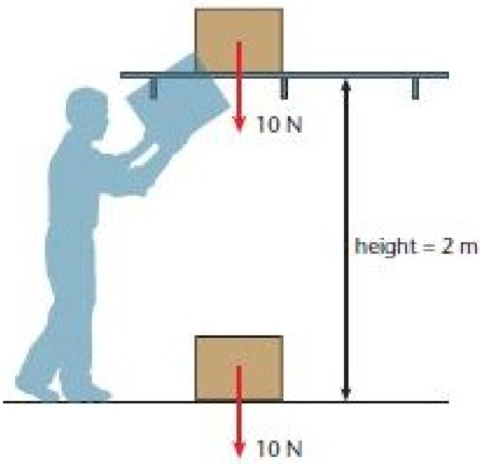
Gravitational Potential Energy and Work

(Syllabus: 9.2.1.2.2)

Gravitational potential energy is equal to the work done to move a mass from infinity (or a distance approaching infinity), that is, outside of the gravitational field of a mass to some point inside the gravitational field of a mass.

On Earth



Gravitational potential energy works in a similar way on all scales. When an object weighing 10 N is lifted onto a 2 m shelf the work done is calculated using the equation:

$$\mathbf{W = m\ g\ h = F \times d}$$

In the example 20 J of work is done. The increase in gravitational potential energy is given by the equation:

$$\mathbf{E_p = m\ g\ h = F \times d}$$

The change in gravitational potential energy is also 20 J.

For a box weighing 10 N falling from a 2m high shelf the work done equals the change in gravitational potential energy equals – 20 J.

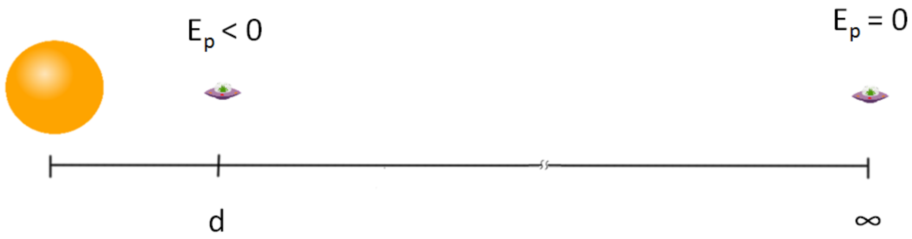
Gravitational potential Energy on Earth is usually defined in terms of the Earth's surface, with an object on the surface of the Earth being thought of as having zero Gravitational potential Energy. This is not particularly satisfactory even on the Earth. If the box from the diagram was pushed into a deep hole, it's Gravitational Potential Energy would become negative, and would become more negative the deeper the hole into which it was pushed. We can get around this by defining Gravitational potential Energy relative to the centre of the Earth.

In order to be useful outside of the near Earth environment we need a more general definition.

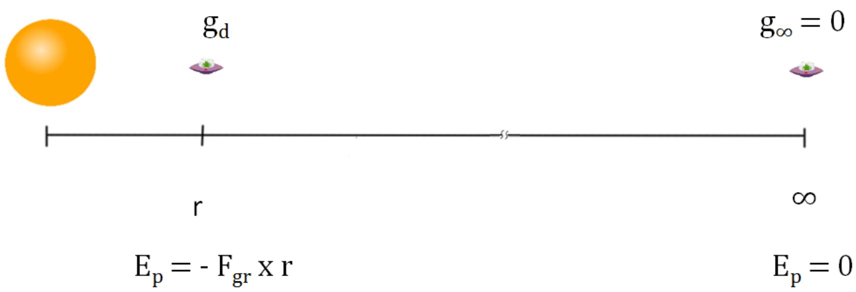
Gravitational Potential Energy and Space

(NSW HSC Syllabus: 9.2.1.2.3)

Gravitational potential Energy is defined as being relative to a point outside the gravitational field of a mass, that is, at an infinite distance from the mass. As in the case of a box falling off a shelf Gravitational Potential Energy decreases as the distance from a mass decreases. Gravitational Potential Energy is, by definition, always negative, as this is a consequence of defining it as being zero at infinity.



Gravitational Potential Energy at any point in Space, relative to a mass (star), such as the one illustrated, is equal to the work done to move a mass (spaceship) from infinity to that point. Just as for a box falling from a shelf the work done is negative, as is the change in Gravitational Potential Energy.



Let us imagine that a space ship (mass m) moves from infinity to some point a distance r from the centre of mass of a star (mass M). The Gravitational potential Energy of the space-ship at r is equal to the work done to move from infinity to r. Just as for a box falling off a shelf the work done is negative.

We can write this in the form of an equation as:

$$E_{pr} = \Delta W$$

Or more precisely:

$$E_{pr} = \Delta W = \int_{\infty}^{r} F_{gr}\, dr$$

The Gravitational Potential Energy at r is equal to the integral (sum of all of the small changes) of work done to move from infinity to r.

The change in potential energy or the work done to move from ∞ to r is not dependant on the path taken, meaning that we are concerned only with the final and initial values, so this can be rewritten as:

$$E_{pr} = \Delta W = E_{p(final)} - E_{p(initial)}$$

This is the same as calculating the change in Gravitational Potential Energy as for a box falling off a shelf, only instead of defining ground level as the reference (zero level) we are defining infinity as the reference point.

For a box falling off a shelf the change in the value of acceleration due to gravity between its initial and final positions is negligible, as therefore is the change in the force due to gravity acting on it. For our space ship, however, this is not the case.

At infinity:

$$\mathbf{g = g_{\infty} = 0}$$

Therefore:

$$\mathbf{F_{g\,\infty} = mg_{\infty} = 0}$$

At d:

$$\mathbf{g = g_d}$$

Therefore:

$$\mathbf{F_{gr} = mg_r}$$

Our equation for the Gravitational Potential Energy of the space-ship at point r thus becomes:

$$\mathbf{E_p = E_{p(final)} - E_{p(initial)}}$$

Therefore:

$$\mathbf{E_{pr} = - F_{gr} \times d - (- F_{g\,\infty} \times \infty)}$$

Or:

$$\mathbf{E_p = - F_{gr} \times d}$$

E_p is negative as the force due to gravity is directed towards the centre of mass (M), and negative work is done on the space ship (m).

Or more generally:

$$\mathbf{E_p = - F_g \times r}$$

Where:

r = distance from the centre of mass (m)

F_g= Force due to Gravity (N)

E_p=Gravitational Potential Energy (J)

Gravitational Potential Energy and Space

When you raise a box weighing 10 N by 2 metres you increase its gravitational potential energy relative to the Earth by 20 J, you also increase the Earth's gravitational potential energy relative to the box by 20 J (Newton's Third Law).

Gravitational potential energy affects both masses, and is the result of the gravitational force resulting from the presence of both masses.

Gravitational potential energy is also related to the distance between two masses, and decreases as the masses get closer together.

The force due to gravity (F_g) is not constant, gravitational field strength decreases with the square of the distance from the centre of mass of an object.

$$F_g \propto \frac{1}{r^2}$$

The force due to gravity exerted by a mass decreases with the square of distance from that mass.

The value of acceleration due to gravity at a distance r in metres from a planet mass M in kg can be calculated using the formula:

$$g = G\frac{M}{r^2}$$

Where g is acceleration due to gravity (ms^{-2}), G is the Universal Gravitational Constant ($6.67 \times 10^{-11}\ Nm^2kg^{-2}$), M is the mass of the planet or other body and r is the distance from the centre of mass of that body (m).

The force due to gravity (F_g) or the weight of an object of mass m can be calculated using the formula:

$$F_g = mg$$

We can combine these two equations, by substituting the expression for g in the first equation for g in the second equation. It becomes:

$$\mathbf{F_g = G\frac{mM}{r^2}}$$

Or, in more General terms, to calculate the force due to gravity between any two masses the equation becomes:

$$\boldsymbol{F_g = G\frac{m_1 m_2}{r^2}}$$

This relationship is Newton's Law of Universal Gravitation.

The constant of proportionality, G, is called the Universal Gravitational Constant (G = 6.67 x 10^{-11} N m^2 kg^{-2}).

Newton's Law of universal Gravitation states:

"Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

We can substitute from Newton's Law of Universal Gravitation into our equation for E_p.

Therefore:

$\mathbf{E_p = - F_g \times r}$

Substituting for F_g:

$$\boldsymbol{E_p = -G\frac{m_1 m_2}{r^2} \times r}$$

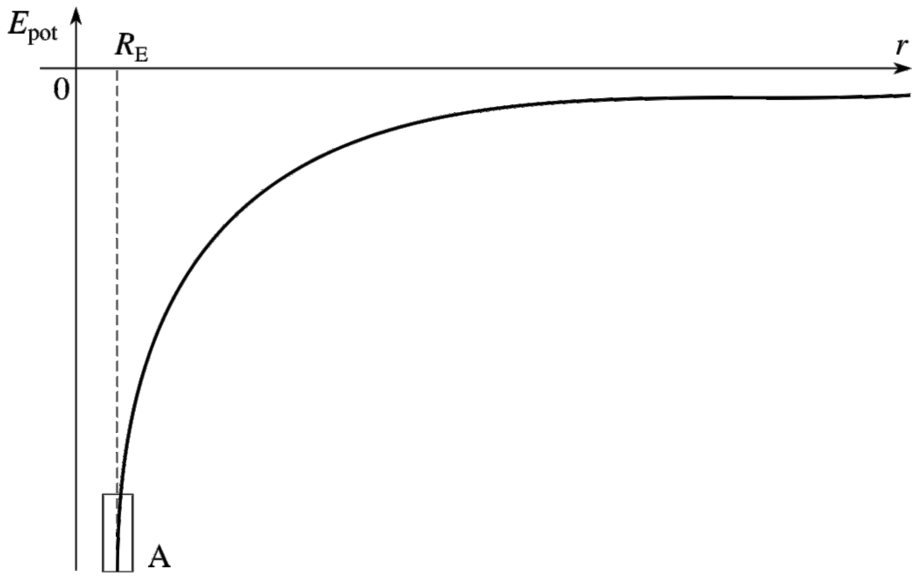
$$\boldsymbol{E_p = -G\frac{m_1 m_2}{r}}$$

Definition:

(NSW HSC Syllabus: 9.2.1.2.3)

Gravitational Potential Energy is equal to the work done to move an object from outside the gravitational field of a mass, to some point within the gravitational field of a mass, the work done is negative as it involves a decrease in potential energy.

Gravitational Potential Energy Graph



Near the Earth's surface the variation in the Earth's gravitational field is small, so that $E_p = mgh$ is a good approximation (box A).

As an object gets further away the variation in g from the Earth's surface becomes more significant.

The formula:

$$E_p = -G\frac{m_1 m_2}{r}$$

is far more accurate for distant objects.

Summary

Gravitational Potential Energy is equal to the work done to move an object from outside the gravitational field of a mass, to some point within the gravitational field of a mass, the work done is negative as it involves a decrease in potential energy.

Mathematically

$$E_p = -G\frac{m_1 m_2}{r}$$

Where: E_p = Gravitational Potential Energy (J)

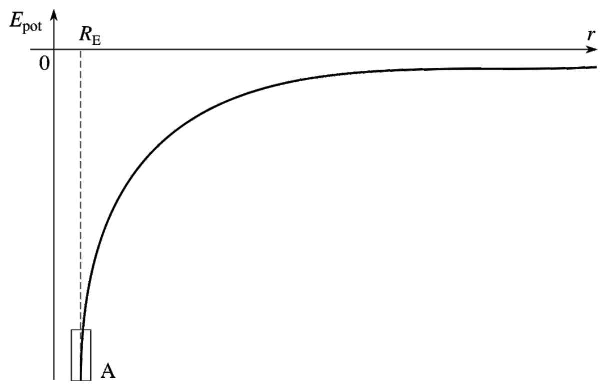
m_1 and m_2 = the masses in kg.

G = The Universal Gravitational Constant

= 6.67 x 10^{-11} N m^2 kg^{-2}

r = the distance between the centres of mass (m)

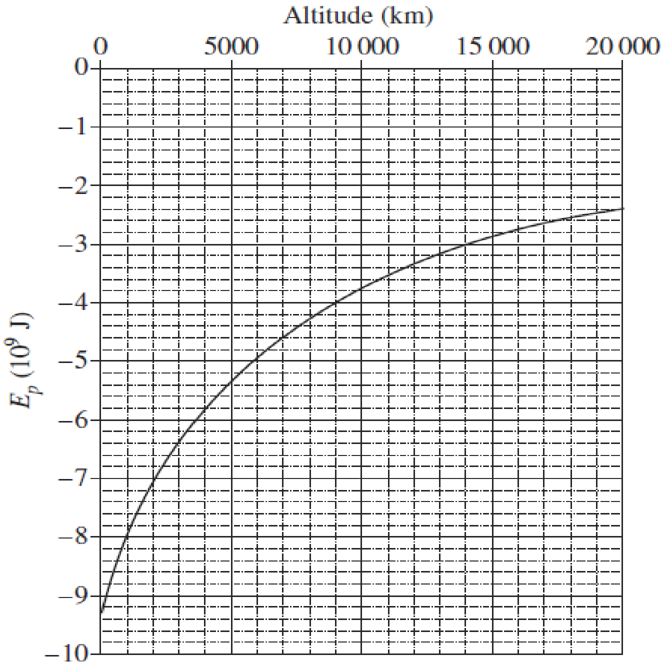
Graphically



Problems:

1, (based on 2012 HSC question 4)

The graph below shows how the gravitational potential energy (E_p) of a satellite changes with altitude.



The satellite descends from 14 000 km to 4000 km, gravitational potential energy changes by?

(a) -8.8×10^9 J

(b) -2.8×10^9 J

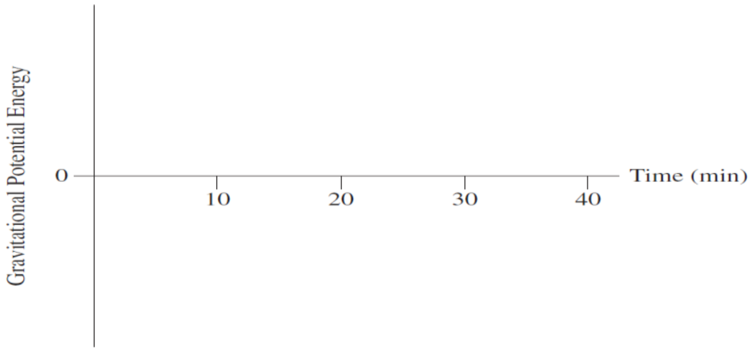
(c) 2.8×10^9 J

(d) 8.8×10^9 J

2, (based on 2014 HSC question 27c)

A space probe is launched from Earth and sent to Neptune. The probe takes 10 minutes to reach an orbit 188 km above the surface of the Earth and it stays in this orbit for several hours.

On the axes below sketch the changes in gravitational potential energy during the first 40 minutes of its journey.



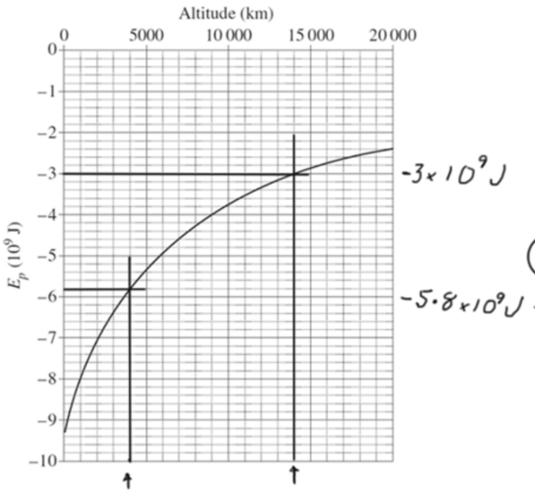
3, (Based on 2006 HSC question 18)

An object is stationary in space at a distance 10 000 km from the centre of a planet. It is determined that 1.0 MJ of work is necessary to move the object to a stationary point 20 000 km from the centre of the planet. Calculate how much more work needs to be done to move the object to a stationary point 80 000 km from the centre of the planet.

<u>**Answers:**</u>

1, b.

The graph below shows how the gravitational potential energy (E_p) of a satellite changes with altitude.



The satellite descends from 14 000 km to 4000 km, gravitational potential energy changes by?

(A) -8.8×10^9 J

(B) -2.8×10^9 J

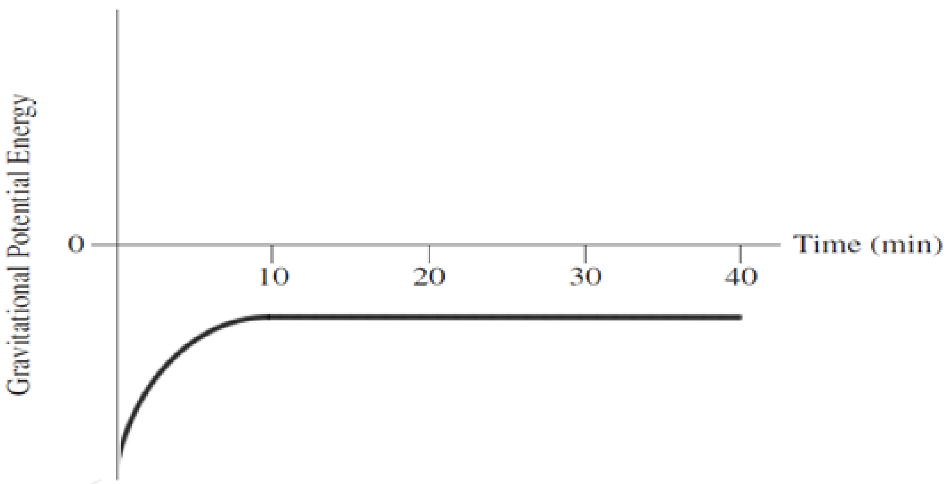
(C) 2.8×10^9 J

(D) 8.8×10^9 J

final − initial = answer

-5.8×10^9 J − (-3.0×10^9 J) = -2.8×10^9 J

2,



3,

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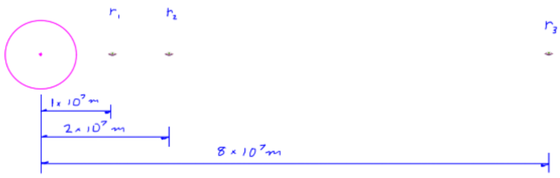
$r_1 = 10,000\,km = 1\times10^7\,m$

$r_2 = 20,000\,km = 2\times10^7\,m$

$r_3 = 80,000\,km = 8\times10^7\,m$



Work $r_1 \rightarrow r_2 = 1.0\,MJ$

Work $r_2 \rightarrow r_3 = ?$

$$\Delta W = -G\frac{mM}{r_2} - \left(-G\frac{mM}{r_1}\right)$$

$$= GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$1\times10^6 = GmM\left(\frac{1}{1\times10^7} - \frac{1}{2\times10^7}\right)$$

$$= \frac{GmM}{2\times10^7}$$

$$\therefore GmM = 1\times10^6 \times 2\times10^7$$

$$= 2\times10^{13}$$

$$\Delta W = -G\frac{mM}{r_3} - \left(-G\frac{mM}{r_2}\right)$$

$$= GmM\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$= 2\times10^{13}\left(\frac{1}{2\times10^7} - \frac{1}{8\times10^7}\right)$$

$$= 750000$$

$$= 0.75\,MJ$$