ALGEBRAIC TECHNIQUES

Rules of Indices	*	*	*
Useful Facts	*	*	*
Negative & Rational Indices	*	*	*

Manipulating Surds	*	*
Rationalising the Denominator	*	*

$a^2 - b^2 =$

$x^2 + bx + c =$

$x =$

$y = A(x + B)^2 + C$ has turning point

Discriminant $b^2 - 4ac$ positive $b^2 - 4ac$ **N**egative $b^2 - 4ac$ zer**O**

Simultaneous Equations

Quadratic Inequalities $a > 0$ $ax^2 + bx + c < 0$ $ax^2 + bx + c > 0$

Graphical Inequalities $f(x) < g(x)$

$f(x) < 0$

$f(x) \geq 0$

Binomial Expansion $(a - bx)^n$ The term with x^p

The expansion only converges to $(a - bx)^n$ as $n \to \infty$ if

Binomial Coefficient	${}^nC_0 =$	${}^nC_1 =$	${}^nC_2 =$	${}^nC_3 =$
a powers ↓				
$(-bx)$ powers ↑				

The Factor Theorem

Division

Partial Fractions

$X \sim$

$X =$

Conditions (1) (2)

(3) (4)

Individual Probability:

$P(X \leq 10)$ 8 9 10 11 12 $P(X \geq 10)$ 8 9 10 11 12

$P(X < 10)$ 8 9 10 11 12 $P(X > 10)$ 8 9 10 11 12

Null Hypothesis H_0 :

Alternative Hypothesis

H_1 :

H_1 :

H_1 :

Conclusion (1) (2) (3)

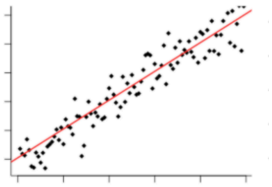
Critical Values

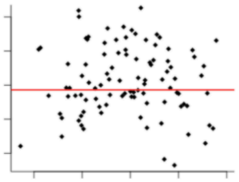
Critical Region

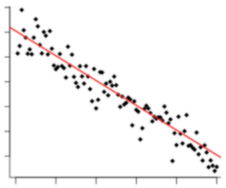
Acceptance Region

Significance Level

Current Bun Problems







Regression equation can only be used to predict

More Reliable Prediction *

*

*

.....

Null Hypothesis H_0 :

Alternative Hypothesis

H_1 :

H_1 :

H_1 :

α	
	n

Conclusion

(1) (2) (3)

DIFFERENTIATION

Notation

1st Principles

Tangent

Normal

Increasing

Stationary Point

Decreasing

Minimum Point

Concave

Maximum Point

Convex

Point of Inflection

$\frac{\mathrm{d}}{\mathrm{d}x}\sin x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}\tan x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}\sec x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}\cos x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}\cot x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{cosec}\, x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}e^{kx} =$

Fact:

$\frac{\mathrm{d}}{\mathrm{d}x}\ln x =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}a^{x} =$

Proof:

$\frac{\mathrm{d}}{\mathrm{d}x}x^{n} =$

Proof:

CHAIN RULE Implicit Connected Rates Parametric

Product Rule

Quotient Rule

If a and b are both positive:

↕ ↕

Exponential Model

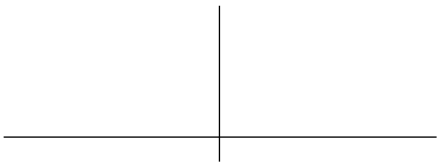
Exponential Growth

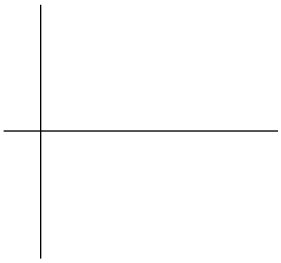
Exponential Decay

Population Growth

Population Decay

Graphs:

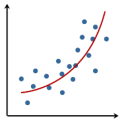




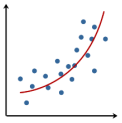
Useful facts:

Laws of Logs:

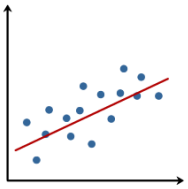
$y = e^{kx}$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x} =$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x} =$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x}$

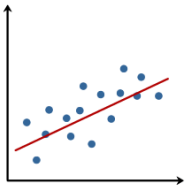


$y = ax^m$



$y = ab^x$





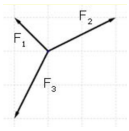
FORCE ACCELERATION

NEWTON'S 1st LAW

NEWTON'S 2nd LAW

NEWTON'S 3rd LAW

Resultant Force



Equilibrium

Weight

Tension

Friction

Thrust

Normal Reaction

Inextensible String

Smooth Pulley/Bead

Light String

Improvements

*

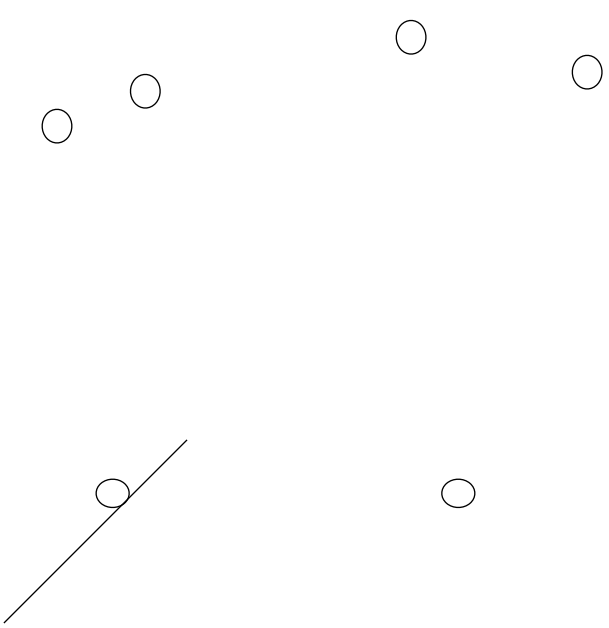
*

*

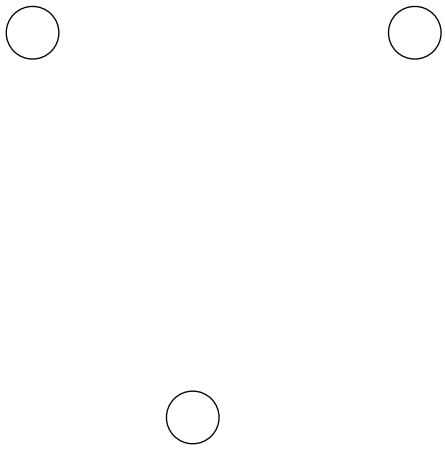
*

FORCE ACCELERATION

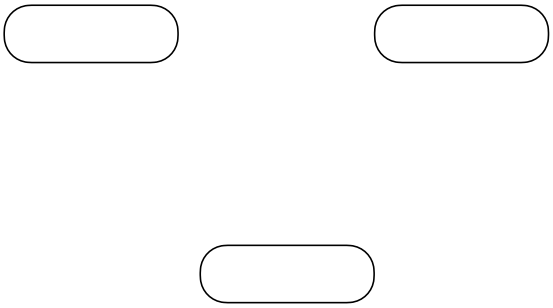
Connected Particles - Pulley



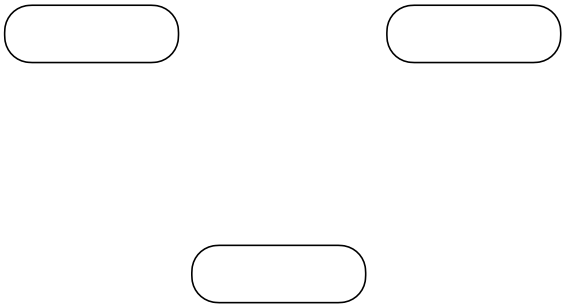
Pulley



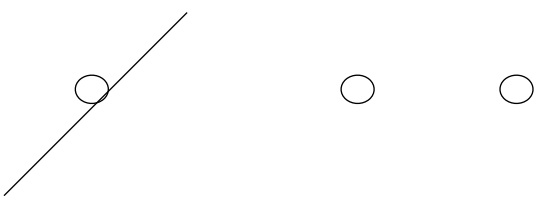
Connected Particles - Lift



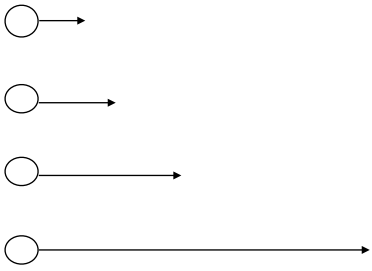
Connected Particles - Tow Bar



Resolving Forces



Friction



AS 67 Pulley Problems	AS 68 Car-Trailer Lift Problems	AS 69 Change in Motion	A 51 Resolving Forces	A 52 Smooth Bead Problems	A 53 Friction

FUNCTIONS

DOMAIN:

RANGE:

VERTICAL LINE TEST:

HORIZONTAL LINE TEST:

INVERSE FUNCTIONS

*
*
*
*
*

(1) (2) (3) (4) (5)

COMPOSITE FUNCTIONS

MODULUS FUNCTIONS $|f(x)| = f(x)$ $|f(x)| = -f(x)$

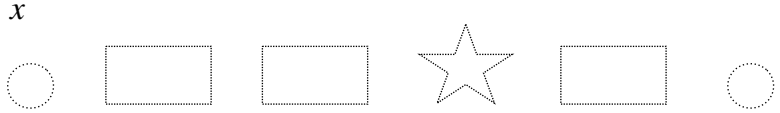
SKETCHING MOD FUNCTIONS $y = |f(x)|$ $y = f(|x|)$

PARAMETRIC EQUATIONS Domain Range

CONVERSION TO CARTESIAN

No trig Trig

THE JOURNEY OF x



GRAPH TRANSFORMATIONS

$f(x+a)$ $\quad$ $f(ax)$ $\quad$ $f(-x)$

$f(x)+a$ $\quad$ $af(x)$ $\quad$ $-f(x)$

Vertical asymptotes

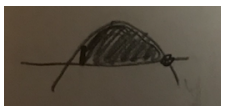
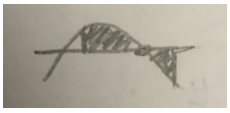
Horizontal asymptotes

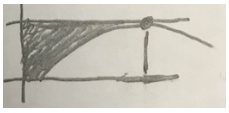
$y=\frac{1}{x}$	$y=a^x$ $\quad$ $y=e^x$	$y=\log_a(x)$ $\quad$ $y=\ln(x)$
$y=\frac{1}{x^2}$	$y=A(x+B)^2+C$	$y=A(x+B)^3+C$
$y=(2x-1)(x+3)^2$	$y=(2x-1)(x+3)^2(1-x)$	$y=\frac{3x+1}{x-2}$

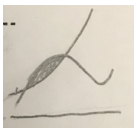
INTEGRATION +c

Notation

Find c





GDA - What was differentiated? - The 10

* * *

* * *

* * * *

$\int f'(x)\sin(f(x))\ \mathrm{d}x$

$\int f'(x)(f(x))^n\ \mathrm{d}x$

$\int f'(x)e^{f(x)}\ \mathrm{d}x$

$\int \frac{\text{linear or higher}}{ax+b}\ \mathrm{d}x$

$\int \frac{\text{lower power}}{\text{factorised thing}}\ \mathrm{d}x$

$\int \frac{f'(x)}{f(x)}\ \mathrm{d}x$

TRIG^2 $\int \cos^2 x\ \mathrm{d}x$ $\int \tan^2 x\ \mathrm{d}x$

$\int \sin^2 x\ \mathrm{d}x$ $\int \cot^2 x\ \mathrm{d}x$

Substitution

Parts

Parametric

Differential Equations

Gradient Distance Midpoint

Line with gradient m through (a, b):

Parallel

Perpendicular

Directly Proportional

Inversely Proportional

Equation of circle centre (a, b), radius r

Circle Language: Segment Sector Chord

Circle facts: (1) (2) (3)

Cosine Rule Sine Rule

Area of a Triangle 1 Area of a Triangle 2

Arc Length Sector Area Ice Cream

PROBABILITY

Tree Diagrams

Venn Diagrams

Lists of Possible Outcomes

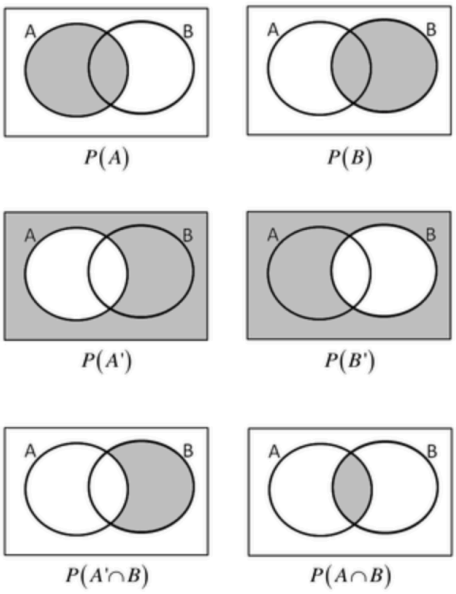
Probability Distribution

The discrete random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 2, 4, 6 \\ k(x-2) & x = 8 \\ 0 & \text{otherwise} \end{cases}$$

x				
$P(X = x)$				

Useful formula:



If A and B are MUTUALLY EXCLUSIVE,

If A and B are INDEPENDENT

The probability of B 'given that' A

Typical Questions: *

*

*

Independent Events * *

NOTATION & PROOF

Number Sets Notation

$x \in \mathbb{N}$

$x \in \mathbb{Z}$

$x \in \mathbb{Q}$

$x \in \mathbb{R}$

Interval Notation

$x \in [a, b]$

$x \in (a, b)$

$x \in [a, b)$

$x \in (a, b]$

$\{x \in \mathbb{R} : x < a\} \cup \{x \in \mathbb{R} : x > b\}$

$\{x \in \mathbb{R} : a < x\} \cap \{x \in \mathbb{R} : x > b\}$

Thing 1 $\Rightarrow$ Thing 2

Thing 1 $\Leftrightarrow$ Thing 2

Thing 1 $\Leftarrow$ Thing 2

Thing 1 $\equiv$ Thing 2

Proof by Exhaustion

Proof by Contradiction

1

2

3

4

Proof by Counter-Example

SAMPLING

Population
Sample

Census
Survey

Sampling Frame

Sampling Unit

	Mean & Variance	Median & IQR
Uses all the data		
Not affected by outliers		

Simple Random Sampling

Unbiased Representative
Cheap Quick Easy

Stratified Sampling

Unbiased Representative
Cheap Quick Easy

Systematic Sampling

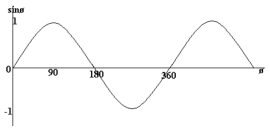
Unbiased Representative
Cheap Quick Easy

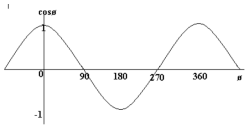
Opportunity Sampling

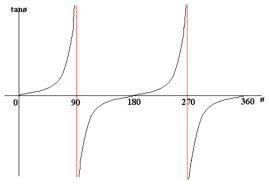
Unbiased Representative
Cheap Quick Easy

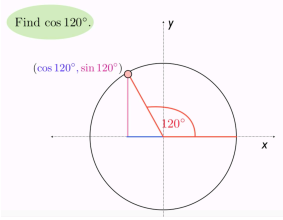
Quota Sampling

Unbiased Representative
Cheap Quick Easy









Trig equation $\sin(\ldots) = a$ (or cos or tan)

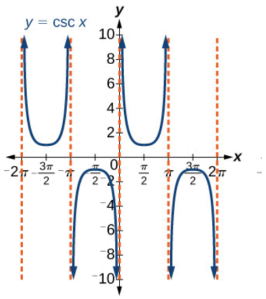
(1) (2) (3) (4)

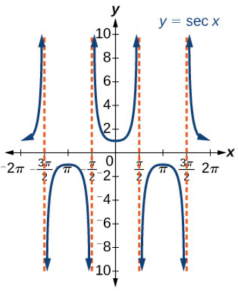
and finally …

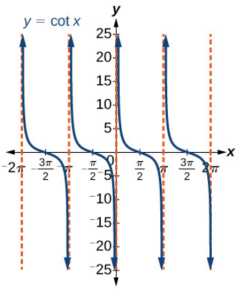
2π rads π rads $\frac{\pi}{2}$ rads 1 rad

When θ is small and measured in radians:

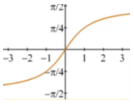
$\text{cosec}x =$ $\sec x =$ $\cot x =$

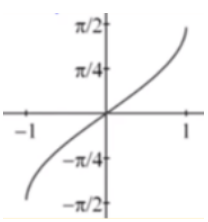


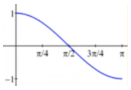


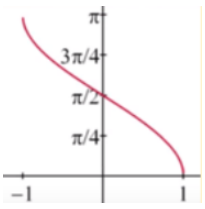


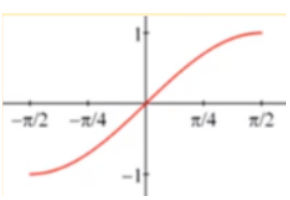
$\arcsin x$ $\arccos x$ $\arctan x$











A 32 Understanding sin, cos, tan	A 33 Solve a Trig Equation	A 13 Reciprocal Trig Functions	A 18 Inverse Trig Functions	A 18a Understanding arcsin, arccos, arctan	A 19 Small Angle Approximations

VARIABLE ACCELERATION

The change in displacement from time t_1 to time t_2

The change in velocity from time t_1 to time t_2

The change in displacement from time t_1 to time t_2 is $\int_{t_1}^{t_2} \mathbf{v}(t)\ \mathrm{d}t.$

The change in velocity from time t_1 to time t_2 is $\int_{t_1}^{t_2} \mathbf{a}(t)\ \mathrm{d}t.$

Notation

Vector Magnitude

Parallel Vectors

Unit Vector Parallel to …..

Vector $\overrightarrow{AB}$

Colinear Points

Angle between vectors

Q divides $\overrightarrow{AB}$ in the ratio $x : y$

	Parallelogram	Rectangle
Trapezium	Rhombus	Square
Similar Triangles	Isosceles Triangle	Right Angled Triangle

VECTORS (MECHANICS)

$F = ma$

s u v a t

Position

Velocity

The Velocity Vector

"Travelling Parallel to ….."

Speed

Direction

α from horizontal

to **i**

to **j**

α from horizontal

to **i**

to **j**

α from horizontal

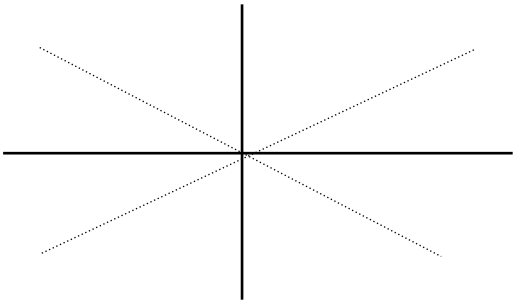
to **i**

to **j**

α from horizontal

to **i**

to **j**



vt, suvat & PROJECTILES

Displacement | Velocity | Acceleration

suvat

*
*
*
*

Proof

Most Common Mistake

PROJECTILES

Horizontally | Vertically

Cartesian Flight Equation (I)

(II)

Time of Flight From Ground ↕ | Time to Greatest Height ↕ | Horizontal Range From Ground ↔