KEY

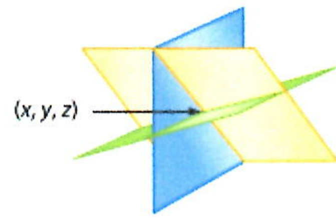
3.4: Systems of Equations in Three Variables

Equations in three variables are graphed as ***planes***. As was the case when graphing lines, there may be:

- ***one solution*** expressed as an ordered triple, (x, y, z)
- ***no solution, ø***
- ***an infinite number of solutions***

One Solution

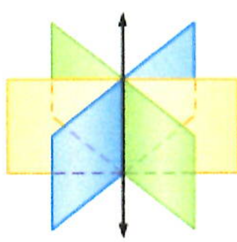
The three individual planes intersect at a specific point.



Infinitely Many Solutions

The planes intersect in a line.

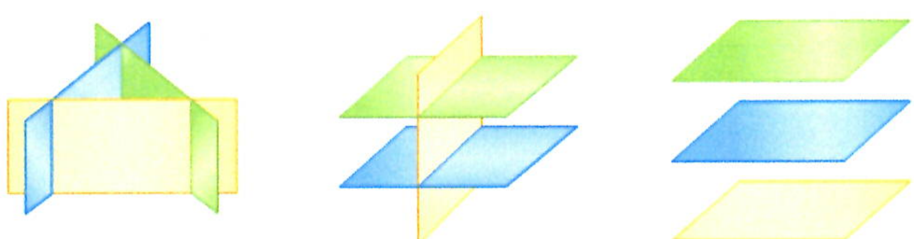
Every coordinate on the line represents a solution of the system.



The planes intersect in the same plane.

Every equation is equivalent.
Every coordinate in the plane represents a solution of the system.

No Solution There are no points in common with all three planes.



Steps to Solving Systems with Three Variables

1) Pick two equations, and eliminate one variable from them.
   You'll be left with one equation, having 2 variables. Let's call this equation "YAY"
2) Pick another pair of equations, and eliminate the same variable you eliminated in step 1.
   You'll be left with another equation, which we'll call "MATH". "MATH" will have the same 2 variables as "YAY".
3) Pair the equations "YAY" and "MATH", to solve for the 2 variables therein.
4) Substitute those two values into one of the original equations, to find the third variable.

Ex#1: Please solve the following system.

(1) $x-3y+z=22$
(2) $2x-2y-z=-9$
(3) $x+y+3z=24$

(1) $x-3y+\cancel{z}=22$
(2) $2x-2y-\cancel{z}=-9$

$3x-5y=13$ "YAY!"

(2) $3(2x-2y-z=-9)$
(3) $x+y+3z=24$

$6x-6y-\cancel{3z}=-27$
$x+y+\cancel{3z}=24$

$7x-5y=-3$ "MATH!"

$3x-5y=13$
$-1(7x-5y=-3)$

$3x-\cancel{5y}=13$
$-7x+\cancel{5y}=3$

$-4x=16$
$x=-4$

$3x-5y=13$
$3(-4)-5y=13$
$-12-5y=13$
$+12 \qquad +12$
$-5y=25$
$y=-5$

$x-3y+z=22$
$-4-3(-5)+z=22$
$11+z=22$
$z=11$

$\boxed{(-4,-5,11)}$

Ex#2: Please solve the following system.

$$x+y+z=1$$
$$x+y-z=3$$
$$2x+2y+z=3$$

$$\begin{aligned} x+y+\cancel{z}&=1\\ x+y-\cancel{z}&=3\\ \hline 2x+2y&=4 \end{aligned}$$
$$\frac{2x+2y}{2}=\frac{4}{2}$$
$$\boxed{x+y=2}$$

$$\begin{aligned} x+y-\cancel{z}&=3\\ 2x+2y+\cancel{z}&=3\\ \hline 3x+3y&=6 \end{aligned}$$
$$\frac{3x+3y}{3}=\frac{6}{3}$$
$$\boxed{x+y=2}$$

Same line; infinite points of intersection

So, we can begin with x

$(x, \ , \ )$

and from $x+y=2$

$$\begin{aligned} \cancel{x}+y&=2\\ -\cancel{x}\quad&\ \ -x\\ \hline y&=2-x \end{aligned}$$

So, $(x, 2-x, \ )$

plug into equation(s),

$$x+y+z=1$$
$$\cancel{x}+2-\cancel{x}+z=1$$
$$2+z=1$$
$$z=-1$$

Same results if we used any other equation. So,

$$\boxed{(x, 2-x, -1)}$$

is the line of intersection

Ex#3: Please solve the following system.

① $x+y+z=2$

② $3x+3y+3z=14$

③ $x-2y+z=4$

$$\begin{aligned} ①\quad x+y+z&=2\\ ③\ -1(x-2y+z&=4)\\ \hline \cancel{x}+y+\cancel{z}&=2\\ -\cancel{x}+2y-\cancel{z}&=-4\\ \hline 3y&=-2\\ y&=-2/3 \end{aligned}$$

(plug into ① & ②)

$$x+\frac{-2}{3}+z=2$$
$$3x+3\left(\frac{-2}{3}\right)+3z=14$$

$$x-\frac{2}{3}+z=2$$
$$3x-6+3z=14$$

then simplify both

$$-3\left(x+z=2\frac{2}{3}\right)$$
$$3x+3z=20$$

$$\begin{aligned} -\cancel{3x}-\cancel{3z}&=-8\\ \cancel{3x}+\cancel{3z}&=20\\ \hline 0&=12 \end{aligned}$$

No solution

$\varnothing$