National
Qualifications
2017

2017 Mathematics Paper 2

Higher

Finalised Marking Instructions

SQA

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General marking principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The illustrative scheme covers methods which are commonly seen throughout the marking. The generic scheme indicates the rationale for which each mark is awarded. In general, markers should use the illustrative scheme and only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the detailed marking instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x=1$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(k) **Horizontal/vertical marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ find mid-point of BC	$\bullet^1$ $(6,-1)$	
			$\bullet^2$ calculate gradient of BC	$\bullet^2$ $-\frac{2}{6}$	
			$\bullet^3$ use property of perpendicular lines	$\bullet^3$ 3	
			$\bullet^4$ determine equation of line in a simplified form	$\bullet^4$ $y=3x-19$	4

Notes:

1. $\bullet^4$ is only available as a consequence of using a perpendicular gradient **and** a midpoint.
2. The gradient of the perpendicular bisector must appear in simplified form at $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be awarded.
3. At $\bullet^4$, accept $3x-y-19=0$, $3x-y=19$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
1.	(b)	$\bullet^5$ use $m=\tan\theta$	$\bullet^5$ 1	
		$\bullet^6$ determine equation of AB	$\bullet^6$ $y=x-3$	2

Notes:

4. At $\bullet^6$, accept $y-x+3=0$, $y-x=-3$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
1.	(c)	$\bullet^7$ find x or y coordinate	$\bullet^7$ $x=8$ or $y=5$	2
		$\bullet^8$ find remaining coordinate	$\bullet^8$ $y=5$ or $x=8$	

Notes:

Commonly Observed Responses:

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>2.</td><td>(a)</td><td></td><td>Method 1

•¹ know to use $x=1$ in synthetic division

•² complete division, interpret result and state conclusion</td><td>Method 1

•¹ $\begin{array}{r|rrrr} 1 & 2 & -5 & 1 & 2 \\ \hline & 2 & & & \end{array}$

•² $\begin{array}{r|rrrr} 1 & 2 & -5 & 1 & 2 \\ & & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & 0 \end{array}$
Remainder $=0$ $\therefore (x-1)$ is a factor</td><td>2</td></tr>
<tr><td></td><td></td><td></td><td>Method 2

•¹ know to substitute $x=1$

•² complete evaluation, interpret result and state conclusion</td><td>Method 2

•¹ $2(1)^3-5(1)^2+(1)+2$

•² $=0$ $\therefore (x-1)$ is a factor</td><td>2</td></tr>
<tr><td></td><td></td><td></td><td>Method 3

•¹ start long division and find leading term in quotient

•² complete division, interpret result and state conclusion</td><td>Method 3

•¹ $\begin{array}{r|l} & 2x^2 \\ \hline (x-1) & 2x^3-5x^2+x+2 \end{array}$

•² $\begin{array}{r|l} & 2x^2-3x-2 \\ \hline (x-1) & 2x^3-5x^2+x+2 \\ & \underline{2x^3-2x^2} \\ & \quad -3x^2+x \\ & \quad \underline{-3x^2+3x} \\ & \qquad -2x+2 \\ & \qquad \underline{-2x+2} \\ & \qquad\quad 0 \end{array}$
remainder $=0$ $\therefore (x-1)$ is a factor</td><td>2</td></tr>
</table>

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(1)=0$ so $(x-1)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the 0 from any method linked to the word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the zero or boxing the zero without comment
   - '$x=-1$ is a factor', '$(x+1)$ is a factor', '$(x+1)$ is a root', '$x=1$ is a root', '$(x-1)$ is a root' '$x=-1$ is a root'.
   - the word 'factor' only with no link

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(b)		$\bullet^3$ state quadratic factor	$\bullet^3$ $2x^2-3x-2$	
			$\bullet^4$ find remaining factors	$\bullet^4$ $(2x+1)$ and $(x-2)$	
			$\bullet^5$ state solution	$\bullet^5$ $x=-\frac{1}{2}, 1, 2$	3

Notes:

4. The appearance of "$=0$" is not required for $\bullet^5$ to be awarded.
5. Candidates who identify a different initial factor and subsequent quadratic factor can gain all available marks.
6. $\bullet^5$ is only available as a result of a valid strategy at $\bullet^3$ and $\bullet^4$.
7. Accept $\left(-\frac{1}{2},0\right)$, $(1,0)$, $(2,0)$ for $\bullet^5$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			•1 substitute for y	•1 $(x-2)^2+(3x-1)^2=25$ or $x^2-4x+4+(3x)^2-2(3x)+1=25$	
			•2 express in standard quadratic form	•2 $10x^2-10x-20=0$	
			•3 factorise	•3 $10(x-2)(x+1)=0$	
			•4 find x coordinates	•4 •4 $x=2$ •5 $x=-1$	
			•5 find y coordinates	•5 $y=6$ $y=-3$	5

Notes:

1. At •3 the quadratic must lead to two distinct real roots for •4 and •5 to be available.
2. •2 is only available if '$=0$' appears at •2 or •3 stage.
3. If a candidate arrives at an equation which is not a quadratic at •2 stage, then •3, •4 and •5 are not available
4. At •3 do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 10.
5. •3 is available for substituting correctly into the quadratic formula.
6. •4 and •5 may be marked either horizontally or vertically.
7. For candidates who identify **both** solutions by inspection, full marks may be awarded provided they justify that their points lie on **both** the line and the circle. Candidates who identify **both** solutions, but justify only one gain 2 out of 5.

Commonly Observed Responses:

Candidate A	Candidate B
$(x-2)^2+(3x-1)^2=25$ •1 ✓ $10x^2-10x=20$ •2 ✗ $10x(x-1)=20$ •3 $\boxed{✓2}$ $x=2 \quad x=3$ •4 ✗ $y=6 \quad y=9$ •5 $\boxed{✓2}$	Candidates who substitute into the circle equation only •1 ✓ •2 ✓ •3 ✓ •4 ✓ Sub $x=2$: $y^2-2y-24=0$, $(y-6)(y+4)=0$, $y=6$ or $\cancel{y=-4}$ Sub $x=-1$: $y^2-2y-15=0$, $(y+3)(y-5)=0$, $y=-3$ or $\cancel{y=5}$ $(2,6)$ $(-1,-3)$ •5 ✗

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		**Method 1** •1 identify common factor •2 complete the square •3 process for c and write in required form	**Method 1** •1 $3(x^2+8x......$ stated or implied by •2 •2 $3(x+4)^2......$ •3 $3(x+4)^2+2$	3
			Method 2 •1 expand completed square form •2 equate coefficients •3 process for b and c and write in required form	**Method 2** •1 $ax^2+2abx+ab^2+c$ •2 $a=3,\ \ 2ab=24,\ \ ab^2+c=50$ •3 $3(x+4)^2+2$	3

Notes:

1. $3(x+4)^2+2$ with no working gains •1 and •2 only; however, see Candidate G.
2. •3 is only available for a calculation involving both multiplication and subtraction of integers.

Commonly Observed Responses:

Candidate A $3\left(x^2+8x+\frac{50}{3}\right)$ •1 ✓ $3\left(x^2+8x+16-16+\frac{50}{3}\right)$ •2 ^ further working is required	**Candidate B** $3x^2+24x+50=3(x+8)^2-64+50$ •1 ✗ •2 ✗ $=3(x+8)^2-14$ •3 $\boxed{✓2}$
Candidate C $ax^2+2abx+ab^2+c$ •1 ✓ $a=3,\ \ 2ab=24,\ \ b^2+c=50$ •2 ✗ $a=3,\ \ b=4,\ \ c=34$ $3(x+4)^2+34$ •3 $\boxed{✓1}$	**Candidate D** $3\left(\left(x^2+24x\right)+50\right)$ •1 ✗ $3\left(\left(x+12\right)^2-144\right)+50$ •2 $\boxed{✓1}$ $3(x+12)^2-382$ •3 $\boxed{✓1}$

Question	Generic scheme	Illustrative scheme	Max mark

Candidate E	Candidate F
$a(x+b)^2+c=ax^2+2abx+ab^2+c$ $\bullet^1$ ✓ $a=3,\ 2ab=24,\ ab^2+c=50$ $\bullet^2$ ✓ $b=4,\ c=2$ $\bullet^3$ ✓ $\bullet^3$ is awarded as all working relates to completed square form	$ax^2+2abx+ab^2+c$ $\bullet^1$ ✓ $a=3,\ 2ab=24,\ ab^2+c=50$ $\bullet^2$ ✓ $b=4,\ c=2$ $\bullet^3$ ✗ $\bullet^3$ is lost as no reference is made to completed square form
Candidate G $3(x+4)^2+2$ Check: $3\left(x^2+8x+16\right)+2$ $=3x^2+24x+48+2$ $=3x^2+24x+50$ **Award 3/3**	**Candidate H** $3x^2+24x+50$ $=3(x+4)^2-16+50$ $\bullet^1$ ✓ $\bullet^2$ ✓ $=3(x+4)^2+34$ $\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)		$\bullet^4$ differentiate two terms $\bullet^5$ complete differentiation	$\bullet^4$ $3x^2+24x....$ $\bullet^5$ $....+50$	**2**

Notes:

3. $\bullet^4$ is awarded for any two of the following three terms: $3x^2$, $+24x$, $+50$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.	(c)		**Method 1** •6 link with (a) and identify sign of $(x+4)^2$ •7 communicate reason **Method 2** •6 identify minimum value of $f'(x)$ •7 communicate reason	**Method 1** •6 $f'(x)=3(x+4)^2+2$ and $(x+4)^2 \geq 0 \ \forall x$ •7 $\therefore 3(x+4)^2+2>0 \Rightarrow$ always strictly increasing **Method 2** •6 eg minimum value =2 or annotated sketch •7 $2>0 \therefore (f'(x)>0) \Rightarrow$ always strictly increasing	2

Notes:

4. Do not penalise $(x+4)^2>0$ or the omission of $f'(x)$ at •6 in Method 1.
5. Responses in part (c) must be consistent with working in parts (a) and (b) for •6 and •7 to be available.
6. Where erroneous working leads to a candidate considering a function which is not always strictly increasing, only •6 is available.
7. At •6 communication should be explicitly in terms of the given function. Do not accept statements such as "$(\text{something})^2 \geq 0$", "something squared ≥ 0". However, •7 is still available.

Commonly Observed Responses:

Candidate I	Candidate J
$f'(x)=3(x+4)^2+2$ $3(x+4)^2+2>0 \Rightarrow$ strictly increasing. Award 1 out of 2	Since $3x^2+24x+50=3(x+4)^2+\frac{166}{50}$ and $(x+4)^2$ is >0 for all x then $3(x+4)^2+\frac{166}{50}>0$ for all x. Hence the curve is strictly increasing for all values of x. •6 ✓ •7 ✓1

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•1 identify pathway •2 state $\overrightarrow{PQ}$	•1 $\overrightarrow{PR}+\overrightarrow{RQ}$ stated or implied by •2 •2 $-3\mathbf{i}-4\mathbf{j}+5\mathbf{k}$	2

Notes:

1. Award •1 $(9\mathbf{i}+5\mathbf{j}+2\mathbf{k})+(-12\mathbf{i}-9\mathbf{j}+3\mathbf{k})$.
2. Candidates who choose to work with column vectors and leave their answer in the form $\begin{pmatrix}-3\\-4\\5\end{pmatrix}$ cannot gain •2.
3. •2 is not available for simply adding or subtracting vectors within an invalid strategy.
4. Where candidates choose specific points consistent with the given vectors, only •1 and •4 are available. However, should the statement 'without loss of generality' precede the selected points then marks •1, •2, •3 and •4 are all available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		•3 interpret ratio •4 identify pathway and demonstrate result	•3 $\frac{2}{3}$ or $\frac{1}{3}$ •4 $\overrightarrow{PR}+\frac{2}{3}\overrightarrow{RQ}$ or $\overrightarrow{PQ}+\frac{1}{3}\overrightarrow{QR}$ leading to $\mathbf{i}-\mathbf{j}+4\mathbf{k}$	2

Notes:

5. This is a 'show that' question. Candidates who choose to work with column vectors must write their final answer in the required form to gain •4. $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ does not gain •4.
6. Beware of candidates who fudge their working between •3 and •4.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Candidate A - legitimate use of the section formula	Candidate B - BEWARE - treating P as the origin
$\overrightarrow{PS} = \dfrac{n\overrightarrow{PQ} + m\overrightarrow{PR}}{m+n}$ $\overrightarrow{PS} = \dfrac{2\overrightarrow{PQ} + \overrightarrow{PR}}{3}$ $\bullet^3$ ✓ $\overrightarrow{PS} = \dfrac{2\begin{pmatrix}-3\\-4\\5\end{pmatrix}}{3} + \dfrac{\begin{pmatrix}9\\5\\2\end{pmatrix}}{3}$ $= \begin{pmatrix}-2\\-8/3\\10/3\end{pmatrix} + \begin{pmatrix}3\\5/3\\2/3\end{pmatrix}$ $= \begin{pmatrix}1\\-1\\4\end{pmatrix}$ $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ $\bullet^4$ ✓	$2\overrightarrow{QS} = \overrightarrow{SR}$ $3\mathbf{s} = 2\mathbf{q} + \mathbf{r}$ $\bullet^3$ ✓ $3\mathbf{s} = 2\begin{pmatrix}-3\\-4\\5\end{pmatrix} + \begin{pmatrix}9\\5\\2\end{pmatrix}$ $\mathbf{s} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ $\bullet^4$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
5.	(c)		**Method 1**	**Method 1**	
			•5 evaluate $\overrightarrow{PQ}.\overrightarrow{PS}$	•5 $\overrightarrow{PQ}.\overrightarrow{PS}=21$	
			•6 evaluate $\lvert\overrightarrow{PQ}\rvert$	•6 $\lvert\overrightarrow{PQ}\rvert=\sqrt{50}$	
			•7 evaluate $\lvert\overrightarrow{PS}\rvert$	•7 $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$	
			•8 use scalar product	•8 $\cos QPS=\dfrac{21}{\sqrt{50}\times\sqrt{18}}$	
			•9 calculate angle	•9 $45\cdot6°$ or $0\cdot795$ radians	5
			Method 2	**Method 2**	
			•5 evaluate $\lvert\overrightarrow{QS}\rvert$	•5 $\lvert\overrightarrow{QS}\rvert=\sqrt{26}$	
			•6 evaluate $\lvert\overrightarrow{PQ}\rvert$	•6 $\lvert\overrightarrow{PQ}\rvert=\sqrt{50}$	
			•7 evaluate $\lvert\overrightarrow{PS}\rvert$	•7 $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$	
			•8 use cosine rule	•8 $\cos QPS=\dfrac{\left(\sqrt{50}\right)^2+\left(\sqrt{18}\right)^2-\left(\sqrt{26}\right)^2}{2\times\sqrt{50}\times\sqrt{18}}$	
			•9 calculate angle	•9 $45\cdot6°$ or $0\cdot795$ radians	5

Notes:

7. For candidates who use $\overrightarrow{PS}$ **not equal to** $\mathbf{i}-\mathbf{j}+4\mathbf{k}$ •5 is not available in Method 1 or •7 in Method 2.
8. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. However, $\sqrt{1^2-1^2+4^2}$ leading to $\sqrt{16}$ indicates an invalid method for calculating the magnitude. No mark can be awarded for any magnitude arrived at using an invalid method.
9. •8 is not available to candidates who simply state the formula $\cos\theta=\dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$.

   However, $\cos\theta=\dfrac{\overrightarrow{PQ}.\overrightarrow{PS}}{\lvert\overrightarrow{PQ}\rvert\times\lvert\overrightarrow{PS}\rvert}$ or $\cos\theta=\dfrac{21}{\sqrt{50}\times\sqrt{18}}$ is acceptable. Similarly for Method 2.
10. Accept answers which round to $46°$ or $0\cdot8$ radians.
11. Do not penalise the omission or incorrect use of units.
12. •9 is only available as a result of using a valid strategy.
13. •9 is only available for a single angle.
14. For a correct answer with no working award 0/5.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Candidate C - Calculating wrong angle $\overrightarrow{QP}.\overrightarrow{QS}=29$ •5 ✘ $\lvert\overrightarrow{QP}\rvert=\sqrt{50}$ •6 $\boxed{✓1}$ $\lvert\overrightarrow{QS}\rvert=\sqrt{26}$ •7 $\boxed{✓1}$ $\cos P\hat{Q}S=\frac{29}{\sqrt{50}\times\sqrt{26}}$ •8 $\boxed{✓1}$ $P\hat{Q}S = 36\cdot5$ •9 ✘ strategy incomplete For candidates who continue, and use the angle found to evaluate the required angle, then all marks are available.	**Candidate D- Calculating wrong angle** $\overrightarrow{PS}.\overrightarrow{QP}=-21$ •5 ✘ $\lvert\overrightarrow{QP}\rvert=\sqrt{50}$ •6 ✓ $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$ •7 ✓ $\cos\theta=\frac{-21}{\sqrt{50}\times\sqrt{18}}$ •8 $\boxed{✓1}$ $\theta = 134\cdot4$ •9 ✘ strategy incomplete For candidates who continue, and use the angle found to evaluate the required angle, then all marks are available.
Candidate E From (a) $\overrightarrow{PQ}=-21\mathbf{i}-14\mathbf{j}+\mathbf{k}$ $\overrightarrow{PQ}.\overrightarrow{PS}=-3$ •5 $\boxed{✓1}$ $\lvert\overrightarrow{PQ}\rvert=\sqrt{638}$ •6 $\boxed{✓1}$ $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$ •7 ✓ $\cos Q\hat{P}S=\frac{-3}{\sqrt{638}\times\sqrt{18}}$ •8 $\boxed{✓1}$ $Q\hat{P}S = 91\cdot6$ •9 $\boxed{✓1}$	**Candidate F** From (a) $\overrightarrow{PQ}=21\mathbf{i}+14\mathbf{j}-\mathbf{k}$ $\overrightarrow{PQ}.\overrightarrow{PS}=3$ •5 $\boxed{✓1}$ $\lvert\overrightarrow{PQ}\rvert=\sqrt{638}$ •6 $\boxed{✓1}$ $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$ •7 ✓ $\cos Q\hat{P}S=\frac{3}{\sqrt{638}\times\sqrt{18}}$ •8 $\boxed{✓1}$ $Q\hat{P}S = 88\cdot4$ •9 $\boxed{✓1}$
Candidate G From (b) $\overrightarrow{PS}=-4\mathbf{i}-3\mathbf{j}+\mathbf{k}$ $\overrightarrow{PQ}.\overrightarrow{PS}=3$ •5 ✘ $\lvert\overrightarrow{PQ}\rvert=\sqrt{50}$ •6 ✓ $\lvert\overrightarrow{PS}\rvert=\sqrt{26}$ •7 $\boxed{✓1}$ $\cos Q\hat{P}S=\frac{3}{\sqrt{50}\times\sqrt{26}}$ •8 $\boxed{✓1}$ $Q\hat{P}S = 85\cdot2$ •9 $\boxed{✓1}$	

Question			Generic scheme	Illustrative scheme	Max mark
6.			•¹ substitute appropriate double angle formula •² express in standard quadratic form •³ factorise •⁴ solve for $\sin x°$ •⁵ solve for x	•¹ $5\sin x - 4 = 2\left(1 - 2\sin^2 x\right)$ •² $4\sin^2 x + 5\sin x - 6 = 0$ •³ $(4\sin x - 3)(\sin x + 2)$ •⁴ $\sin x = \frac{3}{4}$, •⁵ $\sin x = -2$ •⁵ $x = 0\cdot848,\ 2\cdot29,\ \cancel{\sin x = -2}$	5

Notes:

1. •¹ is not available for simply stating $\cos 2x = 1 - 2\sin^2 x$ with no further working.
2. In the event of $\cos^2 x° - \sin^2 x°$ or $2\cos^2 x° - 1$ being substituted for $\cos 2x$, •¹ cannot be awarded until the equation reduces to a quadratic in $\sin x°$.
3. Substituting $1 - 2\sin^2 A$ or $1 - 2\sin^2 \alpha$ for $\cos 2x$ at •¹ stage should be treated as bad form provided the equation is written in terms of x at •² stage. Otherwise, •¹ is not available.
4. '$=0$' must appear by •³ stage for •² to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at •² stage for •² to be awarded.
5. $5\sin x + 4\sin^2 x - 6 = 0$ does not gain •² unless •³ is awarded.
6. $\sin x = \frac{-5 \pm \sqrt{121}}{8}$ gains •³.
7. Candidates may express the equation obtained at •² in the form $4s^2 + 5s - 6 = 0$ or $4x^2 + 5x - 6 = 0$. In these cases, award •³ for $(4s - 3)(s + 2) = 0$ or $(4x - 3)(x + 2) = 0$. However, •⁴ is only available if $\sin x$ appears explicitly at this stage.
8. •⁴ and •⁵ are only available as a consequence of solving a quadratic equation.
9. •³, •⁴ and •⁵ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. •⁵ is not available to candidates who work in degrees and do not convert their solutions into radian measure.
11. Accept answers which round to $0\cdot85$ and $2\cdot3$ at •⁵ eg $\frac{49\pi}{180}, \frac{131\pi}{180}$.
12. Answers written as decimals should be rounded to no fewer than 2 significant figures.
13. Do not penalise additional solutions at •⁵.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Candidate A		Candidate B	
$\bullet^1$ ✓ $\bullet^2$ ✓		$\bullet^1$ ✓	
$(4s-3)(s+2)=0$	$\bullet^3$ ✓	$4\sin^2 x+5\sin x-6=0$	$\bullet^2$ ✓
$s=\frac{3}{4},\ s=-2$	$\bullet^4$ ✗	$9\sin x-6=0$	$\bullet^3$ ✗
$x=0\cdot848,\ 2\cdot29$	$\bullet^5$ ✓	$\sin x=\frac{2}{3}$	$\bullet^4$ $\boxed{✓2}$
		$x=0\cdot730,\ 2\cdot41$	$\bullet^5$ $\boxed{✓2}$

Candidate C		Candidate D	
$5\sin x-4=2\left(1-2\sin^2 x\right)$	$\bullet^1$ ✓	$5\sin x-4=2\left(1-2\sin^2 x\right)$	$\bullet^1$ ✓
$4\sin^2 x+5\sin x=6$	$\bullet^2$ $\boxed{✓2}$	$4\sin^2 x+5\sin x-6=0$	$\bullet^2$ ✓
$\sin x\left(4\sin x+5\right)=6$	$\bullet^3$ $\boxed{✓2}$	$4\sin^2 x+5\sin x=6$	
$\sin x=6,\ \ 4\sin x+5=6$	$\bullet^4$ ✗	$\sin x\left(4\sin x+5\right)=6$	$\bullet^3$ $\boxed{✓2}$
no solution, $\sin x=\frac{1}{4}$		$\sin x=6,\ \ 4\sin x+5=6$	$\bullet^4$ ✗
$x=0\cdot253,\ 2\cdot89$	$\bullet^5$ ✗	no solution, $\sin x=\frac{1}{4}$	
		$x=0\cdot253,\ 2\cdot89$	$\bullet^5$ ✗

Candidate E - reading $\cos 2x$ as $\cos^2 x$			
$5\sin x-4=2\cos^2 x$	$\bullet^1$ ✗		
$5\sin x-4=2\left(1-\sin^2 x\right)$			
$2\sin^2 x+5\sin x-6=0$	$\bullet^2$ $\boxed{✓1}$		
$\sin x=\frac{-5\pm\sqrt{73}}{4}$	$\bullet^3$ $\boxed{✓1}$		
$\sin x=0\cdot886,\ \cancel{\sin x=-3\cdot386}$	$\bullet^4$ $\boxed{✓1}$		
$x=1\cdot08,\ 2\cdot05$	$\bullet^5$ $\boxed{✓1}$		

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ write in differentiable form •² differentiate one term •³ complete differentiation and equate to zero •⁴ solve for x	•¹ $\ldots - 2x^{\frac{3}{2}}$ **stated or implied** •² $\frac{dy}{dx} = 6\ldots$ or $\frac{dy}{dx} = \ldots - 3x^{\frac{1}{2}}\ldots$ •³ $\ldots - 3x^{\frac{1}{2}} = 0$ or $6\ldots = 0$ •⁴ $x = 4$	4

Notes:

1. For candidates who do not differentiate a term involving a fractional index, either •² or •³ is available but not both.
2. •⁴ is available only as a consequence of solving an equation involving a fractional power of x.
3. For candidates who integrate one or other of the terms •⁴ is unavailable.

Commonly Observed Responses:

Candidate A - differentiating incorrectly		Candidate B - integrating the second term	
$y = 6x - 2x^{\frac{3}{2}}$	•¹ ✓	$y = 6x - 2x^{\frac{3}{2}}$	•¹ ✓
$\frac{dy}{dx} = 6 - 3x^{\frac{5}{2}}$	•² ✓	$\frac{dy}{dx} = 6 - \frac{4}{5}x^{\frac{5}{2}}$	•² ✓
$6 - 3x^{\frac{5}{2}} = 0$	•³ ✗	$6 - \frac{4}{5}x^{\frac{5}{2}} = 0$	•³ ✗
$x = 1{\cdot}32$	•⁴ $\boxed{✓1}$	$x = 2{\cdot}24$	•⁴ ✗

Question			Generic scheme	Illustrative scheme	Max mark
7.	(b)		•[5] evaluate y at stationary point •[6] consider value of y at end points •[7] state greatest and least values	•[5] 8 •[6] 4 and 0 •[7] greatest 8, least 0 **stated explicitly**	**3**

Notes:

4. The only valid approach to finding the stationary point is via differentiation. A numerical approach can only gain •[6].
5. •[7] is not available to candidates who do not consider both end points.
6. Vertical marking is not applicable to •[6] and •[7].
7. Ignore any nature table which may appear in a candidate's solution; however, the appearance of $(4,8)$ at a nature table is sufficient for •[5].
8. Greatest $(4,8)$; least $(9,0)$ does not gain •[7].
9. •[5] and •[7] are not available for evaluating y at a value of x, obtained at •[4] stage, which lies outwith the interval $1 \le x \le 9$.
10. For candidates who **only** evaluate the derivative, •[5],•[6] and •[7]are not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] find expression for u_1 •[2] find expression for u_2 and express in the correct form	•[1] $5k-20$ •[2] $u_2 = k(5k-20)-20$ leading to $u_2 = 5k^2 - 20k - 20$	**2**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)		$\bullet^3$ interpret information	$\bullet^3$ $5k^2 - 20k - 20 < 5$	
			$\bullet^4$ express inequality in standard quadratic form	$\bullet^4$ $5k^2 - 20k - 25 < 0$	
			$\bullet^5$ determine zeros of quadratic expression	$\bullet^5$ $-1, 5$	
			$\bullet^6$ state range with justification	$\bullet^6$ $-1 < k < 5$ with eg sketch or table of signs	4

Notes:

1. Candidates who work with an equation from the outset lose $\bullet^3$ and $\bullet^4$. However, $\bullet^5$ and $\bullet^6$ are still available.
2. At $\bullet^5$ do not penalise candidates who fail to extract the common factor or who have divided the quadratic inequation by 5.
3. $\bullet^4$ and $\bullet^5$ are only available to candidates who arrive at a quadratic expression at $\bullet^3$.
4. At $\bullet^6$ accept "$k > -1$ and $k < 5$" or "$k > -1$, $k < 5$" together with the required justification.
5. For a trial and error approach award 0/4.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			**Method 1** •[1] state linear equation •[2] introduce logs •[3] use laws of logs •[4] use laws of logs •[5] state k and n	**Method 1** •[1] $\log_2 y = \frac{1}{4}\log_2 x + 3$ •[2] $\log_2 y = \frac{1}{4}\log_2 x + 3\log_2 2$ •[3] $\log_2 y = \log_2 x^{\frac{1}{4}} + \log_2 2^3$ •[4] $\log_2 y = \log_2 2^3 x^{\frac{1}{4}}$ •[5] $k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	5
			Method 2 •[1] state linear equation •[2] use laws of logs •[3] use laws of logs •[4] use laws of logs •[5] state k and n	**Method 2** •[1] $\log_2 y = \frac{1}{4}\log_2 x + 3$ •[2] $\log_2 y = \log_2 x^{\frac{1}{4}} + 3$ •[3] $\log_2 \frac{y}{x^{\frac{1}{4}}} = 3$ •[4] $\frac{y}{x^{\frac{1}{4}}} = 2^3$ •[5] $k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	5

Question			Generic Scheme	Illustrative Scheme	Max Mark
			Method 3	**Method 3** **The equations at •¹, •² and •³ must be stated explicitly.**	
			•¹ introduce logs to $y = kx^n$	•¹ $\log_2 y = \log_2 kx^n$	
			•² use laws of logs	•² $\log_2 y = n\log_2 x + \log_2 k$	
			•³ interpret intercept	•³ $\log_2 k = 3$	
			•⁴ use laws of logs	•⁴ $k = 8$	
			•⁵ interpret gradient	•⁵ $n = \frac{1}{4}$	**5**
			Method 4	**Method 4**	
			•¹ interpret point on log graph	•¹ $\log_2 x = -12$ **and** $\log_2 y = 0$	
			•² convert from log to exp. form	•² $x = 2^{-12}$ **and** $y = 2^0$	
			•³ interpret point and convert	•³ $\log_2 x = 0, \quad \log_2 y = 3$ $x = 1, \quad y = 2^3$	
			•⁴ substitute into $y = kx^n$ and evaluate k	•⁴ $2^3 = k \times 1^n \Rightarrow k = 8$	
			•⁵ substitute other point into $y = kx^n$ and evaluate n	•⁵ $2^0 = 2^3 \times 2^{-12n}$ $\Rightarrow 3 - 12n = 0$ $\Rightarrow n = \frac{1}{4}$	**5**

Notes:

1. Markers must not pick and choose between methods. Identify the method which best matches the candidates approach.
2. Treat the omission of base 2 as bad form at •¹ and •³ in Method 1, at •¹ and •² for Method 2 and Method 3, and at •¹ in Method 4.
3. '$m = \frac{1}{4}$' or 'gradient $= \frac{1}{4}$' does not gain •⁵ in Method 3.
4. Accept 8 in lieu of 2^3 throughout.
5. In Method 4 candidates may use $(0,3)$ for •¹ and •² followed by $(-12,0)$ for •³.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Candidate A	Candidate B
With no working. Method 3: $k=8$ $\bullet^4$ ✓ $n=\frac{1}{4}$ $\bullet^5$ ✓ **Award 2/5**	With no working. Method 3: $n=8$ $\bullet^4$ ✗ $k=\frac{1}{4}$ $\bullet^5$ ✗ **Award 0/5**

Candidate C	Candidate D
Method 3: $\log_2 k = 3$ $\bullet^3$ ✓ $k=8$ $\bullet^4$ ✓ $n=\frac{1}{4}$ $\bullet^5$ ✓ **Award 3/5**	Method 2: $\log_2 y = \frac{1}{4}\log_2 x + 3$ $\bullet^1$ ✓ $\log_2 y = \log_2 x^{\frac{1}{4}} + 3$ $\bullet^2$ ✓ $y = x^{\frac{1}{4}} + 3$ $\bullet^3$ ✗ $\bullet^4$ ✗ $k=1,\ n=\frac{1}{4}$ $\bullet^5$ ✗ **Award 2/5**

Candidate E	
Method 2: $y=\frac{1}{4}x+3$ $\log_2 y = \frac{1}{4}\log_2 x + 3$ $\bullet^1$ ✓ $\log_2 y = \log_2 x^{\frac{1}{4}} + 3$ $\bullet^2$ ✓ $\frac{y}{x^{\frac{1}{4}}} = 3$ $\bullet^3$ ^ $\bullet^4$ ✗ $y = 3x^{\frac{1}{4}}$ $\bullet^5$ $\boxed{✓1}$ **Award 3/5**	

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		**Method 1** •¹ calculate m_{AB} •² calculate m_{BC} •³ interpret result and state conclusion	**Method 1** •¹ $m_{AB} = \frac{3}{9} = \frac{1}{3}$ see Note 1 •² $m_{BC} = \frac{5}{15} = \frac{1}{3}$ •³ $\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	**3**
			Method 2 •¹ calculate an appropriate vector e.g. $\overrightarrow{AB}$ •² calculate a second vector e.g. $\overrightarrow{BC}$ and compare •³ interpret result and state conclusion	**Method 2** •¹ $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ see Note 1 •² $\overrightarrow{BC} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} \therefore \overrightarrow{AB} = \frac{3}{5}\overrightarrow{BC}$ •³ $\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	**3**
			Method 3 •¹ calculate m_{AB} •² find equation of line and substitute point •³ communication	**Method 3** •¹ $m_{AB} = \frac{3}{9} = \frac{1}{3}$ •² eg, $y - 1 = \frac{1}{3}(x - 2)$ leading to $6 - 1 = \frac{1}{3}(17 - 2)$ •³ since C lies on line A, B and C are collinear	

Notes:

1. At •¹ and •² stage, candidates may calculate the gradients/vectors using any pair of points.
2. •³ can only be awarded if a candidate has stated “parallel”, “common point” and “collinear”.
3. Candidates who state “points A, B and C are parallel” or “m_{AB} and m_{BC} are parallel” do not gain •³.

Question	Generic scheme	Illustrative scheme	Max mark

Commonly Observed Responses:

Candidate A	Candidate B	Candidate C
$m_{AB} = \frac{3}{9} = \frac{1}{3}$ •1 ✓ $m_{BC} = \frac{5}{15}$ •2 ^ $\Rightarrow$ AB and BC are parallel, B is a common point, hence A, B and C are collinear. •3 $\boxed{✓1}$	$\begin{pmatrix} 9 \\ 3 \end{pmatrix}$ •1 ✓ $\begin{pmatrix} 15 \\ 5 \end{pmatrix} \therefore \overrightarrow{AB} = \frac{5}{3}\overrightarrow{BC}$ •2 ✗ $\Rightarrow$ AB and BC are parallel, B is a common point, hence A, B and C are collinear. •3 $\boxed{✓1}$	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ •1 ✓ $\overrightarrow{BC} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} = 5\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ •2 ✓ $\therefore \overrightarrow{AB} = \frac{5}{3}\overrightarrow{BC}$ ignore working subsequent to correct statement at •2. $\Rightarrow$ AB and BC are parallel, B is a common point, hence A, B and C are collinear. •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		•⁴ find radius •⁵ determine an appropriate ratio •⁶ find centre •⁷ state equation of circle	•⁴ $6\sqrt{10}$ •⁵ e.g. 2:3 or $\frac{2}{5}$ (using B and C) or 3:5 or $\frac{8}{5}$ (using A and C) •⁶ $(8,3)$ •⁷ $(x-8)^2+(y-3)^2=360$	4

Notes:

4. Where the correct centre appears without working •⁵ is lost, •⁶ is awarded and •⁷ is still available. Where an incorrect centre or radius **from working** then •⁷ is available. However, if an incorrect centre or an incorrect radius appears ex nihilo •⁷ is not available.
5. Do not accept $(6\sqrt{10})^2$ for •⁷.

Commonly Observed Responses:

Candidate D Radius $=6\sqrt{10}$ •⁴ ✓ Interprets D as midpoint of BC •⁵ ✗ Centre D is $(9\cdot5,\ 3\cdot5)$ •⁶ $\boxed{✓2}$ $(x-9\cdot5)^2+(y-3\cdot5)^2=360$ •⁷ $\boxed{✓1}$	**Candidate E** Radius $=3\sqrt{10}$ •⁴ ✗ Interprets D as midpoint of AC •⁵ ✗ Centre D is $(5,\ 2)$ •⁶ $\boxed{✓2}$ $(x-5)^2+(y-2)^2=90$ •⁷ $\boxed{✓1}$
Candidate F Radius $=\sqrt{10}$ •⁴ ✗ Interprets D as midpoint of AC •⁵ ✗ Centre D is $(5,\ 2)$ •⁶ $\boxed{✓2}$ $(x-5)^2+(y-2)^2=10$ •⁷ $\boxed{✓2}$	**Candidate G** Radius $=6\sqrt{10}$ •⁴ ✓ $\frac{CD}{BD}=\frac{3}{2}$ or simply $\frac{3}{2}$ •⁵ ✓ Centre D is $(11,\ 4)$ •⁶ ✗ $(x-11)^2+(y-4)^2=360$ •⁷ $\boxed{✓1}$

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		**Method 1** •1 substitute for $\sin 2x$ •2 simplify and factorise •3 substitute for $1-\cos^2 x$ and simplify	**Method 1** •1 $\dfrac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •2 $\sin x\left(1-\cos^2 x\right)$ •3 $\sin x\times\sin^2 x$ leading to $\sin^3 x$	3
			Method 2 •1 substitute for $\sin 2x$ •2 simplify and substitute for $\cos^2 x$ •3 expand and simplify	**Method 2** •1 $\dfrac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •2 $\sin x-\sin x\left(1-\sin^2 x\right)$ •3 $\sin x-\sin x+\sin^3 x$ leading to $\sin^3 x$	3

Notes:

1. •1 is not available to candidates who simply quote $\sin 2x = 2\sin x\cos x$ without substituting into the expression given on the LHS. See Candidate B
2. In method 2 where candidates attempt •1 and •2 in the same line of working •1 may still be awarded if there is an error at •2.
3. •3 is not available to candidates who work throughout with A in place of x.
4. Treat multiple attempts which are not scored out as different strategies, and apply General Marking Principle (r).
5. On the appearance of $\text{LHS}=0$, the first available mark is lost; however, any further marks are still available.

Commonly Observed Responses:

Candidate A	**Candidate B**
$\dfrac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x=\sin^3 x$ •1 ✓ $\sin x-\sin x\cos^2 x=\sin^3 x$ •2 ^ $1-\cos^2 x=\sin^2 x$ •3 ✗ $\sin^2 x=\sin^2 x$ In proving the identity, candidates must work with both sides independently ie in each line of working the LHS must be equivalent to the line above.	$\text{LHS}=\dfrac{\sin 2x}{2\cos x}-\sin x\cos^2 x$ $\dfrac{\sin 2x}{2\cos x}=\dfrac{2\sin x\cos x}{2\cos x}$ $=\sin x$ $\sin x-\sin x\cos^2 x$ •1 ✓ $\sin x\left(1-\cos^2 x\right)$ •2 ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		•[4] know to differentiate $\sin^3 x$ •[5] start to differentiate •[6] complete differentiation	•[4] $\frac{d}{dx}(\sin^3 x)$ •[5] $3\sin^2 x....$ •[6] $...\times\cos x$	**3**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]