Mock Test 1

Part A: Knowledge and Understanding (21 marks)

1) The function $s(t) = 5t(t + 1)$ describes the distance (in km) that a truck has travelled after a time t (in hours) for $0 \le t \le 5$. Calculate the average velocity of the car from $t = 2$ to $t = 2.1$ (4 marks)

2) Evaluate the following limits, if they exist. If they do not exist, state why. (2 marks each)

a) $\lim\limits_{x \to 4} 4$

b) $\lim\limits_{x \to \frac{1}{5}} 3\sqrt{5x + 8}$

c) $\lim\limits_{x \to 0} \frac{x-6}{x}$

d) $\lim\limits_{x \to -1} \frac{x^2-4x-5}{x^2-2x-3}$

e) $\lim\limits_{x \to 0} \frac{\frac{3}{\sqrt{x+9}}-1}{x}$

f) $\lim\limits_{x \to -2} f(x)$, where $f(x) = \begin{cases} 6 & x = -2 \\ -6 & x \ne -2 \end{cases}$

3) Determine $\lim\limits_{x \to 0} \frac{\sqrt[3]{x-1000}+10}{x}$ (5 marks)

Part B: Application (26 marks)

1) Determine the values of x for which the function $f(x) = \frac{12}{2x^2-9x-18}$ is discontinuous. (4 marks)

2) Determine the values of x for which the function $f(x) = \frac{\sqrt{3x-6}}{x-5}$ is continuous.
(3 marks)

3) Suppose that you can learn $N(t) = 64t - t^3$ words of a new language after t hours of study time.

   a) how many word have been learned at time $t = 2h$? (1 marks)

   b) what is the rate in words per hour at which the student is learning at $t = 2h$? (5 marks)

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4) Determine $\lim\limits_{x \to -3} \frac{f(x) - 4x}{[f(x)]^2}$ given $\lim\limits_{x \to -3} f(x) = 4$ using properties of limits (3 marks)

5) Determine $\lim\limits_{x \to \frac{7}{2}} \frac{|2x - 7|(x + 3)}{2x - 7}$ algebraically. Illustrate results in a graph. (6 marks)

6) Given the function below

$$f(x) = \begin{cases} -(x+1)^2 + 7 & x < 2 \\ \frac{x}{2} - 6 & 2 \le x < 6 \\ -4 & x \ge 6 \end{cases}$$

   a) Sketch the function. (3 marks)

   b) Find $\lim\limits_{x \to 2} f(x)$ (1 mark)

Part C: Thinking (10 marks)

1) Find constants a and b such that f(x) is continuous for all values of x. (5 marks)

$$f(x) = \begin{cases} x^2 + bx - a & x < 5 \\ 11 & x = 5 \\ ax + 1 & x > 5 \end{cases}$$

2) Find the point on the parabola $f(x) = 3x^2 - 5x - 4$ where the slope of the tangent is horizontal. (5 marks)

Part D: Communication (10 marks)

1) Does the value of a function at a point have to exist in order for the limit to exist at that point? (2 marks)

2) Sketch the graph of a function that satisfies the following conditions. Is your graph the only possible graph? (4 marks)

$$f(4) = -2 \qquad \lim\limits_{x \to 4^-} f(x) = 4 \qquad \lim\limits_{x \to 4^+} f(x) = 0$$

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3) Describe what happens if $\lim\limits_{x \to a} f(x) = f(a)$ for a function $f(x)$. (1 mark)

4) A function s(t) represents the position of an object based on time t in seconds.
Determine what the following represent in the context of the problem. (3 marks)

  a) $\lim\limits_{h \to 0} \dfrac{s(3+h)-s(3)}{h}$ b) s(3)   c) $\dfrac{s(22)-s(3)}{19}$