't change the melody mode (this includes the altered harmony pattern in the bridge).



Here's how to picture the keyboard pattern identified by the anchor notation. Turning this around, it shows how to work out core-based shapes on the keyboard that may then be identified by the anchor notation.



A walking-bass version presented in class is translated here into chromatic scale notation. The bass line doesn't attempt to capture the sequence of the chordal harmony, only to walk through the governing scale in a way that complements the melody. Some out-of-scale notes at the ends of bars provide transitions to in-scale notes in the next bar. The rhythm is steady 4/4.

7↗1↗2 |

🎵 | ↘1↘7↘6↘3 | ↗5↘p5↘4↘1 | ↗3 , — | — 3↗7↘p7 |
▼ ↘1↘5↗1↘3 ↗4 ↗1 ↗4↘7 ↗p7↘4↗p7↘4 ↗6↘3↘6↗p3

🎵 | ↘6 , — | — 6↗3↘p3 | ↘2 , — | — ↘7↗1↗2 |
▼ ↘2↘6↗2↗p3 ↘2↗6↘2↘p2 ↘2↗6↘2↘p6 ↘5↘2↗5↘p2

---

repeat bars 1-4 …

🎵 | ↘6, 6↗3↘2 | ↘1 ↗2 — | ↘1, — | — |
▼ ↘2↗6↘2↘p2 ↘2↘p6↘5↘7 ↘1↗3↗4↗p2 ↗5↗7↗1↘p6

==========

🎵 | 3↗p6↗7↗2 |↘1, — | ↘p6↗7↗2↗4 | ↘3, — |
▼ ↘7↘4↘3↗p6 ↗6↘p6↗6↗p7 ↗7↗4↘3↘p7 ↘6↘3↘6 ↗p7

🎵 | ↘3↗p6↗7↗2 |1↘7↘6↘3 |↘2 , — | — ↗7↗1↗2 |
▼ ↘7↗4↘3↗p6 ↗6↘p6↗6↗p3 ↘2-2↗6↘p7 ↗5↗7↗2↘7

==========

repeat bars 1-4 …

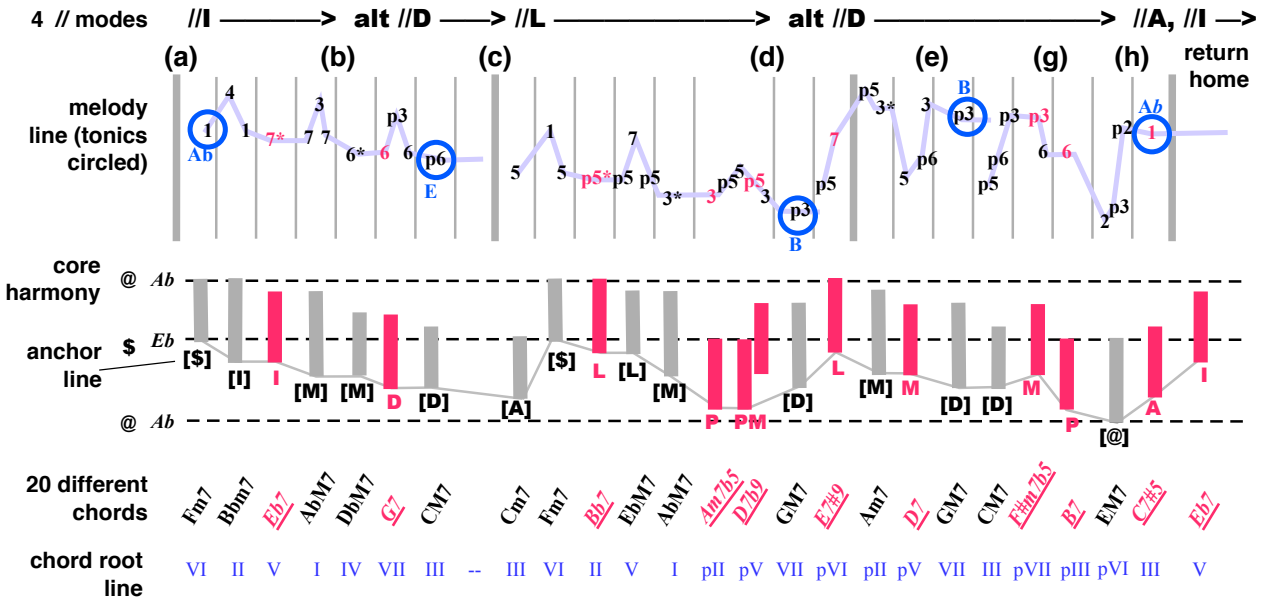
🎵 | ↘6*↗3↘2 |↘1—↗2— |↘1, — | — |
▼ return to chordal harmony

TONIC CHANGES IN CLASSICAL MODES

The example is *All The Things You Are* (original in 4 flats). I learned this strikingly beautiful piece early in my musical adventure and found it's simple keyboard patterns easy to play, but the relationship between its melodic and harmonic changes difficult to understand in terms of the written chords. I drew the following picture to try to understand the changes (it covers only the first 24 bars because this is where all the changes occur). The vertical bars represent building blocks going up from the anchor line, bringing forward the simplicity of the core harmony. The square brackets are an earlier notation for fifos that left sizes open.

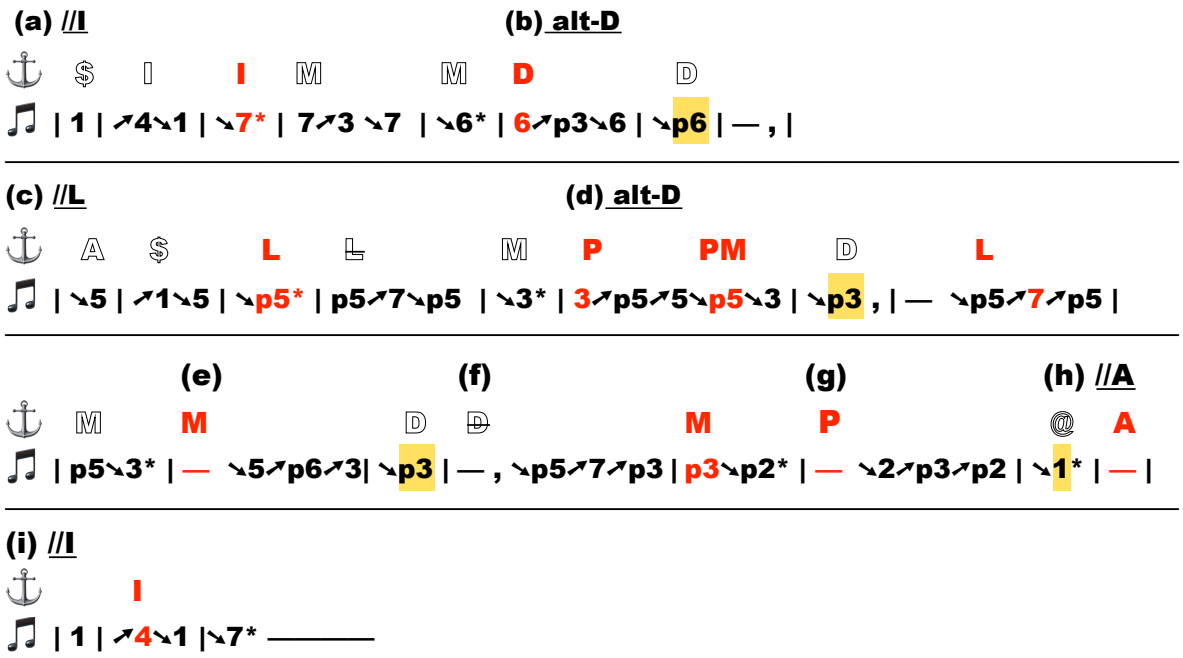
The difficult-to-understand changes appears first in phrase (b) and reappear again in phrases (d)-(g). The harmony in these phrases is purely ornamental, chosen to sound good but not to determine melody scales (like the harmony developed earlier for *Over The Rainbow*). The melody of the piece tells you what's going on. All the phrases except the ones just identified are in classical modes identified by harmony tritones **I, L** and **A**. The other phrases are all from a single **alt //D** mode that's identified in (b) by tritone **D** (its only appearance).

The home tonic scale of the written piece is Ab major. The melody line visibly resolves to two secondary tonics, namely **p6**=E in (b) and **p3**=B in (d) and (e), both of which are in the **alt //D** mode that provides the melody notes of these phrases. The corresponding tonic scales are rotations of this mode to start on these tonics but the rotations don't have to notated because the notes are defined by the mode and the tonic changes are identified by the melody line.



Here follows a summary of melody and core harmony that's all you need to play the harmony as a series of octave shapes based on the core (double the anchor line an octave up). The harmony is so varied that putting more shape variety into it by adjusting the octave shapes is hardly necessary. The harmony consists of tritones from chords containing them (red text) plus fifos faired in between them. The core harmony is remarkably simple, easy to remember, and to play because everything is fifos morphing into each other, and into and from tritones. Alternatively, the chord root line may be played under the core to get voicings of the written chords without referring to the chord symbols.

Here is a shorthand summary of melody plus core harmony of the first 24 bars where all the changes occur. The visited tonics are highlighted in yellow. This piece is particularly well suited to using octaves shapes for 3-part harmony.



(a) The mode is **//I** because melody sequence in scale order is **1-3-4-7** and the harmony tritone is **I** (**4-7**), both of which are from it.

(b) The melody identifies **alt-D** which is **//D** with all non-tritone notes different, namely **p2-p3-3-p5-p6-6-7**. The scale is important because its mixed minor-major tonality yields alternate tonalities in phrases (d)-(g).

(c) The melody sequence in scale order is **1-3-p5-5-7-1**, which is visibly from mode **//L** (Lydian major). The harmony tritones **P** and **PM** are ornamental but sound right (one note of each is from this mode).

(d) This phrase and phrases (e)-(g) that follow it are all from mode **alt-D** of (b). Phrases (d)-(e) each open with secondary tonic **p3**=B from this mode. The mode has mixed minor-major tonality, which accounts for the different tonalities, namely minor in (d), (f) and (g) and major in (e). The single appearances of outside notes **5** in (e) and **2** in (g) are from context and anticipate resolution to the home tonic. The harmony tritones **L**, **M** and **P** are ornamental.

A POSTER-CHILD FOR GENERAL TONIC CHANGES

The piece *Laura* (original has home tonic C) sounds beautifully simple to the ear. Its strong chromaticism (all the notes of the piano's chromatic scale) makes the changes initially mystifying considering the simplicity of its sound. The following shorthand summary brings forward the simplicity on the keyboard. Yellow highlighting identifies melody tonics visited on the way to the home tonic at the end (for the original home tonic **1**=C these are **3**=E, **5**=G, **2**=D and **p3**=Eb). As shown on the next page, modes **//DL**, **//PM** and **//AI** of the home tonic provide the main scales of the secondary tonics. The secondary tonic scales are rotations of these modes to start on the different tonics, but there's no need to think in these terms to play the piece because the only difference is the order of the notes. The melody line is given. Playing an improvised melody line requires only knowing the home tonic modes. Only the tritone core is shown because it governs the changes. Adding core and enrichment fifos is accomplished in ways that have been explained and illustrated in previous chapters and examples. The home tonic of C of the original makes a caution necessary: don't confuse tritone anchor **D**=**p3** with letter note D=**2**.

//DL **//PM**

⚓ DL

🎵 | 7* — | — ↘p7↗7↘p7↘p5↗5 | ↗6* ↘3 — , | — |

//PM **//DI**

⚓ PM D

🎵 | ↗6* — | — ↘p6↗6↘p6↘3↗4 | ↗5 — , | — |

//D

⚓ I

🎵 | — ↘p3↗4 | — ↘p3↗4↘p3↗4 | ↗5*↘2 , | — ↗p3↗5↗p7 |

//DL **//AI**

⚓ D DL AL L A AI

🎵 | ↗p3*↘p6— | — ↗6— | ↗2, | — ↗3↘1↗2 |

repeat bars 1-8

//AI

⚓ AI L

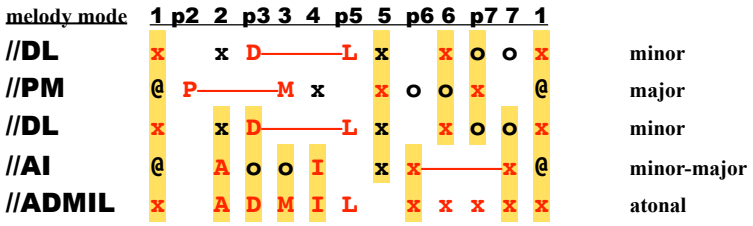
🎵 | — ↗4↗5— | — ↘3↗4↗p6↗7 | ↗2↘1↗2 , | — ↘7↗1 |

⚓: **//ADMIL**, 🎵: **//I**

⚓ AL DL AI

🎵 | ↗p3*, — | — ↗3↘p3↘2↘6↗7 | ↗1, — | — |

Here follows a summary of the parallel modes, listed in the order in which they appear. Yellow highlighting brings forward to the eye the thing that makes the piece easy to play, namely the large number of piano keys shared from one mode to the next.



JOINING SHORT FRAGMENTS FROM DIFFERENT SCALES

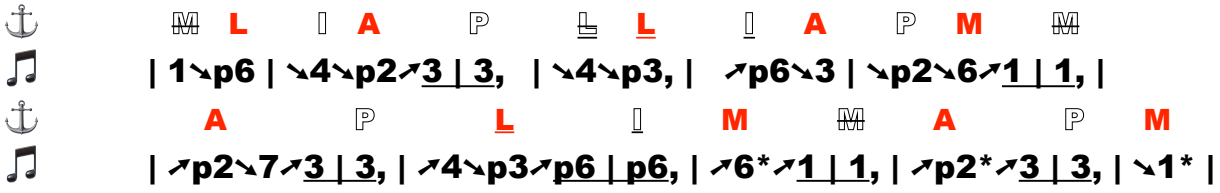
Coltrane's famous jazz classic *Giant Steps* cycles rapidly and repeatedly through three Ionian modes of tonics G (one sharp), Eb (3 flats) and B (4 sharps) that are a major third apart and differ by four notes. The harmony for each tonic is Ionian II-V-I (often with II omitted). The melody line is a sequence of short fragments from these scales that combine into an unusual major scale of tonic F# with ten notes and no pitch center. The piece is complex by any measure in these terms.

The written home tonic is F# (5 sharps for the Ionian mode) but could just as well be Gb (6-flats for the Ionian mode). The nominal slight pitch difference between F# and G*b* is an artifact of music notation, there's no difference on the piano (see Appendix B for more on this). Given that E*b* is one of the original Ionian tonics, it seems more natural to me to represent the home tonic by G*b* than F#. This means five keys are competing for attention in the mind: 1 sharp, 4 sharps, 5 sharps, 3 flats and 6 flats. The only way music notation can represent the differences, namely by accidentals, is at a very low conceptual level, providing no sense of scales as coherent musical entities. The piece is only approachable in these terms by expert pianists.

Yet the piece is essentially simple seen in terms of a whole-tone scale of tonic F# with mode signature **AML** where the letters are the anchors relative to F#=**1** of the tritones of V-chords D7, F#7 and Bb7 of the respective Ionian tonics. The parallel modes of the home tonic identified by the tritones are **alt-A**, **//M** and **alt-L**.

	A	**M**	**L**	**x**	**x**	**x**
	1	**-2**	**-3**	**-p5**	**-p6**	**-p7**	**-1**
Ionian tonics	**p2**		**4**		**6**		
	G		**B**		**Eb**		

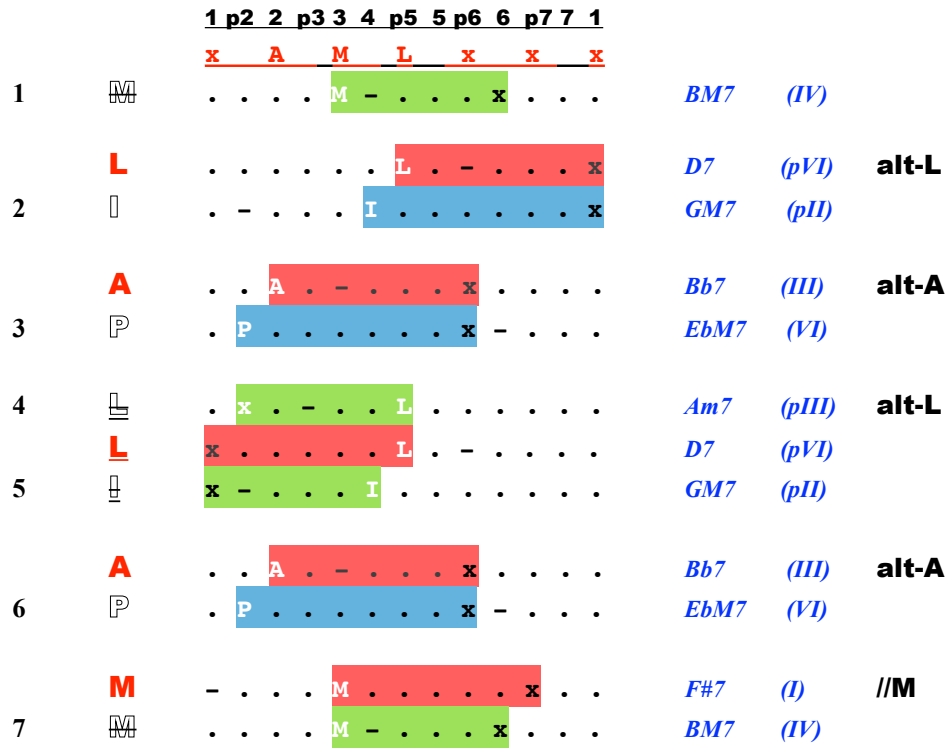
The result of the piecing together is a melody line in the whole tone scale, with inserted passing notes that briefly include the Ionian tonics.



The melody sequences **1↘p6↘4↘p2** and **p6↘3↘p2↘6** go down by the thirds of classical modes (major-minor-major) but resolve by jumping up a minor third to notes not in the modes. The resolutions establish the home tonic (bars 7 and 13) and major tonality of it (bars 3 and 9). The final three bars set up a repeat. The three tritones provide the basic harmony, with transitions between them provided by fifos that fit the very simple flow, without reference to classical modes or chord symbols. Playing this with octave shapes for the core harmony (double the treble notes an octave down) mutes tritone dissonance and adds depth. The harmony is so varied that this provides the essence of the sound of the piece.

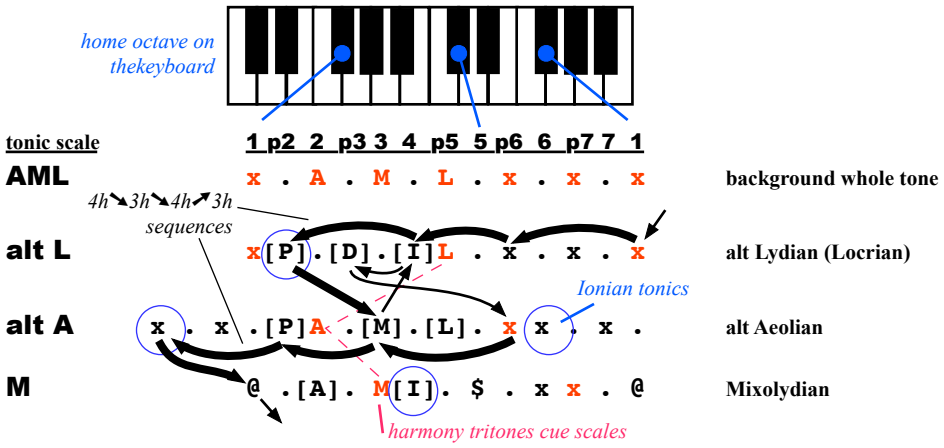
On one hand, the rapid changes of the Ionian modes makes playing the harmony as sliding shapes following chord root lines a job only for expert pianists. On the other hand, anyone can play this simple

core flow in which building blocks morph and slide by small intervals into each other. Shapes that voice the written chords are easily obtained, if desired, without reference to the details of the chord symbols, by inverting selected building blocks and adding the root line underneath.



Parallel Classical Modes

The next figure traces the melody line through the modes of the home tonic. The tritone-fifo sequences in the anchor line are transpositions of Ionian **I-**𝕄 resolution sequences. As the melody cycles through these parallel modes, it creates the 10-note melody scale. The square brackets are an earlier notation for fifo anchors that leaves sizes open.



Variations

Improvisations can use double-tritone modes of the melodic minor mode, derived from the **AML** whole tone scale by replacing any note by the pair of notes a half tone up and down from it. One of these double-tritone modes can replace two consecutive single-tritone modes, thus slowing the rapid pace of context changes and providing a simpler base for variations. Playing the original in these or other parallel modes requires some out-of-scale passing notes, but these are easy to find by ear once you've learned the piece. The **ML** variation suggests the possibility of a blues variation.

FROZEN IMPROVISATION

The hauntingly beautiful piece *Chelsea Bridge* by Billy Strayhorn is, to me, an example of "frozen improvisation." I once heard a classical musician say that all written music is frozen improvisation, and the transitions of this piece seem to me to speak to that. It's basic structure is parallel Ionian-Aeolian-Ionian with chromatic transitions from major to minor and from minor back to major. The piece is complex in music notation even without the chromatic transitions because it moves between written Db major (5 flats) and written C# minor (4 sharps). The complexity is multiplied by making a change of written key-signature from 5 flats to 4 sharps in the middle and then returning to 5 flats at the end via accidentals relative to 4 sharps. Add the chromatic transitions and the result is a confusing muddle of many more than twelve note symbols relative to the two written key signatures. The piece is musically sophisticated by any measure, but the sophistication is much simpler to understand and remember in the following terms. Explanations follow.

(a) 𝅘𝅥𝅮: //I (harmony on repeat)

⚓ M D A P

𝅘𝅥𝅮 ||: — 6↗7, ↗1↗2, ↗3↗4↗5 |

(b) transition→ **(c) //IP→**

⚓ LA ML LA ML P

𝅘𝅥𝅮 | ↗p6—, ↗6↘p5 | p5, ↗5↗p6 | p6, ↗6↘p5—, ↗5↘p2 | p2 |

(d) //A→

⚓ I M M D A

𝅘𝅥𝅮 | ↗3 ↘1—, | ↗3↗5—, | ↘3↘1— :|| — 5↗p6 |

(e) transition→

⚓ A A

𝅘𝅥𝅮 | ↗p7* | p7↘1↗p3↗5↗p7↗2 , ↗p3 | ↘2↘p7↗5↘p3↘1 ↘7↗p5↘2 |

(f) alt-IP

P I

| ↘p2, — ↘p3 ↗4 | ↗5↗p6↗p3— ↘p2↘7 | ↘p7 ↗p2—, ↘7↘p7 |

(g) transition

⚓ M

𝅘𝅥𝅮 | ↘6, ↘p5↗6↗p2, ↘6 ↗p2 |

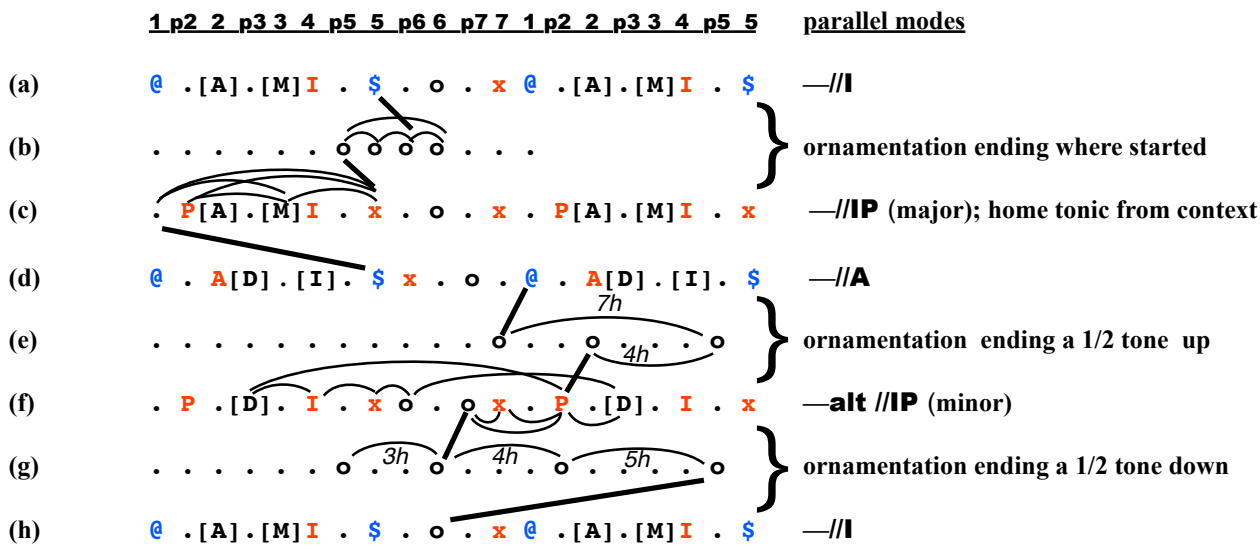
(h) 𝅘𝅥𝅮: //I

⚓ ML D A P

𝅘𝅥𝅮 | ↗p5, ↘6↗7, ↗1↗2, ↗3↗4↗5 |

The following figure traces the melody line through the different modes, skipping the details of segments (a), (d) and (h) because their melody lines are all straight Ionian or Aeolian. The square brackets are an earlier notation for fifo anchors that leaves sizes open.

Ionian-Aeolian-Ionian melody transitions



Segments (b), (e) and (g) seem like improvised ornamentation that performs transitions between well-defined scales, namely the two classical modes **//I** and **//A** and the non-classical modes **//IP** and **alt-IP** (recall that the former is a pseudo mode consisting of the Ionian mode with the tonic raised a half tone, and the latter is a minor mode with all non-tritone notes different).

Segment (b) has a simple, thin and ambiguous melody line. It fits the bitonal interpretation provided by the written chords, in which parallel modes of the melodic minor identified by **LA** and **ML** alternate (the corresponding transposed master tonics are, respectively, up a minor third and up a half tone). However, the full scales are not used by the melody line. The **AML** whole tone scale provides a shared background.

Segment (e) and (g) are transitions that set up and execute a return to Ionian of tonic **1** (Db) via Aeolian of tonic **7** (C). Keep this in mind and some confusing details of these bars in the written music become obvious on the keyboard.

A FAMOUSLY DIFFICULT BALLAD

The beautiful Strayhorn piece *Lush Life* is melodically and harmonically rich, and challenging to play from the written music because there are often two or more chords per bar, many of them chromatic relative to the written key signature of five flats (major tonic Db). The integrated melody and harmony are summarized in PKP terms next. Explanations follow.

[A] Verse, //A.MI

```
      @  A     M  A      M  A     M  I    A   D
5 ||: ↗1*↗2* | ↗3*↘2* | ↗ 3*↘2* | ↗3*↗4*↗5*↗p6*|
                                  1_________  2_________
  I    I              A             I   M        A   D
|↗p7*—↘p6↗p7↘p6|↘5*↗p7↘p3*|↘2*, — ↘5 :|| ↘2*, — ↗7 |
```

[B] Bridge, //IP melody (major), then //I; transitional harmony

```
  I    P     A    P                    DL    I    P    A   P              DL
| ↘7*↗ p2* | ↗3*↗5* |↗7↗p2↘6 ↗7*| 7 — 7 | ↘7* ↗p2* |↗3*↗5* | ↗7↗1↘6↗7* |
   I              M    MP              DL              AI
|↘6↗7↘5↗6↗7↘3 | 3 — | — ↗5↘4↘3 |↘2 | — 2*↗3↘1 | 5 — | — |
```

[C] Chorus, //ADMI

```
      〈M〉  I          〈M〉  I                    D  A    M  L  I
(a) | 5↗6↗p7↗p3↘p6|↘5↗6↗7↗p3↘p7|↘6↘5↗1↗3 ↗4 ↗5 |↘p7 — ↘p6 |
                                                 M  D       P    A
(b) |      repeat bars 1-2 of (a)      |↘6↘5↗1↗3 ↗5 ↗p6 |↘7 — 1 |
        A  L                 DL              M      A  P  L  I
(c) | ↗2↗3↗4* — ↘p3 |↘2↗3↗4*↘p3 |↘p6↗p7↗7* — ↘p7 |↘p6↗p7↗7*↘6 |
                                                 DI  AI  P
(d) |      repeat bars 1-2 of (a)      |↘6↘5↗1↗3 ↗4 ↗5 |↗6 — ↘3 |
        A     L   I                      DL          P
(e) | ↗5↘4↗5*↘4 |↘3↘2↗3↘7 | ↗2↘1↗2 — 2↘1 | 1↘p7↗6 — ↘3 |
        A     L   I                  A  D  M  M  A  D  M
(f) | ↗5↘4↗5*↘4 | ↘3*↘2↗3↘5 | ↗p7↗7↗1↗p2 | ↗2↗p3↗3 |
```


This sees the piece as being in parallel modes of the Db home tonic of the written 5-flats key signature, with no secondary tonics. There are no voicing extensions of the building blocks because the intent is to play octave shapes, which provide depth without adding different notes (the piece is so melodically and harmonically rich that the voicings don't have to add variety). Harmony fifos are mostly omitted for simplicity, except for characteristic sequences of them in the verse and in the ending bars of the chorus.

The piece can be seen as daisy-chained, short segments of different secondary tonics, but this is unhelpful because there are no solid melodic resolutions to any tonic. Resolutions to the home tonic are mostly implied by the harmony.

Verse [A]

The joint melody/harmony scale up to the second ending is the major family scale with mode signature **A.MI**. The harmony provides the **A** and **I** tritones and the melody provides the **A** and **M** tritones. The melody is in the **A.M** parallel major mode of the melodic minor (**1-2-3-4-5-p6-p7**), with one out-of-scale passing note (**p3**). The first and second endings of the written piece are each three bars long but the difference is a tiny variation in the melody line not recorded here. The melody starts on tonic **1** and comes to rest on note **2**, in the first ending as a return to the tonic, and in the second ending as a move towards **p2** of the bridge's **//IP** mode.

Bridge [B]

The melody line is in the **//IP** non-classical mode in the first six bars (**p2-2-3-4-5-6-7**) and in the **//I** classical mode in the next eight bars (**1-2-3-4-5-6-7**). The **//IP** mode is, visibly, a relative mode of a melodic minor mode with tonic a whole tone up from the home tonic, but there's no point in viewing it this way because the melody line in the first six bars doesn't resolve to it. The harmony is ornamental relative to these melody lines but anticipates what's to come. It's main thread is **I-M-D-A** going down by half tones (omitting the second tritone of the double tritones) thus introducing the highly chromatic **//ADMI** scale that dominates the chorus.

Chorus [C]

There are six 4-bar segments. The master scale for the entire segment is **//ADMI** (**1-2-p3-3-4-5-p6-6-p7-7**). The **P** and **L** tritones are ornamental.

(a),(b),(d) These three segments use the full **//ADMI** scale for both melody and harmony. The melody lines are identical up to the middle of the third bar, where they change to provide different transitions (a)-(b), (b)-(c) and (d)-(e).

(c) A variation.

(d) Returns to the pattern of (a) and (b) except for the ending.

(e) The melody line is straight Ionian. The harmony is contrasting minor in the **AD.I** scale with **P** and **L** ornamentation.

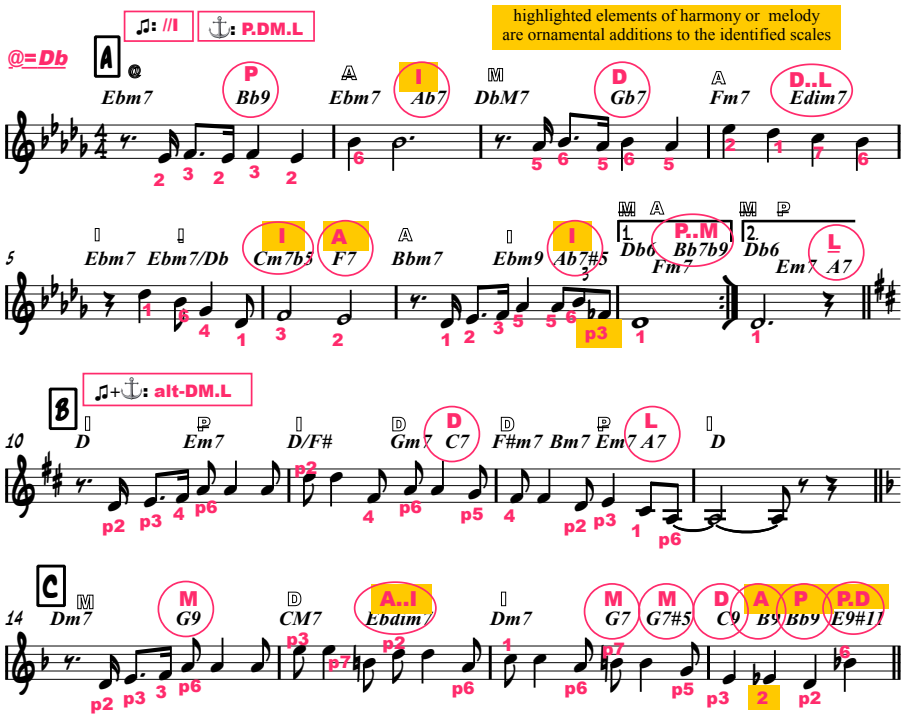
(f) The melody and harmony are solidly major, in the **A.MI** scale with which the piece began. The chromatic run of the melody line in the final two bars is a signature of the piece.

CHAPTER 5: ANNOTATED MUSIC NOTATION

A BELOVED JAZZ STANDARD

Body and Soul, an annotated chart for which is given below, is an often-played jazz standard with a beautiful melody line and interesting harmony. The actual scales and chords relative to the home tonic are identified in PKP terms, as summarized next (remember that the vertical alignment of chord roots and anchors meanly only same notes, not same chords). Thinking in terms of RN symbols for the chord roots avoids possible confusion between their letter notes and anchor symbols. The RN roots are not included in the annotations on the chart to avoid clutter, but are important to keep in mind.

C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C
I	*pII*	*II*	*pIII*	*III*	*IV*	*pV*	*V*	*pVI*	*VI*	*pVII*	*VII*
1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**
@	**P**	**A**	**D**	**M**	**I**	**L**	**$**	**.**	**.**	**.**	**.**



scales

A	♫	//I =	1-2-3-4-5-6-7-1
	⚓	P.DM.L =	1-p2-p3-3-p5-5-6-p7-1
B,C	♫+⚓	alt-DM.L =	1-p2-p3-3-4-p5-p6-6-p7

The piece moves between three different written key signatures but doesn't stick to any of them. The interpretation shown is obscured by the multiple key signatures, and is not the only possible one, but it's both simple and logical on the keyboard and to the ear.

The difference between the harmony scales shown for A and B is 3 notes in contrast to 5 notes in key-signature terms. There's no scale difference between B and C, only slightly different choices of notes of the same scale. The difference between the melody scales of A and B is 4 notes, which is less than the key-signature difference. The return from C to A reverses the transition from A to B. The easy-to-follow logic of this interpretation is summarized below.

Section A. The melody line of A determines the home tonic. The melody mode is visibly **//I** (Ionian), except for one departure. Minor note **p3** appears as a passing note leading to the first and second endings. It's appearance here is a harbinger of mixed minor/major tonality to come in both melody and harmony.

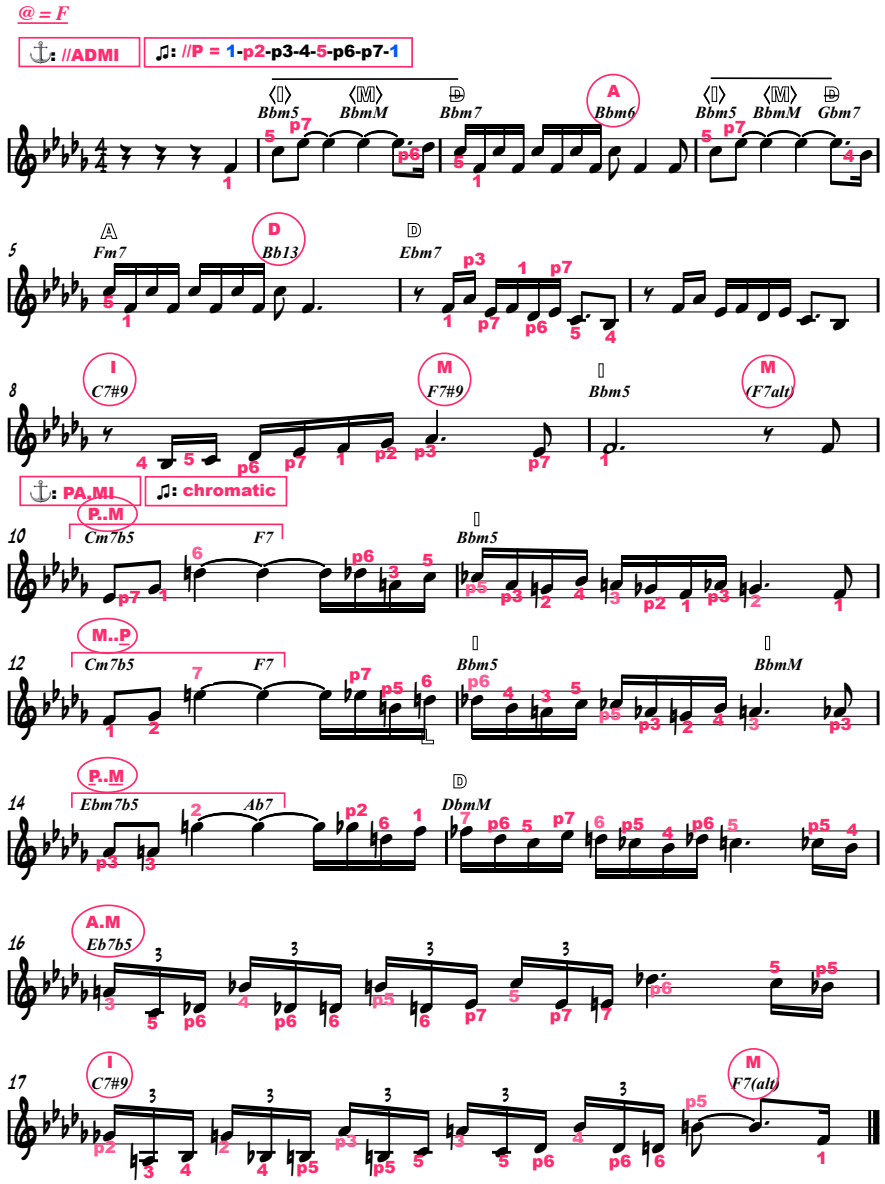
The harmony scale, shown as the 8-note diminished scale **P.DM.L** (atonal), adds contrasting richness to the bland melody scale. The harmony scale is nominally the full chromatic scale, because all the tritones of **PADMIL** appear, but this gives no sense of harmonic organization. The diminished scale links this section to the following sections in a clear and simple way (shared tritone content). Not only that, it highlights the strong contrast between the melody and harmony of A.

Sections B, C. The melody line of B is visibly in the Locrian mode of the home tonic identified by mode signature **alt-L** (Lydian with all non-tritone notes different: **1-p2-p3-4-p5-p6-p7-1**). This can be understood as a tonic change from Ionian of the home tonic to Aeolian of a tonic a major third down, but the logic is simpler if you forget about the key signatures and the tonic changes they might imply and look only at what happens on the keyboard. The harmony of B adds tritones **D** and **M** to the melody tritone **L**, suggesting the blues scale **alt-DM.L** as the governing scale of both melody and harmony of B. This scale is the union of the melody scales of B and C (each is an exact sub-scale). It's also related by shared tritone content to the harmony scale identified for A. Think of the latter as a tritone substitute scale for **//DM.L** that's the scale counterpart of tritone substitute chords. In this interpretation, the final note of the B melody line (**p6**) is not a secondary tonic, only a resting point in a tour of the shared scale.

A HAUNTING, STRONGLY CHROMATIC PIECE

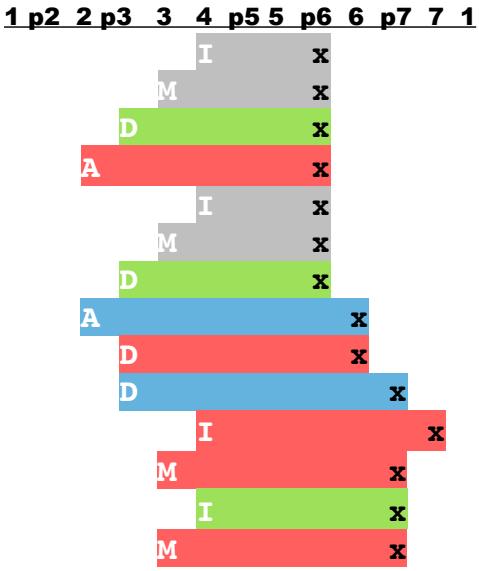
The Peacocks, an annotated chart for which is shown below, is one of the most haunting jazz pieces I have heard or learned to play. It's flow sounds so "right" that jazz improvisations rarely stray far from it. The actual scales and chord roots relative to home tonic F are identified in PKP terms as follows.

F	*F#/Gb*	*G*	*G#/Ab*	*A*	*A#/Bb*	*B*	*C*	*C#/Db*	*D*	*D#/Eb*	*E*
I	*pII*	*II*	*pIII*	*III*	*IV*	*pV*	*V*	*pVI*	*VI*	*pVII*	*VII*
1	**p2**	**2**	**p3**	**3**	**4**	**p5**	**5**	**p6**	**6**	**p7**	**7**
@	**P**	**A**	**D**	**M**	**I**	**L**	**$**	**.**	**.**	**.**	**.**



The home tonic is determined to be F by the final notes of bars 9 and 17 (actually bars 8 and 16, not counting the lead-in bar). In bars 1-9, the combination of the 5 flats key signature, the home tonic and the lack of accidentals in the melody line identifies the melody scale as Phrygian (mode signature **//P**). The melody line in bar 8 runs up through all the notes of this scale starting a fourth up (fifth down) from the tonic.

Harmony in bars 2-9 contrasts strongly with the melody. The core harmony in these bars, shown next, is visibly from the atonal **ADMI** scale. Only the anchors of the grey-shaded minor and major thirds are specified because these intervals are not in the building-block set, but the choices shown are obvious ones from context, sound exactly right to ear, and voice the written chords in context.



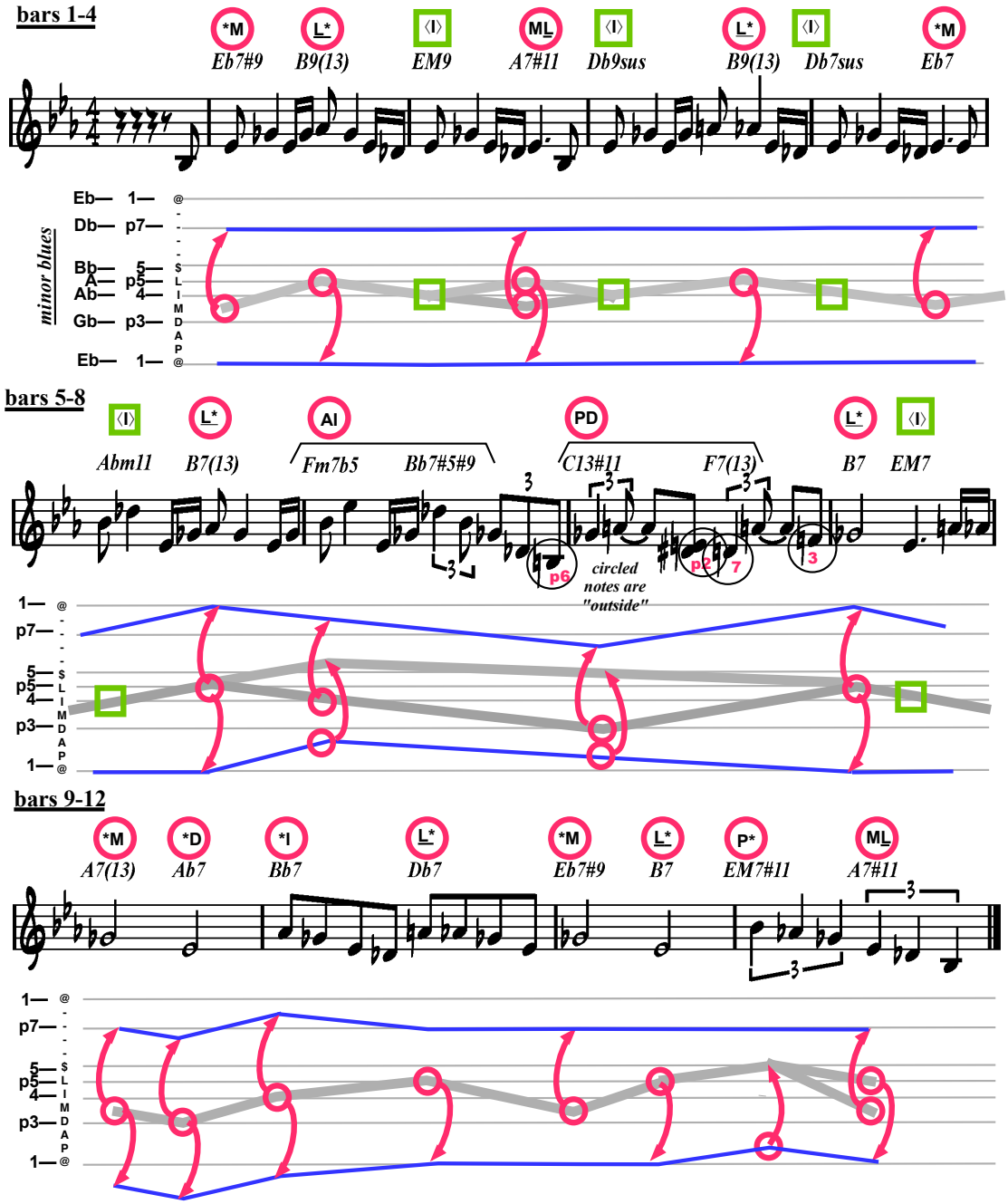
The melody of the second section consists mainly of individually simple, descending patterns from the full chromatic scale that are prompted by the harmony.

A piece that looks forbiddingly complex in music notation is extraordinarily simple on the keyboard.

A POSTER CHILD FOR COMPLEX CHORD PROGRESSIONS

The piece *Goodbye Pork Pie Hat* is a haunting Eb blues written by Mingus in memory of Lester Young. Its complex chord progression was footnoted in the opening chapter. The annotated chart below is shown in a format that puts building blocks in insets below the staff, aligned vertically with melody. This is helpful to the eye but impractical for annotating written music. The same thing in the separate tabular format used in the rest of this document is shown at the end.

The annotations above the staff are in a style that can be handwritten: circles outlining tritone anchors, boxes outlining fifo anchors (green for fourths, blue for fifths of which there are none here).



The melody line is very simple — all in the 6-note minor blues scale except for the "outside" notes outlined in bars 6-7. The telling tritone content of the anchor line reveals simple harmonic patterns that are difficult to see in the written music because they're hidden in a combination of multiple different chord types on a creative root line by bassist Mingus. The chord symbols are sometimes made more complex than necessary by including melody notes (#9, 13, 9, #5 are from melody or nearby harmony). The core harmony is easily understood independently of the creative root line, which may be added later as a bass line, if desired. The roots internal to the harmony shapes shown are omitted here (dashes show where).

All tritones are un-inverted in the core harmony except **L**, which is always inverted. Fixed voicing lines a minor seventh apart in many bars are determined by the double tritone **ML** in bars 2 and 12. Bars 6-7 are an ornamental departure from **DM.L** blues, announced and terminated by octave shapes based on **L**. The double tritones within the ornamental segment, which are each extracted from a pair of complex chords, cue the melody lines in these bars. Continuation of the tritone octave shapes in bars 8-10 seems natural. The five fifo chords are all non-resolution chords from the minor Aeolian or Phrygian modes of the home tonic (which differ by one note). Only the Ionian anchor is specified for the core fifos, leaving the shape to be completed in a visibly obvious way as stacked fourths. The Phrygian mode appears explicitly in the chord root of bars 2 and 8 (which is omitted from the shapes) and in the tritone of bar 12 (which is included).

The harmony is summarized next in the tabular form used in the rest of this document.

		1	p2	2	p3	3	4	p5	5	p6	6	p7	7	1	
		@	.	A	D	M	I	L	$	.	x	x	.	@	
1	*M	@	.	.	.	M	.	.	o	.	.	x	.	.	*I-7#9*
	L*	@	.	.	.	.	.	L	.	–	.	x	.	.	*pVI-9(13)*
2	〈▯〉	@	–	.	.	.	I	.	.	.	.	x	.	.	*pII-M9*
	ML	@	.	.	.	M	.	L	.	.	.	x	.	.	*pV-7#11*
3	〈▯〉	@	.	–	.	.	I	.	.	.	.	x	.	.	. *II-9sus*
	L*	@	.	.	.	.	.	L	.	–	.	x	.	.	*pVI-9(13)*
4	〈▯〉	@	.	–	.	.	I	.	.	.	.	x	.	.	*II-7sus*
	*M	@	.	.	.	M	.	.	.	.	.	x	.	.	*I-7*
5	〈▯〉	@	.	.	.	.	I	.	.	.	.	x	.	.	*IV-m11*
	L*	@	.	.	.	.	.	L	.	.	.	.	.	@	*pVI-7(13)*
6	AI	.	.	A	.	.	I	.	.	x	.	.	x	.	*II-m7b5, V-7#5#9*
7	PD	.	P	.	D	.	.	.	$	.	x	.	.	.	*VI-13#11, II-7(13)*
8	L*	@	.	.	.	.	.	L	.	.	.	.	.	@	*pVI-7*
	〈▯〉	@	–	.	.	.	I	.	.	.	.	x	.	.	*pII-M7*
9	*M	x	.	.	.	.	.	M	.	–	.	.	x	.	.	*pV-7(13)*
	*D	x	.	.	.	.	.	D	.	–	.	.	x	.	.	*IV-7*
10	*I	x	.	.	.	.	.	I	.	–	.	.	.	x	.	*V-7*
	L	@	.	–	.	.	.	L	.	.	.	x	.	.	*II-7*
11	*M	@	.	.	.	M	.	.	.	.	.	x	.	.	*I-7#9*
	L*	@	.	.	.	.	.	L	.	–	.	x	.	.	*bVI-7*
12	P*	.	P	.	.	.	.	.	$	.	.	.	x	.	*pII-M7#11*
	ML	@	.	.	.	M	.	L	.	.	.	x	.	.	*bV-7#11*

CHAPTER 6: OBSERVATIONS & CONCLUSIONS

Music notation is here to stay and must be lived with, but this this doesn't preclude alternate views of piano music from a different perspective. I looked for an alternate view of harmony and found one based on intervals.

The central role of tritones in PKP tends to seem inappropriate to people savvy in music notation, for several reasons. In general, it goes against received musical wisdom. More specifically, tritones cannot even be written down in music notation until a key signature is established, so imagining them as identifiers of scales is difficult. Anything involving multiple tritones is complex in music notation because it implicitly brings in other key signatures — the more tritones, the more complexity. The very concept of fifos is unconventional and even more so is the concept that they can be treated as morphed tritones, understood from context. The payoff is insight into deep structure that is difficult to see in music notation. Appreciating these things requires stepping outside the box of music notation, at least momentarily.

Thinking in notes and thinking in intervals *is* different in kind. Many experts who have learned the hard way to think in notes apparently find thinking in intervals too alien to contemplate. Comments from them have run the gamut from "obviously wrong" to "wrong headed" to "naively simplistic" to "overwhelmingly complex." One expert said I had "found a tritone hammer and saw everything as a nail" as if the very existence of tritones as useful size tools was unthinkable. The book *Modalogy*, devotes 50 or so complex pages to parallel modes without ever mentioning the possibility that tritones might simplify things by providing defining notes.

Because I started out seeing intervals and notes as dual views of the same thing, I came to find it natural to switch back and forth. I have made my best effort in this document to convey the simplicity of this. I believe, based on my personal experience as an adult beginner, that any beginner could easily learn to think this way.

I know from personal experience that PKP helps with learning and remembering new pieces, knowing while playing where to go next from where you are (and how you got there), recovering from getting lost, experimenting with harmonic and melodic variations, and improvising. It turns the conventional relationship between practicing and understanding on its head: instead of practicing being required to develop understanding, understanding guides practicing.

The jumpy chord root lines in many of the examples demonstrate that referring chords to constantly moving roots creates misleading complexity. This complexity is both conceptual (the underlying, smoothly flowing core harmony is obscured) and actual (playing the jumpy harmony is technically difficult because all the fingers must be lifted and moved by large intervals). It's easier to learn the core harmony and introduce the jumps later, if desired, via inversions, which are always easy on the keyboard, given an anchor line.

GENERAL BENEFITS

PKP annotations above the staff tell you many things. Combined with context provided by the melody line, they tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not right at any point by identifying tonic scales in play in a very immediate way. They tell you about transitions between major and minor tonality and between different tonics. The provide a basis for enrichment and improvisation.

Scale, modes and chords are seen as part of a continuum expressed in the same terms. This seems to me to be a very powerful benefit, compared to seeing everything beyond classical modes as

special cases identified by accidentals (which includes chromatic chords with suffixes that imply accidentals). The representations of scales and chords in terms of tritone clusters are unfamiliar to expert pianists but the interval stacks that they imply on the keyboard are completely familiar.

With PKP, playing music is guided by note-based symbology on the music page (one dual view) plus PKP annotations above the staff (the other dual view) that suggest core harmony and make changing tonic scales and tonics directly visible to the eye in terms of changing dictionary words.

A scale dictionary of less than a page covers much ground. The PKP dictionary provides a novel, high-level, conceptual view of tonic scales that's easier to hold in the mind's eye than the very much larger number of spelled out tonic scales in music notation. The dictionary covers 12x39=468 scales of 12 different tonics in music notation (more than is covered by scale dictionaries such as *The Source*). Words of 1-4 letters from the 6-letter alphabet identify, by tritone content, 39 tonic scales/ modes with 7 or more notes, 1-4 tritones, and no intervals larger than a minor third. The count of 39 scales/modes includes 11 single scales covered by single words and 4x7=28 parallel modes covered by transpositions of 1-2 letter master words (only the master mode is identified in the dictionary). A selected home tonic anchors all of this to the keyboard.

The words are analogous to the words of biological DNA in the sense that they identify deep structure without spelling out the details. Words of biological DNA are "expressed" as proteins, the building blocks of life. **PADMIL** words are "expressed" as building blocks of scales and harmony. Biological DNA is "extracted" from biological samples and interpreted by sophisticated machines called "sequencers." **PADMIL** scales and building blocks are "extracted" and "sequenced" from musical lines by sophisticated machines called human eyes. Biological DNA is "inherited" from parents. **PADMIL** words are "inherited" from general knowledge about them encoded in the dictionary.

Symmetry breaking in the scale dictionary and symmetry breaking in core harmony are two sides of the same coin. In the dictionary, it yields scales with fewer tritones. In harmony, it releases the tension of dissonance to produce consonance. Seeing the simplicity of this requires forming mental images of it relative to the schematic tonic octave that are independent of specific assignments of the octave to the keyboard.

Understanding harmony is possible independently of practicing. Keyboard symmetries centering around tritones enable high-level representations of harmony that are simple in their own terms and deep in harmonic meaning in any terms.

Music is freed from its key-signature straightjacket. Harmony scales are understood at a glance from dictionary words aggregated by eye, instead of having to be figured out or remembered from key signatures, accidentals and chromatic chord symbols. Tonic changes are no more difficult than any other changes.

Anchor lines that define harmonic cores replace chord root lines as the fundamental musical lines of harmony. Chord roots are visibly secondary elements in the flow of the music because inversions move them into the body of the chord, where they become no more than another chord note that may be omitted if implied by context.

Sight reading anchor lines provides deep context. Anchor lines tell you where to go next from where you are and how you got there. They tell you what notes will sound right or not by identifying tonic scales in play. They tell you about transitions between major and minor tonality and between different tonics.

Multiple explicit or implicit key signatures become a non-issue. This is true whether the key

signatures imply tonic changes or only scale changes for the same tonic. Tonic changes amount to no more than moving (conceptually) a tonic pointer within the home octave and reinterpreting the alphabet letters relative to it, without changing any notation.

Chord symbols become a non-issue. Chord complexities that result from banging square pegs (chord symbols) into round holes (places in scales where they don't quite fit) are misleading. Chords described by chord symbols are, on the keyboard, almost universally composed of combinations of PKP's two kinds of building blocks, of which only tritones are always core. A melody line plus an anchor line above the staff imply a voicing of an unidentified chord progression. Specific chords are identified by assigning roots.

Learning blues prepares you for anything music notation can throw at you. Blues is an example of "folk music" that evolved independently of music notation and therefore tends to be regarded as naive when measured against its depth and complexity. This condescending view is unwarranted. Blues is harmonically sophisticated in a highly organized way that makes it, to my mind, a better starting point than classical modes of key signatures for understanding many forms of music.

Sharps and flats are not needed. Building blocks are the same number of half tones no matter how they are notated. This enables PKP to dispense with sharps and flats without any loss of musical accuracy. Expert pianists tend to think they are still needed to identify context because they have learned to use them for that, but note positions relative to tonics are sufficient.

The simple flow of harmony in harmonic cores, in which building blocks slide or morph into other building blocks, is universal. Nothing constrains harmony to be played this way but the flow is so natural and easy to play that much is to be gained by learning pieces this way and then transitioning to other playing styles if desired.

Tritone-intensive harmonic cores of much music provide so much harmonic variety that voicing them requires only adding depth. Depth is easily added by doubling the treble line of the harmonic core an octave down, or the bass line an octave up, and then making adjustments to move points on the line closer to the core, if desired.

No more piano wizardry is required to have fun playing these kinds of music than the ability to sight-read melody lines in music notation. Anyone with eyes to see intervals on the keyboard can infer harmonic cores from anchor lines annotated above the staff.

The simple way harmony is represented provides great freedom in choosing how to play it. Playing styles may range between the extremes of smoothly flowing harmony within the octave of the home tonic of piece to jumpy harmony in which all harmony notes go up from a chord root line. Improvisation follows naturally and is fun even if not expertly done.

The simplification provided by the alphabet is different in kind from simplifying chord symbols. Any sophistication in the harmony is in the tonic scales of origin of the chords.

Mode signatures provide an above-the-staff markup notation for the concept of "modal jazz" which is otherwise a concept without a notation.

PKP annotations above the staff are not always necessary. With experience, straightforward chord progressions may be sight-read in PKP terms.

REFERENCES

1. Barta, *The Source: The Dictionary of Contemporary and Traditional Scales*, Hal Leonard (1995), for helping me to be sure I was not missing important scales.
2. Mingus Fakebook, Hal Leonard (1991) for the poster-child-for-misleading-complexity chord progression of *Goodbye Porkpie Hat*.
3. *The Real Book, Sixth Edition* (Hal Leonard) for *Giant Steps*.
4. *The Ultimate Jazz Fakebook*, Wong, Hal Leonard (1988) for *All the Things You Are.*
5. *The Standards Real Book*, Sher Music (2000) for *I Got Rhythm.*
6. Mehegan, *Jazz Improvisation 1: Tonal and Rhythmic Principles*, Watson-Guptil (1984), for Roman-numeral chord notation, and for teaching me (unintentionally) that it is not a solution for complex chromatic chord progressions, but a problem if pushed beyond its basic function of specifying chord root lines.
7. Eskelin, *Lies My Music Teacher Told Me*, Stage Three Publishing (1994) for insight into the nature of scales and musical "perfection," and for encouraging me to think outside the box.
8. Dmitri Tymoczko, *A Geometry of Music* (2011) for stimulating discussions of how to think about music from different angles.
9. Mark Levine, *The Jazz Theory Book*, Sher Music Co. (1995) for providing examples of well known jazz scales and harmonic forms using them in conventional notation against which to verify PKP coverage.
10. George Russell, *The Lydian Chromatic Concept of Tonal Organization*, http://www.georgerussell.com/lc.html, for making me aware that PKP covers the concept, because nothing is changed by replacing the Ionian mode by the Lydian mode as the default reference major mode for any piece of music.
11. Edward Frenkel, *Love and Math: The Heart of Hidden Reality*, Perseus (2013) (on Kindle), for many insights into the usefulness of having different views of complex things.
12. Jeff Brent with Schell Barkley, *Modalogy — Scales, Modes & Chords: the Primordial Building Blocks of Music,* Hal Leonard (2011), for the most comprehensive treatment I have found of the conceptual essence of scales, modes and chords as determined by music notation, presented at a higher level of abstraction than note symbols. The result is accurate but overwhelmingly complex.
13. Ross W. Duffin, *How Equal Temperament Tuning Ruined Harmony (and Why You Should Care)*, W.W. Norton (2007), for an understanding of the piano's deviation from musical perfection.
14. Stephon Alexandar, *The Jazz of Physics: The Secret Link Between Music and The Structure of the Universe*, Basic Books (2016). This is an amazing book. The author is a jazz musician and physicist who speaks of doing physics as informing the playing of jazz and playing jazz as informing the doing of physics. Both require rising above formalisms to form conceptual representations. This resonates strongly with my own ideas.

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I did not take this musical journey alone. I received comments and help from many people over the decade or so the ideas were germinating and consolidating.

Music theorist Paul Steinbeck encouraged me to continue writing at a time when I was becoming discouraged about finding a way of bringing my ideas before the music community. Jazz pianist, teacher and composer Taylor Eigsti has been an inspiration to me. Although I have never been a piano student of his, I have learned much from him in sporadic discussions in person and by email. I am deeply grateful for his willingness to take time away from a busy schedule to engage in these discussions. A short series of piano lessons from SF jazz pianist Michael Parsons helped me to see more clearly the relationship between my ideas and standard jazz-piano practice.

Thanks to SMT (Society for Music Theory) members Charise Hastings, Peter Shultz and Neil Newton for insightful email comments and encouragement following announcements of my website on an SMT mailing list. Thanks to music professor Robert Rawlins for email encouragement, and for helpful examples and comments. Aaron Blumenfeld and Susan Muscarella gave helpful courses at the Jazz School in Berkeley (now the Jazz Institute) that provided many examples to chew on (including, from Susan Muscarella's course, some interesting variations on *Summertime* in D minor that helped me understand ornamental scales). Amateur pianist and Jazz School Board Chair Susan Brand, and concert pianist and music entrepeneur Robert Taub, provided early encouragement.

My piano teacher in San Francisco for several years, Ken Fishler, provided inspiration and how-to information on chord voicings, while patiently tolerating and responding to my question-everything approach. My first piano teacher in Ottawa, Canada, Sally Robinson, started me off right as an adult beginner by helping me to learn favorite harmonically sophisticated pieces by following her fingers on the keyboard without understanding anything about what I was doing except that it sounded right — being able to play these pieces, even if clumsily and by rote, enabled understanding to seep in gradually. I think that without this particular way of starting out — fingers on the keyboard first, written music later — I would not have started thinking about keyboard harmony in the way I did. It forced to my attention the large gap between simplicity on the keyboard and complexity of the full notation that represents it.

I wore out the patience of many music professionals by bombarding them with unsolicited email requests for comments on my ideas. I would like to thank four in particular, who politely responded to numerous emails in spite of being uncomfortable with my ideas: Harry Likas, Dmitri Tymoczko, Jeff Brent and Daniel Glover. Their criticisms helped me change my explanations without changing my mind.

Thanks to friends Marva Black, Mike Budde, Peter Marchant and Selinda Spugies for helpful insights on aspects of this material that helped me to understand how to get the ideas across better. Thanks to singer extraordinaire and friend Lorna Kollmeyer for helping me figure out how to organize piano accompaniment in guitar-strumming style.

My wife Sheila's accurate musical ear helps me know what does and does not sound good on the piano, in addition to making my life generally interesting. My grandsons Joshua and Ethan Feiber provided encouragement and comments; Joshua set up the website www.pianotheoryman.com as birthday gift.

SOME COMMENTS FROM READERS

The following comments provide a kind of history of the development of PKP (the dates on the left identify when the commenters read different drafts of this material). The unconventionality of putting tritones on center stage has tended to get in the way of expert musicians accepting the ideas at all, let alone seeing them as simple, which has not been helped by the fact that it took me a long time to find the simple way of explaining the ideas that appears in this document. I continue to think the ideas are important for learning the piano in a less restrictive manner than is conventional but have not yet been able to find an audience. At the very least, the more recent comments show that the PKP method is more than just a fantasy of an over-enthusiastic amateur.

(2016) Taylor Eigsti more recently than below. "… a fascinating and in-depth look at various ways that keyboard shapes can lead to a whole new way to look at notation and the piano."

(2011) Paul Steinbeck. Assistant Professor of Music Theory; Washington University, St. Louis

"The hook ... , at least in my opinion, is that it's possible to attain a deep understanding of chords (and their constituent intervals) without recourse to Western notation. This has direct consequences for physical patterning, fingerings, etc. Essentially, your method combines the utility of a play-by-ear approach with the depth of a mathematically-informed theory of music."

(2009) Robert Rawlins. University Music Department Chair (Rowan University); jazz musician; teacher; author of several books on jazz

"I became aware of Raymond Buhr's novel method for analyzing and voicing chromatic chord progressions in 2008 through a draft of a paper he wrote on the subject. I have kept up to date on developments of the method and we have had many email exchanges discussing issues of interpretation and application. I am a member of his intended target audience—a jazz musician who is not a pianist who needs to work out harmonic patterns on the piano from time to time. I am also a music teacher who has actually tried out aspects of his method on students. I can vouch from personal experience for his method's helpfulness in dealing with complex chromatic chord progressions."

(2009) Susan Brand. Board Chair, The Jazz School in Berkeley; amateur pianist

"When Raymond Buhr consulted me about his theory of chromatic chord progressions, I was immediately struck by his ability to analyze and attempt to simplify this complex subject. Mr. Buhr brings a unique perspective and a great deal of enthusiasm, depth of understanding and originality of viewpoint to the subject. Over the years I have watched the continuous work that he has put into editing and rethinking his work. He has had ongoing consultations with many knowledgeable musicians/ teachers/editors and all have contributed to the development of the method described in this book. His ideas offer a way of understanding musical theory that will add greatly to the field and will be extremely helpful to musicians and music educators."

(2008) Taylor Eigsti. Jazz pianist, composer, teacher, former Artist in Residence at The Jazz School in Berkeley

"Through the brilliant lens of an engineer, Raymond Buhr has laid out an analysis of harmony that is a unique and complex look at the right-brain from the left-brain's perspective."

APPENDIX A: UNCONVENTIONAL ELEMENTS

TERMINOLOGY & NOTATION

- **anchor:** identifies a building block by the position of its bottom end relative to the home tonic
- **anchor set:** set of tritone and fifo anchors that define a scale by construction
- **anchor line:** anchor sequence written above the staff (outlined by circles for tritones & boxes for fifos)
- **alphabet: PADMIL** identifies anchors by the first letters of the names of classical modes
- **building block:** tritones or fifos (fifths or fourths) anchored at alphabet positions
    - tritone or fifo anchors identified, respectively, by **PADMIL** or PADMIL
    - default direction is up from anchor; underline indicates down
    - tritone is same size either way; default fifo size is fifth, strikethrough indicates fourth
    - uncommitted fifo anchors are represented by fifo anchors in angle brackets
- **chromatic scale: 1-p2-2-p3-3-4-p5-5-p6-6-p7-7-1**
- **context:** provided by mode signatures for melody and harmony plus flow of both
- **core:** sequence of building blocks of harmony identified by an anchor line
- **family:** a set of sub-scales of a family scale defined by a single mode signature
- **fifo:** fifth or fourth that are opposite inversions (add up to an octave)
- **flow:** formed by morphs and slides of building blocks
- **frame:** defined by the tonic **@** and pitch center **$** of a tonic octave (plus sometimes by tonality)
- **mode signature:** word identifying a tritone cluster and implying an anchor set via the scale dictionary
- **pattern:** organized arrangement of intervals on the keyboard or over time
- **pitch center:** note identified by a fifth/fourth octave split, symbolized by **$**
- **morph:** small change in the size of a building block while holding one end fixed
- **outside:** not in a given tonic scale (as distinct from "chromatic" meaning not in a key-signature scale)
- **shape:** combination of building blocks
- **slide:** size-preserving movement of a building block
    - **wobbly slide:** combined morph and slide
- **symmetry:** same keyboard-interval sequence going up and down a shape
- **phlat:** prefix **p** identifying chromatic-scale notes in the whole tone gaps of the major scale of a tonic
- **tonic pointer:** suffix of form **@t** attached to an anchor symbol to indicate a secondary reference tonic
- **word:** set of alphabet letters with optional dots indicating skipped letters

ABOUT THE ALPHABET

A special, boldfaced font for the **PADMIL** alphabet distinguishes these uses of the letters from other uses in music (note symbols "A" and "D", RN symbol "I"). Letters **A** and **D** cannot be confused with letter notes because they identify adjacent piano keys; the different letters are never mixed together in the use of PKP. That said, when the music includes letter notes "A" or "D" avoiding confusion requires effort. I have found the effort worthwhile. I thought of substituting the Greek letters epsilon (**Є**) and delta (**Δ**) for **A** and **D** and the English letter **Y** for **I**, to yield **PЄΔMYL**, pronounced almost the same and therefore easy to remember as having the same meaning (epsilon represents "Ae" of Aeolian). However, the mix of different alphabets introduces a different kind of confusion. I have not been able to think of anything better than **PADMIL**.

QUESTIONS AND ANSWERS LEADING TO THE ALPHABET

Why does every piano key require many different note symbols (the note itself, a sharped lower note, a flatted higher note, cancellation by a natural of a sharp or flat on the note itself)? The fundamental principles of the physics of sound determine that the pitches of the notes of overlapped octaves of different tonics cannot all be exactly aligned. The piano ignores the principles by

aligning them. This is technically incorrect but works because the pitch differences are tiny and half tones are dissonant intervals that sound much the same to human ears when slight errors are made in their pitch sizes. Equal temperament tuning gives the same scales of different tonics the same shape to human ears. This is not to say everyone likes equal temperament tuning (see *Lies My Music Teacher Told Me* for observations on how it can affect singers accompanied by the piano, and *How Equal Temperament Tuning Ruined Music* for a thoroughly negative view).

Why is there no simple representation of scales relative to any tonic "do" analogous to the singer's scale do-re-mi-fa-so-la-ti-do? A key signature identifies relative modes with the same notes and different tonics, so it inherently cannot identify a single tonic, it can only offer a choice of multiple possible tonics that use the same notes. Not only that, a tonic is sometimes assigned different note symbols for a major or minor scale, leading to misleadingly complex representations of simple major/minor transitions (for example, Db for a major tonic and C# for a minor tonic, both of which are the same black piano key).

Why do chord symbols have to specify all the notes of a chord when some of them are determined by context in tonal music? It's the nature of chord symbols — they identify full chords independently of context. There's a weak concept of core harmony in the common use of "guide intervals" as starting points for voicing chords, but the guide intervals are known by degree numbers (e.g., 3rd-7th) that only identify relative positions of notes going up from chord roots, which change from chord to chord. The same note has different degree numbers in different chord scales and in the tonic scales of origin of the chords. The relationship between a written chord progression and the piano keys to play different voicings of it is indirect, intricate and complex in these terms.

Why are many chord progressions dauntingly complex for all but wizards? Part of the answer was just given. Here's the other part. Chord symbols are borrowed from classical modes and adjusted to fit elsewhere by various tweaks, the most common of which is adding suffixes that identify offsets from the standard note positions. This is like "banging square pegs into round wholes" — the results tend to be messy.

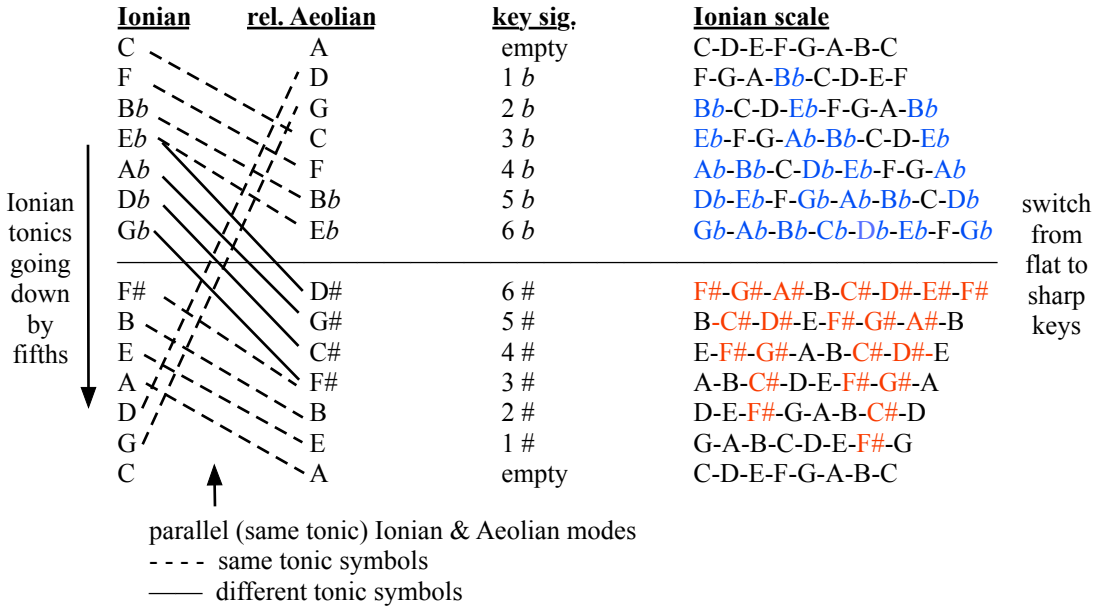
Why not use the chromatic scale to identify anchors of building blocks, thus dispensing with the alphabet? The answer has several parts: the double meaning as notes sometimes and anchors of building blocks at other times would be hopelessly confusing; the scale provides twelve possible anchors, which is double the number needed; and the alphabet provides readable words.

APPENDIX B: ABOUT SCALES

KEY-SIGNATURE SCALES

The following summary of the standard interpretation of the major and natural minor tonic scales of key signatures (Ionian and Aeolian modes) is a helpful reference. It shows the sharped notes of sharp scales in red and the flatted notes of flat scales in blue. Although there are only five black piano keys, scales with six flats or sharps exist because of the scale-spelling rule that the same letter note cannot appear on both sides of a half-tone interval. For example, note B is C*b* in a 6-flats scale.

Major (Ionian) and relative minor (Aeolian) scales are shown on the same line because they have the same key signature. Parallel minor scales (diagonal lines) have the same tonic, which does not necessarily mean the same note symbol because the same piano key may be represented by different note symbols (e.g., the parallel D*b* major and C# sharp minor scales, which have the same black-key tonic on the piano, formally have slightly different tonics indicated by these different note symbols).



Avoiding multiple written key signatures by using accidentals to identify implicit key changes replaces one kind of complexity by another because accidentals have many more uses than this (e.g., ornamental passing notes, or identifying scales that originate in no key signature). Sorting out the possibilities requires considerable head-scratching at a very low conceptual level.

Key-signature-based music notation has stood the test of time and is here to stay because of the huge legacy of music written in it. But the piano has also stood the test of time without needing adjustable piano keys to play the nominally slightly different pitches identified for each piano key by music notation.

CHROMATIC SCALE

The chromatic-scale notation may appear to, but doesn't, violate a fundamental spelling rule for the scales of key signatures, namely that the same note symbol cannot appear twice in a scale. For example, the Ionian and Aeolian scales of any tonic are **1-2-3-4-5-6-7-1** and **1-2-p3-4-5-p6-p7-1** in this notation, neither of which repeats symbols. Sequences that repeat symbols, such as **4-p5-5**, reflect

the same thing in music notation, except more simply (e.g., this sequence could be F-G ♭-G or F-F♯-G of a C blues). Experts develop rules of thumb about sharps and flats as indicators of what comes next and see this way of representing scales as omitting these clues. The clues are present in a different way in the positions of the scale notes relative to a home tonic and to each other.

SCALES AS INTERVAL STACKS

The simplest possible spelling out of a piano scale is as a sequence of inter-piano-key intervals. For classical modes, the intervals are either half tones or whole tones (two half tones). Other scales include intervals of minor thirds (3 half tones). The numbers **1**, **2** and **3** identify these intervals. Any scale may be spelled out as a stack of these numbers that adds up to twelve. For example, the Ionian mode is identified by **|2212|221|** and the Aeolian mode by **|2122|122|**, where the vertical lines indicate the scale frame. The Ionian stack also identifies inter-note interval sequence of the singer's solfege scale **do-re-mi-fa-so-la-ti-do**. The interval stacks are the same for parallel or relative modes, which aids clear thinking about the nature of modes and the relationships between them. For example, the interval stack of the Aeolian mode is visibly a rotation of that of the Ionian mode.

The Ionian and Aeolian modes are visibly determined by symmetric shapes (same interval sequence going up or down) highlighted next: **|2122|122|** and **|2122|122|**. The highlighted shapes are formed of a tritone with attached half tones outside or inside it. Symmetric shapes that reveal underlying simplicity are lurking everywhere in music, obscured by music notation. They become important for multi-tritone scales. The importance of symmetry should not be surprising — music is complex and symmetric shapes are well known to be helpful in dealing with many kinds of complexity in other fields than music.

This useful notation is an adaption of a standard notation called "Figured Bass Notation" (Wikipedia) for identifying harmony by annotating bass notes on a staff with number stacks that represent counts of scale steps going up from the bass notes. The adaption replaces counts of scale steps with counts of half tones, and lists the stacks horizontally instead of vertically.

APPENDIX C: ABOUT CHORDS

Chords identified by chord symbols of any kind boil down to combinations of the two kinds of building blocks. Here follow some examples. Two fifos (fifths or fourths) form major-7, minor-7 or major-6 chords. One fifo and one tritone form dominant-7, half-diminished-7 (a.k.a. minor-7- ♭5) or minor-6 chords. Three fifos or two fifos and a tritone form 9th, 1lth and 13th extensions of these chords. When tonic scales depart from key signatures, the same kinds of building blocks are available from the scales but now more than one tritone is available. For example, two tritones form diminished-7 or dominant-7- ♭5 chords. Two tritones and one fifo (that may share a note with a tritone) form variations of other chord types such as dominant-7♭9. The exceptions prove the rule. An augmented fifth (8 half tones or two major thirds) that are "building blocks" of minor-major-7 chords are combinations of scale fifos or tritones with their inner notes omitted). Simpler triad chords are, structurally, truncated combinations of combinations of two building blocks.

ANCHOR LETTERS FROM TRITONE CHORDS

As described in the body, tritones are the agents of context change. They are directly visible in melody lines if you see them in terms of keyboard intervals between successive notes. They may be extracted from chord progressions using the following table of tritone chords. In either case, they're represented by anchor letters from the **PADMIL** alphabet that identify the position of their nearest notes above the home tonic.

The following table of tritone chords summarizes the simplest examples of chords with tritones in different positions. Only variations that alter tritone content are included. For example, R7, R9, R7#9, R13 and R9(13) are all variations of R7 with the same tritone content and so are all represented in the table by R7 (variations are left to context). Diminished seventh chords (dim7) have no counterpart in classical modes. Sus chords are not shown because they have no tritones. They are typically V-7♯3 chords. A Vsus-V-I progression is a substitute for a II-V-I progression in which one note is altered between the first two steps.

offset of tritone bass note above root R	suffixes on root symbol R										
	7	7♯9(13)	9♭13 (or ♯5)	7♭9	7♭5 (or ♯4 or ♯11)	dim7 (or o7)	m7♭5	m6	M7(11)	♭9	M7♯11 (or ♭5)
fourth									x		
major third	x	x	x	x	x						
minor third		x				x		x			
whole tone			x								
half tone				x						x	
0					x	x	x				x

Inverting the tritone of a chord into the home octave (if it isn't already there) determines its PKP anchor. For example, The tritone of a V-7 chord is partially outside the home octave but its inversion

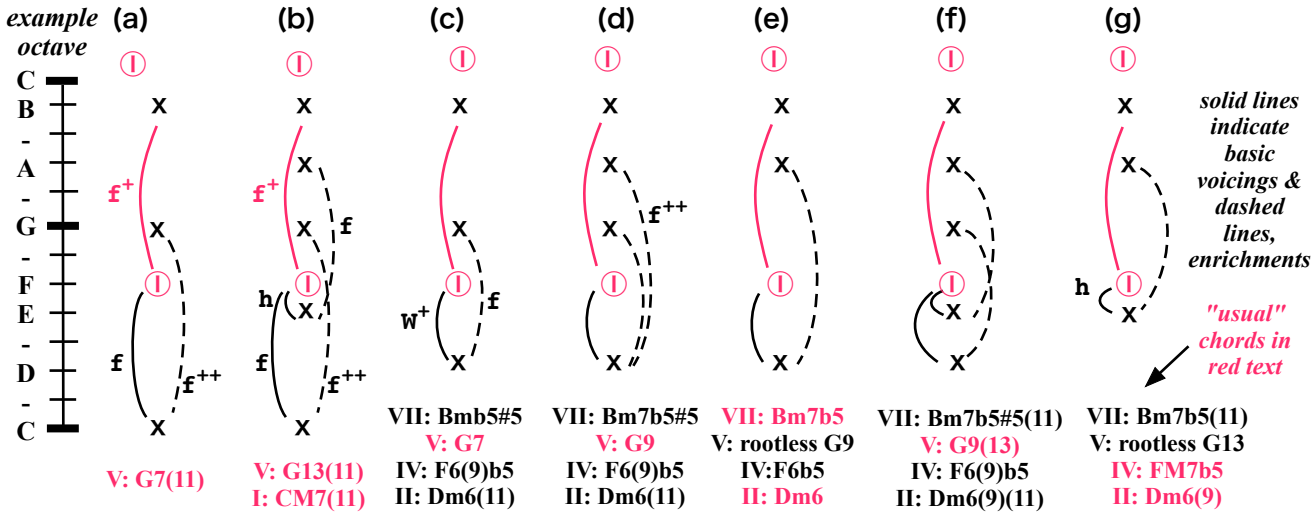
is within it. The bass note of its inversion is a fourth above the home tonic which makes its anchor **I**.

Double tritones are the normal upper limit for tritone content of chords generally encountered above the staff in fake books, lead sheets and sheet music.

MISLEADINGLY COMPLEX CHORD SYMBOLS

Misleadingly complex chord symbols follow from "banging square pegs into round holes" (fitting basic chord symbols into places in scales where they don't quite fit). This is illustrated next by the startling variety of single-tritone chords from the Ionian mode that may be voiced by the single tritone plus a carefully selected scale note below or above the tritone (or both). The symbols **f**, **f+** and **f++** stand for fourth, tritone (augmented fourth) and fifth (this was prior to my decision to use numeric suffixes or prefixes for these intervals of keyboard shapes).

Enrichments by overlapped fifos are important when the chords are played by themselves but the notes they add are visibly fundamental scale notes (the pitch center and a whole tone above it) in the context of the flow of a chord progression, which the ear tends to hear even if the notes aren't sounded. A bass note a half tone below the tritone is equivalent to a fourth above it (same note), yielding the interval stack **f-f+**. The result is an "all fourths" shape. Seventh or sixth chords voiced by all fourths shapes have extended or altered chord symbols. This is in contrast to the "all thirds" shapes of seventh chords identified by basic chord symbols.



The figure and these examples are specific but the concepts are general. Different voicings of different tritone chords may created in a very simple way by specifying a bass or treble note that implies visibly obvious enrichment fifos from the tonic scale. The tonic scale is understood from tritone content. The enrichments are without reference to chord symbols; they're usually so obvious that no extra notation is needed.

Well formed voicings of chord progressions follow from morphing tritones into fifos. For example, for tonic C, the fifo-tritone sequences in (a) and (c) voice chords as follows:

- (a) **f-f+** identifies a 3-note voicing of Dm7(11)-G7 or G7♯3-G7, both equivalent in the flow to Dm7-G7.
- (c) a different **f-f+** is a different 3-note voicing of Dm7(11)-G7 or G7♯3-G7, also equivalent in

the flow to Dm7-G7.

FIGURED BASS NOTATION; EXTENDED CHORDS

Figured bass notation provides a simple representation of chords from the the highly regular scales of classical modes (7 notes, no adjacent half tones). It represents chords by stacks of numbers going up from a bass note, in which each number is a count of the scale steps to the next note up. The stacks are annotated on a staff as literal stacks of numbers (vertical lists). I write them here as horizontal lists separated by plus signs. Translating interval stacks into figured bass notation (and vice versa) for classical modes is easy because **W+** and **W++** are always two scale steps and **W** is always one scale step (e.g., a seventh chord is **2+2+2** and a sixth chord is **2+2+1**).

Figured bass notation can be helpful for understanding extended chords of classical modes that follow from splitting chord steps of size **2** into **1+1** sequences and then moving the added note up an octave. The **+4** on top of two of the extended chords on the right is a consequence of avoiding **1+1+1** sequences in the in-place forms. The corresponding chord symbols have degree-number suffixes 6, 9, 11 or 13 added to basic symbols to identify the extensions. Extension suffixes altered by sharps or flats are needed to represent chords from scales that are not classical modes.

chord type	(2) = split	in place	extended
seventh	2+2+2	2+2+2	–
ninth	(2)+2+2	1+1+2+2	2+2+2+2
eleventh (seventh+11th)	2+(2)+2	2+1+1+2	2+2+2+2+4
thirteenth (ninth+13th)	(2)+2+(2)	1+1+2+1+1	2+2+2+2+2+4

APPENDIX D: ABOUT PARALLEL MODES

MODES FROM MODALOGY

The non-classical parallel modes as presented in the book *Modalogy* are summarized next, except with notes in numeric-chromatic-scale notation, instead of in the RN (Roman Numeral) notation with sharps and flats used in *Modalogy*. Corresponding PKP mode signatures are shown on the left.

The purpose is twofold. One is to verify the PKP view of modes relative to *Modalogy*. The other is to illustrate the complexity of the conventional representations. For example, the tritone anchored by **D** that is a component of many of the minor modes is understood as **p3-6** in the symbolic chromatic scale, and that's it. Inversions in different places in harmony are left to context. In *Modalogy*, tritone anchor **p3** is ♭III or ♯II and tritone anchor **6** is VI or 𝄫VII, and that's only the anchors.

The alphabet soup of names and the intricate details of the scales makes them very difficult to distinguish functionally, or to remember as enumerated scales. Seeing the possibility that tritones might be fundamental scale-defining objects for these modes is effectively impossible in these terms. This impossibility is demonstrated by a discussion in *Modalogy* of defining and non-defining notes of the many and various modes that never mentions tritones.

The unique mode signatures of PKP are novel. Green highlighting brings forward to the eye that some modes are of opposite tonality to the master mode, or of mixed tonality.

Parallel Modes of the Melodic Minor |WhWW|WWh|

alt-LA	1-2-p3-4-p5-p6-p7-1	Aeolian Diminished (Aeolian- ♭5)
LA	1-2-3-p5-p6-6-7-1	Lydian Augmented (Lydian-♯5)
alt-ML	1-p2-p3-3-p5-p6-p7-1	Jazz Altered (Locrian- ♭4)
ML	1-2-3-p5-5-6-p7-1	Lydian Dominant (Lydian- ♭7)
DI	1-2-p3-4-5-6-7-1	Melodic, or Jazz, Minor (Ionian- ♭3)
AM	1-2-3-4-5-p6-p7-1	Jazz Mixolydian (Mixolydian- ♭6)
PD	1-p2-p3-4-5-6-p7-1	Jazz Phrygian (Phrygian-♯6)

Parallel Modes of the Harmonic Minor |WhWW|hW+h|

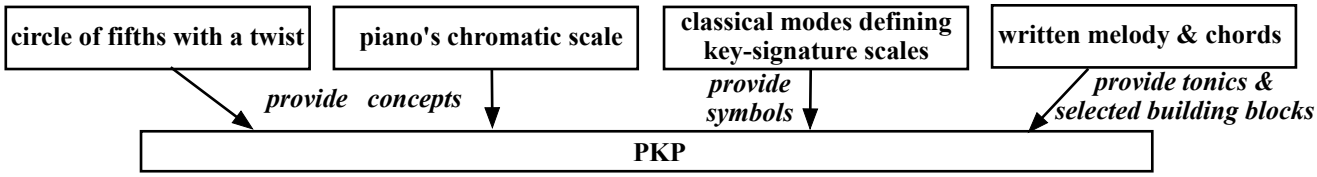
DL	1-2-p3-p5-5-6-p7-1	Romanian, Dorian ♯4, Mishebarakh
alt-DL	1-p2-p3-3-p5-p6-6-1	Leading Tone Minor Diminished, Super Locrian ♭7
LD	1-p3-3-p5-5-6-7-1	Lydian Blues Major, Lydian ♯2
alt-LD	1-p2-p3-4-p5-6-p7-1	Jazz Phrygian Diminished
AI	1-2-p3-4-5-p6-7-1	Harmonic Minor, Aeolian ♮7, Jazz Minor ♭6, Mohammedan
IA	1-2-3-4-p6-6-7-1	Ionian Augmented
PM	1-p2-3-4-5-p6-p7-1	Phrygian Dominant

Parallel Modes of the Harmonic Major |WWhW|hW+h|

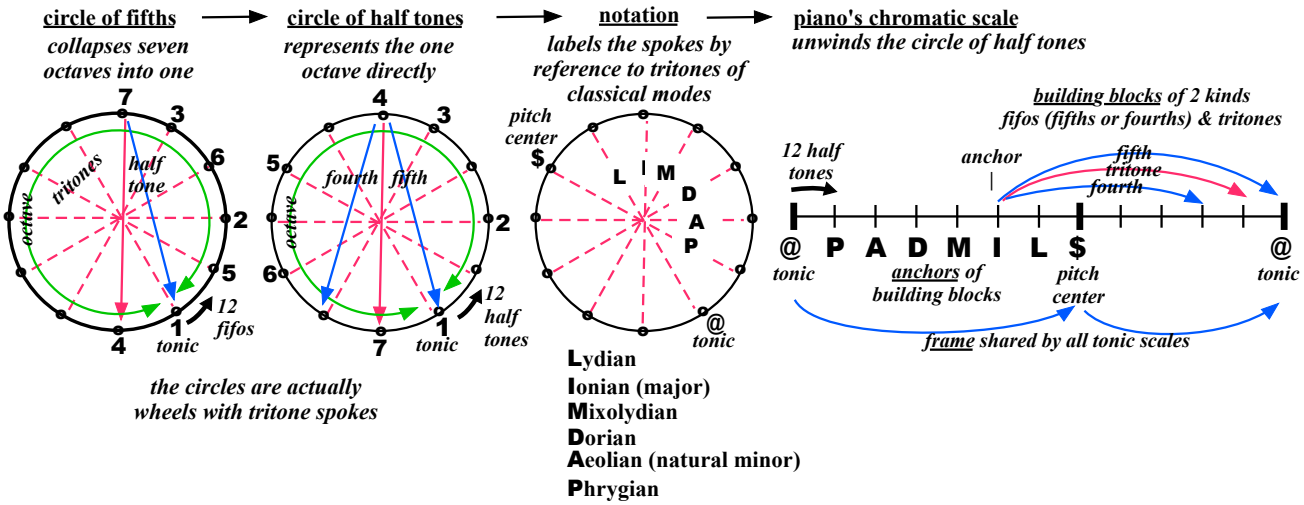
DL	1-p3-3-p5-p6-6-7-1	Lydian Blues Augmented, Lydian Augmented ♯2
alt-DL	1-2-p3-4-p5-6-p7-1	Jazz Minor ♯4, Lydian Diminished
LD	1-2-p3-p5-5-6-7-1	Lydian Melodic Minor, Lydian ♭3
alt-LD	1-p2-p3-4-p5-p6-6-1	Leading Tone Major Diminished, Locrian ♭7, Locrian Diminished-7
AI	1-2-3-4-5-p6-7-1	Harmonic Major
PM	1-p2-3-4-5-6-p7-1	Jazz Phrygian Dominant, Mixolydian ♭2
MP	1-p2-p3-3-5-p6-p7-1	Altered Phrygian Dominant, Phrygian ♭4, Superlocrian ♮5, Superphrygian

APPENDIX E: ABOUT SYMMETRY & SYMMETRY-BREAKING

The elements of PKP are summarized below in a way that provides a link between the concept of symmetry breaking in these pages and in the book *The Jazz of Physics*.



The circle of fifths shown below left presents the basic elements of tonic scales and chord progressions from them in one simple picture. The points going counterclockwise around the circle are 12 notes a fifth apart (7 half tones) across 7 keyboard octaves (12 fifths x 7 half tones = 7 octaves x 12 half tones). The circle is manifestly a wheel with tritone spokes. The notes of the major scale are the numbered points around the half circle delimited by the highlighted tritone spoke (6 half tones).



The circle visibly establishes two kinds of Lego™-like **building blocks** as fundamental elements of music, namely fifths or fourths referred to jointly as **fifos** for simplicity, and tritones. The circle as labeled identifies fifths going up the keyboard through 7 octaves or fourths going down the keyboard through 5 octaves. For the major scale, it identifies a basic chord root line going down by five fifths (3½ octaves) or five fourths (2½ octaves) to the tonic (equivalent to progressively zig-zagging down a fifth and up a fourth within one octave).

The pictorial geometry of the circle of fifths and the musical geometry of its interpretation are different in a way that is counterintuitive (different numbers of octaves going around the circle for fifths and fourths, very different musical sizes of lines of almost the same pictorial length that cut across the circle). This becomes confusing for more general chord progressions of this and other scales.

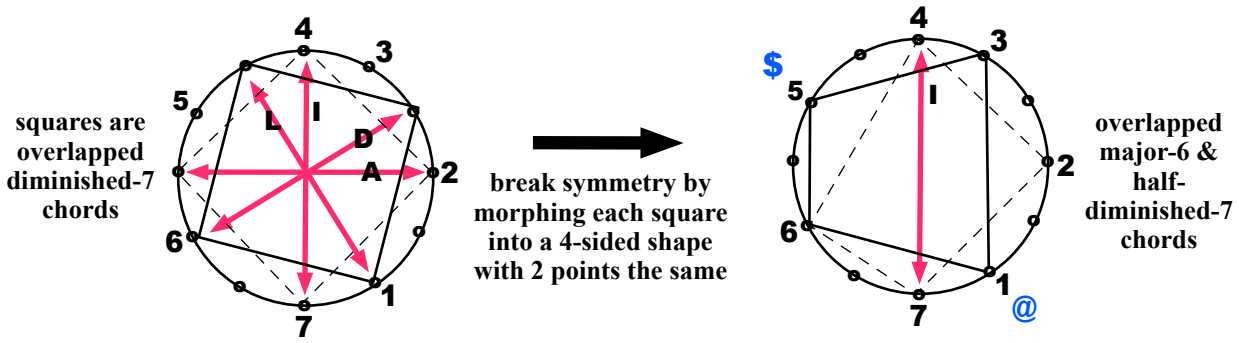
The circle of half tones rearranges the spokes of the circle to put the labeled points around it in scale order. The result is an intuitively natural **@1** between pictorial geometry and musical geometry. Around the circle remains an octave and across any spoke remains a tritone. The half tones across the circle now go around it and the fifos around the circle now go across it.

The end points of all the spokes of either circle visibly identify all the points around it, suggesting a notation that labels spokes (building blocks) instead of points (notes). This simple observation is the twist that leads to PKP's unique combination of simplicity and depth. The spoke labels, called **anchors**, directly identify tritones but also identify fifos. In the chromatic scale that unwinds from the circle, each anchor letter identifies a tritone and two fifos morphed from it. The letters identify relative positions of the anchors in the chromatic scale, and outlining them differently in annotations above the staff distinguishes the different kinds of building blocks (circles for tritones and squares for fifos, e.g., Ⓘ and 🄸). Tritone anchors are unambiguous independently of context and fifo anchors are ambiguous but the ambiguity is generally resolved by context. No anchors are needed in the top pitch half of the octave because inversions cover them.

The letters of the **PADMIL** alphabet mark the relative positions of the anchors of the single tritones of classical modes, but this is only to provide a correspondence between PKP notation and music notation. It does not bind the letters to the classical modes, or to tritones. The letters are in a special boldfaced font to distinguish them from other uses of some of them in music notation/theory (there is no actual possibility of confusion because the different notations are never mixed together).

SYMMETRY BREAKING IN THE CIRCLE OF FIFTHS

According to *The Jazz of Physics*, symmetry-breaking is a deep feature of how both music and the universe work. Here's a view of breaking the symmetry of a diminished scale to yield a major scale that corresponds to an example on page 66 of the book (in the book, the dotted square and its transformation are not shown and no mention is made of tritones). This kind of thing is fascinating to anyone with a mathematical bent but probably not anyone else.



The symmetry-breaking is shown next in progressive steps that morph tritones into fifos.