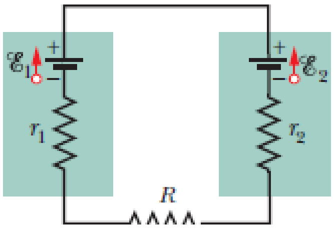
••10 (a) In Fig. 27-28, what value must R have if the current in the circuit is to be 1.0 mA? Take $\mathscr{E}_1 = 2.0$ V, $\mathscr{E}_2 = 3.0$ V, and $r_1 = r_2 = 3.0\ \Omega$. (b) What is the rate at which thermal energy appears in R?



10. (a) We solve $i = (\varepsilon_2 - \varepsilon_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{i} - r_1 - r_2 = \frac{3.0\ \text{V} - 2.0\ \text{V}}{1.0 \times 10^{-3}\ \text{A}} - 3.0\Omega - 3.0\Omega = 9.9 \times 10^2\ \Omega.$$

(b) $P = i^2R = (1.0 \times 10^{-3}\ \text{A})^2(9.9 \times 10^2\ \Omega) = 9.9 \times 10^{-4}$ W.

••12 Figure 27-30 shows a resistor of resistance $R = 6.00\ \Omega$ connected to an ideal battery of emf $\mathscr{E} = 12.0$ V by means of two copper wires. Each wire has length 20.0 cm and radius 1.00 mm. In dealing with such circuits in this chapter, we generally neglect the potential differences along the wires and the transfer of energy to thermal energy in them. Check the validity of this neglect for the circuit of Fig. 27-30: What is the potential difference across (a) the resistor and (b) each of the two sections of wire? At what rate is energy lost to thermal energy in (c) the resistor and (d) each section of wire?

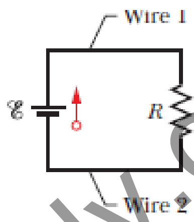


Figure 27-30 Problem 12.

12. (a) For each wire, $R_{wire} = \rho L/A$ where $A = \pi r^2$. Consequently, we have

$$R_{wire} = (1.69 \times 10^{-8}\ \Omega\cdot\text{m})(0.200\ \text{m})/\pi(0.00100\ \text{m})^2 = 0.0011\ \Omega.$$

The total resistive load on the battery is therefore

$$R_{tot} = 2R_{wire} + R = 2(0.0011\ \Omega) + 6.00\ \Omega = 6.0022\ \ \Omega.$$

Dividing this into the battery emf gives the current

$$i = \frac{\varepsilon}{R_{tot}} = \frac{12.0\ \text{V}}{6.0022\Omega} = 1.9993\ \text{A}.$$

The voltage across the $R = 6.00\ \Omega$ resistor is therefore

$$V = iR = (1.9993\ \text{A})(6.00\ \Omega) = 11.996\ \text{V} \approx 12.0\ \text{V}.$$

(b) Similarly, we find the voltage-drop across each wire to be

$$V_{wire} = iR_{wire} = (1.9993\ \text{A})(0.0011\ \Omega) = 2.15\ \text{mV}.$$

(c) $P = i^2R = (1.9993\ \text{A})(6.00\ \Omega)^2 = 23.98\ \text{W} \approx 24.0$ W.

(d) Similarly, we find the power dissipated in each wire to be 4.30 mW.

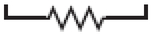


Figure 27-35 Problem 23.

•24 In Fig. 27-36, $R_1 = R_2 = 4.00\ \Omega$ and $R_3 = 2.50\ \Omega$. Find the equivalent resistance between points *D* and *E*. (*Hint:* Imagine that a battery is connected across those points.)

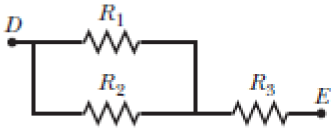
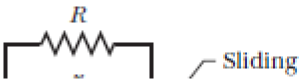


Figure 27-36 Problem 24.

•25 SSM Nine copper wires of length *l* and diameter *d* are connected in parallel to form a single composite conductor of resistance *R*. What must be the diameter *D* of a single copper wire of length *l* if it is to have the same resistance?



$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{12} = \frac{R_1 R_2}{R_1 + R_2}.$$

This situation consists of a parallel pair that are then in series with a single $R_3 = 2.50\ \Omega$ resistor. Thus, the situation has an equivalent resistance of

$$R_{eq} = R_3 + R_{12} = 2.50\Omega + \frac{(4.00\Omega)(4.00\Omega)}{4.00\Omega + 4.00\Omega} = 4.50\Omega.$$

25. **THINK** The resistance of a copper wire varies with its cross-sectional area, or its diameter.

EXPRESS Let *r* be the resistance of each of the narrow wires. Since they are in parallel the equivalent resistance R_{eq} of the composite is given by

$$\frac{1}{R_{eq}} = \frac{9}{r},$$

or $R_{eq} = r/9$. Now each thin wire has a resistance $r = 4\rho\ell / \pi d^2$, where ρ is the resistivity of copper, and $A = \pi d^2/4$ is the cross-sectional area of a single thin wire. On the other hand, the resistance of the thick wire of diameter *D* is $R = 4\rho\ell / \pi D^2$, where the cross-sectional area is $\pi D^2/4$.

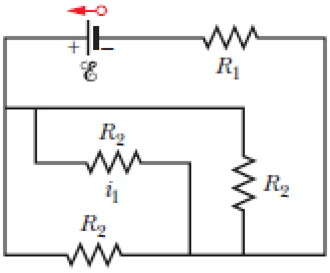
ANALYZE If the single thick wire is to have the same resistance as the composite of 9 thin wires, $R = R_{eq}$, then

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2}.$$

Solving for *D*, we obtain $D = 3d$.

LEARN The equivalent resistance R_{eq} is smaller than *r* by a factor of 9. Since $r \sim 1/A \sim 1/d^2$, increasing the diameter of the wire threefold will also reduce the resistance by a factor of 9.

••29 In Fig. 27-40, $R_1 = 6.00\ \Omega$, $R_2 = 18.0\ \Omega$, and the ideal battery has emf $\mathscr{E} = 12.0$ V. What are the (a) size and (b) direction (left or right) of current i_1? (c) How much energy is dissipated by all four resistors in 1.00 min?



29. (a) The parallel set of three identical $R_2 = 18\ \Omega$ resistors reduce to $R = 6.0\ \Omega$, which is now in series with the $R_1 = 6.0\ \Omega$ resistor at the top right, so that the total resistive load across the battery is $R' = R_1 + R = 12\ \Omega$. Thus, the current through R' is $(12\text{V})/R' = 1.0$ A, which is the current through R. By symmetry, we see one-third of that passes through any one of those 18 Ω resistors; therefore, $i_1 = 0.333$ A.

(b) The direction of i_1 is clearly rightward.

(c) We use Eq. 26-27: $P = i^2R' = (1.0\text{ A})^2(12\ \Omega) = 12$ W. Thus, in 60 s, the energy dissipated is (12 J/s)(60 s) = 720 J.

••34 The resistances in Figs. 27-45*a* and *b* are all 6.0 Ω, and the batteries are ideal 12 V batteries. (a) When switch S in Fig. 27-45*a* is closed, what is the change in the electric potential V_1 across resistor 1, or does V_1 remain the same? (b) When switch S in Fig. 27-45*b* is closed, what is the change in V_1 across resistor 1, or does V_1 remain the same?

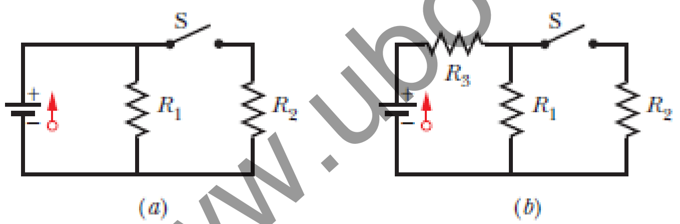


Figure 27-45 Problem 34.

34. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of "voltage going through" a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm's law) the voltages across R_1 and R_3 (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery, which means more current is in R_3, implying its voltage drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across R_1 has decreased a corresponding amount. When the switch was open, the voltage across R_1 was 6.0 V (easily seen from symmetry considerations). With the switch closed, R_1 and R_2 are equivalent (by Eq. 27-24) to 3.0 Ω, which means the total load on the battery is 9.0 Ω. The current therefore is 1.33 A, which implies that the voltage drop across R_3 is 8.0 V. The loop rule then tells us that the voltage drop across R_1 is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

••38 Figure 27-49 shows a section of a circuit. The resistances are $R_1 = 2.0\ \Omega$, $R_2 = 4.0\ \Omega$, and $R_3 = 6.0\ \Omega$, and the indicated current is $i = 6.0$ A. The electric potential difference between points A and B that connect the section to the rest of the circuit is $V_A - V_B = 78$ V. (a) Is the device represented by "Box" absorbing or providing energy to the circuit, and (b) at what rate?

Figure 27-48 Problems 37 and 98.

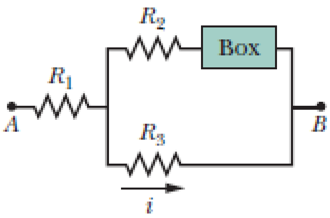


Figure 27-49 Problem 38.

38. (a) The voltage across $R_3 = 6.0\ \Omega$ is $V_3 = iR_3 = (6.0\ \text{A})(6.0\ \Omega) = 36$ V. Now, the voltage across $R_1 = 2.0\ \Omega$ is

$$(V_A - V_B) - V_3 = 78 - 36 = 42\ \text{V},$$

which implies the current is $i_1 = (42\ \text{V})/(2.0\ \Omega) = 21$ A. By the junction rule, then, the current in $R_2 = 4.0\ \Omega$ is

$$i_2 = i_1 - i = 21\ \text{A} - 6.0\ \text{A} = 15\ \text{A}.$$

The total power dissipated by the resistors is (using Eq. 26-27)

$$i_1^2 (2.0\ \Omega) + i_2^2 (4.0\ \Omega) + i^2 (6.0\ \Omega) = 1998\ \text{W} \approx 2.0\ \text{kW}.$$

By contrast, the power supplied (externally) to this section is $P_A = i_A (V_A - V_B)$ where $i_A = i_1 = 21$ A. Thus, $P_A = 1638$ W. Therefore, the "Box" must be providing energy.

(b) The rate of supplying energy is $(1998 - 1638)\ \text{W} = 3.6 \times 10^2$ W.

ance 2, (d) resistance 3, and (e) resist-
ance 4?

••45 ILW In Fig. 27-54, the resistances are $R_1 = 1.0\ \Omega$ and $R_2 = 2.0\ \Omega$, and the ideal batteries have emfs $\mathscr{E}_1 = 2.0$ V and $\mathscr{E}_2 = \mathscr{E}_3 = 4.0$ V. What are the (a) size and (b) direction (up or down) of the current in battery 1, the (c) size and (d) direction of the current in battery 2, and the (e) size and (f) direction of the current in battery 3? (g) What is the potential difference $V_a - V_b$?

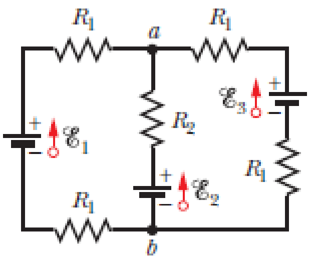


Figure 27-54 Problem 45.

45. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\,\text{V} - 2.0\,\text{V}}{4(1.0\,\Omega) + 2.0\,\Omega} = 0.33\ \text{A}.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A.

(b) The direction of i_1 is downward.

(c) The current through ε_2 is $i_2 = 0.33$ A.

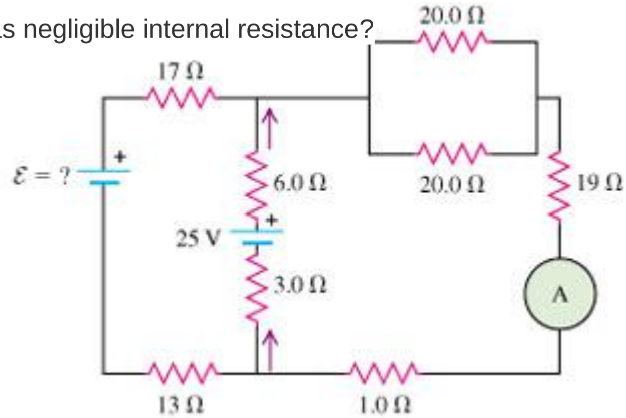
(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\ \text{A})(2.0\ \Omega) + 4.0\ \text{V} = 3.3$ V.

34. In the circuit shown in the figure . the 6.0 Ω resistor is consuming energy at a rate of 23.0 J/s when the current through it flows as shown. Find the current through the ammeter *A*. What are the polarity and emf of the battery ε, assuming it has negligible internal resistance?



26.34. **IDENTIFY:** We first reduce the parallel combination of the 20.0-Ω resistors and then apply Kirchhoff's rules.

SET UP: $P = I^2 R$ so the power consumption of the 6.0-Ω resistor allows us to calculate the current through it. Unknown currents I_1, I_2 and I_3 are shown in Figure 26.34. The junction rule says that $I_1 = I_2 + I_3$. In Figure 26.34 the two 20.0-Ω resistors in parallel have been replaced by their equivalent (10.0 Ω).

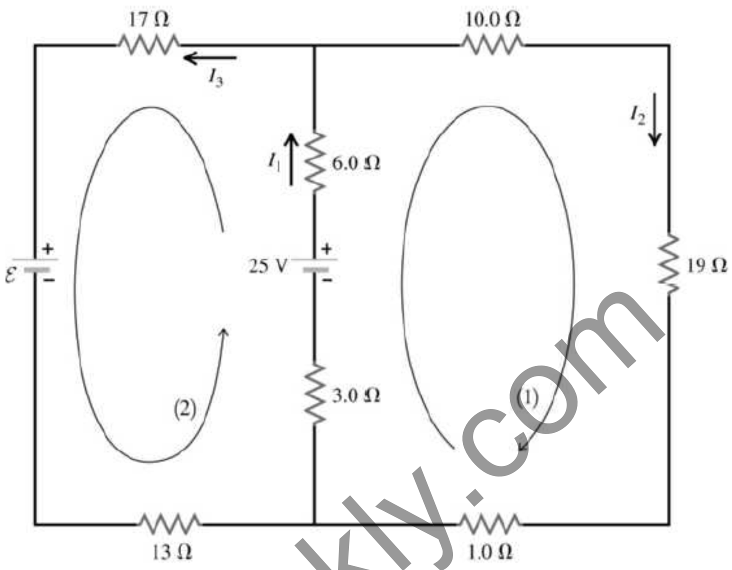


Figure 26.34

EXECUTE: **(a)** $P = I^2 R$ gives $I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{24 \text{ J/s}}{6.0 \ \Omega}} = 2.0$ A. The loop rule applied to loop (1) gives:

$-(2.0 \text{ A})(3.0 \ \Omega) - (2.0 \text{ A})(6.0 \ \Omega) + 25 \text{ V} - I_2(10.0 \ \Omega + 19.0 \ \Omega + 1.0 \ \Omega) = 0.$ $I_2 = \frac{25 \text{ V} - 18 \text{ V}}{30.0 \ \Omega} = 0.233$ A.

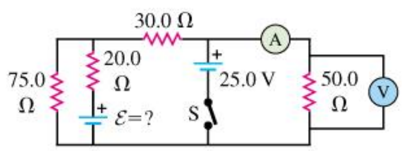
(b) $I_3 = I_1 - I_2 = 2.0 \text{ A} - 0.233 \text{ A} = 1.77$ A. The loop rule applied to loop (2) gives:

$-(2.0 \text{ A})(3.0 \ \Omega + 6.0 \ \Omega) + 25 \text{ V} - (1.77 \text{ A})(17 \ \Omega) - \varepsilon - (1.77 \text{ A})(13 \ \Omega) = 0.$

$\varepsilon = 25 \text{ V} - 18 \text{ V} - 53.1 \text{ V} = -46.1$ V. The emf is 46.1 V.

EVALUATE: Because of the minus sign for the emf, the polarity of the battery is opposite to what is shown in the figure in the problem; the + terminal is adjacent to the 13-Ω resistor.

In the circuit shown in the figure (Figure 1) the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15.0 V. Find the emf E of the battery. What will the ammeter read when the switch is closed?



26.31. **(a) IDENTIFY:** With the switch open, the circuit can be solved using series-parallel reduction.

SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.

EXECUTE: The $30.0\text{-}\Omega$ and $50.0\text{-}\Omega$ resistors are in series, and hence have the same current. Using Ohm's law $I_{50} = (15.0\text{ V})/(50.0\ \Omega) = 0.300\text{ A} = I_{30}$. The potential drop across the $75.0\text{-}\Omega$ resistor is the same as the potential drop across the $80.0\text{-}\Omega$ series combination. We can use this fact to find the current through the $75.0\text{-}\Omega$ resistor using Ohm's law: $V_{75} = V_{80} = (0.300\text{ A})(80.0\ \Omega) = 24.0\text{ V}$ and $I_{75} = (24.0\text{ V})/(75.0\ \Omega) = 0.320\text{ A}$.

The current through the unknown battery is the sum of the two currents we just found:

$$I_{\text{Total}} = 0.300\text{ A} + 0.320\text{ A} = 0.620\text{ A}$$

The equivalent resistance of the resistors in parallel is $1/R_p = 1/(75.0\ \Omega) + 1/(80.0\ \Omega)$. This gives $R_p = 38.7\ \Omega$. The equivalent resistance "seen" by the battery is $R_{\text{equiv}} = 20.0\ \Omega + 38.7\ \Omega = 58.7\ \Omega$. Applying Ohm's law to the battery gives $\varepsilon = R_{\text{equiv}} I_{\text{Total}} = (58.7\ \Omega)(0.620\text{ A}) = 36.4\text{ V}$.

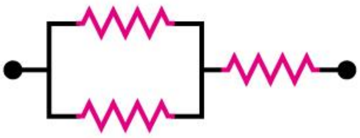
(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the $50.0\text{-}\Omega$ resistor.

SET UP: Take a loop around the right part of the circuit.

EXECUTE: Ohm's law gives $I = (25.0\text{ V})/(50.0\ \Omega) = 0.500\text{ A}$.

EVALUATE: The current through the $50.0\text{-}\Omega$ resistor, and the rest of the circuit, depends on whether or not the switch is open.

Each of the three resistors in the figure ([Figure 1]) has a resistance of $3.0\ \Omega$ and can dissipate at a maximum rate of 48 W without becoming excessively heated.



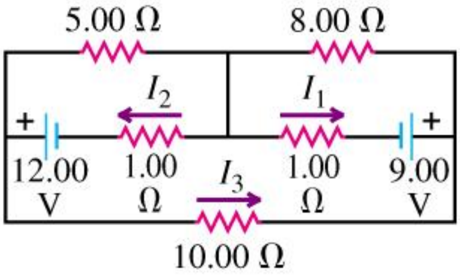
IDENTIFY: Half the current flows through each parallel resistor and the full current flows through the third resistor, that is in series with the parallel combination. Therefore, only the series resistor will be at its maximum power.

SET UP: $P = I^2 R$

EXECUTE: The maximum allowed power is when the total current is the maximum allowed value of $I = \sqrt{P/R} = \sqrt{48\text{ W}/2.4\ \Omega} = 4.47\text{ A}$. Then half the current flows through the parallel resistors and the maximum power is $P_{\max} = (I/2)^2 R + (I/2)^2 R + I^2 R = \frac{3}{2} I^2 R = \frac{3}{2}(4.47\text{ A})^2 (2.4\ \Omega) = 72\text{ W}$.

EVALUATE: If all three resistors were in series or all three were in parallel, then the maximum power would be $3(48\text{ W}) = 144\text{ W}$. For the network in this problem, the maximum power is half this value.

Calculate the three currents I_1,I_2, and I_3 indicated in the circuit diagram shown in the figure (Figure 1) .



26.63. **IDENTIFY:** Apply the junction rule to express the currents through the 5.00-Ω and 8.00-Ω resistors in terms of I_1, I_2 and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.63.

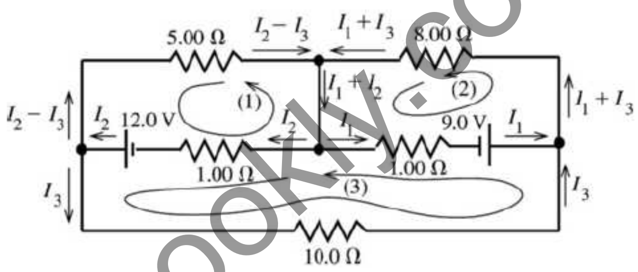


Figure 26.63

The current in each branch has been written in terms of I_1, I_2 and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2(1.00\ \Omega) + (I_2 - I_3)(5.00\ \Omega) = 0$

$I_2(6.00\ \Omega) - I_3(5.00\ \Omega) = 12.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$-I_1(1.00\ \Omega) + 9.00\text{ V} - (I_1 + I_3)(8.00\ \Omega) = 0$

$I_1(9.00\ \Omega) + I_3(8.00\ \Omega) = 9.00\text{ V}$ eq. (2)

Apply the loop rule to loop (3).

$-I_3(10.0\ \Omega) - 9.00\text{ V} + I_1(1.00\ \Omega) - I_2(1.00\ \Omega) + 12.0\text{ V} = 0$

$-I_1(1.00\ \Omega) + I_2(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\text{ V}$ eq. (3)

Eq. (1) gives $I_2 = 2.00\text{ A} + \frac{5}{6}I_3$; eq. (2) gives $I_1 = 1.00\text{ A} - \frac{8}{9}I_3$

Using these results in eq. (3) gives $-(1.00\text{ A} - \frac{8}{9}I_3)(1.00\ \Omega) + (2.00\text{ A} + \frac{5}{6}I_3)(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\text{ V}$

$(\frac{16+15+180}{18})I_3 = 2.00\text{ A}; I_3 = \frac{18}{211}(2.00\text{ A}) = 0.171\text{ A}$

Then $I_2 = 2.00\text{ A} + \frac{5}{6}I_3 = 2.00\text{ A} + \frac{5}{6}(0.171\text{ A}) = 2.14\text{ A}$ and

$I_1 = 1.00\text{ A} - \frac{8}{9}I_3 = 1.00\text{ A} - \frac{8}{9}(0.171\text{ A}) = 0.848\text{ A}.$

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the 5.00-Ω, 8.00-Ω and 10.0-Ω resistors. Going around the loop clockwise:

$-(I_2 - I_3)(5.00\ \Omega) + (I_1 + I_3)(8.00\ \Omega) + I_3(10.0\ \Omega) = -9.85\text{ V} + 8.15\text{ V} + 1.71\text{ V}$, which does equal zero, apart from rounding.