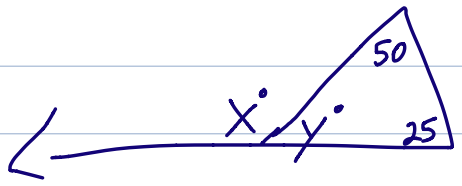
Bell Work ① Find x, y

② Write a sentence explaining what you did and why.



Unit 8. Ch. 8. "More than three Sides"

1. Use a straightedge to draw CONVEX polygons with 4, 5, 6 and 7 sides.

2. In each draw all of the diagonals from ONLY ONE VERTEX.

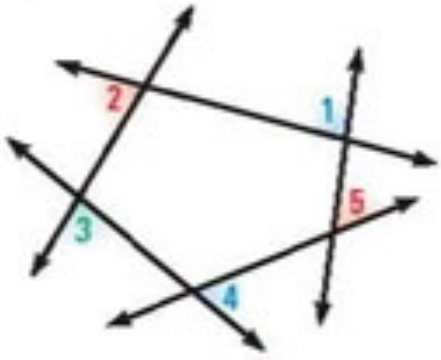
A diagonal is a segment that joins two non-consecutive vertices. Note the number of triangles formed.

3. Make a table:

Polygon.	#of sides.	#of triangles.	Sum of Interior angles
	4		
	5		
	6		
	7		

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Look for a pattern in the last column of the table. What is the sum of the measures of the interior angles of a convex heptagon? a convex octagon? *Explain* your reasoning.
2. Write an expression for the sum of the measures of the interior angles of a convex n-gon.
3. Measure the side lengths in the hexagon you drew. Compare the lengths with those in hexagons drawn by other students. Do the side lengths affect the sum of the interior angle measures of a hexagon? *Explain*.



STEP 1 **Shade** one exterior angle at each vertex.

A. Does the interior angle sum depend on whether the polygon is convex? Test this idea by drawing a few non-convex polygons (like the one at right) on your paper and determine if it matters whether the polygon is convex. Explain your findings.

B. Find the sum of the interior angles of a 100-gon. Explain your reasoning.

1. What is the sum of the interior angles of a quadrilateral?

2. What is the sum of the exterior angles of an octagon?

1. How many sides does a polygon have if the sum of its interior angles is 1260°?

1. What is the measurement of an interior angle of a regular nonagon?

2. What is the measurement of an exterior angle of a regular triangle?

1. An exterior angle of a regular polygon measures 36°. How many sides does the polygon have?

2. An interior angle of a regular polygon measures 60°. How many sides does it have?

Name _____ Date _____

LESSON 8.1

Practice B

For use with the lesson "Find Angle Measures in Polygons"

Find the sum of the measures of the interior angles of the indicated convex polygon.

1. Hexagon **2.** Dodecagon **3.** 11-gon

4. 15-gon **5.** 20-gon **6.** 40-gon

The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.

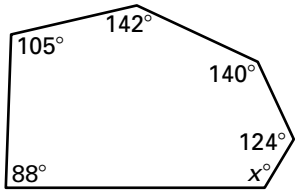
7. 180° **8.** 540° **9.** 900°

10. 1800° **11.** 2520° **12.** 3960°

13. 5040° **14.** 5940° **15.** 8640°

Find the value of *x*.

16.



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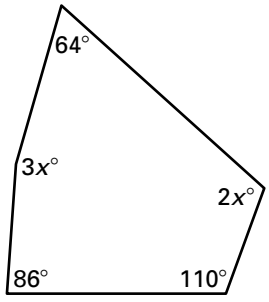

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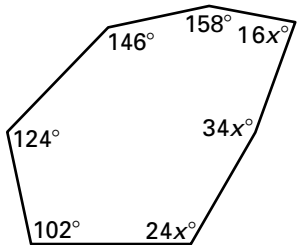

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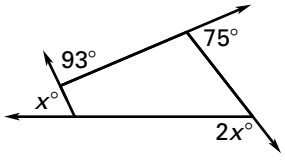

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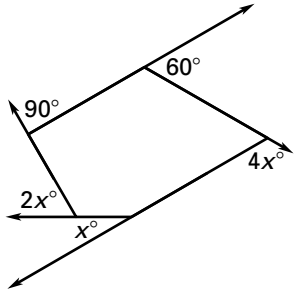

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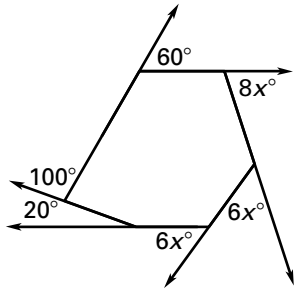


22. What is the measure of each exterior angle of a regular nonagon?

23. The measures of the exterior angles of a convex quadrilateral are 90°, 10*x*°, 5*x*°, and 45°. What is the measure of the largest exterior angle?

24. The measures of the interior angles of a convex octagon are 45*x*°, 40*x*°, 155°, 120°, 155°, 38*x*°, 158°, and 41*x*°. What is the measure of the smallest interior angle?

Find the measures of an interior angle and an exterior angle of the indicated polygon.

25. Regular triangle **26.** Regular octagon **27.** Regular 16-gon

28. Regular 45-gon **29.** Regular 60-gon **30.** Regular 100-gon

Name _____ Date _____

LESSON 8.1

Practice B *continued*

For use with the lesson "Find Angle Measures in Polygons"

In Exercises 31–34, find the value of *n* for each regular *n*-gon described.

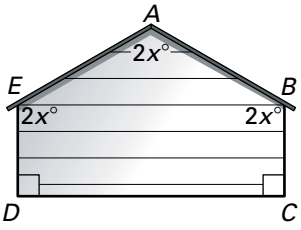
31. Each interior angle of the regular n-gon has a measure of 140°.

32. Each interior angle of the regular n-gon has a measure of 175.2°.

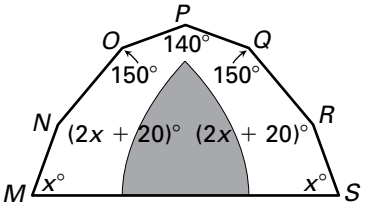
33. Each exterior angle of the regular n-gon has a measure of 45°.

34. Each exterior angle of the regular n-gon has a measure of 3°.

35. **Storage Shed** The side view of a storage shed is shown below. Find the value of x. Then determine the measure of each angle.



36. **Tents** The front view of a camping tent is shown below. Find the value of x. Then determine the measure of each angle.



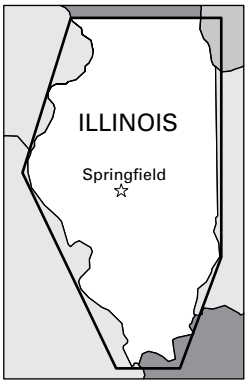
37. **Proof** Because all the interior angle measures of a regular n-gon are congruent, you can find the measure of each individual interior angle. The measure of each interior angle of a regular n-gon is $\frac{(n-2) \cdot 180}{n}$. Write a paragraph proof to prove this statement.

Name _____ Date _____

Problem Solving Workshop: Mixed Problem Solving

For use with the lessons "Find Angle Measures in Polygons", "Use Properties of Parallelograms", and "Show that a Quadrilateral is a Parallelogram"

1. **Multi-Step Problem** The shape of Illinois can be approximated by a polygon, as shown.



   a. How many sides does the polygon have?

   b. What is the sum of the measures of the interior angles of the polygon?

   c. What is the sum of the measures of the exterior angles of the polygon?

2. **Multi-Step Problem** Use a compass, protractor, and a ruler for parts (a)–(c) below.

   a. Draw a large angle that measures 140°. Mark congruent lengths on the sides of the angles.

   b. From the end of one of the congruent lengths in part (a), draw the second side of another angle that measures 140°. Mark another congruent length along this new side.

   c. Continue to draw angles that measure 140° with congruent sides until a polygon is formed. How many sides does the polygon have?

   d. Verify the answer to part (c) by using mathematics.

3. **Open-Ended** Draw a quadrilateral that has one pair of congruent sides and one pair of parallel sides but is not a parallelogram.

4. **Gridded Answer** A convex octagon has interior angles with measures 132°, 134°, 146°, 147°, 150°, 158°, x°, and $2x$°. Find the value of x.

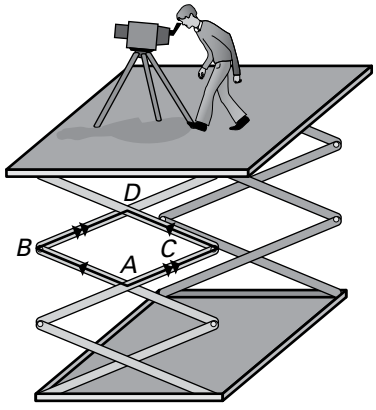
5. **Extended Response** The vertices of quadrilateral $JKLM$ are $J(-2, 4)$, $K(5, 7)$, $L(3, -1)$, and $M(-4, -4)$.

   a. Use what you know about slopes of lines to prove that $JKLM$ is a parallelogram. *Explain* your reasoning.

   b. Use the distance formula to show that $JKLM$ is a parallelogram. *Explain*.

6. **Short Response** The measure of an angle of a parallelogram is 18 degrees less than 5 times the measure of an adjacent angle. *Explain* how to find the measures of all the interior angles of the parallelogram.

7. **Extended Response** Photographers can use scissor lifts for overhead shots. The crossing beams of the lift form parallelograms that move together to raise and lower the platform. Use the diagram of parallelogram $ABCD$ below.



   a. What is $m\angle C$ when $m\angle A = 120°$?

   b. Suppose you decrease $m\angle A$. What happens to $m\angle C$?

   c. Suppose you increase $m\angle A$. What happens to AD?

   d. Suppose you decrease $m\angle A$. What happens to the overall height of the scissor lift? *Explain*.

Lesson 8.1 Find Angle Measures in Polygons

Teaching Guide

1. equilateral triangle; 60° **2.** square; 90°
3. regular pentagon; 108°
4. regular hexagon; 120°
5. regular heptagon; about 129°
6. regular octagon; 135°

Practice Level A

1. 1440° **2.** 1980° **3.** 2700° **4.** 2880°
5. 3600° **6.** 4140° **7.** 5040° **8.** 5760°
9. 6660° **10.** 9 **11.** 14 **12.** 20 **13.** 28
14. 32 **15.** 44 **16.** 93 **17.** 127 **18.** 85
19. 71 **20.** 52 **21.** 36 **22.** 150° **23.** 24°
24. 120°; 60° **25.** 144°; 36° **26.** 156°; 24°
27. 162°; 18° **28.** 168°; 12° **29.** 170°; 10°
30. 4 **31.** 5 **32.** 8 **33.** 10 **34.** 4 **35.** 6
36. 9 **37.** 12 **38. a.** 4; quadrilateral **b.** 360° **c.** 360° **39.** 135° **40.** 135°; 45°

Practice Level B

1. 720° **2.** 1800° **3.** 1620° **4.** 2340°
5. 3240° **6.** 6840° **7.** triangle **8.** pentagon
9. heptagon **10.** dodecagon **11.** 16-gon
12. 24-gon **13.** 30-gon **14.** 35-gon **15.** 50-gon
16. 121 **17.** 56 **18.** 5 **19.** 64 **20.** 30 **21.** 9
22. 40° **23.** 150° **24.** 114° **25.** 60°; 120°
26. 135°; 45° **27.** 157.5°; 22.5° **28.** 172°; 8°
29. 174°; 6° **30.** 176.4°; 3.6° **31.** 9 **32.** 75
33. 8 **34.** 120 **35.** $x = 60$; $m\angle A = m\angle B = m\angle E = 120°$ and $m\angle C = m\angle D = 90°$
36. $x = 70$; $m\angle M = m\angle S = 70°$, $m\angle N = m\angle R = 160°$, $m\angle O = m\angle Q = 150°$, and $m\angle P = 140°$
37. Let A be a regular n-gon and $x°$ be the measure of each interior $\angle$. Then the sum of the interior $\triangleleft$ is $n \cdot x°$. By the Polygon Interior $\triangleleft$ Theorem, the sum of the measures of the interior $\triangleleft$ of A is $(n - 2) \cdot 180°$.
So, $n \cdot x° = (n - 2) \cdot 180°$, or $x = \frac{(n - 2) \cdot 180}{n}$.

Practice Level C

1. 3420° **2.** 5940° **3.** 8640° **4.** 10 **5.** 19
6. 23 **7.** 38 **8.** 47 **9.** 51 **10.** 83 **11.** 17
12. 8 **13.** 16 **14.** 13 **15.** 3 **16.** 140° **17.** 105°
18. 105° **19.** about 128.6°; about 51.4°
20. 150°; 30° **21.** about 158.8°; about 21.2°
22. 172.8°; 7.2° **23.** about 174.9°; about 5.1°
24. about 177.1°; about 2.9° **25.** 24 **26.** 150
27. 72 **28.** 30 **29.** No; The polygon would have 14.4 sides which is not possible.
30. Yes; The polygon would have 18 sides.
31. Yes; The polygon would have 72 sides.
32. Yes; The polygon would have 30 sides.
33. always **34.** never **35.** always
36. sometimes
37. $x = 13.5$; $m\angle A = 117°$, $m\angle B = 86°$, $m\angle C = 134°$, $m\angle D = 86°$, $m\angle E = 117°$
38. $x = 16$; $m\angle F = 80°$, $m\angle G = 160°$, $m\angle H = 145°$, $m\angle J = 130°$, $m\angle K = 145°$, $m\angle L = 160°$, $m\angle M = 80°$ **39.** By the Polygon Interior Angles Theorem, $(n - 2) \cdot 180°$ = the sum of the measures of the interior angles. By the definition of a regular polygon, all interior angles are congruent. Thus, the sum of the measures of the interior angles is $(n \cdot x)°$. This means that $(n - 2) \cdot 180° = (n \cdot x)°$. Multiplying $(n - 2)$ by 180 gives $180n - 360 = nx$. Subtracting nx from both sides and adding 360 to both sides gives $180n - nx = 360$. Factoring n out of the left side of the equation gives $n(180 - x) = 360$. Dividing both sides by $(180 - x)$ gives $n = \frac{360}{180 - x}$.

Study Guide

1. 1800° **2.** pentagon **3.** 71 **4.** 43°

Problem Solving Workshop: Worked Out Example

1. 140°, 40° **2.** An interior angle of a nonagon is 20° larger than an interior angle of a hexagon. An exterior angle of a hexagon is 20° larger than an interior angle of a nonagon. **3.** 135°, 45° **4.** 55

Challenge Practice

1. a. $\frac{1}{4}x^2 \tan 72°$ square units
b. $\frac{x^2}{4}(\tan 72°) + x^2(\sin 72°) + x^2(\cos 72°)(\sin 72°)$ square units
c. $x^2(\sin 72°) + x^2(\cos 72°)(\sin 72°)$ square units

2. a. $\frac{1}{4}s^2$ square units **b.** $\frac{1}{2}\left(s^2 + \sqrt{2}s^2 + \frac{s^2}{2}\right)$ square units