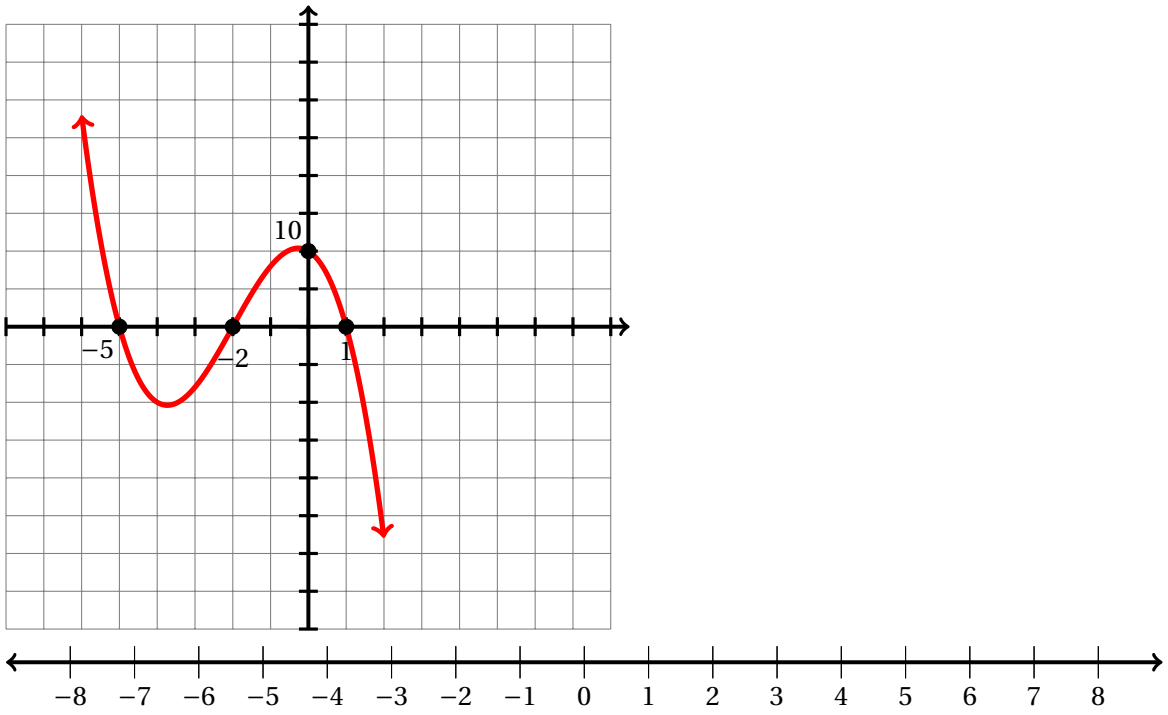
Unit 5 Unit Study Guide Answer Key Version B*

Q1 Given the function $f(x) = -3x^2 + 4x - 5$, evaluate $f(-4)$.

Solution:

-4	-3	4	-5
		12	-64
	-3	16	-69

Q2 Draw a sketch of the graph $f(x) = -(x-1)(x+5)(x+2)$. Label all intercepts in the graph.



Q3 Which of the following is a zero of the polynomial $x^2 + 2x + 10$?

Ⓐ $-1 - i$
Ⓑ $\boxed{-1 + 3i}$
Ⓒ $-3 + 2i$
Ⓓ $-2 + i$

Solution:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(10)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 40}}{2}$$

$$\frac{-2 \pm i\sqrt{36}}{2} \Rightarrow \frac{-2}{2} \pm \frac{6i}{2} \Rightarrow -1 \pm 3i$$

Q4 Which of the following is a factor of $12x^4 - 20x^3 - 6x^2 - 2x - 4$?

Ⓐ $x+3$ Ⓒ $3x+5$

Ⓑ $\boxed{x-2}$ Ⓓ $4x+6$

Solution:

	12	−20	−6	−2	−4	
−3		−36	168	−486	1464	so $(x-3)$ is not a factor
	12	−56	162	−488	1460	

	12	−20	−6	−2	−4	
2		24	8	4	4	so $(x-2)$ IS a factor
	12	4	2	2	0	

Q5 One of the factors of $56x^2 - 13x - 3$ is $(7x+1)$. What is another factor?

Solution: $8x + -3$

The Work:

$$
\begin{array}{r}
8x-3 \\
7x+1\overline{)\ 56x^2-13x-3} \\
\underline{-56x^2\ -8x\ \ \ } \\
-21x-3 \\
\underline{\ \ 21x+3} \\
0
\end{array}
$$

Q6 Factor $g(x) = 2x^3 + 7x^2 - 18x - 63$ completely.

Solution: $(x-3)(x+3)(2x+7)$

The Work:

(1) $(2x^3+7x^2)+(-18x-63) \Rightarrow x^2(2x+7)+9(-2x-7)$

(2) $x^2(2x+7)+{}^{-}9(2x+7) \Rightarrow (x^2-9)(2x+7)$

(3) $(2x+7)(x-3)(x+3)$

Q7 Given the polynomial $w(x) = -2x^3 + 2x^2 + 40x$

(a) State the number of zeroes the polynomial is expected to have.

Solution: The highest power variable is x^3 so there are 3 zeroes.

(b) Determine the zeroes of the polynomial.

Solution:

$$-2x(x^2 + x + 20) \qquad \text{x-factor not shown}$$

The $-2x$ will give a zero of 0.
$x^2 - x - 20$ will factor into $(x-5)(x+4)$
The last two zeroes are 5 and -4.

(c) How many of the zeroes of w are complex zeroes?

Solution: There are no complex zeroes.

Q8 The expression $(x-4)$ is a factor of the polynomial expression $3x^3-17x^2+24x-16$.

(a) State the number of zeroes the polynomial is expected to have.

Solution: There are 3 zeroes for this polynomial.

(b) Determine the zeroes of the polynomial.

Solution:

$$\begin{array}{r} 3x^2-5x+4 \\ x-4\overline{)\;3x^3-17x^2+24x-16} \\ \underline{-3x^3+12x^2} \\ -5x^2+24x \\ \underline{5x^2-20x} \\ 4x-16 \\ \underline{-4x+16} \\ 0 \end{array} \quad \Rightarrow (x-4)(3x^2-5x+4)$$

The quadratic cannot factor so use the quadratic formula.

$$\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(4)}}{2(3)} \Rightarrow \frac{5\pm\sqrt{25-48}}{6} \Rightarrow \frac{5\pm i\sqrt{23}}{6}$$

So the other two zeros are $\boxed{\frac{5-i\sqrt{23}}{6}}$ and $\boxed{\frac{5+i\sqrt{23}}{6}}$

(c) How many of the zeroes are complex zeroes?

Solution: There are 2 complex zeroes for this polynomial.

Q9 Describe the zeroes of the function $h(x) = x^4 - 18x^2 + 81$ and their multiplicity knowing that $(x+3)$ is a root of the polynomial.

Solution: -3 with multiplicity two and 3 with multiplicity two

WORK

$$
\begin{array}{r}
x^3 - 3x^2 - 9x + 27 \\
\hline
x+3\,\big)\; x^4 - 18x^2 + 81 \\
\underline{-x^4 - 3x^3} \\
-3x^3 - 18x^2 \\
\underline{3x^3 + 9x^2} \\
-9x^2 \\
\underline{9x^2 + 27x} \\
27x + 81 \\
\underline{-27x - 81} \\
0
\end{array}
\qquad \Rightarrow \text{Use grouping on the answer}
$$

$x^2(x-3) + {}^{-}9(x-3) \Rightarrow (x^2-9)(x-3)$

Notice that (x^2-9) is a different of square so $(x^2-9) \Rightarrow (x-3)(x+3)$

So the entire factored form is $\boxed{(x-3)(x-3)(x+3)(x+3)}$ and you use this to get the zeroes.

Q10 The polynomial $f(x) = x^4 + x^3 - 7x^2 - x + 6$ has two zeroes equal to 1 and 2.

(a) Use the two given zeroes to find the remaining two factors of f.

Solution: The remaining factors are $\boxed{(x+3) \text{ and } (x+1)}$

WORK:

$$
\begin{array}{r}
x^3 + 2x^2 - 5x - 6 \\
\hline
x-1\,\big)\; x^4 + x^3 - 7x^2 - x + 6 \\
\underline{-x^4 + x^3} \\
2x^3 - 7x^2 \\
\underline{-2x^3 + 2x^2} \\
-5x^2 - x \\
\underline{5x^2 - 5x} \\
-6x + 6 \\
\underline{6x - 6} \\
0
\end{array}
$$

$$
\begin{array}{r}
x^2 + 4x + 3 \\
\hline
x-2\,\big)\; x^3 + 2x^2 - 5x - 6 \\
\underline{-x^3 + 2x^2} \\
4x^2 - 5x \\
\underline{-4x^2 + 8x} \\
3x - 6 \\
\underline{-3x + 6} \\
0
\end{array}
$$

Now factor $x^2+4x+3 \Rightarrow (x+3)(x+1)$
so the missing factors are $\boxed{(x+3) \text{ and } (x+1)}$

(b) Based on your answer to the previous part, what are the remaining zeroes not already provided in the problem.

Solution: The remaining zeroes are -3 and -1.